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PERFORMANCE PREDICTIONS OF SILICA-GEL
DESICCANT DEHUMIDIFIERS

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NOMENCLATURE

Abbreviation	Term
a, c, d	Coefficients in equation (1.45)
a_1, a_2, a_w	Width of process channel, width of cooling channel, twice the thickness of silica gel sheet respectively, meters
b	Ratio of heat transfer coefficients
C_2, C_3, C_4, C_5	Constants defined by equations (1.98), (1.99), (1.96 and (1.101) respectively
c_a, c_t, c_s, c_m	Specific heat of aluminum, Teflon, silica gel and water moisture respectively, kJ/kg °C
c_{WR}	Some reference specific heat, kJ/kg °C
COP	Coefficient of performance
f	Mass fraction of silica gel in the dehumidifier
g	Mass fraction of Teflon in the silica gel sheet
H_1, H_2, H_P, H_w	Enthalpy of process air, cooling air, purging air and desiccant wall respectively kJ/kg
h_1, h_2, h_P	Heat transfer coefficient for process air, cooling air and purging air respectively, kJ/m ² sec °C
H	Height of the dehumidifier, meters
ΔH_w	Integral heat of wetting, kJ/kg desiccant
I, J, K	Nodal point variables in the finite difference scheme
K_y	Mass transfer coefficient, kg/m ² sec
$\dot{m}_1, \dot{m}_2, \dot{m}_P$	Mass flowrates of process air, cooling air and purging air respectively, kg/sec
m_w	Mass of the dehumidifier, kg
n	Number of channels
Q	Heat of sorption, kJ/kg H ₂ O

Abbreviation	Term
RH	Relative humidity of process air
T_1, T_2, T_p, T_w	Temperature of the process air, cooling air, purging air and the desiccant wall respectively, °C
T_o, T_{2i}, T_{pi}	Inlet temperatures of sorbing air, cooling air and purging air respectively, °C
T_D	Inlet temperature of desorbing air, °C
T_A	Ambient air temperature, °C
T_f	Temperature function defined by equation (1.42)
$t_A, t_{AF}, \Delta t_A$	Time variable, total sorption time, and time increment respectively, sec
$t, t_F, \Delta t$	Nondimensional time variable, total non-dimensional sorption time, nondimensional time step respectively
t_B, t_C	Time variables defined by equations (1.6) and (1.73) respectively, sec
t_{FP}	Nondimensional purging time
u_1, u_2, u_p	Velocity of the process air, cooling air and the purging air respectively, m/sec
V	Volume of the dehumidifier, m ³
$x_A, x_{AF}, \Delta x_A$	Space variable in the direction of process air flow, length of process channel and length of element respectively, meters
$x, x_F, \Delta x$	Nondimensional space variable in the direction of process air flow, nondimensional process channel length and nondimensional x-step size respectively
Y	Absolute humidity of process air, kg H ₂ O/kg dry air
Y_o	Inlet absolute humidity of process air, kg H ₂ O/kg dry air
Y_D	Inlet absolute humidity of desorbing air, kg H ₂ O/kg dry air

Abbreviation	Term
Y_w	Absolute humidity of the air in equilibrium with the wall during any sorption process, kg H ₂ O/kg dry air
$Y_A, Y_{AF}, \Delta Y_A$	Space variable in the direction of cooling air flow, length of the cooling channel and length of element respectively, meters
$y, y_F, \Delta y$	Nondimensional space variable in the direction of cooling air flow, nondimensional length of the cooling channel and nondimensional y-step size respectively
$\lambda_1, \lambda_2, \lambda_3, \lambda_4$	Parameters defined by equations (1.19), (1.14), (1.12) and (1.33) respectively
a_w, β_w	Coefficients in equation (1.39)
ϕ	Ratio of vapor pressure of H ₂ O in air to the atmospheric pressure
ϕ_s	Ratio of saturation vapor pressure of H ₂ O in air to the atmospheric pressure
ρ_1, ρ_2, ρ_w	Density of process air, cooling air and desiccant wall respectively, kg/m ³

ABSTRACT

The report describes the analysis of a cross-cooled desiccant dehumidifier using silica gel in the form of sheets. This unit is the principal component of solar powered desiccant air conditioning system. The mathematical model has first been formulated describing the dynamics of the dehumidifier. The model leads to a system of nonlinear coupled heat and mass transfer equations for the sorption processes and linear heat transfer equations for the purging processes. The model accounts for the gas film resistance and for the moisture diffusion in the desiccant. The governing equations are solved by a finite difference scheme to obtain periodic steady state solutions. The accuracy of the theoretical predictions is ascertained by comparing them with the experimental results. The performance of the dehumidifier, for a chosen set of initial conditions and dehumidifier parameters, has also been given.

CHAPTER I

INTRODUCTION

This report describes the details of the theoretical predictions of the performance of solar powered cross-cooled desiccant dehumidifiers. The dehumidifier is the principal component of desiccant air conditioning system. The function of the dehumidifier is to remove moisture from the air to be conditioned. Hot air from the solar collector is used to periodically regenerate the dehumidifier.

The desiccant considered for this analysis is silica gel. The main advantage of using silica gel is that it can be regenerated at low temperatures (65°C). Hence, silica gel dehumidifiers can utilize the air delivered by simple flat plate collectors for regeneration. However, silica gel has good adsorption properties only at low temperatures. The adsorption capacity decreases as the desiccant temperature increases. Therefore it is necessary to keep the temperature of the desiccant bed low during adsorption. Roy and Gidaspo¹ have shown that this can be achieved by the technique of cross-cooling.

In cross-cooled dehumidifiers, the process air is passed through channels containing desiccant material and the cooling air is passed at right angles to the direction of the process air flow through intermittently arranged cooling channels (Figure 1.1). Furthermore, the desiccant in the process channel may either be packed or made into sheet form and bonded to the walls of the process channels. The latter

Figure 1.1 Schematic of a Cross Flow Desiccant Dehumidifier

arrangement gives good thermal contact with the cooling stream and also reduces the pressure drop in the process channel. Perkari² described the details of desiccant sheet making which, in addition to having micropores, have also macropores formed by the silica gel particles and the bonding medium.

For the present study, cross-cooled dehumidifiers with silica gel sheets have been considered. The unit can either be of fixed bed type or of rotating type. If the fixed bed type is used in a cooling system, two dehumidifiers are needed; one being regenerated while the other is adsorbing. In a rotating system, a portion of the unit will adsorb while the remaining portion is desorbing.

Sorption Process

The process of exchanging moisture and heat between the process air and a desiccant is called the sorption process. When the moisture is given up by the process air to the desiccant, the process is called adsorption and when the moisture is given back to the hot air from the desiccant, the process is called desorption. During adsorption, the heat is released by the sorbate and therefore adsorption is exothermic. Similarly, heat is supplied to the sorbate during desorption. This heat is usually called the heat of sorption.

Purging Process

A dehumidifier undergoes cyclically adsorption and desorption at a predetermined cycle time. The desiccant bed is usually hot at the end of desorption and thus is not immediately ready for subsequent adsorption. It requires precooling.

Similarly, the desiccant bed at the end of adsorption is cool and needs heating before desorption begins. The process of cooling the desorbed bed and heating the adsorbed bed is known as purging.

In fixed bed dehumidifiers, purging is accomplished by passing ambient air through the hot bed and directing the air delivered from this bed to the cold bed, thus transferring heat from one unit to the other. During the period of purging, no useful cooling of process air takes place. This duration must be carefully designed. Excessive purging time renders the two beds better prepared for the next process but the cooling capacity averaged over both active and idle periods is reduced.

Literature Survey

The work of earlier investigators deals mostly with adiabatic sorption processes and centers on a once through adsorption process only, without considering periodic steady state operation.

Hubard³ performed extensive studies on equilibrium data of silica gel. Most of his data were obtained from Taylor's study⁴ and the remaining data were determined by a dynamic air saturation technique involving relative humidity control solutions.

Bullock and Threlkeld⁵ studied the dehumidification of moist air by adiabatic adsorption both theoretically and experimentally. For the numerical solution, they used a modified Euler method and a Predictor-Corrector method.

Chi and Wasan⁶ have analysed both isothermal and adiabatic adsorption processes by solving the governing equations in a manner similar to the numerical schemes of Bullock and Threlkeld. They have also reported the experimental results of air-water-silica gel systems and Helium-methane-activated carbon systems.

Lunde and Kester⁷ have extended the basic BET approach to apply to gas adsorption in finite layers of porous materials. They have obtained closed form solutions to the resulting equations and applied them to published data for the adsorption of water by silica gel, to predict a wide range of adsorption isotherms.

Banks, Close and MacLaine-Cross⁸ have presented an approximate analogy approach to solve the governing equations from the well known heat transfer alone solutions. Comparison between the analogy solutions and the numerical nonlinear solutions has been made and shown to be adequate.

Holmberg⁹ has studied theoretically the performance of adiabatic dehumidifiers under periodic steady state operations. An iterative numerical scheme has been used to match the transfer conditions between adsorption and desorption. They have also shown the influence of matrix heat and moisture capacities on temperature and humidity.

Rousseau et al¹⁰ are in the process of developing a Solar Powered Adiabatic Desiccant Cooling System. They are using a sorbent bed model, developed by Hwang¹¹, and prepared an integrated system-load computer model to predict the perfor-

mance of the cooling system. They include purging between the sorption process and obtained a design COP of 0.52.

Roy and Gidaspow¹ solved the system of governing equations for cross-cooled dehumidifiers by the method of Green's functions and presented results of the adsorption process for various degrees of cooling.

Clark, Mills and Buchberg¹² have studied both theoretically and experimentally the performance of fixed silica gel beds for the dehumidification of air in an air conditioning system. They found considerable discrepancies between the experimental and predicted results which are attributed to the neglect of solid side mass transfer resistance and the inaccuracy of the gas side transfer coefficient correlations.

The present work deals with a cross-cooled adsorption process, purging and an adiabatic desorption process. The analysis is for the periodic steady state operation of the dehumidifier.

CHAPTER II

MATHEMATICAL FORMULATION

The basic unit of a cross cooled dehumidifier is shown in Figure 1.1. The wall, separating the process and cooling channels, includes a desiccant material which is capable of adsorbing (and desorbing) H_2O from the process stream. The wall will exchange heat with both the process and the cooling streams. Thus the process stream exchanges both mass and heat with the separating wall while the cooling stream exchanges only heat with the wall. The rate of exchange of heat and mass between the wall and the two fluid streams is described by transfer coefficients which can be calculated from Nusselt and Sherwood numbers.

Model for the Sorption Process

The dynamics of sorption can be described by the conservation equations and transfer rate equations for heat and mass.

