NUCLEAR COLLISIONS AT SEVERAL TENS OF MeV PER NUCLEUS*

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PREFACE

Nuclear beams with energies of several tens of MeV per nucleon will soon be available at a number of research centers around the world. Such beams offer a tool for probing new aspects of nuclear structure and dynamics. As the energy is raised models and concepts developed for the relatively well-studied lower-energy domain will be pressed to their limits and are likely to grow obsolete as novel phenomena enter the scene. The first part of these notes deals with the theory of ordinary damped collisions; the testing of such theories is one important aspect of the research with higher beam energies. Another is the search for truly novel phenomena and the second part of these notes contains some more speculative material on that aspect.

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PART I: ORDINARY DAMPED COLLISIONS

When two heavy nuclei collide with energies of several MeV per nucleon a large fraction of their initial translational energy is converted into intrinsic excitation (hence the term "damped collisions"). However, although strongly perturbed and highly excited, the two final fragments do show a large resemblance with the initial nuclei; this fact indicates that a binary configuration has been maintained throughout the entire collision process. The dynamics of the intermediate complex can therefore be discussed in terms of the degrees of freedom associated with a dinuclear system.

Although the final fragments resemble the initial nuclei they are by no means identical to them: typical mass and charge widths amount to several units. This implies that a substantial number of nucleons are exchanged in such a collision. The transfer of a nucleon is generally associated with a dissipation of energy and momentum and in fact simple estimates suggest that this mechanism is an important, if not dominant, agency for the damping of the dinuclear motion.

In the following we shall indicate how the effects of nucleon transfer in nuclear collisions may be explored in a simple model. The aim is to obtain an impression of the general features associated with the transfer-induced transport in nuclear collisions. It should be emphasized from the outset that although we focus our attention on the transfer mechanism we do not wish to preclude the coexistence of other important mechanisms, such as the excitation of various collective modes in the nuclei. In fact, the dynamical interplay between the different

coexisting mechanisms forms a fascinating subject for future study.

I.1. The Model

In the present discussion we take advantage of the approximate validity of the independent-particle model of nuclei. We thus assume that the nucleons move nearly independently in the nuclear one-body mean field. The occupation probabilities of the single-particle states in the nucleus are then taken to be as in Fermi-Dirac gases.

$$\begin{aligned} f^A(\epsilon_a) &= (1 + e^{(\epsilon_a - \epsilon_A)/\tau})^{-1} \\ f^B(\epsilon_b) &= (1 + e^{(\epsilon_b - \epsilon_B)/\tau})^{-1} . \end{aligned} \tag{1}$$

For simplicity we consider here only one type of nucleon - the generalization is straightforward. In (1) ϵ_a is the energy of the nucleon in the projectile-like nucleus A and ϵ_b is the energy of the nucleon in the target-like nucleus B; the corresponding Fermi energies are denoted by ϵ_A and ϵ_B . The above assumption need not be good at the very early stages of the collision but this is less important. At the time when good communication is established between the two nucleides they are typically excited by several MeV and (1) appears a reasonable description. Furthermore, due to the good communication, we assume that the (time-dependent) temperature τ is the same in the two collision partners.

The two nuclei move with velocities $\vec{U}_A$ and $\vec{U}_B$. These velocities refer to those parts of the nuclei which are in the interaction zone rather than to the nuclear centers.

When the communication between A and B has been established the transfer of nucleons is possible. In line with the mean-field independent-particle description we shall assume that the nucleons are transferred individually rather than as correlated clusters (although we do not wish to preclude that such transfers might also occur and even play an important role). The driving force for the nucleon transfer arises partly from the difference in the Fermi levels, $F = \epsilon_B - \epsilon_A$, and partly from the relative velocity of the two gases, $\vec{U} = \vec{U}_A - \vec{U}_B$. The transfer of a nucleon creates a one-particle one-hole type excitation of the intrinsic nuclear system. It can be shown¹⁾ that the energy of this exciton amounts to

$$\omega = F - \vec{U} \cdot \vec{p} . \quad (2)$$

Here $\vec{p} = \frac{1}{2} (\vec{p}_a + \vec{p}_b)$ where $\vec{p}_a$ is the momentum of the nucleon relative to A and $\vec{p}_b$ is its momentum relative to B. It is an important assumption that once a nucleon has been transferred it is quickly accepted as an equal member of the recipient nucleus and no memory of its heritage remains. By this assumption the multiple transfers can be considered as markovian and the process can be treated by standard transport theory. The relevance of transport theory to damped nuclear collisions was first recognized by Nörenberg.²⁾

