Dr. 1922-4

DOE/MC/08216-1440, Vol. I
(DE83017035)

SIMULATOR FOR UNCONVENTIONAL GAS RESOURCES MULTI-DIMENSIONAL MODEL "SUGAR-MD"

Volume I: Reservoir Model Analysis and Validation

September 1983

Work Performed Under Contract No. : DE-AM21-78MC08216

For
U. S. Department of Energy
Morgantown Energy Technology Center
Morgantown, WV

By
Science Applications, Inc.
Morgantown, WV

TECHNICAL INFORMATION CENTER
UNITED STATES DEPARTMENT OF ENERGY

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

This report has been reproduced directly from the best available copy.

Available from the National Technical Information Service, U. S. Department of Commerce, Springfield, Virginia 22161.

Price: Printed Copy A04
Microfiche A01

Codes are used for pricing all publications. The code is determined by the number of pages in the publication. Information pertaining to the pricing codes can be found in the current issues of the following publications, which are generally available in most libraries: *Energy Research Abstracts (ERA)*; *Government Reports Announcements and Index (GRA and I)*; *Scientific and Technical Abstract Reports (STAR)*; and publication NTIS-PR-360 available from NTIS at the above address.

**SIMULATOR FOR UNCONVENTIONAL GAS RESOURCES
MULTI-DIMENSIONAL MODEL
"SUGAR-MD"**

Volume I: Reservoir Model Analysis and Validation

September 1983

Work Performed Under Contract No. : DE-AM21-78MC08216

For
U. S. Department of Energy
Morgantown Energy Technology Center
P.O. Box 880
Morgantown, WV 26505

By
Science Applications, Inc.
Chestnut Ridge Professional Building, Suite 8
Morgantown, WV 26505

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

SUGAR-MD

VOLUME I

TABLE OF CONTENTS

	<u>Page</u>
INTRODUCTION.	I-1
SUMMARY	I-1
DESCRIPTION OF TWO-DIMENSIONAL (2D) MODEL	I-2
REFERENCES.	I-8
APPENDIX I-A - Fluid Flow Equations	I-A-1
APPENDIX I-B - Induced Fracture Capability.	I-B-1
APPENDIX I-C - Model Validation and Performance	I-C-1
APPENDIX I-D - Comparison of Model Results with Laboratory Experiments.	I-D-1

INTRODUCTION

In recent years energy shortages have prompted both Government and private industry to seek and develop unconventional oil and natural gas resources. The U.S. Department of Energy's (DOE) Eastern Gas Shales Project (EGSP) has been in existence for approximately 5 years and has as one of its goals the commercial development of some of the more organic-rich Devonian shale reservoirs underlying the Appalachian, Michigan, and Illinois Basins. A part of this effort involves the development of mathematical models with predictive capability so that various alternative production schemes can be investigated and accurate forecasts of additional recovery can be made.

Most Devonian shale reservoirs are believed to consist of very tight porous shale formations which contain a network of randomly distributed natural fractures. Under these conditions, the fractures may provide most of the permeability to gas flow but contribute very little to the overall storage capacity. By comparison, the matrix of the shale may provide most of the storage capacity of the shale but has very low permeability.

Gas transport in the Devonian shale matrix can be by three primary mechanisms: Fickian diffusion, Knudsen diffusion, or Darcy flow -- depending upon the pore diameter, the mean free path of the free gas molecules, and the existence of a pressure gradient. Knudsen diffusion is the term applied to diffusion in pores with small diameters and/or at low pressure when collisions of gas molecules with pore walls are more frequent than intermolecular collisions. Also, it is believed that physical desorption is a contributing factor to gas production (1,2,23).

Although several studies on Devonian shale modeling have been presented (3-6), a careful review of the results indicates that much work remains to attain adequate predictive capability. The two-dimensional numerical model with induced fracture capability presented here is believed to be a significant step in advancing the state-of-the-art in forecasting performance of Devonian shale reservoirs. It is based on the mathematical model presented by Kucuk and Sawyer (7) and is an extension of the work presented in a previous SAI report (8).

SUMMARY

The Department of Energy (DOE), Morgantown Energy Technology Center (METC), has been supporting the development of flow models for Devonian shale gas reservoirs. The broad objectives of this modeling program are:

1. To develop and validate a mathematical model which describes gas flow through Devonian shales.
2. To determine the sensitive parameters that affect deliverability and recovery of gas from Devonian shales.
3. To recommend laboratory and field measurements for determination of those parameters critical to the productivity and timely recovery of gas from the Devonian shales.

4. To analyze pressure and rate transient data from observation and production gas wells to determine reservoir parameters and well performance.
5. To study and determine the overall performance of Devonian shale reservoirs in terms of well stimulation, well spacing, and resource recovery as a function of gross reservoir properties such as anisotropy, porosity and thickness variations, and boundary effects.

Previously, a mathematical model describing gas flow through Devonian shales and the software for a two-dimensional numerical model for multi-well performance was completed and placed into operation (8). The two-dimensional flow model is a useful tool for studying multiple well reservoirs in the absence of induced fractures. However, additional validation work was required to:

1. Demonstrate the practical capability to simulate an induced fracture.
2. Validate the model based on recent analytical model developments (9) and laboratory studies conducted by the Battelle Columbus Laboratory (23).

Once validated with induced fracture capability, the two-dimensional numerical model (SUGAR-MD) provides a powerful tool for studying multi-well behavior, including the determination of the effects of well spacing, stimulation treatments, and variation in reservoir properties (e.g., directional permeability).

The purpose of this work was to extend the model to include induced fracture capability and perform additional validation work, maintaining full capability regarding Klinkenberg effects, desorption, and shale matrix parameters. The two-dimensional flow model has been successfully extended to provide the induced fracture capability and validated by both analytical (7,9) and laboratory models. In addition, a suite of runs has been made to demonstrate the performance of a dual porosity reservoir produced by a well containing an induced fracture with and without fracture permeability anisotropy. It is believed that this model will prove to be an important step in developing accurate predictive capability for the Devonian shales as well as other unconventional gas resources.

DESCRIPTION OF TWO-DIMENSIONAL MODEL

Mathematical Formulation

The flow equations that are the mathematical basis of the model are presented here. It is assumed that gas transport to producing wells in Devonian shale reservoirs occurs through a natural fracture system into which matrix blocks of contrasting physical properties deliver contained gas. That is, the matrix acts as a uniformly distributed gas source in a fracture medium. Gas desorption from pore walls will be treated as a uniformly distributed source within the matrix blocks.

Assumptions made in the development of the mathematical model are as follows:

1. The reservoir has single-phase gas flow.
2. The reservoir is at isothermal conditions.

3. Gravity effects are negligible.
4. Surface diffusion of adsorbed molecules on the walls of the pores is negligible.

Gas flow through the fracture system can be described by the following equation:

$$\nabla \cdot \left[\rho \frac{k_f}{\mu} \nabla p_f \right] + w_m(p_f, t) = \frac{\partial}{\partial t} (\phi_f \rho) \quad (1)$$

where

ρ = density, g/cm³
 k_f = fracture permeability, Darcy's
 μ = viscosity, cp
 ϕ_f = fracture porosity
 p_f = fracture pressure, atm
 t = time, sec
 w_m = mass flow rate per volume of shale matrix element, g/sec/cm³

The equation of state for real gas is given by:

$$\rho = \frac{M}{RT} \frac{p_f}{z} \quad (2)$$

where

M = molecular weight, g-moles
 R = universal gas constant, cm³-atm/g-mole °K
 T = reservoir temperature, °K
 z = real gas deviation factor

Substitution of Equation 2 into Equation 1 yields:

$$\nabla \cdot \left[\frac{k_f}{\mu} \frac{p_f}{z} \nabla p_f \right] + \frac{p_f}{z} q_m = \frac{\partial}{\partial t} \left(\phi_f \frac{p_f}{z} \right) \quad (3)$$

where

q_m = volumetric flow rate per volume of shale matrix element, cm³/sec/cm³

$$= - \frac{A_m}{V_m} \frac{k_m}{\mu} \left(\frac{\partial p_m}{\partial n} \right)_{\text{surface}}$$