Consider the sorption process on an element of size Δx_A and Δy_A as shown in Figure 1.2. The conservation of mass of H_2O in the wall and in the process stream gives

The rate of increase of moisture in air in this element
 + the rate of increase of moisture in the bed in this element
 + the net outflow of moisture from this element = 0

$$\rho_1 \Delta x_A \Delta y_A a_1 \frac{\partial Y}{\partial t_A} + f \rho_w \Delta x_A \Delta y_A a_w \frac{\partial w}{\partial t_A} + \frac{\dot{m}_1 \Delta y_A}{n y_{AF}} \left[\left(Y + \frac{\partial Y}{\partial x_A} \Delta x_A \right) - Y \right] = 0$$

Figure 1.2 Model of a Differential Element

$$\rho_1 a_1 \frac{\partial Y}{\partial t_A} + f \rho_w a_w \frac{\partial w}{\partial t_A} + \frac{\dot{m}_1}{n y_{AF}} \frac{\partial Y}{\partial x_A} = 0$$

Using the relations $\dot{m}_1 = \rho_1 a_1 Y_{AF} u_1 n$ and $\dot{m}_w = \rho_w x_{AF} Y_{AF} a_w n$, we get

$$\dot{m}_1 \left[\frac{1}{u_1} \frac{\partial Y}{\partial t_A} + \frac{\partial Y}{\partial x_A} \right] + \frac{f \dot{m}_w}{x_{AF}} \frac{\partial w}{\partial t_A} = 0 \quad (1.1)$$

The mass transfer rate of H_2O between the process stream and the wall is given by

$$f \rho_w \Delta x_A \Delta y_A a_w \frac{\partial w}{\partial t_A} = 2 K_Y \Delta x_A \Delta y_A (Y - Y_w)$$

$$\frac{f \dot{m}_w}{n x_{AF} Y_{AF}} \frac{\partial w}{\partial t_A} = 2 K_Y (Y - Y_w) \quad (1.2)$$

The conservation of energy in the wall, in the process stream and in the cooling stream gives

The rate of increase of process air enthalpy
 + the rate of increase of cooling air enthalpy
 + the rate of increase of the bed enthalpy
 + the net outflow of enthalpy in process channel
 + the net outflow of enthalpy in cooling channel = 0

$$\rho_1 \Delta x_A \Delta y_A a_1 \frac{\partial H_1}{\partial t_A} + \rho_2 \Delta x_A \Delta y_A a_2 \frac{\partial H_2}{\partial t_A} + \rho_w \Delta x_A \Delta y_A a_w \frac{\partial H_w}{\partial t_A}$$

$$+ \frac{\dot{m}_1 \Delta Y_A}{n y_{AF}} \left[\left(H_1 + \frac{\partial H_1}{\partial x_A} \Delta x_A \right) - H_1 \right]$$

$$+ \frac{\dot{m}_1 \Delta x_A}{n x_{AF}} \left[\left(H_2 + \frac{\partial H_2}{\partial y_A} \Delta y_A \right) - H_2 \right] = 0$$

Using the relations $\dot{m}_1 = \rho_1 a_1 y_{AF} u_1 n$, $\dot{m}_2 = \rho_2 a_2 x_{AF} u_2 n$ and $\dot{m}_w = \rho_w x_{AF} y_{AF} a_w n$, we get

$$\frac{\dot{m}_1}{y_{AF}} \left[\frac{1}{u_1} \frac{\partial H_1}{\partial t_A} + \frac{\partial H_1}{\partial x_A} \right] + \frac{\dot{m}_2}{x_{AF}} \left[\frac{1}{u_2} \frac{\partial H_2}{\partial t_A} + \frac{\partial H_2}{\partial y_A} \right] + \frac{\dot{m}_w}{x_{AF} y_{AF}} \frac{\partial H_w}{\partial t_A} = 0 \quad (1.3)$$

The energy transfer rate between the process stream and the wall is given by

The rate of increase of process air enthalpy +
the net outflow of enthalpy in process channel
= The rate of heat released by the sorbed vapor +
the rate of heat transfer from the wall

$$\begin{aligned} & \rho_1 \Delta x_A \Delta y_A a_1 \frac{\partial H_1}{\partial t_A} + \frac{\dot{m}_1 \Delta y_A}{n y_{AF}} \left[(H_1 + \frac{\partial H_1}{\partial x_A} \Delta x_A) - H_1 \right] \\ &= 2K_Y \Delta x_A \Delta y_A (Y_w - Y) \frac{\partial H_1}{\partial Y} + 2h_1 \Delta x_A \Delta y_A (T_w - T_1) \\ & \frac{\dot{m}_1}{n y_{AF}} \left[\frac{1}{u_1} \frac{\partial H_1}{\partial t_A} + \frac{\partial H_1}{\partial x_A} \right] = 2K_Y (Y_w - Y) \frac{\partial H_1}{\partial Y} + 2h_1 (T_w - T_1) \quad (1.4) \end{aligned}$$

The energy transfer rate between the cooling stream and the wall is given by

The rate of increase of cooling air enthalpy +
the net outflow of enthalpy in the cooling channel
= The rate of heat transfer from the wall

$$\begin{aligned} & \rho_2 \Delta x_A \Delta y_A a_2 \frac{\partial H_2}{\partial t_A} + \frac{\dot{m}_2 \Delta x_A}{n x_{AF}} \left[(H_2 + \frac{\partial H_2}{\partial y_A} \Delta y_A) - H_2 \right] \\ &= 2h_2 \Delta x_A \Delta y_A (T_w - T_2) \end{aligned}$$

$$\frac{\dot{m}_2}{n x_{AF}} \left[\frac{1}{u_2} \frac{\partial H_2}{\partial t_A} + \frac{\partial H_2}{\partial y_A} \right] = 2h_2 (T_w - T_2) \quad (1.5)$$

Now a new time variable t_B can be defined for convenience in the following form:

$$t_B = t_A - \frac{x_A}{u_1} - \frac{y_A}{u_2} \quad (1.6)$$

Then we have

$$\frac{\partial}{\partial t_A} = \frac{\partial}{\partial t_B} \frac{\partial t_B}{\partial t_A} = \frac{\partial}{\partial t_B} \quad (1.7)$$

Further

$$\begin{aligned} \left. \frac{\partial}{\partial x_A} \right|_{t_A} &= \frac{\partial}{\partial t_B} \frac{\partial t_B}{\partial x_A} + \left. \frac{\partial}{\partial x_A} \right|_{t_B} + \frac{\partial}{\partial y_A} \frac{\partial y_A}{\partial x_A} \\ &= -\frac{1}{u_1} \frac{\partial}{\partial t_B} + \left. \frac{\partial}{\partial x_A} \right|_{t_B} \end{aligned} \quad (1.8)$$

Similarly

$$\begin{aligned} \left. \frac{\partial}{\partial y_A} \right|_{t_A} &= \frac{\partial}{\partial t_B} \frac{\partial t_B}{\partial y_A} + \frac{\partial}{\partial x_A} \frac{\partial x_A}{\partial y_A} + \left. \frac{\partial}{\partial y_A} \right|_{t_B} \\ &= -\frac{1}{u_2} \frac{\partial}{\partial t_B} + \left. \frac{\partial}{\partial y_A} \right|_{t_B} \end{aligned} \quad (1.9)$$

To avoid ambiguity, in equations (1.8) and (1.9), the variable held constant in a particular differentiation, among the two time variables, is shown explicitly.

Substituting equations (1.7) and (1.8) in equation (1.1),

$$\dot{m}_1 \frac{\partial Y}{\partial x_A} + \frac{f m_w}{x_{AF}} \frac{\partial w}{\partial t_B} = 0$$

Using equation (1.2) in this equation

$$\dot{m}_1 \frac{\partial Y}{\partial x_A} = 2nK_y Y_{AF} (Y_w - Y) \quad (1.10)$$

Multiplying throughout by $\frac{1}{h_1} \frac{\partial H_1}{\partial T_1}$ and rearranging

$$\frac{\dot{m}_1}{2n h_1 Y_{AF}} \frac{\partial H_1}{\partial T_1} \frac{\partial Y}{\partial x_A} = \lambda_3 (Y_w - Y) \quad (1.11)$$

where

$$\lambda_3 = \frac{K_y}{h_1} \frac{\partial H_1}{\partial T_1} \quad (1.12)$$

Equation (1.2) can be written as

$$\frac{m_w c_{wR}}{2n x_{AF} Y_{AF} h_1} \frac{\partial w}{\partial t_B} = \lambda_2 (Y - Y_w) \quad (1.13)$$

where

$$\lambda_2 = \frac{K_y c_{wR}}{f h_1} \quad (1.14)$$

and c_{wR} is some reference quantity having the dimensions of specific heat.

Using equations (1.7), (1.8) and (1.10) in equation (1.4)

$$\frac{\dot{m}_1}{n Y_{AF}} \frac{\partial H_1}{\partial x_A} = \frac{\dot{m}_1}{n Y_{AF}} \frac{\partial Y}{\partial x_A} \frac{\partial H_1}{\partial Y} + 2h_1 (T_w - T_1)$$

Rewriting $\frac{\partial H_1}{\partial x_A}$ as $\frac{\partial H_1}{\partial T_1} \frac{\partial T_1}{\partial x_A} + \frac{\partial H_1}{\partial Y} \frac{\partial Y}{\partial x_A}$, we get

$$\frac{\dot{m}_1}{nY_{AF}} \frac{\partial H_1}{\partial T_1} \frac{\partial T_1}{\partial x_A} = 2h_1 (T_w - T_1)$$

$$\frac{\dot{m}_1}{2nh_1 Y_{AF}} \frac{\partial H_1}{\partial T_1} \frac{\partial T_1}{\partial x_A} = T_w - T_1 \quad (1.15)$$

Also using equations (1.7) and (1.9) in equation (1.5)

$$\frac{\dot{m}_2}{nx_{AF}} \frac{\partial H_2}{\partial Y_A} = 2h_2 (T_w - T_2)$$

Rewriting $\frac{\partial H_2}{\partial Y_A}$ as $\frac{\partial H_2}{\partial T_2} \frac{\partial T_2}{\partial Y_A}$, we get

$$\frac{\dot{m}_2}{2nh_2 x_{AF}} \frac{\partial H_2}{\partial T_2} \frac{\partial T_2}{\partial Y_A} = T_w - T_2 \quad (1.16)$$

Now, since $H_w = H_w(T_w, w)$

$$\frac{\partial H_w}{\partial t_A} = \frac{\partial H_w}{\partial T_w} \frac{\partial T_w}{\partial t_A} + \frac{\partial H_w}{\partial w} \frac{\partial w}{\partial t_A}$$

Using equations (1.7) and (1.2)

$$\frac{\partial H_w}{\partial t_A} = \frac{\partial H_w}{\partial T_w} \frac{\partial T_w}{\partial t_B} + \frac{\partial H_w}{\partial w} \frac{2nx_{AF}Y_{AF}K_Y}{f\dot{m}_w} (Y - Y_w) \quad (1.17)$$

Using equations (1.4), (1.5) and (1.17) in equation (1.3)

$$2nK_Y (Y_w - Y) \frac{\partial H_1}{\partial Y} + 2nh_1 (T_w - T_1) + 2nh_2 (T_w - T_2)$$

$$+ \frac{\dot{m}_w}{x_{AF}Y_{AF}} \frac{\partial H_w}{\partial T_w} \frac{\partial T_w}{\partial t_B} + \frac{2nK_Y}{f} (Y - Y_w) \frac{\partial H_w}{\partial w} = 0$$

Dividing throughout by $2nh_1$ and rearranging,

$$\begin{aligned} \frac{m_w}{2nh_1 x_{AF} y_{AF}} \frac{\partial H_w}{\partial T_w} \frac{\partial T_w}{\partial t_B} &= (T_1 - T_w) + \frac{h_2}{h_1} (T_2 - T_w) \\ &+ \frac{K_y}{h_1} (y - y_w) \left[\frac{\partial H_1}{\partial Y} - \frac{1}{f} \frac{\partial H_w}{\partial w} \right] \\ &= (T_1 - T_w) + b(T_2 - T_w) + \lambda_1 (y - y_w) \end{aligned} \quad (1.18)$$

where

$$\lambda_1 = \frac{QK_y}{h_1} \quad (1.19)$$

and

$$Q = \frac{\partial H_1}{\partial Y} - \frac{1}{f} \frac{\partial H_w}{\partial w} \quad (1.20)$$

In the literature, Q is called the heat of sorption and $\partial H_w / \partial w$ is called the differential heat of wetting.