Consider for the moment a flat contact geometry with the area σ . The rates of transfer between the two nuclei are then given by

$$\begin{aligned} \dot{A}^+ &= \sigma \int \frac{d\vec{p}}{h} \frac{|\vec{v}_z|}{2} \bar{f}^A(\vec{p}) f^B(\vec{p}) \\ \dot{A}^- &= \sigma \int \frac{d\vec{p}}{h} \frac{|\vec{v}_z|}{2} f^A(\vec{p}) \bar{f}^B(\vec{p}) \end{aligned} \quad (3)$$

for transfer into and out of A, respectively. Here the occupation factors f give the probability that a nucleon is initially present in the donor nucleus with the momentum $\vec{p}$ and the blocking factors $\bar{f} = 1-f$ give the probability that such a state is available in the recipient nucleus. The transfer rate is proportional to the magnitude of the velocity in the direction normal to the contact surface as simple classical arguments would suggest.³⁾

The above expressions for the basic transfer rates form the core of the model. If the geometry of the interaction zone is gently curved the proper generalization of (3) can be accomplished by application of the proximity method.⁴⁾ The ensuing formulas make it relatively simple to study the dynamical role of nucleon transfer in nuclear collisions. Such studies, based on direct numerical simulation, are presently under way.⁵⁾

For the general discussion of the transport problem it is convenient to reduce the master equation implied by (3) to its Fokker-Planck approximation. The characteristic quantities are then the transport coefficients, which govern the rate of change of the mean values of the macroscopic variables and their covariances. They can be determined by following the short-term evolution of a system which has been prepared with specified sharp values of the macroscopic

variables. If we were to consider only the particle number A we would have

$$\begin{aligned} V_A(A) &= \frac{d}{dt} A = \dot{A}^+ - \dot{A}^- \\ 2D_{AA}(A) &= \frac{d}{dt} \sigma_A^2 = \dot{A}^+ + \dot{A}^- \end{aligned} \quad (4)$$

for the drift and diffusion coefficients, respectively, and the dynamical evolution of the probability distribution for a given mass partition, $P(A;t)$, would be governed by

$$\frac{\partial}{\partial t} P = - \frac{\partial}{\partial A} V_A P + \frac{\partial^2}{\partial A^2} D_{AA} P. \quad (5)$$

This only serves as a simple illustration. In reality we wish to consider the simultaneous evolution of several interrelated macroscopic variables.

1.2. Nearly Degenerate Limit

Until now, most of the experimental studies of damped nuclear collisions have been carried out at relatively low energies, with the nuclei meeting each other with kinetic energies of a few MeV per nucleon. Since this energy is small in comparison with the intrinsic kinetic energies of the nucleons the collective motion is relatively slow and the nuclei acquire only modest excitation. Under these circumstances only the nucleons near the Fermi surface take part in the exchange and the entire treatment simplifies considerably. In the following we shall specialize to this limit and thus assume

$$U \ll v_F, F, \tau, \omega \ll T_F. \quad (6)$$

Then the occupation factors can be written in the approximate form¹⁾

$$\begin{aligned} \bar{f}^A f^B &\approx \nu(\omega) \delta(\epsilon - \epsilon_F) \\ f^A \bar{f}^B &\approx \nu(-\omega) \delta(\epsilon - \epsilon_F) \end{aligned} \quad (7)$$

where

$$\nu(\omega) = \omega(1 - e^{-\omega/\tau})^{-1}. \quad (8)$$

Due to the appearance of the δ -function in (7) the energy integration in (3) is trivial and only the directional integration remains. It is useful to introduce the flux-weighted directional average of a function $g(\Omega)$ by

$$\langle g \rangle \equiv \frac{1}{2\pi} \int d\Omega |\cos\theta| g(\Omega) \quad (9)$$

where the polar axis is perpendicular to the interaction surface between A and B.⁶⁾

With these simplifications the transfer rates (3) reduce to

$$A^{\pm} \approx N^{\pm}(\epsilon_F) \langle \nu(\pm\omega) \rangle_F \quad (10)$$

where the subscript F has been attached to the flux average to indicate that only the particles in the Fermi surface should be considered. The overall transfer rate is governed by the quantity

$N'(\epsilon_F)$ which is the differential one-body current of nucleons transferred at the Fermi surface:⁴⁾

$$N'(\epsilon_F) = \frac{\partial}{\partial T_F} N(\epsilon_F), \quad N(\epsilon_F) = \frac{1}{4} \rho \bar{v} \sigma. \quad (11)$$

Here $\frac{1}{4} \rho \bar{v}$ is the one-way flux in standard nuclear matter. The expression for $N(\epsilon_F)$ holds for a flat, fully open contact surface; in general the current $N(\epsilon_F)$ depends sensitively on the geometry of the interaction zone and may be difficult to calculate. For a certain family of dinuclear configurations simple estimates can be obtained by use of the proximity method.⁴⁾