A_m = surface area of matrix element, cm²
 V_m = volume of element, cm³

Equation 3 describes gas flow through the fractured shale reservoir with a source term which is the contribution from the shale matrix. Gas transport through the matrix is also described by the diffusivity equation with a source

term due to desorption of gas from the pore walls of the matrix. The following equation describes the motion of gas through the matrix:

$$\nabla \cdot \left[\rho_m \frac{k_{gm}}{\mu} \nabla p_m \right] + w_d = \frac{\partial}{\partial t} (\phi_m \rho_m) \quad (4)$$

where

$$\begin{aligned} k_{gm} &= \text{permeability to gas, Darcy's} \\ w_d &= \text{desorption rate, g/sec/cm}^3 \text{ shale} \\ p_m &= \text{matrix pressure, atm} \\ \phi_m &= \text{matrix porosity} \end{aligned}$$

The rate of desorption can be expressed as:

$$w_d = - M \frac{dc_d}{dp_m} \left(\frac{\partial p_m}{\partial \tau} \right) \quad (5)$$

where

$$\begin{aligned} c_d &= \text{concentration of gas at the surface of pore} \\ &\quad \text{walls, mole/cm}^3 \text{ shale} \\ \frac{dc_d}{dp_m} &= \text{slope of gas desorption isotherm curve,} \\ &\quad \text{mole/cm}^3 \text{ shale/atm} \end{aligned}$$

The gas permeability of a porous medium usually exceeds the liquid permeability of the same medium. The difference in these permeabilities is due to the phenomenon known as gas slippage, which is related to the mean free path of the gas molecules relative to pore diameter. Consequently, the gas permeability of a porous medium should be a function of the temperature, pressure, and the nature of the gas. Klinkenberg (11) developed the relationship between gas permeability of a porous medium to a nonreactive liquid, viz.,

$$k_g = k_\ell \left(1 + \frac{b}{p_m} \right) \quad (6)$$

where b is the Klinkenberg factor which is constant for a given gas and a given porous medium at a constant temperature. A graph of k versus $1/p_m$ should result in a straight line with an intercept of k_ℓ and a slope of bk_ℓ . Thus, gas permeability is greater at low pressures. The Klinkenberg factor, b , has been given by Jones and Owens (12) for tight sands and more recently for Devonian shale (see Appendix D) by Coutant (23).

$$b = \frac{1}{30,000 \bar{r}} \quad (7)$$

where $\bar{r}$ is the mean pore radius in cm and b is in atmospheres.

With all other parameters constant, k_g is a function of mean pore pressure and pore diameter for a given gas and a given reservoir temperature. Since pore diameters are small for Devonian shales, b is expected to be large. Therefore, the Klinkenberg effect or slippage factor may be significant in determining production performance of Devonian shale wells.

Substitution of Equations 2, 5, and 6 into Equation 4 yields:

$$\nabla \cdot \left[\frac{k_{\ell}}{\mu} \frac{p_m}{z} \left(1 + \frac{b}{p_m} \right) \nabla p_m \right] = \phi_d \frac{\partial p_m}{\partial t} + \phi_m \frac{\partial}{\partial t} \left(\frac{p_m}{z} \right) \quad (8)$$

where

$$\phi_d = \frac{dc_d}{dp} RT, \text{ dimensionless}$$

The source term in Equation 3 can be determined from the solution of Equation 8. An iterative finite-difference solution technique is used to solve Equation 3 and Equation 8 simultaneously. Detailed development of the numerical procedure is presented in Appendix A.

Recent analytical model development of Raghavan (9), together with the analysis of interference data from the Offset Well Test Program (24) suggests that, in some instances, Devonian shale well performance may behave like the simpler Warner & Root (16) model. In this formulation, it is assumed that flow is pseudo-steady-state in the shale matrix; i.e., $\partial p_m / \partial t$ is constant. This means that it is only necessary to consider the matrix element as a single "block" of some shape (cylinder, sphere, slab) with a single average pressure p_m at any point in time. In this case, q_m in Equation 3 is given by:

$$q_m = \alpha \frac{k_{gm} A_{sm}}{\mu_{Bg}} (\bar{p}_m - p_f) \quad (9)$$

where α is a "shape factor" depending on the geometry of the element and A_{sm} is the surface area of the element. Shape factors are discussed in detail in a recent paper by Peaceman (17). The results of his theoretical development, which are summarized in Appendix A, are given below:

Geometry	Shape Factor (α)
Cylinder, radius = a	$4/a$
Sphere, radius = a	$5/a$
Slab, thickness = $2a$	$3/a$

Pseudo-steady-state flow has been incorporated into the numerical model as an option to provide additional flexibility. Slippage is included in the k_{gm} in Equation 9; desorption is also considered in this option, but is restricted to a linear isotherm.

Wellbore Options

1. Skin Factor

For radial flow simulations, skin damage may be handled via a damaged zone permeability obtained from the equation presented by Craft and Hawkins (13):

$$S = (k - k_d)/k_d \cdot \ln(r_{skin}/r_w) \quad (10)$$

where k is the undamaged permeability and k_d is the permeability of a damaged zone between r_w and r_{skin} . It is convenient to take Node 1 to be within the wellbore and Node 2 to be the damaged zone. Then, for a given skin factor, S , one can solve Equation 10 for the value of k_d , using r_{skin} as the external radius of Node 2 in the simulator. The value thus obtained, when input into the simulator as the permeability of Node 2, will result in well performance corresponding to a dimensionless skin factor of S .

2. Wellbore Storage

Although no explicit provision for wellbore storage exists in SUGAR-MD, the effects of afterflow can be readily observed by appropriately modifying porosity values in the input data for subroutine PARM. For example, when a small grid-block is used (e.g., .25 ft x .25 ft in cartesian coordinates), it is only necessary to assign a porosity value so that the array VP in the simulator contains the proper (desired) wellbore volume; that is, if V_w is the wellbore volume, then we must input the value of porosity (ϕ) which satisfies:

$$(\phi) * dx * dy * dz = V_w \quad (11)$$

This may be done for any number of nodes by utilizing the "special porosity values" option in the routine PARM. It should be noted that the pressure thus calculated by the simulator will be the average wellbore pressure, assuming that the grid-block dimensions are close to actual wellbore dimensions.

In order to actually see the "afterflow" rates, each node at which wellbore storage is to occur should also be specified as a "history node" as discussed in the User's Manual (Volume II of this report). This will assure that, upon shut-in, the afterflow rates are printed at each well.

3. Induced Fracture Options

An induced vertical fracture can be adequately represented by appropriately defined productivity indices with the variable rate and/or variable pressure node options. For a uniform flux constant rate fracture, the production rate is prorated over the length of the fracture; i.e., if Q is the specified total rate and L_f is the fracture length, then for a fracture oriented in the x-direction:

$$Q_{i,j} = Q \times \frac{\Delta x_i}{L_f} \quad (12)$$

$$\text{where } Q_{i,j} = PI \left(\frac{1}{\mu_g B_g} \right) (P_{i,j} - P_{wf}) \quad (13)$$

and PI is the specified productivity index defined by:

$$PI = \frac{.006328 \Delta x_{i,hk}}{\Delta y/2} \quad (14)$$

For an infinite conductivity, constant pressure fracture with specified P_{wf} , Equations 13 and 14 are again used with the rate $Q_{i,j}$ calculated. The total production rate is then given by the sum of the individual node rates:

$$Q = \sum_{i,j} Q_{i,j} \quad (15)$$

Further details are given on induced fracture capability in Appendix B.