Finally define nondimensional independent variables in the form,

$$x = \frac{2nh_1 y_{AF} x_A}{m_1 (\partial H_1 / \partial T_1)} \quad (1.21)$$

$$y = \frac{2nh_2 x_{AF} y_A}{m_2 (\partial H_2 / \partial T_2)} \quad (1.22)$$

$$t = \frac{2nh_1 x_{AF} y_{AF} t_B}{m_w^c w_R} \quad (1.23)$$

The quantities $\partial H_1 / \partial T_1$ and $\partial H_2 / \partial T_2$ are the specific heat of moist process air and moist cooling air respectively. The enthalpy of moist air can be expressed as

$$H_1 = 1.0046465 T_1 + Y(2467.4304 + 1.8837122 T_1) \quad (1.24)$$

in S.I. units

$$H_1 = 0.24 T_1 + Y(1061 + 0.45 T_1) \text{ in British Units}^5$$

During the sorption process, Y is a variable and therefore the specific heat of process air is not a constant. However, the range of its variation is always small and hence can be treated as constant. The absolute humidity of the cooling air does not, however, change during the sorption process and hence $(\partial H_2 / \partial T_2)$ is exactly constant.

Using equations (1.21), (1.22), (1.23) at $x_A = x_{AF}$, $y_A = y_{AF}$ and $t_B = t_{BF}$ respectively and defining the corresponding values of x , y and t as x_F , y_F and t_F , we get

$$x_F = \frac{2nh_1 x_{AF} y_{AF}}{\dot{m}_1 (\partial H_1 / \partial T_1)} \quad (1.25)$$

$$y_F = \frac{2nh_2 x_{AF} y_{AF}}{\dot{m}_2 (\partial H_2 / \partial T_2)} \quad (1.26)$$

$$t_F = \frac{2nh_1 x_{AF} y_{AF} t_{BF}}{\dot{m}_w C_{WR}} \quad (1.27)$$

Thus, x_F , in addition to being the nondimensional process channel length, is also the inverse of the nondimensional process air mass flow rate. Also y_F , in addition to being the nondimensional cooling channel length, is also the inverse of the nondimensional cooling air mass flow rate. Similarly, t_F , in addition to being the nondimensional sorption time,

is also the inverse of the nondimensional mass of the dehumidifier unit.

Using equations (1.21), (1.22) and (1.23) in equations (1.11), (1.13), (1.15), (1.16) and (1.18), we get

$$\frac{\partial Y}{\partial x} = \lambda_3 (Y_w - Y) \quad (1.28)$$

$$\frac{\partial w}{\partial t} = \lambda_2 (Y - Y_w) \quad (1.29)$$

$$\frac{\partial T_1}{\partial x} = T_w - T_1 \quad (1.30)$$

$$\frac{\partial T_2}{\partial y} = T_w - T_2 \quad (1.31)$$

$$\lambda_4 \frac{\partial T_w}{\partial t} = (T_1 - T_w) + b(T_2 - T_w) + \lambda_1 (Y - Y_w) \quad (1.32)$$

where

$$\lambda_4 = \frac{(\partial H_w / \partial T_w)}{C_{WR}} \quad (1.33)$$

The system of equations (1.28) through (1.32) has been solved by Roy and Gidaspo¹ for the case of once through processes (with given initial bed conditions) by the method of Greens' functions. They neglected the resistance in the sheet and used the equilibrium data of Bullock and Threlkeld⁵. The present analysis differs from that of Roy and Gidaspo in several ways: (1) The resistance in the sheet is accounted. (2) Solutions have been obtained for periodic steady state conditions. (3) Finite difference schemes have been used with large saving in computer time.

Specific Heat of the Wall

The parameter λ_w given in equation (1.33) contains the term $(\partial H_w / \partial T_w)$ which is the specific heat of the wall. Assume that the wall is constructed of metal sheets over which desiccant sheets are glued. Such desiccant sheets can be manufactured by mixing desiccants with Teflon and ammonium-bicarbonate into cake form, rolling it into the desired thickness and finally baking in an oven. The details of the sheet making are given by Perkari². In our analysis, f represents the mass fraction of the desiccant material to the total mass of the dehumidifier and g represents the fraction of Teflon to the mass of desiccant material. Typically, a dehumidifier can be constructed having a value of ' f ' about 0.5 and the desiccant sheets can be made with a ' g ' value of 0.05.

If m_w is the total mass of the dehumidifier in its dry condition, then fm_w would be the mass of desiccant material and fgm_w would be the mass of Teflon in the desiccant sheet. Thus, the mass of supporting metals would be $m_w(1-f-g)$. For an average H_2O concentration w , the mass of H_2O contained in the sheet is fwm_w .

Further, if c_s is the specific heat of the desiccant material, c_a is the specific heat of the supporting metals, c_t is the specific heat of Teflon and c_m is the specific heat of liquid water, then the enthalpy balance will give,

$$m_w H_w = \left[m_w(1-f-g)c_a + m_w f g c_t + m_w f c_s + m_w f w c_m \right] T_w + f \Delta H_w m_w$$

where ΔH_w represents the integral heat of wetting given in kJoules per kg of desiccant.

Dividing throughout by m_w , we get

$$H_w = T_w \left[(1-f-fg)c_a + fgc_t + fc_s + fwc_m \right] + f\Delta H_w \quad (1.34)$$

Differentiating with respect to T_w ,

$$\frac{\partial H_w}{\partial T_w} = (1-f-fg)c_a + fgc_t + fc_s + fwc_m + f \frac{\partial \Delta H_w}{\partial T_w}$$

Close and Banks¹³ have shown that the quantity $(\partial \Delta H_w / \partial T_w)$ is always less than 2% of $(\partial H_w / \partial T_w)$ and thus can be neglected. Also Bullock and Threlkeld⁵ have fitted the data of Ewing and Bauer¹⁴ for the integral heat of wetting to a polynomial in terms of w only, thus showing that ΔH_w is independent of T_w . Therefore, for our analysis, we can neglect the term $\partial \Delta H_w / \partial T_w$ in the expression for $\partial H_w / \partial T_w$, and we get

$$\frac{\partial H_w}{\partial T_w} = (1-f-fg)c_a + fgc_t + fc_s + fwc_m \quad (1.35)$$

It can also be noticed that $(\partial H_w / \partial T_w)$ is a function of w and therefore λ_u is also a function of w . Since w is a dependent variable, λ_u makes the system nonlinear.

Equilibrium Relations of Desiccant Materials

As was mentioned before, the system of equations governing the dynamics of sorption has to be solved along with the equilibrium relations of the desiccant material. Different desiccant materials have different equilibrium characteristics.

A given desiccant is said to be in equilibrium with its surrounding air when (1) the temperature of the desiccant is equal to the dry bulb temperature of the surrounding air and (2) there is no exchange of H_2O between the dessicant and the surrounding air by sorption processes.

There is, in general, a single value of moisture content in the desiccant material when it is in equilibrium with air of given dry bulb temperature and humidity. This statement can be expressed as

$$w_{eq} = w_{eq}(T_w, Y_w) \quad (1.36)$$

Here, T_w represents the temperature of air in equilibrium which is also the same as the temperature of the desiccant and Y_w represents the absolute humidity of air in equilibrium.

Using the psychrometric relations for moist air, the equation can also be written as

$$w_{eq} = w_{eq}(T_w, RH) \quad (1.37)$$

where RH represents the relative humidity of moist air whose dry bulb temperature is T_w and absolute humidity is Y_w .

In the literature, the equilibrium moisture content w_{eq} is often given as a function of the temperature and vapor pressure of moisture³. For computational purposes, these relations are also made into algebraic equations^{5,12}.

For some desiccants, when the equilibrium relation is expressed in the form given by equation (1.37), w_{eq} is usually a weak function of T_w . Hence, as an approximation (though

it is not strictly correct), the equilibrium of such desiccants is given as

$$w_{eq} = w_{eq}(RH)$$

Equation (1.37) can also be written as

$$RH = RH(w_{eq}, T_w) \quad (1.38)$$

and in particular, it is often convenient to write equation (1.38) as

$$RH = \alpha_w(T_w)w_{eq} + \beta_w(T_w)w_{eq}^2 + \dots \quad (1.39)$$

Usually, two terms in the RHS of equation (1.39) are enough to express the equilibrium to a very high degree of accuracy.

It has been shown at IIT that the equilibrium relations of the desiccant sheets are different from the equilibrium relations of the desiccants used to make the sheets. The distributions of H_2O in the sheets which have micro and macropores and in granular desiccant which sorbs H_2O only in micropores are indeed different. This and other still unknown reasons may be the causes for the different equilibrium relations.

Preliminary test results of silica gel sheets manufactured at IIT using Syloid-63 silica gel were found to have equilibrium characteristics which are about 20% less than the equilibrium values of Syloid-63 gel at a temperature of $21.11^\circ C$. For higher temperatures, the equilibrium values of sheets are not available. Hubbard³ has presented equilibrium data

of silica gel for a range of temperatures. For the present analysis, the equilibrium values obtained here at 21.11°C are used for that temperature and necessary corrections were made for higher temperatures, following the trends of Hubbard's data. The data prepared in this way is shown in figure 1.3.

The sheet equilibrium data given in figure 1.4 show the value of w_{eq} with Y_w as the argument and T_w as the parameter. Figure 1.5 gives the value of w_{eq} with T_w as the argument and Y_w as the parameter.

Using the results of figure 1.3, functional relations were formed in the form of equation (1.39) taking two terms on the RHS. This lead to the relation

$$RH = (143.31 + 1.68678 T_w) w_{eq} + (231.74 - 2.64006 T_w) w_{eq}^2 \quad (1.40)$$

where RH is the relative humidity of the air in equilibrium with the desiccant.

The absolute humidity of air in equilibrium can be found using the psychrometric relations

$$Y_w = \frac{0.622 RH}{T_f - RH} \quad (1.41)$$

where the quantity $T_f = T_f(T_w)$ is given by

$$T_f = 10^s \quad (1.42)$$

where

$$s = 4.21429 - \frac{7.5 T_w}{237.3 + T_w} \quad (1.43)$$

Figure 1.3 Equilibrium Characteristics of Silica Gel Sheet Showing Constant Temperature Lines in w-RH Plot

Figure 1.4 Equilibrium Characteristics of Silica Gel
Sheet Showing Constant Temperature Lines
in w - Y_w Plot

Figure 1.5 Equilibrium Characteristics of Silica Gel Sheet Showing Constant Absolute Humidity Lines in w - T_w Plot

Relation Between Average Concentration and Surface Concentration

The variable w in equation (1.29), represents the average concentration of H_2O per unit mass of desiccant and is a function of time, t , and space variables x and y . One would expect a concentration distribution over the thickness of desiccant sheet and/or a concentration distribution in the structure of each desiccant particle. This distribution of concentration is governed by the diffusion of H_2O into the micro and macropores of the desiccant particles and of the sheet. As of today, the exact mechanism of diffusion is not well understood, though it has been established by experiments² that in the process of sorption, diffusion resistance in the desiccant sheet is playing an important role.