From the knowledge of the basic transfer rates it is straightforward to derive the appropriate expressions for the transport coefficients for a given set of macroscopic variables $\{\mathcal{C}\}$ (assumed to be additive such as e.g. the particle number A and the momentum $\vec{p}$). The drift coefficient vector $\vec{V}$ represents net rate of change of the variables, hence

$$\begin{aligned} V_{\mathcal{C}} &= N'(\epsilon_F) \langle [1(\omega) - 1(-\omega)] \mathcal{C}(\vec{p}) \rangle_F \\ &= N'(\epsilon_F) \langle \omega \mathcal{C}(\vec{p}) \rangle_F. \end{aligned} \quad (12)$$

The corresponding diffusion coefficient tensor $\overleftrightarrow{D}$ represents the rate of increase in the covariances, hence

$$\begin{aligned}
 D_{\mathcal{L}_1 \mathcal{L}_2} &= N'(\epsilon_F) \left\langle \frac{1}{2} \{1(\omega) + 1(-\omega)\} \mathcal{L}_1(\vec{p}) \mathcal{L}_2(\vec{p}) \right\rangle_F \\
 &= N'(\epsilon_F) \left\langle \frac{\omega}{2} \coth\left(\frac{\omega}{2T}\right) \mathcal{L}_1(\vec{p}) \mathcal{L}_2(\vec{p}) \right\rangle_F.
 \end{aligned}
 \tag{13}$$

These expressions appear immediately plausible when one considers the fact that the differential transition rate from B to A, at the energy ϵ , is given by $N'(\epsilon) \bar{f}^A f^B$ and the rate for the opposite direction is given by $N'(\epsilon) f^A \bar{f}^B$. (The situation corresponds to a random walk where the net gain is the difference in the number of steps taken and the variance is the total number of steps.) The cancellation of the blocking factors $\bar{f}$ in the expression for the drift coefficients V is a general reflection of the fact that this quantity can be represented in terms of one-body operators and hence is insensitive to correlations among the particles.

I.3. The Dinucleus

The expressions (12) and (13) have been written for arbitrary additive observables $\{\mathcal{L}\}$. Let us now consider the case of actual interest, the dinucleus. For simplicity we shall restrict our attention to the following variables

$$\mathcal{C} = \{Z, N, P, \vec{L}, \vec{S}_A, \vec{S}_B\}
 \tag{14}$$

where Z and N are the proton and neutron numbers of the projectile-like partner A, P and $\vec{L}$ are the radial and angular momenta at the relative dinuclear motion, and $\vec{S}_A$ and $\vec{S}_B$ are the individual angular

momenta carried by the dinuclear partners A and B.

It is simplest to consider the particle numbers Z and N. From (12) and (13) we obtain

$$\begin{aligned} V_Z &= N'_Z F_Z, & V_N &= N'_N F_N \\ D_{ZZ} &= N'_Z \tau^*, & D_{NN} &= N'_N \tau^* \end{aligned} \quad (15)$$

and $D_{ZN} = 0$. Here N'_Z and N'_N are the differential one-body currents of protons and neutrons, respectively, and $F_Z = -\partial\mathcal{H}/\partial Z$ and $F_N = -\partial\mathcal{H}/\partial N$ are the corresponding driving forces. The "effective temperature" τ^* is given by

$$\tau^* \equiv \left(\frac{\omega}{2} \coth \left(\frac{\omega}{2\tau^*} \right) \right)_F. \quad (16)$$

It is the average energy stored in the elementary transfer modes of excitation. The appearance of τ^* is a characteristic feature of the model. It should be noted that the transport coefficients (15) satisfy a generalized Einstein relation $DF = V\tau^*$ in accordance with the fluctuation-dissipation theorem. A general discussion of the implications of the fluctuation-dissipation theorem on low-energy nuclear dynamics has been made by Hofmann and Siemens.⁷⁾ The transport coefficients (15) make it possible to study the simultaneous transport of charge and mass in nuclear collisions.⁸⁾

By elementary but somewhat more complicated calculation it can be shown that the transport coefficients for the various momentum variables are approximately

$$\begin{aligned}
 v_{PP} &= -2m N \dot{\vec{R}} \\
 D_{PP} &\approx 2m N \tau^* \\
 \vec{v}_L &= -m N \dot{\vec{R}} \times \vec{U} \\
 \vec{v}_{S_A} &= -\frac{a}{R} \vec{v}_L, \quad \vec{v}_{S_B} = -\frac{b}{R} \vec{v}_L \\
 \vec{D}_{LL} &\approx m N R^2 \tau^* \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
 \vec{D}_{LS_A} &= -\frac{a}{R} \vec{D}_{LL}, \quad \vec{D}_{LS_B} = \frac{b}{R} \vec{D}_{LL} \\
 \vec{D}_{S_A S_B} &\approx -\frac{ab}{R^2} \vec{D}_{LL} + m N \rho_{\text{eff}}^2 \tau^* \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
 \vec{D}_{S_A S_A} &= \vec{D}_{S_A S_B} (b \rightarrow a) \\
 \vec{D}_{S_B S_B} &= \vec{D}_{S_A S_B} (a \rightarrow b).
 \end{aligned} \tag{18}$$

Here $N = N_Z + N_N$ denotes the average one-body nucleon current from one partner to the other, $\vec{R}$ is the position of A relative to B, and a and b are the distances from the (the centers of) A and B to the interaction zone where the transfers occur. The off-diagonal diffusion coefficients coupling the above momentum variables to the particle numbers have been omitted here since they are often (but not always) negligible. Likewise, the explicit appearance of τ^* is only approximate and may not always be quantitatively accurate.