REFERENCES

1. Thomas, J., Jr., and R. R. Frost, "Internal Surface Area and Porosity in Eastern Gas Shales from the Sorption of Nitrogen, Carbon Dioxide, and Methane," Proceedings, First Eastern Gas Shales Symposium, U.S. DOE, MERC/SP-77/5, March 1978.
2. Frost, R. R., and J. Thomas, Jr., "Use of Internal Surface Area and High-Pressure Methane Sorption Data to Estimate Capacity for Gas Production from the New Albany Shale Group," Preprints, Second Eastern Gas Shales Symposium, U.S. DOE, METC/SP-77/6, Vol. I, October 1978.
3. Holditch, S. A., and W. L. Laufer, "The Analysis of Fractured Gas Wells Using Reservoir Simulation," Paper No. SPE 7473, 53rd Annual Fall Technical Conference and Exhibition SPE-AIME, Houston, TX, October 2-4, 1978.
4. Kuuskraa, V. A., J. P. Brashear, T. M. Doscher, and L. E. Elkins, "Enhanced Recovery of Unconventional Gas," Main Report, Vol. II, (for DOE, Contract No. EF-77-C-01-2705), Lewin and Associates, Inc., February 1978.
5. Springer, T. E., and E. B. Nuckols, "Cottageville Gas Field Correlation Analyses for Reservoir Modeling," Proceedings, Third Eastern Gas Shales Symposium, U.S. DOE, METC/SP-79/6, October 1-3, 1979.
6. Intercomp, "Analysis of Gas Production from Eastern Gas Shales," Final Report to U.S. DOE (Contract EW-78-C-21-8485), November 1979.
7. Kucuk, F., and W. K. Sawyer, "Modeling of Devonian Shale Gas Reservoir Performance," Proceedings Third Eastern Gas Shales Symposium, U.S. DOE, METC/SP-79/6, October 1-3, 1979.
8. Science Applications, Inc. (SAI), "Modeling of Devonian Shale Gas Reservoirs, Final Report to U.S. DOE (Task 16, Contract No. DE-AT21-78MC08216), July 31, 1980.
9. Raghavan, R., "Unsteady Flow to a Well Produced at Constant Pressure in a Fractured Reservoir," Paper SPE 9902 presented at California Regional Meeting, March 1981.
10. Sawyer, W. K., J. Alam, and W. D. Rose, "Utilizing Well Interference Data in Unconventional Gas Reservoirs," Paper No. SPE 8940, First SPE/DOE Unconventional Gas Recovery Symposium, Pittsburgh, PA, May 18-20, 1980.
11. Klinkenberg, L. J., "The Permeability of Porous Media to Liquids and Gases," Drilling and Production Practice (1941), pp. 200-213.
12. Jones, F. O., and W. W. Owens, "A Laboratory Study of Low Permeability Gas Sands," SPE Paper 7551, presented at SPE Symposium on Low Permeability Gas Reservoirs, SPE-AIME, Denver, CO, May 20-22, 1979.
13. Craft, B. C., and M. F. Hawkins, Applied Petroleum Reservoir Engineering, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1959.

14. Young, D. M., Iterative Solution of Large Linear Systems, Academic Press, 1971.
15. Price, H. S., and K. H. Coates, "Direct Methods in Reservoir Simulation," SPE Preprint No. 4278, January 1973.
16. Warren, T. E., and P. J. Root, "The Behavior of Naturally Fractured Reservoirs," Soc. Pet. Eng. J., September 1963.
17. Peaceman, D. W., "Convection in Fractured Reservoirs -- The Effect of Matrix-Fissure Transfer on the Instability of a Density Inversion in a Vertical Fissure," Soc. Pet. Eng. J., October 1976.
18. Raghavan, R., University of Tulsa, Private Communication, December 1980.
19. Morse, R. A., and W. D. Von Gonten, "Productivity of Vertically Fractured Wells Prior to Stabilized Flow," J. Pet. Tech., July 1972.
20. Sawyer, W. K., J. C. Mercer, and K. H. Frohne, "Determination of Fracture Extent by Simulation of Gas Well Pressure and Production Data," DOE/METC TPR-76/8.
21. Gringarten, A. C., H. J. Ramey, Jr., and R. Raghavan, "Pressure Analysis for Fractured Wells," Soc. Pet. Eng. J., August 1974.
22. Locke, C. D., and W. K. Sawyer, "Constant Pressure Injection Test in a Fractured Reservoir -- History Match Using Numerical Simulation and Type Curve Analysis," paper SPE 5594 presented at 50th Annual Fall Meeting of the Society of Petroleum Engineers of AIME, Dallas, TX, Sept. 28 - Oct. 1, 1975.
23. Coutant, R. W., "Mechanisms of Natural Gas Release From Devonian Shale," Battelle Columbus Laboratories, SAI Task No. 212, July 23, 1981.
24. Science Applications, Inc., "Analysis of Devonian Shale Multi-Well Interference Tests in Meigs County, Ohio," Final Report to U.S. DOE (Contract No. AC21-81MC16283), February 23, 1982.

APPENDIX A

Fluid Flow Equations and Concepts

The following discussion details the development of the fluid flow equations used in the model development. Although the model presently is written in two dimensions, the concepts, for completeness, are presented in three dimensions.

Reservoir Nodes

With reference to Figure A-1 and the basic definitions:

F = flux, rate of flow of quantity being conserved/unit cross-sectional area normal to the direction of flow

C = concentration/unit volume

q = input rate/unit volume,

we see that the fundamental conservation relationship:

$$\text{Net Input} = \text{Net Change}, \quad (\text{A1})$$

when expressed over a time interval, Δt , is:

$$\begin{aligned} & \left\{ (F_x)_{x-\frac{\Delta x}{2}} - (F_x)_{x+\frac{\Delta x}{2}} \right\} \Delta y \Delta z + \left\{ (F_y)_{y-\frac{\Delta y}{2}} - (F_y)_{y+\frac{\Delta y}{2}} \right\} \Delta x \Delta z + \\ & \left\{ (F_z)_{z-\frac{\Delta z}{2}} - (F_z)_{z+\frac{\Delta z}{2}} \right\} \Delta x \Delta y = q(\Delta x \Delta y \Delta z) = \Delta x \Delta y \Delta z \left(\frac{C_{t+\Delta t} - C_t}{\Delta t} \right) \end{aligned} \quad (\text{A2})$$

Here F_x , F_y , and F_z are in the positive x , y , and z directions, respectively, and we have divided both sides by Δt .

Assuming the x , y , and z axes are the principal axes of conductivity, fluid flow in porous media, heat conduction in solids, and diffusion all have flux components of the form:

$$F_s A_s = q_s = -K_s A_s \frac{\Delta U_s}{\Delta s} \quad (\text{A3})$$

where s represents one of the coordinate axes and K_s is a general conductivity term. Using the differential form:

$$F_s = -K_s \frac{\partial U_s}{\partial s} \quad (\text{A4})$$

in Equation A2, dividing by $\Delta x \Delta y \Delta z$, and expressing C in terms of U gives equation of the form:

$$\nabla \cdot K \nabla U = \alpha \frac{\partial U}{\partial t} \quad (\text{A5})$$

The most direct approach to developing a finite-difference model, however, takes a different route.

FIGURE A-1. Volume Element Representations -- Cartesian Coordinates

(a) Coordinate Convention

(b) Finite-Difference Notation

FIGURE A-2. Volume Element Representations -- Polar Cylindrical Coordinates

Defining the coefficient of ΔU_s in Equation A3 as the transmissibility across the appropriate face of a volume element as illustrated in Figures A-1 and A-2:

$$TS = \frac{K A_s}{\Delta s} \quad (A6)$$

we see from Equation A3, that

$$(F_x)_{(x-\frac{\Delta x}{2})} A_{(x-\frac{\Delta x}{2})} = TX_{(x-\frac{\Delta x}{2})} (U_{x-\Delta x} - U_x) \quad (A7a)$$

$$- (F_x)_{(x+\frac{\Delta x}{2})} A_{(x+\frac{\Delta x}{2})} = TX_{(x+\frac{\Delta x}{2})} (U_{x+\Delta x} - U_x). \quad (A7b)$$

Similar results hold for the y and z terms. Setting $Q = q (\Delta x \Delta y \Delta z)$ and $V = \Delta x \Delta y \Delta z$, and using the finite difference notation of Figure A-1, Equation A2 thus becomes:

$$\begin{aligned} & TZ_{k-\frac{1}{2}} (U_{k-1} - U_k) + TY_{j-\frac{1}{2}} (U_{j-1} - U_j) + \\ & TX_{i-\frac{1}{2}} (U_{i-1} - U_i) + TX_{i+\frac{1}{2}} (U_{i+1} - U_i) + \\ & TY_{j+\frac{1}{2}} (U_{j+1} - U_j) + TZ_{k+\frac{1}{2}} (U_{k+1} - U_k) + Q = \Delta V \left\{ \frac{C^{n+1} - C^n}{\Delta t} \right\}. \end{aligned} \quad (A8)$$

Here, most subscripts i, j, and k, have been suppressed for brevity and a superscript has been added to denote the time level on the right-hand side. All U's on the left-hand side are taken at time level n+1. Each block to block transmissibility is the harmonic mean of the two values.