Many of the earlier investigators^{5,6,15,1} have neglected the diffusion in the desiccant and thus considered that the H_2O concentration at any location and time in the desiccant is uniform and equal to the average value given by equation (1.29). The experiments conducted at IIT have shown that the results of models with uniform concentration cannot match the experimental results¹⁶. One such comparison showing a large discrepancy between predicted and experimental results is shown in figure 1.6. Hence, the diffusion in the desiccant must be incorporated into the model and thus allow for a non-uniform concentration distribution at every t , x and y . On the other hand, since the mechanism of diffusion is not yet completely understood, a complete mathematical model

Figure 1.6 Comparison of Experimental Data and Theoretical Predictions of Models with Uniform Concentrations in Desiccant.

could not be included. This subject is currently under active research at the Chemical Engineering Department of IIT.

In the meantime, an attempt was made to include the effect of diffusion in an indirect way. Figure 1.7 shows qualitatively the solution of the diffusion equation in a channel with prescribed flux on one wall and no flux on the other wall. Under periodic steady state operating conditions, the concentration at the top surface of the desiccant will vary widely between adsorption and desorption, whereas the concentration at the adiabatic surface (which may be the bottom of the sheet of desiccant or the center of a single particle of desiccant) will have very little variation during cycling. Figure 1.7(a) shows the concentration profiles during the desorption process and figure 1.7(b) shows the profiles during adsorption. If diffusion were neglected, the concentration would be uniform and the profiles will be as shown in figure 1.7(c). Figure 1.7(d) shows concentration profiles, with linear variation of concentration in the direction of diffusion. This profile, though not an exact solution of the diffusion process, accounts for non-uniform concentration and thus considers indirectly the diffusion in the sheet.

The concentration at the top surface is also the equilibrium concentration, since it is this surface which is adjacent to the process air at the temperature of T_w and absolute humidity of Y_w .

Assuming that the concentration profile given in figure

(a) EXPECTED PROFILE DURING DESORPTION

(b) EXPECTED PROFILE DURING ADSORPTION

(c) EARLIER MODEL WITH UNIFORM CONCENTRATION

(d) REVISED MODEL WITH LINEARLY VARYING
CONCENTRATION

Figure 1.7 Concentration Profiles in the Desiccant

1.7(d) would be adequate to predict the performance of the dehumidifier, we can now find the relation between the average concentration of moisture w and the top surface concentration or the equilibrium concentration w_{eq} .

Referring to figure 1.8, we see that

$$w_{eq} = w - (w_{mid} - w) = 2w - w_{mid} \quad (1.44)$$

Equation (1.44) is valid both when $w_{eq} < w$ and when $w_{eq} > w$.

Concentration at Most Remote Desiccant Site for Periodic Steady State Operation

Apparently equation (1.44) gives a direct relation between w and w_{eq} ; however, the value of w_{mid} is still unknown. Considering all the nodal points in (t, x, y) domain, it is reasonable to assume that the value of w_{mid} will be the value of concentration averaged over t , x and y directions. As an example, if the sorption takes place in a dehumidifier, between the two moisture levels of say, 0.04 kg H₂O/kg of desiccant and 0.12 kg H₂O/kg of desiccant, under a periodic steady state operation, then w_{mid} can be considered as $(0.04 + 0.12)/2 = 0.08$ kg H₂O/kg of desiccant.

Later, after solving the governing equations, the effect of w_{mid} is studied by perturbing this value on each side. The results indicated that the performance of dehumidifier is relatively insensitive to the value of w_{mid} .

Heat of Sorption

At this point, it is necessary to determine the expression for the heat of sorption given by equation (1.20) by

$$w - w_{eq} = w_{mid} - w$$

$$\Rightarrow w_{eq} = 2w - w_{mid}$$

Figure 1.8 Relation Between the Surface Concentration and the Average Concentration

evaluating the terms $(\partial H_1/\partial Y)$ and $(\partial H_w/\partial T_w)$.

The enthalpy of moist air can be satisfactorily expressed as

$$H_1 = aT_1 + Y(d + cT_1) \quad (1.45)$$

In S.I. Units

$$a = 1.0046465 \text{ kJ/kg dry air } ^\circ\text{C}$$

$$d = 2467.4304 \text{ kJ/kg H}_2\text{O}$$

$$c = 1.8837122 \text{ kJ/kg H}_2\text{O } ^\circ\text{C}$$

Differentiation of equation (1.45) with respect to Y gives

$$\frac{\partial H_1}{\partial Y} = d + cT_1 \quad (1.46)$$

The enthalpy of the wall is given by equation (1.34) as

$$H_w = T_w \left[(1-f-fg)c_a + fgc_t + fc_s + fwc_m \right] + f\Delta H_w$$

Differentiating with respect to w, we obtain

$$\frac{\partial H_w}{\partial w} = fc_m T_w + f \frac{\partial \Delta H_w}{\partial w} \quad (1.47)$$

Substitution of equations (1.46) and (1.47) in equation (1.20) gives

$$Q = (d + cT_1) - c_m T_w - \frac{\partial \Delta H_w}{\partial w} \quad (1.48)$$

Close and Banks¹³ have shown that the quantity $(\partial \Delta H_w/\partial w)$ can

be calculated using

$$\frac{\partial \Delta H_w}{\partial w} = \frac{\partial H_1}{\partial Y} \left[1 - \frac{\partial \ln \phi}{\partial \ln \phi_s} \right]_w \quad (1.49)$$

where ϕ is the ratio of vapor pressure of H_2O in the air to the atmospheric pressure and ϕ_s is the ratio of saturation vapor pressure to the atmospheric pressure.

From the psychrometric relations, the ratio of vapor pressure of H_2O in moist air to the atmospheric pressure is given by,

$$\phi = \frac{RH}{T_f} \quad (1.50)$$

where RH is the relative humidity of air and T_f is a temperature function given by equations (1.42 and (1.43) as

$$T_f = 10^s$$

where

$$s = 4.21429 - \frac{7.5 T_w}{237.3 + T_w}$$

Equation (1.40) can be written as,

$$RH = \alpha_w w_{eq} + \beta_w w_{eq}^2 \quad (1.51)$$

where

$$\alpha_w = 143.31 + 1.68678 T_w$$

$$\beta_w = 231.74 - 2.64006 T_w$$

Substitution of equation (1.40) into equation (1.50) gives

$$\phi = \frac{1}{T_f} (\alpha_w w_{eq} + \beta_w w_{eq}^2) \quad (1.52)$$

Then

$$\ln \phi = \ln(\alpha_w w_{eq} + \beta_w w_{eq}^2) - \ln T_f$$

Differentiating with respect to T_w

$$\frac{\partial \ln \phi}{\partial T_w} = \frac{\alpha'_w w_{eq} + \beta'_w w_{eq}^2}{\alpha_w w_{eq} + \beta_w w_{eq}^2} - \frac{T'_f}{T_f}$$

Using equation (1.50) for RH = 100

$$\phi_s = \frac{100}{T_f} \quad (1.53)$$

or

$$\ln \phi_s = \ln 100 - \ln T_f \quad (1.54)$$

Differentiating equation (1.42) with respect to T_w , we get

$$\frac{T'_f}{T_f} = \frac{-4098.0258}{(237.3 + T_w)^2} \quad (1.55)$$

Differentiation of both sides of equation (1.54) with respect to T_w gives

$$\frac{\partial \ln \phi_s}{\partial T_w} = - \frac{T'_f}{T_f} \quad (1.56)$$

Now

$$\begin{aligned}
 \left. \frac{\partial \ln \phi}{\partial \ln \phi_s} \right|_w &= \frac{\partial \ln \phi}{\partial T_w} \frac{\partial T_w}{\partial \ln \phi_s} = \left[\frac{\alpha'_w w_{eq} + \beta'_w w_{eq}^2}{\alpha_w w_{eq} + \beta_w w_{eq}^2} - \frac{T_f}{T_f} \right] \left(-\frac{T_f}{T_f} \right) \\
 &= -\frac{T_f}{T_f} \frac{\alpha'_w w_{eq} + \beta'_w w_{eq}^2}{\alpha_w w_{eq} + \beta_w w_{eq}^2} + 1 \\
 &= \frac{(237.3 + T_w)^2 (\alpha'_w w_{eq} + \beta'_w w_{eq}^2)}{4098.0258 (\alpha_w w_{eq} + \beta_w w_{eq}^2)} + 1 \quad (1.57)
 \end{aligned}$$

Substituting equation (1.57) in equation (1.49)

$$\begin{aligned}
 \frac{\partial \Delta H_w}{\partial w} &= \frac{\partial H}{\partial Y} \left[-\frac{(237.3 + T_w)^2 (\alpha'_w w_{eq} + \beta'_w w_{eq}^2)}{4098.0258 (\alpha_w w_{eq} + \beta_w w_{eq}^2)} \right] \\
 &= -(d + cT_1) \left[\frac{(237.3 + T_w)^2 (\alpha'_w w_{eq} + \beta'_w w_{eq}^2)}{4098.0258 (\alpha_w w_{eq} + \beta_w w_{eq}^2)} \right] \quad (1.58)
 \end{aligned}$$

Using equation (1.58) in equation (1.48)

$$Q = (d + cT_1) \left[1 + \frac{(237.3 + T_w)^2 (\alpha'_w w_{eq} + \beta'_w w_{eq}^2)}{4098.0258 (\alpha_w w_{eq} + \beta_w w_{eq}^2)} \right] - c_m T_w \quad (1.59)$$

From equation (1.59), we can see that for a given desiccant sheet, Q is a function of process stream temperature, equilibrium moisture content and the temperature of the sheet.

Figure 1.9 shows the value of Q for a range of wall temperatures and moisture content for process stream temperatures of 30°C and 60°C.

Governing Equations

We can now conveniently collect all the governing differential and algebraic equations derived earlier. They

Figure 1.9 Heat of Sorption for the Silica Gel Sheet

are:

$$\frac{\partial w}{\partial t} = \lambda_2 (Y - Y_w) \quad (1.29)$$

$$\lambda_4 \frac{\partial T_w}{\partial t} = (T_1 - T_w) + b(T_2 - T_w) + \lambda_1 (Y - Y_w) \quad (1.32)$$

$$w_{eq} = 2w - w_{mid} \quad (1.44)$$

$$RH = \alpha_w(T_w)w_{eq} + \beta_w(T_w)w_{eq}^2 \quad (1.40)$$

$$Y_w = 0.622 RH / (T_f - RH) \quad (1.41)$$

$$\frac{\partial T_1}{\partial x} = T_w - T_1 \quad (1.30)$$

$$\frac{\partial Y}{\partial x} = \lambda_3 (Y_w - Y) \quad (1.28)$$

$$\frac{\partial T_2}{\partial y} = T_w - T_2 \quad (1.31)$$

where

$$\lambda_1 = QK_y/h_1 \quad (1.19)$$

$$\lambda_2 = K_y c_{wR} / (fh_1) \quad (1.14)$$

$$\lambda_3 = K_y (\partial H_1 / \partial T_1) / h_1 \quad (1.12)$$

$$\lambda_4 = (\partial H_w / \partial T_w) / c_{wR} \quad (1.33)$$

$$b = h_2 / h_1$$

$$\alpha_w(T_w) = 143.31 + 1.68678 T_w$$

$$\beta_w(T_w) = 231.74 - 2.64006 T_w$$

$$T_f = 10^S \quad (1.42)$$

$$s = 4.21429 - 7.5 T_w / (237.3 + T_w) \quad (1.43)$$

$$x = 2nh_1 y_{AF} x_A / (\dot{m}_1 (\partial H_1 / \partial T_1)) \quad (1.21)$$

$$y = 2nh_2 x_{AF} y_A / (\dot{m}_2 (\partial H_2 / \partial T_2)) \quad (1.22)$$

$$t = 2nh_1 x_{AF} y_{AF} t_B / (m_w c_{wR}) \quad (1.23)$$

$$t_B = t_A - x_A / u_1 - y_A / u_2 \quad (1.6)$$

Inlet and Initial Conditions

Five of the governing equations listed above are differential equations and for their solutions, we need to prescribe inlet and initial conditions.