The drift coefficients for P and $\vec{L}$ are recognized as the radial and tangential components of the window friction^{3,9)} and can be derived on simple classical grounds due to their one-body character. The vanishing

of the zz -component of D_{LL} is a trivial reflection of angular momentum conservation ($\vec{L}$ is always perpendicular to $\vec{R}$). The diffusion tensors for the intrinsic angular moments contain an additional term resulting from off-axis transfers (such transfers may change the K quantum numbers in A and B and hence $\vec{S}_A$ and $\vec{S}_B$ need not remain perpendicular to $\vec{R}$); this term depends explicitly on the geometrical size of the interaction zone as measured by the effective neck radius ρ_{eff} (and it is therefore typically smaller than the first term by a factor ρ_{eff}^2/R^2).

The degradation of the initial macroscopic energy leads to excitation of the microscopic degrees of freedom in the two nucleides. The generated heat Q is of primary interest since it characterizes the state of the intrinsic system (through the temperature $\tau = (Q/a)^{1/2}$, $a \approx (A+B)/8$ MeV). It is therefore convenient to include Q in the set of macroscopic variables considered. This can be done although the intrinsic energy is not an additive variable. The drift coefficient, equal to the energy dissipation rate, can be calculated from the loss of macroscopic energy,

$$\begin{aligned} V_Q &= \dot{Q} \\ &= F_Z V_Z + F_N V_N + mN(2 \dot{R}^2 + U_t)^2. \end{aligned} \quad (19)$$

The various diffusion coefficients involving Q can be obtained approximately by use of the generalized Einstein relation (with the driving force $F_Q = -\partial \mathcal{H}/\partial Q$ equal to unity):

$$\begin{aligned} D_{QQ} &\approx V_a \tau^* \\ D_{\mathcal{C}Q} &\approx V \tau^* \end{aligned} \quad (20)$$

This completes the derivation of the transport coefficients for the disphere. No arbitrary parameters enter in the expressions (although the form factors depend delicately on the details of the interaction zone and therefore are difficult to estimate accurately). The theory thus implies that certain specific relations exist between the different macroscopic variables. This feature may be particularly useful when trying to determine from experiment the relative importance of the particle-transfer mechanism in damped nuclear collisions.

1.4. Confrontation with Experiment

In order to determine the relative importance of the nucleon transfer mechanism in damped nuclear collisions, and verify the specific structure implied by the above formulas, it is necessary to confront the theory with experiment. This task is made difficult by the fact that the transport process depends delicately on the details of the interaction zone whose dynamics is still only poorly understood. One may try to circumvent this complication, which presents an interesting problem by itself, by correlating a number of different observables. We shall discuss here one example of such an approach, namely the relation between the energy loss and the mass dispersion. It was first suggested by Huizenga et al.¹⁰⁾ that this relation may be used to elucidate the basic nature of the transfer mechanism.

The basic idea is the following: Each nucleon transfer induces an energy loss ω given by (2). The transfer rate is equal to the rate of increase in the dispersion σ_A^2 , by ordinary random-walk theory. Hence the rate of energy loss can be written

$$\frac{d}{dt} T \approx -\dot{Q} = -\omega_{\text{ave}} \frac{d}{dt} \sigma_A^2 \quad (21)$$

where ω_{ave} is the appropriate average exciton energy and $T = \frac{1}{2} \mu U^2$ is the kinetic energy of the relative motion. We restrict our attention to nearly peripheral, partially damped collisions so that the relative velocity $\vec{U}$ is predominantly tangential.

If there were no intrinsic motion of the nucleus prior to their transfer they would contribute an excitation energy $\omega = \frac{1}{2} m U^2 = \frac{m}{\mu} T$ (neglecting the relatively small contribution from driving force F acting on asymmetric systems). Therefore, Huizenga et al.¹⁰⁾ have introduced the quantity

$$\alpha \equiv -\frac{\mu}{m} \frac{1}{T} \frac{dT}{d\sigma_A^2} \quad (22)$$

which can be extracted empirically from the relation between the kinetic energy loss and the mass dispersion obtained in a given experiment. (It is here important that α indeed turns out to be experimentally well-defined.) It follows that if all the dissipation were induced by transfer of particles initially at rest then α would be unity.