For Darcy flow of a gas through a porous media of permeability k (.006328 · md) and porosity ϕ , we have:

$$K = \frac{k}{\mu B_g} \quad (A9)$$

$$C = \phi \rho_g / \rho_{gsc} = C/B_g \quad (A10)$$

where μ is gas viscosity, ρ_{gsc} is gas density at standard conditions, and ρ_g is gas density given by:

$$\rho_g = \frac{M}{RT} \cdot \frac{P}{Z} \cdot \frac{\text{lbm}}{\text{cu ft}} \quad (A11)$$

Since our dependent variable is pressure, and the gas formation volume factor may be written as:

$$B_g = \frac{P_{sc} T}{T_{sc}} \cdot \frac{z}{p} \cdot \frac{\text{cu ft}}{\text{scf}} \quad (A12)$$

equation A8 becomes:

$$TZ_{k-\frac{1}{2}} (p_{k-1} - p_k) + TY_{j-\frac{1}{2}} (p_{j-1} - p_j) +$$

$$\begin{aligned}
& TX_{i-\frac{1}{2}} (p_{i-1} - p_i) + TX_{i+\frac{1}{2}} (p_{i+1} - p_i) + \\
& TY_{j+\frac{1}{2}} (p_{j+1} - p_j) + TZ_{k+\frac{1}{2}} (p_{k+1} - p_k) + \\
& Q_{sc} + Q_m = \phi \Delta V \frac{T_{sc}}{p_{sc} T} \left\{ \frac{(p/z)^{n+1} - (p/z)^n}{\Delta t} \right\}
\end{aligned} \tag{A13}$$

where each term has units of scf/day. The nonlinear term on the right-hand side may be expanded as follows:

$$\frac{p^{n+1}}{z^{n+1}} - \frac{p^n}{z^n} \simeq p^n \left(\frac{1}{z^*} - \frac{1}{z^n} \right) + \frac{1}{z^*} (p^{n+1} - p^n) \tag{A14}$$

where z^* represents an approximation to z^{n+1} .

Using $z^* = z^n$ to start, Equation A13 may be solved by any of the standard techniques (e.g., LSOR, ADIP, direct methods). Then using the new pressures, new z^* 's may be calculated and the process repeated until no further change is observed. Normally only two to four iterations are required for convergence.

Matrix Element (Unsteady-State Flow)

Gas flow through a "matrix" element, including the Klinkenberg effect, can be described by:

$$\nabla \left[\frac{k_m}{\mu} \frac{p_m}{z} \left(1 + \frac{b}{p_m} \right) \nabla p_m \right] = \frac{dc_D}{dp} \left[RT \frac{\partial p_m}{\partial t} + \phi_m \frac{\partial}{\partial t} \left(\frac{p_m}{z} \right) \right] \tag{A15}$$

where c_D is adsorbed gas concentration, moles/cm³, T is in degrees K, R is the gas constant in cm³-atm/g mole degrees K, and other terms are in Darcy units.

Solution of Equation A15 may be accomplished by first writing the finite-difference analog. Subscript " ℓ " will be used to distinguish the matrix equations from the reservoir finite-difference equations. The Klinkenberg term can be accounted for by first calculating effective gas permeability:

$$k_{gm} = k_m (1 + b/p_m)$$

and then determining transmissibilities in the normal manner. The additional term, due to desorption in the "matrix," may easily be handled. Thus, the finite-difference analog of Equation A15 is:

$$\begin{aligned}
& TR_{\ell-\frac{1}{2}}^{n+1} \left(p_{\ell-1}^{n+1} - p_{\ell}^{n+1} \right) + TR_{\ell+\frac{1}{2}}^{n+1} \left(p_{\ell+1}^{n+1} - p_{\ell}^{n+1} \right) = \\
& \Gamma_{\ell} p_{\ell}^n \left[\left(\frac{1}{z_{\ell}^*} - \frac{1}{z_{\ell}^n} \right) + \frac{1}{z_{\ell}^*} \left(p_{\ell}^{n+1} - p_{\ell}^n \right) \right] + \\
& \left[\frac{V_{\ell}}{\Delta t} \left(\frac{dc}{dp} \right)_{p_{\ell}}^{**} \left(p_{\ell}^{n+1} - p_{\ell}^n \right) \right]
\end{aligned} \tag{A16}$$

where $V_\ell = 2\pi k \bar{r}_\ell (r_{\ell+1} - r_\ell)$, $**$ denotes the average of the latest value of p_ℓ and p_ℓ^n , and z^* represents an approximation to z^{n+1} .

In Equation A16, TR is the interblock transmissibility which, for a cylindrical element, is defined as follows:

$$TR_{\ell-\frac{1}{2}}^{n+1} = \pi h \frac{T_{sc}}{p_{sc} T} TM_{\ell-\frac{1}{2}} (p_{\ell-1}^{n+1} - p_\ell^{n+1}) \quad (A17)$$

where

$$TM_{\ell-\frac{1}{2}} = \frac{k_{\ell-1} k_\ell}{\left[k_\ell (\mu z)_{\ell-1} \ln \left(\frac{r_\ell}{\bar{r}_{\ell-1}} \right) \right] + \left[k_{\ell-1} (\mu z)_\ell \ln \left(\frac{\bar{r}_\ell}{r_\ell} \right) \right]} \quad (A18)$$

In this case, we have only a one-dimensional problem and it is easy to see that a tri-diagonal system is present; that is,

$$a_\ell p_{\ell-1}^{n+1} + b_\ell p_\ell^{n+1} + c_\ell p_{\ell+1}^{n+1} = d_\ell \quad (A19)$$

where

$$a_\ell = TR_{\ell-\frac{1}{2}}^* \quad (A20a)$$

$$b_\ell = -TR_{\ell-\frac{1}{2}}^* - TR_{\ell+\frac{1}{2}}^* - \frac{\Gamma_\ell}{z_\ell^*} - \frac{V_\ell}{\Delta t} \left(\frac{dc}{dp} \right)^{**} p_\ell \quad (A20b)$$

$$c_\ell = TR_{\ell+\frac{1}{2}}^* \quad (A20c)$$

and

$$d_\ell = - \left[\frac{\Gamma_\ell}{z_\ell^n} + \frac{V_\ell}{\Delta t} \left(\frac{dc}{dp} \right)^{**} \right] p_\ell^n \quad (A20d)$$

The source term, Q_m , in Equation A13 may be written implicitly in terms of matrix and fracture pressures as:

$$Q_m^{n+1} = TR_{L-\frac{1}{2}}^n \left(p_{m, L-1}^n - p_{m, L}^{n+1} \right) \quad (A21)$$

To maintain pressure continuity at the matrix element boundary, we must have:

$$p_{m, L}^{n+1} = p_{i,j,k}^{n+1} \quad (A22)$$

Matrix Element (Pseudo-Steady-State Flow -- Warren and Root (16) Model)

Pseudo-steady-state flow out of a shale matrix element means, by definition, that $\partial \bar{p}_m / \partial t$ is constant. Thus, if Q_m is the withdrawal rate in Mcfd, material balance considerations dictate that we must have:

$$-\frac{Q_m \bar{B}_{gm}}{V_{Bm}} = (\phi c_g)_m \frac{\partial \bar{p}_m}{\partial t} \quad (A23)$$

where V_{Bm} is the bulk volume of the matrix element.

On the other hand, the flow rate must also be related to the difference between matrix and fracture pressures ($\bar{p}_m - p_f$). This is accomplished (16-18) by use of a shape factor, α , in the Darcy equation, giving:

$$\frac{Q_m \bar{B}_{gm}}{V_{Bm}} = \alpha \frac{k_{gm}}{\mu_{gm}} \frac{A_{sm}}{V_{Bm}} (\bar{p}_m - p_f) \quad (A24)$$

where A_{sm} is the surface area of the element, $k_{gm} = .006328 \times md$, and each term has units of reservoir cu ft/day.