Even though the governing equations of the sorption process are common to both adsorption and desorption, the inlet conditions of the process and cooling air would be different for the two processes. We write these inlet conditions as follows:

$$T_1(t, 0, y) = T_0(t) \quad (1.60)$$

$$Y(t, 0, y) = Y_0(t) \quad (1.61)$$

$$T_2(t, x, 0) = T_{2i}(t) \quad (1.62)$$

for adsorption, and

$$T_1(L, 0, y) = T_D(t) \quad (1.63)$$

$$Y(t, 0, y) = Y_D(t) \quad (1.64)$$

$$T_2(t, x, 0) = T_{2id}(t) \quad (1.65)$$

for desorption.

The initial conditions of the wall, under periodic steady state conditions, for a given sorption process will be the final conditions of the wall in the earlier process of the cycle. Considering the cycle of operations as consisting of (1) Desorption Process, (2) Precooling or Purging Process, (3) Adsorption Process and (4) Preheating or Purging Process, the initial conditions can be prescribed as follows:

$$T_w(0, x, y) \text{ at the beginning of adsorption} = T_w(t_{FP}, x, y) \text{ at the end of precooling} \quad (1.66)$$

$$T_w(0, x, y) \text{ at the beginning of desorption} = T_w(t_{FP}, x, y) \text{ at the end of preheating} \quad (1.67)$$

$$w(0, x, y) \text{ at the beginning of adsorption} = w(t_F, x, y) \text{ at the end of desorption} \quad (1.68)$$

$$w(0, x, y) \text{ at the beginning of desorption} = w(t_F, x, y) \text{ at the end of adsorption} \quad (1.69)$$

Model for Purging

Purging is done by sending air through the cooling channels. Specifically during precooling, after desorption, ambient air is passed through the hot bed and thus the sensible heat of the bed is transferred to the air. This air then passes through the cool bed, which had just finished adsorption, thereby preheating it.

The conservation of energy for the wall and for the air used for purging, gives

The rate of increase of purging air enthalpy
 + the net outflow of enthalpy in cooling channel
 + the rate of increase of the bed enthalpy = 0

$$\rho_p \Delta x_A \Delta y_A a_2 \frac{\partial H_p}{\partial t_A} + \frac{\dot{m}_p \Delta x_A}{n x_{AF}} \left[(H_p + \frac{\partial H_p}{\partial y_A} \Delta y_A) - H_p \right] \\ + \rho_w \Delta x_A \Delta y_A a_w \frac{\partial H_w}{\partial t_A} = 0$$

Using the relations $\dot{m}_p = \rho_p a_2 x_{AF} u_p n$, $\dot{m}_w = \rho_w x_{AF} y_{AF} a_w n$, we get

$$\frac{\dot{m}_p}{x_{AF}} \left[\frac{1}{u_p} \frac{\partial H_p}{\partial t_A} + \frac{\partial H_p}{\partial y_A} \right] + \frac{\dot{m}_w}{x_{AF} y_{AF}} \frac{\partial H_w}{\partial t_A} = 0 \quad (1.70)$$

The energy transfer rate between the stream of air and the wall is given by

The rate of increase of purging air enthalpy
 + the net outflow of enthalpy in the cooling channel
 = The rate of heat transfer from the wall.

$$\rho_p \Delta x_A \Delta y_A a_2 \frac{\partial H_p}{\partial t_A} + \frac{\dot{m}_p \Delta x_A}{n x_{AF}} \left[(H_p + \frac{\partial H_p}{\partial y_A} \Delta y_A) - H_p \right]$$

$$= 2h_2 \Delta x_A \Delta y_A (T_w - T_p)$$

$$\frac{\dot{m}_p}{n x_{AF}} \left[\frac{1}{u_p} \frac{\partial H_p}{\partial t_A} + \frac{\partial H_p}{\partial y_A} \right] = 2h_p (T_w - T_p) \quad (1.71)$$

Define

$$t_c = t_A - \frac{y_A}{u_p} \quad (1.72)$$

then

$$\frac{\partial}{\partial t_A} = \frac{\partial}{\partial t_c} \quad (1.73)$$

and

$$\left(\frac{\partial}{\partial y_A} \right)_{t_A} = \frac{\partial}{\partial t_c} \frac{\partial t_c}{\partial y_A} + \left(\frac{\partial}{\partial y_A} \right)_{t_A} = -\frac{1}{u_p} \frac{\partial}{\partial t_c} + \left(\frac{\partial}{\partial y_A} \right)_{t_c} \quad (1.74)$$

Using equations (1.73) and (1.74) in equation (1.71) gives

$$\frac{\dot{m}_p}{2n x_{AF} h_p} \frac{\partial H_p}{\partial y_A} = T_w - T_p \quad (1.75)$$

Using equations (1.71) and (1.73) in equation (1.70)

$$\frac{\dot{m}_w}{2n h_p x_{AF} y_{AF}} \frac{\partial H_w}{\partial t_c} = T_p - T_w \quad (1.76)$$

During the purging process,

$$H_p = H_p(T_p) \text{ only}$$

and

$$H_w = H_w(T_w) \text{ only}$$

Therefore

$$\frac{\partial H_p}{\partial y_A} = \frac{\partial H_p}{\partial T_p} \frac{\partial T_p}{\partial y_A}$$

and

$$\frac{\partial H_w}{\partial t_c} = \frac{\partial H_w}{\partial T_w} \frac{\partial T_w}{\partial t_c}$$

Using these equations in (1.75) and (1.76) respectively

$$\frac{\dot{m}_p (\partial H_p / \partial T_p)}{2n x_{AF} h_p} \frac{\partial T_p}{\partial y_A} = T_w - T_p \quad (1.77)$$

$$\frac{\dot{m}_w (\partial H_w / \partial T_w)}{2n h_p x_{AF} y_{AF}} \frac{\partial T_w}{\partial t_c} = T_p - T_w \quad (1.78)$$

Define nondimensional independent variables as

$$y_p = \frac{2n h_p x_{AF} y_A}{\dot{m}_p (\partial H_p / \partial T_p)} \quad (1.79)$$

$$t_p = \frac{2n h_p x_{AF} y_{AF} t_c}{\dot{m}_w c_{wR}} \quad (1.80)$$

Using equations (1.79) and (1.80) in equations (1.77) and (1.78), we get

$$\frac{\partial T_p}{\partial y_p} = T_w - T_p \quad (1.81)$$

$$\lambda_4 \frac{\partial T_w}{\partial t_p} = T_p - T_w \quad (1.82)$$

where

$$\lambda_4 = (\partial H_w / \partial T_w) / c_{wR} \quad (1.33)$$

The set of equations (1.81) and (1.82) are similar to the classical Schumann problem equations.

Inlet and Initial Conditions

The inlet conditions can be prescribed as

$$T_p(t, x, 0) = T_{pi}(t) \text{ during preheating} \quad (1.83)$$

and

$$T_p(t, x, 0) = T_{pic}(t) \text{ during precooling} \quad (1.84)$$

The initial conditions are:

$$T_w(o, x, y) \text{ at the beginning of preheating} = T_w(t_F, x, y) \text{ at the end of adsorption} \quad (1.85)$$

$$T_w(o, x, y) \text{ at the beginning of precooling} = T_w(t_F, x, y) \text{ at the end of desorption} \quad (1.86)$$

CHAPTER III

SOLUTION OF THE GOVERNING EQUATIONS

Solution for the Sorption Process

The governing equations for the sorption process are (1.28) - (1.29), (1.30), (1.31) and (1.32) [which are given on page 16]. These equations are subject to the initial and inlet conditions given by equations (1.60) through (1.69).

The differential equations (1.28) - (1.32) are written as finite difference equations using the finite difference scheme outlined below:

Define the nodal point variables I, J, K such that

$$t = (I - 1)\Delta t \quad (1.87)$$

$$x = (J - 1)\Delta x \quad (1.88)$$

$$y = (K - 1)\Delta y \quad (1.89)$$

Using the forward difference scheme in (1.29) at $(t, x+\Delta x, y+\Delta y)$ we get

$$\begin{aligned} & \frac{w(t+\Delta t, x+\Delta x, y+\Delta y) - w(t, x+\Delta x, y+\Delta y)}{\Delta t} \\ &= \lambda_2 \left[Y(t, x+\Delta x, y+\Delta y) - Y_w(t, x+\Delta x, y+\Delta y) \right] \end{aligned}$$

or

$$\frac{w(I, J, K) - w(I-1, J, K)}{\Delta t} = \lambda_2 \left[Y(I-1, J, K) - Y_w(I-1, J, K) \right]$$

Rearranging, we get

$$w(I, J, K) = w(I-1, J, K) + \Delta t \lambda_2 [Y(I-1, J, K) - Y_w(I-1, J, K)] \quad (1.90)$$

Using the forward difference scheme in equation (1.32) at $(t, x+\Delta x, y+\Delta y)$ gives

$$\begin{aligned} & \frac{\lambda_4}{\Delta t} [T_w(t+\Delta t, x+\Delta x, y+\Delta y) - T_w(t, x+\Delta x, y+\Delta y)] \\ &= T_1(t, x+\Delta x, y+\Delta y) - T_w(t, x+\Delta x, y+\Delta y) \\ &+ b [T_2(t, x+\Delta x, y+\Delta y) - T_w(t, x+\Delta x, y+\Delta y)] \\ &+ \lambda_1 [Y(t, x+\Delta x, y+\Delta y) - Y_w(t, x+\Delta x, y+\Delta y)] \end{aligned}$$

or

$$\begin{aligned} & \frac{\lambda_4}{\Delta t} [T_w(I, J, K) - T_w(I-1, J, K)] \\ &= T_1(I-1, J, K) - T_w(I-1, J, K) + b [T_2(I-1, J, K) - T_w(I-1, J, K)] \\ &+ \lambda_1 [Y(I-1, J, K) - Y_w(I-1, J, K)] \end{aligned}$$

and

$$\begin{aligned} T_w(I, J, K) &= T_w(I-1, J, K) + \frac{\Delta t}{\lambda_4} [T_1(I-1, J, K) - T_w(I-1, J, K)] \\ &+ \frac{b \Delta t}{\lambda_4} [T_2(I-1, J, K) - T_w(I-1, J, K)] \\ &+ \frac{\lambda_1 \Delta t}{\lambda_4} [Y(I-1, J, K) - Y_w(I-1, J, K)] \quad (1.91) \end{aligned}$$

Using the central difference scheme in equation (1.30) at $(t+\Delta t, x, y+\Delta y)$

$$\frac{T_1(t+\Delta t, x+\Delta x, y+\Delta y) - T_1(t+\Delta t, x-\Delta x, y+\Delta y)}{2\Delta x}$$

$$= T_w(t+\Delta t, x, y+\Delta y) + T_1(t+\Delta t, x, y+\Delta y)$$

or

$$\frac{T_1(I, J, K) - T_1(I, J-2, K)}{2\Delta x} = T_w(I, J-1, K) - T_1(I, J-1, K)$$

and

$$T_1(I, J, K) = T_1(I, J-2, K) + 2\Delta x [T_w(I, J-1, K) - T_1(I, J-1, K)] \quad (1.92)$$