In the present theory where the dissipation is produced by transfer of Fermi-Dirac particles (which have an initial motion and are subject to the blocking effect) we have

$$-\frac{d}{dt} T = -\langle \omega^2 \rangle_F N'(\epsilon_F) \quad (23)$$

and

$$\frac{dT}{dt} \sigma_A^2 \approx 2\tau^* N'(\epsilon_F) \quad (24)$$

as follows from the general expressions (12) and (13) respectively. Hence

it follows that

$$\frac{dT}{d\sigma_A} \approx - \frac{\langle w^2 \rangle_F}{2\tau^*} . \quad (25)$$

For a nearly peripheral collision (where $\vec{U}$ is almost tangential) between nearly symmetric systems (so that F_A can be neglected) we have

$$\langle w^2 \rangle_F \approx \frac{1}{4} U^2 p_F^2 = \frac{1}{2} m U^2 T_F . \quad (26)$$

Consequently we arrive at the following simple estimate

$$\alpha \approx \frac{T_F}{2\tau^*} . \quad (27)$$

It should be noted that if the blocking effect were ignored in the calculation one would arrive at $\alpha \approx 1$.

The above simplistic estimate (27) indicates that α should typically be substantially larger than unity (since usually $\tau^* \ll T_F$). Moreover the formula suggests that α should decrease as the bombarding energy E_{cm} (and hence τ^*) is increased. Both of these features are indeed present in the experimental data where α -values of 2 to 16 have been found and where a clear decrease with E_{cm} has been established in all cases explored.¹¹⁾ We wish to emphasize that these are both features which would not find an explanation within a classical transfer model.

For nearly symmetric systems the value of τ^* is essentially determined by $\frac{1}{2} m U^2$. The formula (27) therefore suggests that the α -values for many different systems should fall on the same "universal" curve when plotted against $\frac{1}{2} m U^2$. Such a behavior is indeed borne out by experiment.

This fact lends support to the employed model, and provides evidence that the mechanism considered, namely the transfer of individual nucleons, plays an essential role as a damping mechanism. However, it must be stressed that the perturbative estimate (27) relies on a number of idealizations and a more refined treatment is called for before a definite comparison can be made.

1.5. Concluding Remarks

We have explored the consequences of the independent-particle idealization for the dynamical properties of the dinucleus; no additional physical assumptions have been introduced. In this way clear and relatively simple results have been derived for the dinuclear transport coefficients. Although the approach treats only uncorrelated microscopic modes of excitation, it is not meant to preclude the possible coexistence of additional mechanisms, such as the interplay with collective dinuclear modes and the special dynamics of the interaction zone. The high specificity of the present results holds promise that a careful confrontation with experiment might indicate conclusively to what extent the predicted behavior is borne out by real nuclei. At the same time the relative importance of other agencies might be established; should they prove important, an appropriate extension of the theory is called for.

PART II. RAISING THE ENERGY

In the preceding derivation of the transport coefficients it was an essential assumption that the excitons created by the individual nucleon transfers are quickly dissipated into more complicated excitations so that the intrinsic nucleonic system remains in quasi-equilibrium throughout the collision; on grounds of the relatively long mean free path it is also assumed that the interior of the nucleus remains homogeneous. At relatively low energies, these assumptions are rather well justified. The theory then makes a number of general predictions about the correlations between the various observables.

But as the energy is raised those assumptions are expected to break down. The tacit assumption that the transferred nucleon is absorbed by its new host nucleus need no longer be generally valid; in the next section we shall discuss the possibility that transferred nucleons under favorable circumstances may be ejected promptly from the dinuclear complex. Furthermore, with the faster macroscopic motion the microscopic relaxation process can no longer be considered as instantaneous and the dynamical description in terms of time-local macroscopic equations of motion becomes dubious. By the same token the assumption of a quasi-equilibrated intrinsic system grows increasingly inadequate and one needs to include additional degrees of freedom describing inhomogeneities in the nuclear interior.

II.1. The energy ladder

It is instructive to start out by putting the energy region considered into the proper perspective in relation to other energy regions of nuclear physics. As the energy of the nuclear beams is steadily raised, a number of important thresholds are being passed. In the low end of the energy scale, where the characteristic energies are of the order of a few MeV, the fine details of the nuclear structure can be studied; this is the domain of nuclear spectroscopy. With increasing energy the individual quantum states lose their significance and the detailed structure of the intrinsic nuclear system largely dissolves.

When the energy has reached a few MeV per nucleon we are in the domain of strongly damped, or deep inelastic, collisions. In this regime, where the excitation per nucleon is still small in comparison with the intrinsic single-particle energies, the important degrees of freedom are a few macroscopic variables, particularly those associated with the nuclear surface. Since the macroscopic velocities are small in comparison with typical intrinsic speeds the communication of disturbances is rather fast and the corresponding macroscopic equations of motion are approximately local in time. The coupling of the macroscopic degrees of freedom to the structureless intrinsic reservoir damps the macroscopic motion and is a source of dynamical fluctuations in these variables.