Equating Equations A23 and A24 gives:

$$\alpha \frac{A_{sm}}{V_{Bm}} (\bar{p}_m - p_f) = -\frac{1}{D_m} \frac{\partial \bar{p}_m}{\partial t} \quad (A25)$$

where D_m is the matrix diffusivity ($k/\phi\mu c$)_m. If we define a function, u , by:

$$u = -\frac{D_m}{\partial \bar{p}_m / \partial t} (\bar{p}_m - p_f) \quad (A26)$$

and make the assumption that $\partial \bar{p}_m / \partial t$ is uniform throughout the matrix element, then it can be shown that:

$$\nabla^2 u = -1 \quad (A27)$$

Moreover, from Equation A25 we find that

$$\alpha = \frac{V_{Bm}}{A_{sm}} \frac{1}{\bar{u}} \quad (A28)$$

Solving Equation A27 for a cylinder of radius a with the boundary conditions $u = 0$ at $r = a$ and $du/dr = 0$ at $r = 0$ gives:

$$\bar{u} = \frac{a^2}{8} \quad (A29)$$

Thus, for cylinders,

$$\alpha = \frac{4}{a} \quad (A30)$$

and Equation A24 may be written:

$$Q_m = \frac{8}{a^2} \frac{k_{gm}}{\mu_{gm} B_{gm}} (\bar{p}_m - p_f) \quad (A31)$$

The equivalent Warren and Root (16) shape factor is:

$$\sigma = \alpha \frac{A_{sm}}{V_{Bm}} \quad (A32)$$

which, for cylinders, is $8/a^2$.

Shape factors for spheres and slabs may be found using similar developments. The results are summarized below:

	<u>Slab</u>	<u>Cylinder</u>	<u>Sphere</u>
α	$3/a$	$4/a$	$5/a$
σ	$3/a^2$	$8/a^2$	$15/a^2$

In the numerical model, the finite-difference analogs of Equations A23 and A24 are used in the following iterative procedure to obtain new fracture and matrix pressures over a time-step.

- Step 1: Use Equation A24 as an implicit source term to solve for fracture pressure (p_f 's).
- Step 2: Use fracture pressures in Equation A24 to calculate Q_m 's.
- Step 3: Use Q_m in Equation A23 to approximate new matrix pressures (p_m 's).
- Step 4: Use new p_f 's and p_m 's in Equation A24 and repeat procedure; iterate until fracture pressure converges to specified tolerance.

APPENDIX B

Induced Vertical Fracture Capability

Having two-dimensional modeling capability for fractured reservoirs, it was deemed desirable to be able to account for the presence of an induced vertical fracture in order to predict the performance of a hydraulically fractured Devonian shale well. Simulation of a vertically fractured well, in general, is not difficult and only requires modification of the finite-difference grid structure and the proper handling of source terms in the simulation equations. The simplest method is to use very small grid-blocks (e.g., 0.25 to 1.0 ft) in the vicinity of the fracture and assign those nodes adjacent to the fracture the desired boundary conditions. For example, if a fracture (wellbore) pressure of 200 psi is desired, those small nodes adjacent to the fracture would be assigned a fixed pressure of 200 psi for the duration of the run. This avoids any oscillation problem and has been used successfully by several authors (19,20).

In cases where computation time is a factor, due either to machine limitations or model complexity, the use of small grid-blocks to represent an induced fracture may, however, be impractical. The dual-porosity simulator (SUGAR-MD) is a prime example in which model complexity makes it impractical on all but the fastest machines available today. In the unsteady-state mode, each grid-block in the two-dimensional model contains representative shale matrix cylinders which require a complete radial flow simulator to obtain the pressure distribution for each cycle of each time-step. Typically, two to five cycles are required for convergence of fracture pressures. Thus, if three cycles are required for convergence, even a 100-grid-block, two-dimensional system requires 300 radial flow simulations (with 7 to 15 cylindrical grid-blocks) as well as three; two-dimensional solutions for the 100-grid-block system to complete one time-step. Added to this is the additional cycling within each radial and two-dimensional simulation due to the nonlinearity introduced by real gas properties. Thus, it is very important to avoid a fine grid structure when using the two-dimensional model in order to keep computation time within reason.

It has been demonstrated in this work that it is not necessary to use a fine grid structure to represent an induced vertical fracture. The grid networks shown in Figures B-1 and B-2 were used in this study to simulate constant rate, uniform flux and constant pressure, infinite-conductivity fractures. These simulations were performed in the single porosity mode and results were compared with analytical solutions from the literature (21,22) which are summarized in Table B-1. The successful validation of induced fracture capability in the single porosity mode with the coarse grids shown is, in itself, a worthwhile accomplishment in that it demonstrates a method of significantly reducing computation time for any simulation work involving hydraulic fractures. Moreover, it permits conducting dual porosity runs using the same grid networks with some degree of confidence.

Analytical solutions are not yet available in the literature which consider both a fractured reservoir and a vertically fractured well. However, the fact that the numerical model developed here has been validated by comparison with analytical model results for both (1) induced fracture capability in the single porosity mode and (2) single well behavior in the dual porosity mode (see Appendix C) lends considerable support for its ability to accurately simulate the performance of a vertically fractured well in the dual porosity mode.

The remainder of this discussion will focus on the actual pressure and rate performance obtained by the simulator (as compared to analytical results) and the method of data entry required to simulate an induced fracture. Figure B-3 shows numerical and analytical results for infinite and finite-acting constant rate cases. The grid shown in Figure B-1 was used for the numerical simulations. It should be noted that here, as well as in the remaining figures, no shifting (type-curve matching) was done to obtain the good agreement that resulted.

Utilizing PVT data that was used as input into the simulator, a separate program was used to generate a corresponding pseudo-pressure table. This pseudo-pressure table was then input, along with the data normally required for the purpose of calculating and printing dimensionless variables according to the following definitions:

$$t_{Dxf} = \left[\frac{.0002637 \text{ k}}{\phi \mu_i c_i x_f^2} \right] t \quad (B1)$$

$$m_D = \left[\frac{kh}{1422T q_{Mcfd}} \right] [m(p_i) - m(p_{wf})] \quad (B2)$$

$$q_D = \left[\frac{1422T}{kh [m(p_i) - m(p_{wf})]} \right] q_{Mcfd} \quad (B3)$$

Here k is permeability in md, t is time in hours, $m(p)$ is pseudo-pressure in psia^2/cp , and x_f is the fracture half-length in feet.

The success of obtaining such accurate results with a 30-foot grid-block is believed to be primarily due to the use of implicit source terms in the two-dimensional simulator finite-difference equations. The source term, "Q", is written as:

$$Q_{ij} = \text{PI} \times \left(\frac{1}{\mu_g B_g} \right) \times \left(p_{ij}^{n+1} - p_{wf} \right) \quad (B4)$$

with either Q_{ij} or p_{wf} specified. In either case, the $n+1$ time level is retained on p_{ij} requiring alteration of both the main diagonal and the right-hand side vector of the penta-diagonal matrix system of linear equations generated by the two-dimensional, finite-difference equation for each node. The point is that p_{ij}^{n+1} in Equation B4 is obtained implicitly along with the new pressure at all other nodes by simultaneous solution of the penta-diagonal system. Then either Q_{ij} or p_{wf} is calculated using Equation B4. This method completely avoids any oscillation problems commonly encountered with explicit source terms. The same technique is also used in SUGAR-MD in the source term (Equation A21) used for maintaining continuity of fracture and matrix pressures at the outer boundary of the cylindrical shale element.

The appropriate productivity index (PI in Equation B4) for "fracture nodes" as illustrated in Figures B-1 and B-2 is as follows:

$$\text{PI} = \frac{.006328 \text{ k} \Delta x h}{\Delta y / 2} \quad (B5)$$

Here Δx , Δy , h , and k are the properties of the particular grid-block used to simulate fracture performance with Δx , Δy , and h in feet and k in md. Of course, k is the permeability orthogonal to the fracture if anisotropy is being considered. In Figure B-1, five nodes are used to simulate a fracture with only one quadrant of the drainage area being simulated. In this case, if p_{wf} is specified, the calculated rate must, of course, be multiplied by four to obtain the total well production rate.

Figure B-2 shows a grid network for representing a vertically fractured well which is offset within the drainage area. In this case, it is convenient to locate the grid in such a manner that the fractured well lies on a grid-line and include as "variable rate nodes" the two rows of adjacent grid-blocks to account for flow from both sides of the fracture.

Figure B-4 shows a comparison of numerical and analytical solutions for the constant fracture pressure case using the grid network similar to that of Figure B-1 and analytical solutions from Reference 22. Good agreement is obtained for both infinite and finite-acting cases. Using the grid network of Figure B-2, the numerical solution shown in Figure B-5 was obtained. An exact comparison with the analytical solution after the onset of boundary effects is not expected in this case. However, it should be noted that, for the dimensions used, the numerical results should (as indicated) yield a higher production rate than the analytical solution for a well centered in the drainage area with $x_e/x_f = 3$.