Using the central difference scheme in equation (1.28) at $(t+\Delta t, x, y+\Delta y)$ gives

$$\begin{aligned} & \frac{Y(t+\Delta t, x+\Delta x, y+\Delta y) - Y(t+\Delta t, x-\Delta x, y+\Delta y)}{2\Delta x} \\ &= \lambda_3 [Y_w(t+\Delta t, x, y+\Delta y) - Y(t+\Delta t, x, y+\Delta y)] \end{aligned}$$

or

$$\frac{Y(I, J, K) - Y(I, J-2, K)}{2\Delta x} = \lambda_3 [Y_w(I, J-1, K) - Y(I, J-1, K)]$$

and

$$Y(I, J, K) = Y(I, J-2, K) + 2\Delta x \lambda_3 [Y_w(I, J-1, K) - Y(I, J-1, K)] \quad (1.93)$$

Finally using the central difference scheme in equation (1.31) at $(t+\Delta t, x+\Delta x, y)$

$$\frac{T_2(t+\Delta t, x+\Delta x, y+\Delta y) - T_2(t+\Delta t, x+\Delta x, y-\Delta x)}{2\Delta y}$$

$$= T_w(t+\Delta t, x+\Delta x, y) - T_2(t+\Delta t, x+\Delta x, y)$$

$$\frac{T_2(I, J, K) - T_2(I, J, K-2)}{2\Delta y} = T_w(I, J, K-1) - T_2(I, J, K-1)$$

and

$$T_2(I, J, K) = T_2(I, J, K-2) + 2\Delta y [T_w(I, J, K-1) - T_2(I, J, K-1)] \quad (1.94)$$

Marching Steps

The following are the sequence of steps that are followed to calculate the values of all dependent variables at a given nodal point (I, J, K)

1. Use equation (1.90) and determine w
2. Use equation (1.91) and calculate T_w
3. Determine the surface concentration w_{eq} using equation (1.44)
4. Knowing T_w and w_{eq} , the relative humidity of air in equilibrium, RH, is found using equation (1.40)
5. Calculate the temperature function T_f from the value of T_w using equations (1.42) and (1.48)
6. Determine Y_w using equation (1.41) since RH and T_w are known
7. Use equation (1.92) to calculate T_1
8. Use equation (1.93) to calculate Y and
9. Use equation (1.94) to calculate T_2 .

This sequence will be followed until the solution marches

to the desired values of I, J, and K.

Solution for the Purging Process

Equations (1.81) and (1.82) govern the process of purging. Using the finite differences, the above two equations become

$$T_w(I, J, K) = T_w(I-1, J, K) + \frac{\Delta t}{\lambda_u} [T_p(I-1, J, K) - T_w(I-1, J, K)]$$

and

$$T_p(I, J, K) = T_p(I, J, K-2) + 2\Delta y [T_w(I, J, K-1) - T_p(I, J, K-1)]$$

One should realize that although the governing equations of the purging process do not contain the variable x (or direction J), T_w is a function of x at $t = 0$ of the purging process and therefore, marching in x direction is necessary. The above two finite difference equations are therefore used for every J to obtain the complete solution.

Comparison Between the Solutions Obtained by Integral Representation and the Present Finite Difference Scheme

The system of governing equations given by equations (1.28) - (1.32) has been solved by Roy and Gidaspow¹ by changing the differential equations into integral equations using Green's functions, for the special case of $w = w_{eq}$ and $\lambda_u = 1.0$. Using $w = w_{eq}$ implies that the model assumes no resistance in the desiccant sheet. The present finite difference program was used to solve a typical example with $w = w_{eq}$ and $\lambda_u = 1.0$, and the results were compared with the solutions obtained from Green's functions technique. The comparison is shown in Table 1.1.

Table 1.1 Comparison of Results Predicted by Finite Difference Scheme and Green's Functions Method

$t_F = 20.0$	$T_O = 23.89$	$Y_O = 0.009$
$x_F = 3.0$	$T_{wi} = 26.67$	$w_i = 0.0775$
$y_F = 3.0$	$T_{zi} = 23.89$	$b = 1.0$

Nondimensional Time	Moisture Removed Finite Difference Scheme	Predicted by Green's Functions Method
0.0	0.00000	0.00000
4.0	0.00714	0.00724
8.0	0.01392	0.01406
12.0	0.02055	0.02070
16.0	0.02702	0.02717
20.0	0.03332	0.03344

It can be seen that the present method predicts correctly the performance of the dehumidifier. The computation time saved is about a factor of 10.

CHAPTER IV

METHODS OF CHECKING THE SOLUTIONS

Integral Mass Balances

It is very important to check the results obtained by the finite difference solutions by any possible method. One method of checking is by making an integral mass balance of the moisture in the process air and in the desiccant sheet. The total moisture change in the process stream should always be equal to the total moisture change in the desiccant wall.

Combining equation (1.28) and (1.29), one gets

$$\frac{\partial w}{\partial t} = -\frac{\lambda_2}{\lambda_3} \frac{\partial Y}{\partial x}$$

Using the definitions of λ_2 and λ_3 ,

$$\frac{\partial w}{\partial t} = -\frac{c_{wR}}{f(\partial H_1 / \partial T_1)} \frac{\partial Y}{\partial x}$$

Performing the triple integration $\int_0^t \int_0^{x_F} \int_0^{Y_F} () dy dx dt$

on both sides,

$$\begin{aligned} \int_0^t \int_0^{x_F} \int_0^{Y_F} \frac{\partial w}{\partial t} dy dx dt &= -\frac{c_{wR}}{f(\partial H_1 / \partial T_1)} \int_0^t \int_0^{x_F} \int_0^{Y_F} \frac{\partial Y}{\partial x} dy dx dt \\ \int_0^{x_F} \int_0^{Y_F} [w(t, x, y) - w(0, x, y)] dy dx &= -\frac{c_{wR}}{f(\partial H_1 / \partial T_1)} \int_0^t \int_0^{Y_F} [Y(t, 0, y) - Y(t, x_F, y)] dy dt \end{aligned}$$

Divide throughout $x_F y_F$ to get

$$\begin{aligned} & \frac{1}{x_F y_F} \int_0^{x_F} \int_0^{y_F} [w(t, x, y) - w(0, x, y)] dy dx \\ &= \frac{C_u}{t y_F} \int_0^t \int_0^{y_F} [Y(t, 0, y) - Y(t, x_F, y)] dy dt \end{aligned} \quad (1.95)$$

$$\text{where } C_u = \frac{c_{wR} t}{f(\partial H_1 / \partial T_1) x_F} \quad (1.96)$$

The L.H.S. of equation (1.95) represents the average moisture change of the desiccant sheet per unit mass of desiccant. The R.H.S. of (1.95) represents the average moisture change of the process air per unit mass of desiccant. If the results of the finite difference solution satisfy equation (1.95) for all t , we may consider the solution to be correct.

Integral Heat Balances for Sorption Process

The results of the finite difference solutions can also be checked by making integral heat balances. This is obtained as follows:

Substituting equations (1.29), (1.30) and (1.31) in equation (1.32), one gets

$$\lambda_4 \frac{\partial T_w}{\partial t} = -\frac{\partial T_1}{\partial x} - b \frac{\partial T_2}{\partial y} + \frac{\lambda_1}{\lambda_2} \frac{\partial w}{\partial t}$$

Performing the triple integration $\int_0^t \int_0^{x_F} \int_0^{y_F} () dy dx dt$ on both sides treating all λ 's as constants gives

$$\begin{aligned}
& \lambda_4 \int_0^t \int_0^{x_F} \int_0^{y_F} \frac{\partial T_w}{\partial t} dy dx dt \\
&= - \int_0^t \int_0^{x_F} \int_0^{y_F} \frac{\partial T_1}{\partial x} dy dx dt - b \int_0^t \int_0^{x_F} \int_0^{y_F} \frac{\partial T_2}{\partial y} dy dx dt \\
&+ \frac{\lambda_1}{\lambda_2} \int_0^t \int_0^{x_F} \int_0^{y_F} \frac{\partial w}{\partial t} dy dx dt
\end{aligned}$$

$$\begin{aligned}
& \lambda_4 \int_0^{x_F} \int_0^{y_F} [T_w(t, x, y) - T_w(0, x, y)] dy dx \\
&= \int_0^t \int_0^{y_F} [T_1(t, 0, y) - T_1(t, x_F, y)] dy dt \\
&+ b \int_0^t \int_0^{x_F} [T_2(t, x, 0) - T_2(t, x, y_F)] dx dt \\
&+ \frac{\lambda_1}{\lambda_2} \int_0^{x_F} \int_0^{y_F} [w(t, x, y) - w(0, x, y)] dy dx
\end{aligned}$$

Multiply throughout by $\frac{c_{wR}}{f x_F y_F}$ and use the definitions of λ_4 , λ_1 and λ_2 from equations (1.33), (1.19) and (1.14) respectively,

$$\begin{aligned}
& \frac{(\partial H_w / \partial T_w)}{f x_F y_F} \int_0^{x_F} \int_0^{y_F} [T_w(t, x, y) - T_w(0, x, y)] dy dx \\
&= \frac{C_2}{t y_F} \int_0^t \int_0^{y_F} [T_1(t, 0, y) - T_1(t, x_F, y)] dy dt \\
&+ \frac{b C_2}{t x_F} \int_0^t \int_0^{x_F} [T_2(t, x, 0) - T_2(t, x, y_F)] dx dt \\
&+ \frac{Q}{x_F y_F} \int_0^{x_F} \int_0^{y_F} [w(t, x, y) - w(0, x, y)] dy dx \quad (1.97)
\end{aligned}$$

where

$$C_2 = \frac{c_{wR}t}{fx_F} \quad (1.98)$$

$$C_3 = \frac{c_{wR}t}{fy_F} \quad (1.99)$$

The term on the LHS of equation (1.97) represents the amount of sensible heat gain by the desiccant wall per unit mass of desiccant. The first term on the RHS of equation (1.97) gives the amount of sensible heat lost by the process air, the second term gives the amount of sensible heat lost by the cooling air and the third gives the amount of heat generated due to the release of heat of sorption.

For the solution obtained by the finite difference scheme to be correct, equation (1.97) must be satisfied for all t .

Integral Heat Balances for Purging Process

The integral heat balance for the purging process is similar to that for the sorption process. Combining equations (1.81) and (1.82)

$$\lambda_w \frac{\partial T_w}{\partial t_p} = - \frac{\partial T_p}{\partial y_p}$$

Performing the triple integration $\int_0^{t_p} \int_0^{x_F} \int_0^{y_{Fp}} () dy_p dx dt_p$ on both sides treating λ_w as a constant gives

$$\lambda_w \int_0^{t_p} \int_0^{x_F} \int_0^{y_{Fp}} \frac{\partial T_w}{\partial t_p} dy_p dx dt_p = - \int_0^{t_p} \int_0^{x_F} \int_0^{y_{Fp}} \frac{\partial T_p}{\partial y_p} dy_p dx dt_p$$

$$\begin{aligned}
& \lambda_4 \int_0^{x_F} \int_0^{y_{Fp}} [T_w(t_p, x, y_p) - T_w(0, x, y_p)] dy_p dx \\
& = \int_0^{t_p} \int_0^{x_F} [T_p(t_p, x, 0) - T_p(t_p, x, y_{Fp})] dx dt_p
\end{aligned}$$

Multiply throughout by $\frac{c_{wR}}{f_{x_F y_{Fp}}}$ to get

$$\begin{aligned}
& \frac{(\partial H_w / \partial T_w)}{f_{x_F y_{Fp}}} \int_0^{x_F} \int_0^{y_{Fp}} [T_w(t_p, x, y_p) - T_w(0, x, y_p)] dy_p dx \\
& = \frac{C_s}{t_p x_{F0}} \int_0^{t_p} \int_0^{x_F} [T_p(t_p, x, 0) - T_p(t_p, x, y_{Fp})] dx dt_p \quad (1.100)
\end{aligned}$$

$$\text{where } C_s = \frac{c_{wR} t_p}{f_{y_{Fp}}} \quad (1.101)$$

The LHS of the equation (1.100) represents the amount of sensible heat gain by the desiccant wall per unit mass of desiccant and the RHS gives the amount of sensible heat loss by the purging air per unit mass of desiccant.