This situation will gradually change when the energy is raised further. When the macroscopic velocities are no longer small in comparison with the intrinsic speeds the intrinsic communication will require a non-negligible time and the retardation terms must be retained in the

equations of motion. At the same time, inhomogeneities are likely to occur in the nuclear interior and additional degrees of freedom are activated.

An important threshold is probably reached when the excitation energy is of the same size as the nuclear binding energy. The nuclear system has then largely lost its cohesiveness and multifragmentation becomes increasingly dominant.

When the excitation energy per nucleon increases above the Fermi kinetic energy the quantal nature of the nucleons is expected to become less important and classical models gain increasing applicability. At the same time, the one-body mean field gives way to direct two-body collisions as the main governor of the nucleonic motion.

At still higher energies it becomes possible to produce pions by direct nucleon-nucleon collisions and we then enter into the mesonic regime. The basic microscopic degrees of freedom are now no longer conserved during the collision process.

The relativistic regime is reached when the kinetic energies are of the same order as the rest masses of the participating baryons. It is then possible to excite the intrinsic states of the baryon and the entire field merges more and more with elementary-particle physics.

One is presently contemplating nuclear beams with energies which are large in comparison with the rest masses. This regime of ultra-relativistic nuclear physics is still unexplored. There are speculations that sufficiently far into this domain the hadrons may "melt" and liberate their intrinsic constituents. They might then form an extended

system of quark matter. The production of such an environment would clearly have a profound bearing on our fundamental understanding of nature.

II.2. The mean free path

The nucleons are fermions. This generally reduces the states accessible in a collision process. At densities and excitations typical of collisions with $E/A \approx T_F$ this effect may appreciably inhibit the occurrence of direct nucleon-nucleon collisions. This is well known from the lower end of the energy scale where the action of the exclusion principle largely eliminates two-body collisions, leaving the one-body mean field as the dominant governor of the nucleonic motion. Since the choice of approximations is very dependent on the nucleon mean free path it would be useful to obtain a semi-quantitative estimate of this quantity over a broad range of situations. We have made such an attempt, as will be described in the following.

Consider a uniform system of nuclear matter with a given temperature τ . The distribution of nucleons in momentum space is then given by

$$P(\vec{p}) = (1 + e^{-(\epsilon - \epsilon_F)/\tau})^{-1} \quad (1)$$

as a consequence of the Fermi-Dirac statistics. Here ϵ is the energy of the nucleon and ϵ_F is the Fermi energy. We now imagine that a nucleon with momentum $\vec{p}_0 = m\vec{v}_0$ is moving through the system. Its motion will be degraded as a consequence of two-body collisions with the nucleons in the medium. If there were no Pauli blocking of the final states, the

collision rate of the intruder nucleon would be given by

$$v_0 = \frac{v_0}{\lambda_0} = \int \frac{d\vec{p}}{h^3} f(\vec{p}) v_{rel} \sigma_{NN}, \quad (2)$$

where λ_0 is the corresponding mean free path. Here σ_{NN} is the collision cross section for two nucleons in free space with the same kinematical conditions and v_{rel} is their relative speed.

However, the probability that a given final momentum $\vec{p}'$ is available is reduced by the factor $\bar{f}(\vec{p}') = 1 - f(\vec{p}')$. Therefore, the actual collision rate is rather

$$\begin{aligned} v &= \frac{v_0}{\lambda} \\ &= \int \frac{d\vec{p}}{h^3} f(\vec{p}) v_{rel} \int d\Omega \bar{f}(\vec{p}_0') \bar{f}(\vec{p}') \frac{d\sigma_{NN}}{d\Omega} \end{aligned} \quad (3)$$

where $d\sigma_{NN}/d\Omega'$ is the differential collision cross section for the process $|\vec{p}_0, \vec{p}\rangle \rightarrow |\vec{p}_0', \vec{p}'\rangle$.

The occurrence of the blocking factors in the collision integral gives rise to a reduction in the actual collision rate and a corresponding increase of the mean free path between collisions. The exact size of this effect depends of course on the variation of $d\sigma_{NN}/d\Omega'$ with angle and energy as well as on the distribution of nucleons in the medium. An approximate indication of the effect can be obtained by assuming that the free nucleon-nucleon collisions are isotropic and energy-independent. The mean free path is then increased by the factor

$$\eta = \frac{\int d\vec{p} f(\vec{p}) v_{rel} \int d\Omega' \bar{f}(\vec{p}_0') \bar{f}(\vec{p}')}{4\pi \int d\vec{p} f(\vec{p}) v_{rel} d\Omega'}. \quad (4)$$