TABLE B-1. Analytical Solutions Used to Validate Induced Fracture Capability (Single-Porosity)

Constant Production Rate (21)				Constant Wellbore Pressure (22)			
$x_e/x_f = 2$		$x_e/x_f = 10^*$		$x_e/x_f = 3$		$x_e/x_f = 10^*$	
t_{Dxf}	P_D	t_{Dxf}	P_D	t_{Dxf}	P_D	t_{Dxf}	P_D
.01	0.1772	.01	0.1772	.0122	3.580	.0120	3.580
.015	0.2171	.015	0.2171	.0196	2.894	.0192	2.946
.02	0.2507	.02	0.2507	.0319	2.382	.0276	2.557
.03	0.3070	.03	0.3070	.0530	1.926	.0450	2.067
.04	0.3545	.04	0.3545	.0897	1.585	.0776	1.671
.05	0.3963	.05	0.3963	.1573	1.281	.1221	1.425
.06	0.4340	.06	0.4340	.2521	1.112	.1921	1.215
.08	0.5007	.08	0.5007	.4343	.9152	.3134	1.036
.10	0.5587	.10	0.5587	.7086	.7943	.4842	0.899
.15	0.6790	.15	0.6790	1.221	.6654	.7758	0.780
.20	0.7756	.2	0.7756	2.065	.5574	1.2658	0.677
.30	0.9261	.3	0.9261	3.370	.4507	2.1415	0.588
.40	1.0417	.4	1.0417	5.399	.3517	3.3695	0.519
.50	1.1355	.5	1.1355	8.494	.2512	5.1128	0.475
.60	1.2145	.6	1.2145	12.66	.1701	8.4944	0.427
.80	1.3427	.8	1.3427	17.23	.1152	15.174	0.384
1.0	1.4447	1.0	1.4447	24.31	.0689	26.619	0.345
1.5	1.6344	1.5	1.6344	31.91	.0405	47.552	0.322
2.	1.85	2.	1.7716	38.25	.0251		
3.	2.28	3.	1.9676				
4.	2.65	4.	2.1080				
5.	3.05	5.	2.2175				
6.	3.45	6.	2.3073				
8.	4.25	8.	2.4494				
10.	5.00	10.	2.5600				
15.	7.00	15.	2.7613				
20.	9.00	20.	2.9045				
30.	12.9	30.	3.1065				
40.	16.8	40.	3.2500				
		50.	3.3614				

* Same as infinite-acting until $t_{Dxf} \approx 50$.

FIGURE B-1. Finite-Difference Grid for Simulation of Fractured Well in Square Drainage Area

FIGURE B-2. Finite-Difference Grid for Simulation of Fractured Well Offset in Drainage Area

I-B-7

FIGURE B-3. Dimensionless Pseudo-Pressure Drop Versus Dimensionless Time -- Comparison of Numerical and Analytical Models for Constant Rate Case

FIGURE B-4. Dimensionless Rate Versus Dimensionless Time -- Comparison of Numerical and Analytical Models for Constant Wellbore Pressure Case

FIGURE B-5. Dimensionless Rate Versus Dimensionless Time -- Simulation of Vertically Fractured Well Offset in Drainage Area

APPENDIX C

Model Validation and Performance

Comparison with Analytical Models

Recent analytical studies by Raghavan, et al., (9) deemed it necessary to revalidate the numerical model. Using parameters for which production decline are available from analytical model results, simulation runs without slippage and gas desorption were made. Figure C-1 shows comparisons of numerical and analytical results for pseudo-steady-state flow in the shale matrix and Figure C-2 shows a similar comparison for unsteady-state flow. In each case, the analytical results are those recently presented by Raghavan (9). The numerical model SUGAR-MD was run using the pseudo-steady-state flow option (MCODE = 5) (see User's Manual) to obtain the reasonably good agreement shown in Figure C-1. The model was run in the normal unsteady-state mode (MCODE = 4) with 15 matrix grid-blocks to obtain the numerical results given in Figure C-2. The reason for the small difference during depletion of the fracture system is due to the fluctuation in the numerical inverter of the LaPlace transform used in the analytical solution.

Model Performance -- Dual Porosity with Induced Vertical Fracture

Figures C-3 through C-7 show production performance of a fractured well in a fractured reservoir for different drainage areas and matrix and fracture properties assuming pseudo-steady-state flow in the shale matrix. These results are being presented for the first time; that is, to our knowledge, no one has presented analytical or numerical results in the literature or in any public forum on production behavior of a vertically fractured well in a naturally fractured (dual porosity) reservoir.

The results are presented in dimensionless form, both to collapse information and to be consistent with analytical presentations such as Raghavan's recent paper (9). The parameter omega is defined by:

$$\omega = \frac{\phi_f c_{tf}}{\phi_m c_{tm} + \phi_f c_{tf}} \quad (C1)$$

where ϕ_f and ϕ_m are the porosities of the fracture system and matrix, respectively. Similarly, c_{tf} and c_{tm} are the total compressibilities of fluids within the fractures and matrix. As an approximation, we can think of omega as the ratio of fracture system storage capacity to total system storage capacity; i.e.,

$$\omega \sim \frac{\phi_f}{\phi_f + \phi_m} \quad (C2)$$

Since compressibilities are not explicitly input into the gas simulator, but rather calculated for each grid-block (fracture or matrix) as the simulation proceeds, Equation C2 was used to determine the values of omega presented in the graphical results. For each run, matrix porosity was held constant at 2 percent and ϕ_f was varied to obtain the ratios 10^{-1} and 10^{-3} , which are believed to cover the practical range of storage ratios for Devonian shale.

The matrix-fissure transfer coefficient, λ , is defined by:

$$\lambda = \alpha \frac{k_m}{k_f} r_w^2 \quad (C3)$$

where α is the shape factor discussed in detail in Appendix A. For cylinders $\alpha = 8/a^2$ where a is the cylinder radius. It has been shown analytically for the constant wellbore pressure case (9) that production decline is a function of the two dimensionless parameters, λ and ω . Thus, in the absence of slippage and desorption, one can compare and discuss analytical and numerical results in terms of these two parameters.

The drainage shape was square and the fracture length was 660 feet for all runs. Results are presented in terms of the dimensionless ratio x_e/x_f , where x_f is one-half the induced fracture length and the drainage area is $(2x_e)^2$.

Figure C-3 shows the calculated production decline for two values of the matrix-fissure transfer coefficient, λ , with $\omega = 10^{-1}$ and a drainage area-fracture half-length ratio of 3.0. In comparing these results with the single porosity performance given in Figure B-4, the following features are important:

1. Dimensionless production rate is slightly higher at early dimensionless times ($t_{Dxf} < 10^{-1}$).
2. Dimensionless production rate as a function of dimensionless time is substantially the same as a single porosity system for about one log cycle ($0.1 < t_{Dxf} < 1.0$).
3. For $t_{Dxf} > 1.0$, any given dimensionless production rate is sustained much longer as compared to a single porosity system.

Note that the larger the transfer coefficient, the greater the production enhancement at intermediate times. Of course, the matrix will also deplete more rapidly for the higher value of λ , thus resulting in a "crossover" which in this case occurs at a t_{Dxf} of about 200.

Figure C-4 shows production performance for a lower value of the dimensionless storage ratio. Otherwise, all input data was identical to that of Figure C-3. Here we note a definite flattening as observed for a nonfractured well (Figure C-1) for the same value of ω . The flattening occurs here for about one-and-one-half log cycles for $\lambda = 10^{-7}$ which was not evident for the larger value of ω in Figure C-3. Thus, it appears that pseudo-steady-state behavior, as observed for nonfractured wells, also occurs for the case where the well is vertically fractured. However, the vertical separation is less (only about one-half log cycle) and the time for separation to occur is much greater, which may make type-curve matching more difficult for fractured wells than for nonfractured wells in fractured reservoirs.

Figure C-5 shows the effect of drainage area size on production response for a fractured well in a fractured reservoir for $\lambda = 10^{-8}$ and $\omega = 10^{-1}$. The shape of the production decline is essentially the same as that for a single porosity system, with the exception of the partial flattening that occurs for x_e/x_f at a dimensionless time of about 35.

Figures C-6 and C-7 show the effect of anisotropy on production decline for two different size drainage areas. A fracture system permeability ratio of $k_x/k_y = 5$ was used with the mean permeability, $\bar{k} = \sqrt{k_x k_y}$, the base case value of 0.1 md. In Figure C-6, a substantial reduction in production rate results from the reduced y-direction permeability. With the larger drainage area (Figure C-7), much less difference is observed.