The solution of the equations (1.81) and (1.82) must satisfy equation (1.100) for all t_p .

CHAPTER V

PERIODIC STEADY STATE OPERATION

Cyclic Process

The dehumidifier performs the following cycling operations described earlier:

- a) Desorption. The moisture in the desiccant is removed by hot air supplied by the solar collector and/or storage tank. At the end of the desorption process, the desiccant bed is very dry and hot.
- b) Precooling. This is a purging process during which ambient air cooled by an evaporative cooler, is passed through the cooling channels to cool the dehumidifier unit. (The air, leaving this dehumidifier, is directed to the adjacent dehumidifier which has to be preheated.) At the end of precooling process, the desiccant bed is very dry and also fairly cool.
- c) Adsorption. The process air is dried and moisture is gained by the desiccant. The processed air is sent to the sensible heat exchanger and then to the evaporative cooler. Cooling air is sent through the cooling channels to keep the bed cool during adsorption. At the end of the adsorption process, the desiccant bed is wet but fairly cool.
- d) Preheating. This is also a purging process during which the air delivered from the adjacent dehumidifier, being pre-cooled, is sent through the cooling channels to heat the unit. At the end of the preheating process, the desiccant bed is wet and fairly hot and is ready for the next desorption process. The cycle is thus completed.

The dehumidifier is said to be operating under periodic steady state condition, when the performance is identical at successive cycles. The initial state of the dessicant bed at the beginning of any operation is not readily known. Both the temperature and the concentration of the wall have non-uniform unknown distributions over the entire bed in x and y direction.

Because of these unknown initial conditions, it is necessary to guess the initial conditions of the bed at the beginning of the cyclic operation. Let the initial conditions of the bed at the beginning of desorption be guessed. Then the desorption performance of the unit can be solved for. The solution will also give the final condition of the desiccant bed which is taken as the initial condition of the bed for the precooling process. The solution of the purging process can now be used to determine the outlet conditions of the purging air and also the final bed conditions. Now, using these results as the initial condition of the bed for adsorption, the adsorption performance of the dehumidifier is found, again using the solution of the sorption process. The final condition of the bed predicted at the end of adsorption is used as the initial condition of the bed for preheating. Finally, the purging process solution is determined, and the final bed conditions are taken as the initial condition of the bed for desorption.

It is highly improbable that this initial condition would be the same as the guess made at the beginning of the

cycle. Therefore, the initial conditions of the bed, predicted by one cycle, must be used as the second improved guess for the next cycle performance. In the same manner, the performance of the unit will be calculated for a sufficient number of cycles until the results of two subsequent cycles are almost identical. The number of cycles necessary to ensure periodic steady state operation depends on the first guess made. For most cases, it takes 4 to 6 cycles.

The process of adsorption is always accompanied with cross-cooling while the process of desorption may or may not involve "cross-heating." It was found that for a given quantity of available hot air from the solar collector or storage tank, it is always advantageous to send it all through the process channels instead of sharing it between the process channels and the cross-cooling channels. If all the regenerating air is passed through the process channels, the desorption process is adiabatic.

Apparently the solution of the sorption process cannot be used for adiabatic desorption process, since $y \rightarrow \infty$ as $\dot{m}_2 = 0$. One possible remedy is to assign a large finite value for y . This however, will require excessive computing time. The alternative way is to declare the value of b as zero. In this case, the solution is insensitive to the value of y and therefore any finite value of y will give the same solution to the desorption process. It is of course necessary to supply a dummy value for the inlet temperature of the cross-heating air and to ignore the results calculated for the

outlet temperatures.

Computer Programs

The numerical solutions of the sorption and purging processes were used to prepare a computer program which can predict the performance of the dehumidifier. The input data needed are the nondimensional variables t_F , x_F , y_F and the inlet conditions of process and cooling streams during adsorption and desorption. The output are the exit conditions of the process and cooling air and also the moisture exchanged in the desiccant bed as a function of time. A copy of the computer program is given in Appendix A.

The performance predicted by the computer model was compared with experimental data obtained at IIT by Mei¹⁶. The inlet conditions from 20 experiments were fed into the computer program and the exit conditions and the overall performance was carefully compared. Mei concluded that the present mathematical model is adequate to predict the dehumidifier performance operating under periodic steady state conditions. Figure 1.10 shows one of the several comparisons between the predicted and experimental results. An error analysis for the experiments shows that the experimental data are accurate within 5%. We therefore conclude that the mathematical model and the solutions are sufficiently accurate.

An Example

Figure 1.11 shows the exit temperatures of the process, cooling and regenerating stream for a selected set of inlet conditions and nondimensional variables. At the beginning

Figure 1.10 Comparison of Experimental and Theoretical Results Showing that the Linearly Varying Concentration Model is Adequate

Figure 1.11 Inlet and Exit Stream Temperatures in a Dehumidifier under Periodic Steady State Conditions

of adsorption, the process air leaves at a fairly low temperature and then it slowly increases. After about a third of the sorption time, the temperature reaches a maximum and later the exit temperature drops continuously. The cooling stream leaves at a high temperature at the beginning and then it decreases continuously. During the later part of the adsorption period, the temperature of the process stream and of the cooling stream are almost the same. The regenerating air initially leaves the dehumidifier at moderate regeneration temperature. Later, the exit temperature reduces, attains a minimum and then it slowly increases until the end of the process.

Figure 1.12 shows the inlet and exit humidities of the process and regenerating streams. The process air leaves very dry a short time after the beginning of sorption, and then the moisture content slowly and continuously increases. On the other hand, the regenerating stream leaves very wet sometime after the beginning of the process and then the humidity drops continuously.

The results of figures 1.11 and 1.12 are also given on a psychrometric chart in figure 1.13. It is interesting to note that the enthalpy of the exiting process air is lower than that of the inlet process air. This advantage is due to cross-cooling. For an adiabatic system, the exit air will leave at a higher enthalpy than the inlet air.

Figure 1.14 shows the average bed moisture concentration as a function of time. For the set of chosen data, the moisture concentration cycled is between 9.3% and 15%.

Figure 1.12 Inlet and Exit Stream Humidities in a Dehumidifier under Periodic Steady State Conditions

Figure 1.13 Inlet and Exit Stream States Shown on a Psychrometric Type Chart

Figure 1.14 Average Concentration of Moisture in the Desiccant Bed Under Periodic Steady State Operation

CHAPTER VI

CONCLUSIONS

1. The dynamic performance of cross-cooled desiccant dehumidifiers operating under periodic steady state conditions can be predicted by formulating an appropriate mathematical model and solving the resulting governing equations.
2. Simple models which consider only gas phase resistance for mass transfer are not adequate to predict the performances. Corrections are required to account for the solid phase resistance to mass flow. With such corrections, the model predicts the performances very closely to the experimental results for a large range of operating conditions. Hence, the solutions of the improved model can be used reliably for further applications.
3. Theoretically predicted results show that sufficient degree of dehumidification is attainable using cross-cooled dehumidifiers for solar powered desiccant air conditioning applications.

APPENDIX A

COMPUTER PROGRAM OF THE CROSS COOLED DEHUMIDIFIER

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C THIS PROGRAM PREDICTS THE PERFORMANCE OF CROSSCOOLED DEHUMIDIFIERS
  DIMENSION TD(201),YD(201),T2IA(201),TD(201),YD(201),T2ID(201),
  *T2IC(201),T2IH(201)
  COMMON/G1/TW(41,41),W(41,41),T2(41,41)
  COMMON/G2/E3,E3,IF,DX,N,CON2,CON3,CON4,WAVI
  COMMON/G3/IFF,CONS
  COMMON/G4/JF,KF,DT,DY,LUW,TWAVI,CON1,CWF,CWR,WMID,F
C DEFINE THE FOLLOWING LOGICAL UNIT NUMBERS FOR READING & WRITING
  LUR=1
  LUW=1
C READ THE INPUT DATA AND CALCULATE THE PRIMARY PARAMETERS
  READ(LUR,13) F,G,DT,DX,DY,TF,XF,YF,TFP,TF,BA,BD,ND,NC,NA,NH
  IF=TF/DT+1.1
  IFF=TFP/DT+1.1
  CALL READIN(ND,LUR,IF,DT,TD,YD,T2ID,TDAV,T2IDAV,YDAV)
  CALL READIN(NC,LUR,IFF,DT,T2IC,T2ICAV)
  CALL READIN(NA,LUR,IF,DT,TD,YD,T2IA,TOAV,T2IAAV,YOAV)
  CALL READIN(NH,LUR,IFF,DT,T2IH,T2IAV)
  JF=XF/DX+1.1
  KF=YF/DY+1.1
  CWR=1.0
  CWF=0.216*(1.-F-F*G)+0.22*F+0.28*F*G
  E3=1.0
  C1=1.033
  QA=2650.0
  QD=2700.0
  N=TF/DT+1.1
  E1A=E3*QA/C1
  E1D=E3*QD/C1
  E2=E3*CWR/C1/F
C COMPUTE THE CONSTANTS INVOLVED IN THE HEAT AND MASS BALANCES
  CON1=CWR/F
  CON2=CON1*TF/XF
  CON3=CON1*TF/YF
  CON4=CON1*DT/XF/C1
  CON5=CON1*TFP/YF
C WRITE THE INPUT DATA AND THE PRIMARY PARAMETERS
  WRITE(LUW,14)TF,XF,TOAV,YOAV,T2IAAV,T2IDAV,TFP,YF,TDAV,YDAV,T2IAV
  *,T2ICAV
C INITIAL GUESSES TO DEFINE THE STATE OF THE DESICCANT BED
  TWAVI=40.0
  WAVI=0.10
  WMID=WAVI-0.03
  DO 11 J=1,JF
  DO 11 K=1,KF
    TW(J,K)=TWAVI
  11 W(J,K)=WAVI
C CYCLING SEVERAL TIMES TO OBTAIN THE PERIODIC STEADY STATE CONDITIONS
  DO 12 IJ=1,10
    WRITE(LUW,15) IJ
    CALL SORBER (TD,YD,T2ID,TDAV,T2IDAV,QD,E1D,BD)
    CALL PURGE (T2IC,T2ICAV)
    CALL SORBER (TO,YO,T2IA,TOAV,T2IAAV,QA,E1A,BA)
  12 CALL PURGE (T2IH,T2IAV)
  13 FORMAT(8F10.5/4F10.5,4I3)