We have calculated this factor as a function of the temperature τ of the medium and for different values of the incident energy T_0 of the intruder nucleon. The result is shown in Fig. 1. In general η decreases when either τ or T_0 is increased. It is noteworthy that even at rather large values of these two parameters η remains appreciably above unity. For example, for a particle in the Fermi surface ($T_0 = T_F$) we find $\eta = 10$ even at a temperature of $\tau \approx 10$ MeV, and at $T_0 = 2T_F$ (which would correspond to a physical approach energy of $T_{\text{beam}} = T_0 - T_F - B \approx 30$ MeV) we still find an η -value of around 3. Since λ_0 would be around 2 fm we expect the actual mean free path $\lambda = \eta\lambda_0$ to remain at least of the order of the nuclear radius in the domain considered. The figure is useful for gaining a quick impression of the blocking effect in a given situation. The calculated result suggests that the exclusion principle remains effective even at rather large temperatures and energies. The nucleons should therefore not be treated as classical particles.

II.3. Promptly emitted particles

The discussion in Part I is based on the tacit assumption that a transferred nucleon is quickly assimilated in its new host. However, and this is particularly true at higher bombarding energies, the equilibration process may be aborted before completion, leading to the prompt emission of one or several particles. Such prompt ejectiles may teach us about the non-equilibrium properties of nuclei, and in particular, carry information on the early collision stage. It is therefore important to study in more detail the further fate of the transferred nucleons.

II.3a. Fermi jets

The particle-hole excitation created by the transfer of a nucleon has a very special structure: it consists of a hole left behind in the down nucleus A, say, and a particle moving through the recipient nucleus B. The velocity of the intruder nucleon follows from the kinematics of the situation,

$$\vec{v}_b = \vec{v}_a + \vec{U}. \quad (5)$$

Here $\vec{v}_a$ is the velocity of the nucleon as seen From A and $\vec{v}_b$ its velocity as seen from B; $\vec{U}$ is the velocity of A relative to B.

At moderately low excitation the intruder nucleon will proceed relatively uninhibited by two-body collisions, due to the action of the exclusion principle. It is then the bouncing around of the nucleon in the one-body nuclear container which provides the mechanism for bringing the nucleon into equilibrium with the other nucleons. However, under special kinematical circumstances the nuclear potential, which has only a finite height, will not be able to reflect the nucleon which may thus escape. One may appreciate that such a process is indeed energetically possible by recalling that the kinetic energy of the intruder nucleon, as seen from the host nucleus, is boosted by an amount which is of the order of UP_F . Therefore, when the relative radial nuclear velocity is sufficiently large, a transferred nucleon may be transmitted right across the recipient nucleus and emerge on the opposite side as an energetic ejectile. Since this phenomenon is a consequence of the kinematical coupling of the relative nuclear motion and the intrinsic nucleonic Fermi motion it has been called a "Fermi jet", although only one or a few nucleons can be

ejected in a given collision. The Fermi jets form a subclass of Promptly Emitted Particles (so called PEPs) which refer to all light particles emitted at the early stage of a nuclear collision, whatever may be their production mechanism.

The Fermi-jet nucleons appear in a rather well-collimated angular region on the sides opposite to the interaction zone between the two nuclei. As time progresses, the dinuclear complex swings around and the relative motion is gradually degraded. The jets therefore appear in a narrow band in angle-energy space, a feature which should help in identifying the phenomenon experimentally. An extensive theoretical study of the Fermi jet mechanism has been carried out by Robel.¹²⁾ An independent investigation has been made by Bondorf et al.^{13,14)}

II.3b. Two-body collisions

Although fairly long, the nucleon mean free path can not be considered as infinite. In fact, in typical situations it is known to be of the same order as the nuclear size. This fact has an impact on the Fermi jets since the transmission of the transferred nucleons is thus obscured. One may attempt to take account of this effect by assuming that the degradation of the nucleonic motion is dominantly due to two-body collisions. At the end of its free path the nucleon then collides with another one from the host nucleus, resulting in the creation of two quasi-free nucleons. Each of these may now propagate onward and possibly escape when reaching the surface. Such secondary ejectiles have been called two-body PEPs as opposed to the one-body Fermi-jet PEPs discussed above.¹⁴⁾ The angular distribution of the two-body PEPs will be distinctly different

from that of the one-body PEPs: they are in general aimed more side-wards than the predominantly forward-backward directed Fermi jets. This characteristic is the combined effect of the geometrical features of the distribution of the nucleons, in coordinate and momentum space.

As an illustration we show in Fig. 2 the calculated distribution of promptly emitted neutrons from the collision of 152 MeV ^{12}C with ^{158}Gd .¹⁴⁾ One-body as well as two-body PEPs have been included; the total contribution of the two-body PEPs is around 30%. The bump around $\theta \approx 110^\circ$ appearing at $E_n = 11$ MeV results almost exclusively from the backward PEPs (i.e., those originating in the target and transmitted through the projectile, in the direction opposite to the beam direction). The theoretical calculation is based on a statistical (Monte Carlo) method in which the fate of many individual nucleons are followed while the dinucleus evolves along an average dynamical trajectory (determined from an equation of the form (4)).