Figure C-8 shows a comparison of unsteady-state and pseudo-steady-state flow in the shale matrix for a vertically fractured shale well with $x_e/x_f = 3$ and w and λ of 10^{-1} and 10^{-8} , respectively. Fifteen grid-blocks were used to solve for the pressure distribution in the cylindrical element in an attempt to approximate the analytically correct solution. The production performance for the two cases is quite similar. The crossover at a dimensionless time of about 60 results in nearly equal cumulative productions. Also, type-curve matching for the $t_{Dxf} < 30$ would give very similar results for either case. Since the model runs much faster for pseudo-steady-state flow (which should be used whenever possible), this is an important observation.

Additional support for use of the pseudo-steady-state model for fractured wells is provided by the results given in Figure C-9. Here the drainage area is very large and there is essentially no difference in the production performance for unsteady-state and pseudo-steady-state flow over six log cycles which covers a real-time range of .01 days to 10,000 days.

Figure C-10 shows a comparison of production performance for $x_e/x_f = 3$, $\lambda = 10^{-8}$ and a low fracture porosity giving $w = 10^{-3}$. In this case, there is a significant difference in performance, particularly at intermediate times. As expected, the pseudo-steady-state model gives a more constant rate at intermediate times. In this case, type-curve matching could help identify the flow regime in the shale matrix.

FIGURE C-1. Comparison of Numerical and Analytical Results for Pseudo-Steady-State Flow in Shale Matrix

FIGURE C-2. Comparison of Numerical and Analytical Results --
Unsteady-State Flow in Shale Matrix

FIGURE C-3. Effect of Matrix-Fissure Transfer Coefficient on Fractured Well Response

FIGURE C-4. Effect of Matrix-Fissure Transfer Coefficient on Fractured Well Response

FIGURE C-5. Effect of Size of Drainage Area on Response of Vertically Fractured Well in Fractured Reservoir

FIGURE C-6. Effect of Anisotropy on Response of Vertically Fractured Well in a Fractured Reservoir

FIGURE C-7. Effect of Anisotropy on Response of Vertically Fractured Well in a Fractured Reservoir

FIGURE C-8. Comparison of Unsteady-State and Pseudo-Steady-State Flow in Shale Matrix for a Vertically Fractured Well

FIGURE C-9. Comparison of Unsteady-State and Pseudo-Steady-State Flow in Shale Matrix for a Vertically Fractured Well

FIGURE C-10. Comparison of Unsteady-State and Pseudo-Steady-State Flow in Shale Matrix for a Vertically Fractured Well

APPENDIX D

Comparison of Model Results With Laboratory Experiments

The following sections consist of: (1) a brief description of laboratory work conducted by the Battelle Columbus Laboratories (23), (2) modifications made in using the numerical model to simulate laboratory flow experiments, and (3) comparison of laboratory and model results.

I. SUMMARY OF LABORATORY DATA

A. Gas Slippage

One of the primary purposes of the Battelle Laboratories study (23) was to ascertain whether or not gas slippage (i.e., molecular flow) is a significant factor in flow and production performance, based on Devonian shale samples from the Meigs County, Ohio, offset well test site. It was found that gas slippage can be very significant in the Devonian shale in terms of the molecular flow contribution.

Coutant (23) presents the following equation for the fractional flow due to molecular effects or gas slippage:

$$f_m = \frac{1}{30,500 \bar{p} \bar{r} + 1} \quad (D1)$$

Here $\bar{p}$ is the mean pressure in atmospheres and $\bar{r}$ is the average pore radius in cm. From the Klinkenberg formulation, we can also determine a molecular flow contribution from the following relationship:

$$f_m^* = \frac{b k_L / P}{k_L (1 + b/p)} \quad (D2)$$

Simplifying Equation D2 and equating f_m and f_m^* gives:

$$b = \frac{1}{30,500 \bar{r}} \quad (D3)$$

To illustrate the use of Equation D3, consider average pore radii of $36\text{\AA}$ and $54\text{\AA}$ which correspond to Battelle's samples BA35 and BA39 from Meigs County, Ohio.

Since the effective gas permeability is:

$$k_g = k_L (1 + b/\bar{p}) \quad (D4)$$

we see that a very significant (pressure-dependent) permeability enhancement results at pressures less than 500 to 700 psi, which are typical of most Devonian shale reservoirs.

It is interesting to note that there are two simple graphical analysis methods for determining the true permeability and slippage factor from gas flow measurements at different pressures. These methods are illustrated by the following equations:

Method 1:

$$k_g = k_L (1 + b/\bar{p})$$

$$k_g = k_L + (k_L b) (1/\bar{p}) \quad (D5)$$

A plot of k_g versus $(1/\bar{p})$ is linear with intercept k_L and slope $(k_L b)$.

Method 2:

$$k_g = k_L (1 + b/\bar{p})$$

$$k_g \bar{p} = k_L \bar{p} + k_L b \quad (D6)$$

A plot of $(k_g \bar{p})$ versus $\bar{p}$ is linear with intercept $(k_L b)$ and slope k_L .

As Coutant (23) plots p_m versus $\bar{p}$ where

$$p_m = (Q\mu L)/(A\Delta\bar{p}) \quad (Q @ 1 \text{ atm}) \quad (D7)$$

and, for gas,

$$k_g = (\bar{Q}\mu L)/(A\Delta\bar{p}) \quad (D8)$$

with $\bar{Q} = Q/\bar{p}$, we see that Method 2 is appropriate for analysis of slippage data in the Battelle report.

B. Adsorption/Desorption

By use of a low-temperature ashing (LTA) procedure, Coutant (23) determined gravimetrically the kerogen content of several Meigs County, Ohio, samples and correlated kerogen content with adsorption gas (methane) as determined in a manner similar to that of Thomas and Frost (1). It was found that virtually all adsorbed gas is contained within the kerogen. Moreover, methane adsorption was recommended as a more sensitive and accurate measure of kerogen content than the tedious LTA method.

Figure D-1 presents adsorption/desorption data for Meigs County, Ohio, samples and also a curve from the data of Thomas and Frost (2) for comparison. The data shown for samples BA39 and BA41 were obtained from Coutant's Figure 12 and the kerogen content given in Table 1 (23). Sample BA41 contained the most kerogen among the 42 samples analyzed. If this is representative of the area, then the maximum adsorbed gas content in Meigs County, Ohio, is similar to the Illinois Basin samples of Thomas and Frost (1).

C. Porosity and Permeability

Porosity measurements were made by Coutant (23) using a conventional BET system. Shale matrix porosities were normally 2 to 3 percent.

Battelle's permeability measurements were, for the most part, made on small plugs mounted in special holders designed for containment pressures of 3000 to 3500 psi. In this manner, actual in-situ stress could be simulated. However, two problems must be considered in using the high-pressure plug permeabilities. First, permeability was found to be very dependent on containment pressure. Since the maximum axial containment pressure in the radial flow apparatus (23) was about 1100 psi, the correction for shale compressibility presented by Coutant must be used to approximate permeability at the lower pressure. Second, the plug sample may not even be representative of the whole core section from which it was cut. These problems were partially eliminated in the simulation study by using the steady-state radial permeability data on sample BA39 given in Coutant's Figure 15.

II. MODEL MODIFICATIONS

To simulate Battelle's unsteady-state radial flow experiments and consider desorption, some coding changes in SUGAR-MD were necessary. In order to handle the boundary conditions properly, it was desired to run the model in the single porosity mode (MCODE = 0). However, no provision for adsorption/desorption was available in this mode. Therefore, the desorption source term was added to the model for the otherwise single-porosity mode and is invoked by specifying MCODE = 6.

With this added feature the model can readily be used to simulate nearly any type of laboratory flow experiment on the Devonian shale. In addition, the model now has added flexibility for field-scale simulations. The following table summarizes the modes of operation now available in SUGAR-MD.