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14  FORMAT(///,1X,'TF = ',F5.1,3X,'XF = ',F5.1,3X,'TD = ',F6.1,3X,
    *'YO = ',F7.4,3X,'T2A = ',F6.1,3X,'T2D = ',F6.1,/,1X,'TFP = ',F5.1,
    *3X,'YF = ',F5.1,3X,'TD = ',F6.1,3X,'YD = ',F7.4,3X,'TFH = ',F6.1,
    *3X,'TFC = ',F6.1,///)
15  FORMAT(30X,'C Y C L E * ',I2,///,1X,'TIME',4X,'T1OUT',7X,'Y1OUT',
    *6X,'T2OUT',6X,'GAIN',10X,'LOSS',9X,'AUGAIN',/)
    STOP
    END
C THIS SUBROUTINE SOLVES THE GOVERNING EQNS OF THE SORPTION PROCESS
  SUBROUTINE SORBER (TO,YO,T2I,TOAV,T2IAV,Q,E1,R)
    DIMENSION T1(41,41),YW(41,41),Y(41,41),T1AV(201),
    *YAV(201),T2AV(201),TO(201),YO(201),T2I(201)
    COMMON/G1/TW(41,41),W(41,41),T2(41,41)
    COMMON/G2/E2,E3,IF,DX,N,CON2,CON3,CON4,WAVI
    COMMON/G4/JF,KF,DT,DY,LUW,TWAVI,CON1,CWF,CWR,WMID,F
    E4=(CWF+WMID*F)/(0.23869*CWR)
    ALOSS=0.0
    GAIN=0.0
    AUGAIN=0.0
    N1=N-1
C MARCHING IN T,X & Y DIRECTIONS BY FINITE DIFFERENCE SCHEMES
    DO 26 I=1,IF
      T=(I-1)*DT
      DO 24 J=1,JF
        T2(J,1)=T2I(I)
        DO 24 K=1,KF
          T1(1,K)=TO(I)
          Y(1,K)=YO(I)
          IF(I.EQ.1) GO TO 21
          TW(J,K)=TW(J,K)+DT*(T1(J,K)-TW(J,K)+R*(T2(J,K)-TW(J,K))
    *E1*(Y(J,K)-YW(J,K)))/E4
          W(J,K)=W(J,K)+DT*E2*(Y(J,K)-YW(J,K))
21      WS=2.*W(J,K)-WMID
          IF(WS.LT.0.0) YW(J,K)=0.0
          IF(WS.LT.0.0) GO TO 22
          RH=(143.31+1.68678*TW(J,K))*WS+(231.74-2.64006*TW(J,K))*WS*WS
          YW(J,K)=.622*RH/(10.*(4.21429-7.5*TW(J,K))/(237.3+TW(J,K)))-RH)
22      IF(J.EQ.1) GO TO 23
          IF(J.EQ.2) T1(J,K)=T1(J-1,K)+DX*(TW(J-1,K)-T1(J-1,K))
          IF(J.EQ.2) Y(J,K)=Y(J-1,K)+DX*E3*(YW(J-1,K)-Y(J-1,K))
          IF(J.GT.2) T1(J,K)=T1(J-2,K)+2.*DX*(TW(J-1,K)-T1(J-1,K))
          IF(J.GT.2) Y(J,K)=Y(J-2,K)+2.*DX*E3*(YW(J-1,K)-Y(J-1,K))
23      IF(K.EQ.1) GO TO 24
          IF(K.EQ.2) T2(J,K)=T2(J,K-1)+DY*(TW(J,K-1)-T2(J,K-1))
          IF(K.GT.2) T2(J,K)=T2(J,K-2)+2.*DY*(TW(J,K-1)-T2(J,K-1))
24      CONTINUE
C CALCULATE THE AVERAGES OF THE TEMPERATURES AND CONCENTRATIONS
    CALL DOUBAV (2,JF,KF,T1,T1AV(I))
    CALL DOUBAV (2,JF,KF,Y,YAV(I))
    CALL DOUBAV (1,JF,KF,T2,T2AV(I))
    IF(I.EQ.1) GO TO 25
    ALOSS=ALOSS+0.5*(YO(I)-YAV(I)+YO(I-1)-YAV(I-1))*CON4
    IF(I.NE.N) GO TO 26
    N=N+N1
    CALL DOUBAV (3,JF,KF,W,WAV)
    GAIN=(WAV-WAVI)
    AUGAIN=0.5*(GAIN+ALOSS)
C WRITE THE COMPUTED RESULTS AS FUNCTIONS OF TIME
25  WRITE(LUW,27) T,T1AV(I),YAV(I),T2AV(I),GAIN,ALOSS,AUGAIN
26  CONTINUE
C COMPUTE THE AVERAGE OVER THE SORPTION TIME TO MAKE HEAT BALANCES
    CALL DOUBAV(3,JF,KF,TW,TWAV)
    CALL SINGAV(IF,T1AV,T1OUT)
    CALL SINGAV(IF,YAV,YOUT)
    CALL SINGAV(IF,T2AV,T2OUT)

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      HGAINW=(TWAV-TWAVI)*CON1*E4
      HGAIN1=(T1OUT-T0AV)*CON2
      HGAIN2=(T2OUT-T2IAV)*E*CON3
      HGAINW=HGAINW+HGAIN1+HGAIN2
      HGEN=AVGAIN*Q
      WRITE(LUW,28) HGAINW,HGAIN1,HGAIN2,HGAINW,HGEN
C REPLACE THE INITIAL GUESSES BY THE END CONDITIONS OF THE RED
      WMID=(WAV+WAVI)/2.
      TWAVI=TWAV
      WAVI=WAV
      N=N1+1
27  FORMAT(F5.2,F10.3,F13.7,F10.3,3F14.7)
28  FORMAT(/,10X,'HEAT GAINED BY THE WALL      =',F10.5,/,10X,
      *'HEAT GAINED BY PROCESS AIR =',F10.5,/,10X,
      *'HEAT GAINED BY COOLING AIR =',F10.5,/,10X,
      *'HEAT GAINED BY WA+FR+CO.   =',F10.5,/,10X,
      *'HEAT GENERATED           =',F10.5,/)
      RETURN
      END
C THIS SUBROUTINE SOLVES THE GOVERNING EQNS OF THE FURGING PROCESS
      SUBROUTINE FURGE(T2I,T2IAV)
      DIMENSION T2I(201),T2AV(201)
      COMMON/G1/TW(41,41),W(41,41),T2(41,41)
      COMMON/G3/IF,CONS
      COMMON/G4/JF,KF,DT,DY,LUW,TWAVI,CON1,CWF,CWR,WMID,F
      E4=(CWF+WMID*F)/(0.23869*CWR)
C INPUT THE INLET CONDITIONS AND MARCH USING FINITE DIFFERENCE SCHEME
      DO 42 I=1,IF
      DO 41 J=1,JF
      T2(J,1)=T2I(I)
      DO 41 K=1,KF
      IF(I.EQ.1) GO TO 40
      TW(J,K)=TW(J,K)+DT*(T2(J,K)-TW(J,K))
40  IF(K.EQ.1) GO TO 41
      T2(J,K)=T2(J,K-1)+DY*(TW(J,K-1)-T2(J,K-1))
41  CONTINUE
42  CALL DOUBAV (1,JF,KF,T2,T2AV(I))
C WRITE THE EXIT STATE AS A FUNCTION OF TIME
      DO 43 I=1,IF,4
      T=(I-1)*DT
43  WRITE(LUW,44) T,T2AV(I)
C COMPUTE THE AVERAGES OVER THE FURGE TIME TO MAKE HEAT BALANCES
      CALL SINGAV(IF,T2AV,T2OUT)
      CALL DOUBAV(3,JF,KF,TW,TWAV)
      HGAIN=(T2IAV-T2OUT)*CONS
      HLOSS=(TWAV-TWAVI)*CON1*E4
      TWAVI=TWAV
      WRITE(LUW,45) HGAIN,HLOSS
44  FORMAT(F10.2,F38.3)
45  FORMAT(/,10X,'HEAT GIVEN TO THE WALL =',F10.5,/,10X,
      *'HEAT GAIN BY THE WALL =',F10.5,/)
      RETURN
      END
C THIS SUBROUTINE CALCULATES THE AVERAGE OF AN ONE-DIMENSIONAL ARRAY
      SUBROUTINE SINGAV(IF,Q,QAV)
      DIMENSION Q(201)
      SUMQ=Q(1)+4.*Q(2)+Q(IF)
      DO 50 I=5,IF,2
50  SUMQ=SUMQ+2.*Q(I-2)+4.*Q(I-1)
      QAV=SUMQ/(3.*(IF-1))
      RETURN
      END
C THIS SUBROUTINE CALCULATES THE AVERAGE OF A TWO-DIMENSIONAL ARRAY
      SUBROUTINE DOUBAV(M,JF,KF,Q,QAV)
      DIMENSION Q(41,41)

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        IF(M-2) 60,62,64
60      SUMQ=Q(1,KF)+4.*Q(2,KF)+Q(JF,KF)
        DO 61 J=5,JF,2
61      SUMQ=SUMQ+2.*Q(J-2,KF)+4.*Q(J-1,KF)
        QAV=SUMQ/(3.*(JF-1))
        RETURN
62      SUMQ=Q(JF,1)+4.*Q(JF,2)+Q(JF,KF)
        DO 63 K=5,KF,2
63      SUMQ=SUMQ+2.*Q(JF,K-2)+4.*Q(JF,K-1)
        QAV=SUMQ/(3.*(KF-1))
        RETURN
64      C=1.
        L=1
        SUMQ=0.0
        DO 66 J=1,JF
        IF(J.EQ.JF) C=1.
        SUMQ=SUMQ+C*(Q(J,1)+4.*Q(J,2)+Q(J,KF))
        DO 65 K=5,KF,2
65      SUMQ=SUMQ+C*(2.*Q(J,K-2)+4.*Q(J,K-1))
        C=3+L
66      L=-L
        QAV=SUMQ/(9.*(JF-1)*(KF-1))
        RETURN
        END
C THIS SUBROUTINE READS THE INLET CONDITIONS OF THE SORPTION PROCESS
SUBROUTINE READIN(N,LUR,IF,DT,TO,YO,T2I,TOAV,T2IAV,YOAV)
DIMENSION T(20),T1(20),Y1(20),T2(20),TO(201),YO(201),T2I(201)
DO 70 J=1,N
70  READ(LUR,74) T(J),T2(J),T1(J),Y1(J)
    TQ(1)=T1(1)
    YQ(1)=Y1(1)
    T2I(1)=T2(1)
    J=2
    DO 73 I=2,IF
    TIME=(I-1)*DT
    GO TO 72
71  J=J+1
72  IF(TIME.GT.T(J)) GO TO 71
    RATIO=(TIME-T(J-1))/(T(J)-T(J-1))
    TO(I)=RATIO*(T1(J)-T1(J-1))+T1(J-1)
    YQ(I)=RATIO*(Y1(J)-Y1(J-1))+Y1(J-1)
73  T2I(I)=RATIO*(T2(J)-T2(J-1))+T2(J-1)
    CALL SINGAV (IF,TO,TOAV)
    CALL SINGAV (IF,T2I,T2IAV)
    CALL SINGAV (IF,YO,YOAV)
74  FORMAT(4F10.5)
    RETURN
    END
C THIS SUBROUTINE READS THE INLET CONDITIONS OF THE PURGING PROCESS
SUBROUTINE READIM(N,LUR,IF,DT,T2I,T2IAV)
DIMENSION T(20),T2(20),T2I(101)
DO 80 J=1,N
80  READ(LUR,84) T(J),T2(J)
    T2I(1)=T2(1)
    J=2
    DO 83 I=2,IF
    TIME=(I-1)*DT
    GO TO 82
81  J=J+1
82  IF(TIME.GT.T(J)) GO TO 81
    T2I(I)=(TIME-T(J-1))/(T(J)-T(J-1))*(T2(J)-T2(J-1))+T2(J-1)
    CALL SINGAV(IF,T2I,T2IAV)
84  FORMAT(2F10.5)
    RETURN
    END

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