Master Code (MCODE)	Type Model	Options	Major Features
0	Single porosity	Slippage	
4	Dual porosity	Slippage, desorption in shale matrix	Unsteady-state flow in shale matrix
5	Dual porosity	Slippage, desorption in shale matrix	Pseudo-steady-state flow in shale matrix
6	Single porosity	Slippage, desorption	

III. COMPARISON OF MODEL AND LABORATORY RESULTS

In order to further validate the numerical model (SUGAR-MD), it was necessary to show that it would, in fact, simulate laboratory flow measurements on the Devonian shale under carefully controlled boundary and initial conditions. Battelle's unsteady-state radial flow experiments on sections of whole core were selected for this purpose. Figures D-2 and D-3 give examples of the type of data involved.

These experiments are described in detail by Coutant (23) and will not be repeated here. Basically, each experiment consisted of (1) an overnight "soaking period" during which the exterior cylindrical surface of the sample was subjected to an applied pressure of approximately 500 psia and (2) a "blowdown period" in which the exterior source of gas was turned off and the center borehole was opened to local atmospheric pressure. During this blowdown period, both the pressure falloff at the exterior of the sample and the borehole flowrate were accurately measured. From a knowledge of the volume of tubing and annulus at the exterior of the sample and the bulk volume and porosity of the sample, the volume of desorbed gas that was produced, if any, could be calculated.

To simulate these experiments, the outer boundary pressure falloff was used as a specified boundary condition. One reason for doing this was to eliminate the problem of the void space in the annulus and tubing. The borehole flowrate was calculated as a function of time and used as the matching parameter. That is, the simulator flowrate was compared to the laboratory flowrate to determine if the model was performing in accordance with the laboratory experiment. Figures D-4 through D-7 give results of the model validation conducted in this manner. The flowrates given are the laboratory flowrates converted from cc/sec to Mcfd and adjusted for the different sample lengths used in the laboratory and in the simulator. A very long length was used in the simulator to obtain workable values of gas-in-place.

Shale density and porosity from Coutant's Table 1 were used and not changed during the course of the simulation runs. Also, using the pore radius from Table D-1, Equation D3 was used to obtain a "b value" which was not varied. Since whole-core permeability data was available for a sample fragment (BA39-3358X), it was selected for matching. Also, sample BA39-3358 indicated desorption from the unsteady flow rate data, whereas sample BA39-3358X did not. Thus, both these laboratory radial flow tests were simulated using the numerical model. Battelle's flow and pressure data are given for these two samples in Tables D-2 and D-3.

Figures D-4 to D-6 give results for sample BA39-3358X. No desorption was used, and the permeability range of .042 to .071 μ d given in Table D-1 was used in the initial runs. This permeability range was determined using the graphical analysis Method 2 described on page I-D-2. As shown in Figure D-4, a minor adjustment in k to .080 μ d gave a fairly good match to the laboratory data. Figure D-5 shows the sensitivity of the match to matrix permeability, and Figure D-6 shows the sensitivity to the slippage parameter. Also, Figure D-6 shows that the laboratory flow rate data could not be matched without slippage by increasing matrix permeability.

Figure D-7 shows results for sample BA39-3358. Desorption was used in the simulator in this case, as the laboratory data indicated a gas recovery about 60 percent greater than the initial free gas content. The adsorption data of sample BA41, shown in Figure D-1, was used as an upper bound since kerogen content could vary considerably within a whole core section. Figure D-7 shows that a fair match was obtained for the first 90 minutes with a matrix permeability of .078 μ d. The lower curve shows the sensitivity to desorption. It was found that without desorption a match could still be obtained by increasing permeability to .090 μ d. This is due to the fact that, with the outer boundary pressure controlled, desorbed gas was replaced by external injection. Thus, unless shale matrix permeability and desorption data are known very accurately for the specific sample being tested, it is virtually impossible to say the model will or will not duplicate laboratory experiments with all parameters specified. However, the fact that the laboratory flow data for sample BA39-3358 could be matched with adsorption and permeability data similar to values determined in the laboratory is quite encouraging.

IV. SUMMARY AND RECOMMENDATIONS

In summary, it has been shown that the model SUGAR-MD will reasonably simulate unsteady-state laboratory flow experiments on Devonian shale with input parameters either the same or similar to data measured in the laboratory on the same type of shale.

The model has previously been validated based on analytical models for fractured reservoirs (7,9) and also has successfully simulated several field tests. Thus, the SUGAR-MD code is considered to be a complete, validated numerical simulator for Devonian shale.

It is recommended that any additional laboratory testing on the Devonian shale be concentrated in the following areas:

1. Steady-state permeability measurements (preferably on whole core samples) to obtain more data on matrix permeability and gas slippage.
2. Unsteady-state flow experiments performed in a manner similar to Coutant (23) with the void space at the upstream side of the sample reduced to a negligible fraction of the pore volume.

Such data, together with porosity and adsorption/desorption data for the same sample, could be used to provide additional testing of the model performance. That is, (a) direct measurements on both slippage and permeability could be input and not varied and (b) simulated pressure falloff at the upstream side of the sample (as well as downstream flow-rate) could be used to compare against laboratory results. However, considering the difficulty of obtaining good reliable laboratory data on all the critical parameters, such additional testing may not be warranted at this time.

TABLE D-1

Summary of Battelle Laboratories Data for Selected Samples

Sample	Average Pore Radius A°	Porosity (%)	Permeability (μ d)	Bulk Density (gm/cc)	Kerogen Content (wt.%)
BA35-3238	36	2.72	.007*	2.684	<0.1
BA39-3358	54	2.26	1.75*	2.527	0.4
BA39-3358X	54	2.26	.042 - .071**	2.527	0.4
BA41-3289	147	8.15	<.0002*	2.497	1.40

* Gas permeability (k_g) from plugs.

** True permeability (k_L) from steady-state radial flow experiment.

TABLE D-2

Unsteady-State Radial Flow Data (23) Indicating Desorption

RUN ID = BA39-3358

P(C) = 1050 psig (axial containment pressure)

V(C) = 753 cc (sample volume)

P(E) = 498 psig (external pressure during 16 hr "soaking" period)

Time (S)	P(A) (psig)	Flow (cc-atm/sec)	V(T) (cc-atm)	V(S) (cc-atm)
666.00	490.	0.6110	428.00	361.00
818.00	480.	0.6050	520.00	386.0
1063.00	460.	0.5820	666.00	398.0
1325.00	440.	0.5560	815.00	413.0
1929.00	400.	0.4970	1133.00	464.0
2354.00	375.	0.4490	1334.00	498.0
2722.00	355.	0.4230	1494.00	524.0
3371.00	325.	0.3730	1753.00	582.0
3920.00	300	0.3340	1947.00	609.0
5092.00	256.	0.2630	2296.00	664.0
7547.00	193.	0.1700	2828.00	774.0
10233.00	147.	0.1110	3205.00	843.0
16703.00	87.	0.0450	3712.00	948.0

P(A) = borehole pressure with external pressure turned off.

TABLE D-3

Unsteady-State Radial Flow Data (23) Indicating No Desorption

RUN ID = BA39-3358X

P(C) = 1130 psig (axial containment pressure)

V(C) = 544 cc (sample volume)

P(E) = 487 psig (external pressure during 16 hr "soaking" period)

Time (S)	P(A) (psig)	Flow (cc-atm/sec)	V(T) (cc-atm)	V(S) (cc-atm)
178.00	480.	0.3510	65.90	18.5
398.00	470.	0.3640	145.00	29.4
605.00	460.	0.3670	220.00	37.1
1027.00	440.	0.3570	373.00	54.4
1461.00	420.	0.3390	524.00	70.3
1896.00	400.	0.3210	667.00	78.0
2643.00	370.	0.2850	894.00	101.5
3518.00	338.	0.2490	1128.00	118.5
4770.00	300.	0.2050	1412.00	145.9
6582.00	255.	0.1570	1741.00	169.8
9698.00	200.	0.0720	2099.00	155.0
13496.00	155.	0.0430	2317.00	68.8

P(A) = borehole pressure with external pressure turned off.

1:jj:529e9

FIGURE D-1. Summary of Adsorption/Desorption Data

FIG RE D-2. Unsteady-State Radial Flow Data (23)

FIGURE D-3. Laboratory Pressure Falloff Data (23)

FIGURE D-4. Comparison of Laboratory and Model Results -- Unsteady-State Radial Flow

FIGURE D-5. Comparison of Laboratory and Model Results -- Unsteady-State Radial Flow

FIGURE D-6. Comparison of Laboratory and Model Results -- Unsteady-State Radial Flow

FIGURE D-7. Comparison of Laboratory and Numerical Model Results -- Unsteady-State Radial Flow