As the energy is increased, the two-body collisions will grow increasingly dominant and the Fermi-jet nucleons will give way to nucleons which are emitted after one or several collisions. The relative importance of the one- and two-body PEPs is illustrated in Fig. 3 for the $\text{C} + \text{Gd}$ case.¹⁴⁾

II.3c. Hot spots?

When the nucleon mean free path λ is of the order of the nuclear radius R additional degrees of freedom are activated and the nuclear interior no longer remains homogeneous. Although the general expressions for the transfer of energy and momentum may still hold, it is now necessary

to also consider the dynamics of the subsequent deposition of these quantities.

When $\lambda \approx R$ there is a large probability that an intruder nucleus will suffer its first collision in the front part of the recipient nucleus. If the flux of intruders is sufficiently high in comparison with the rate at which the local excitation resulting from the two-body collision is being dispersed the subsequent intruders will encounter an extra hot zone upon entering. Since the local mean free path is then diminished, they are therefore more likely to suffer an early collision. We are thus dealing with a self-amplifying process by which the deposition of the intruders' energy in the front part of the host will produce an intransparent region: there will be a sudden phase transition where a relatively cold and transparent medium locally develops a hot and opaque zone. A hot spot has been formed.

Whether such a phenomenon will indeed occur is still an open question. After having indicated, the main line of argument in favor of it I would like to add a few words to the contrary: Although substantially reduced, the mean free path is still quite long, even at fairly high temperatures, if Fig. 1 can be trusted. Since one can hardly speak of a thermalized region with a size less than one mean free path the term "spot" appears somewhat misleading since it would occupy a sizable fraction of the nuclear volume. The size of possible temperature gradients occurring in the system is therefore severely limited and one ought to be very cautious about applying ordinary thermal-conduction theory.

II.4. Multifragmentation

When the beam energy per nucleon is of the same order as the Fermi kinetic energy the energy available for intrinsic excitation is comparable to the total nuclear binding energy. Hence, in principle, it would be energetically possible to totally disassemble the colliding nuclei into their nucleon constituents.

In such a situation the character of the nuclear system is profoundly altered. Rather than dealing with a dynamical situation dominated by two large nucleides, we are now faced with a competition between a large number of widely different fragmentations.

In the ordinary approach to nuclear dynamics the system considered is described in terms of a relatively small number of macroscopic degrees of freedom; in the exit channel these degrees of freedom describe a definite number of fragments, in various states of intrinsic excitation. Within such a framework we are now in the novel situation that the number of macroscopic variables to be dealt with is no longer an approximately conserved quantity. Quite to the contrary, macroscopic degrees of freedom can be created and destroyed in the course of the dynamical evolution as the system fluctuates back and forth between its different accessible fragmentations, ranging from a few hot or fast to many cold and slow fragments.

The theoretical methods presently in use for nuclear dynamics are inadequate for treating such a more complex phenomenon and there is a clear need for substantial formal development before a dynamical theory of such processes can be formulated.

Multifragmentation processes have already been observed experimentally. As an example Fig. 4 shows the result of 91 MeV/n C colliding with a Ag nucleus.¹⁵⁾ A large number of fragments are ejected. Efforts are presently underway to pursue the study of this type of collision in a forthcoming experiment at CERN. Recently it has become possible to study multifragmentation processes at lower energies by the novel streamer chamber developed in Berkeley, as B. G. Harvey will discuss. Again, large parts of the initial nuclei emerge as relatively light fragments. Such pictures give a spectacular impression of the violent and complicated collision processes, the dynamics of which we must now try to understand.

II.5. Concluding remarks

Numerous interesting phenomena may occur in the intermediate energy region and this contribution is not intended to give a complete and balanced view of the field. I have discussed some of the basic features characterizing the new physical situation we are faced with and illustrated with a few specific phenomena which one might expect to occur. We are only just embarking on our venture into this new field and I foresee an exciting period when we try to identify the new phenomena and attempt to understand them.

Finally, I would like to thank the organizers for giving me this unique opportunity to interact with the physicists from this region; it has been a thorough pleasure to participate and I feel that this school has contributed towards a strengthening of the contact between our laboratories.

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Fig. 1. The blocking factor $\eta = \lambda/\lambda_0$ defined by Eq. (4) as a function of the temperature τ , for various values of the kinetic energy T_0 of the intruder nucleon; T_F is the Fermi kinetic energy of the medium.

Fig. 2. The angular distribution of promptly emitted neutrons in the $152 \text{ MeV } ^{12}\text{C} + ^{158}\text{Gd}$ collision, for selected cm energies.⁵

Fig. 3. The energy dependence of the contribution from one and two-body PEPs in the C + Gd case for one selected impact parameter $b = 3.25$ fm.⁵

Fig. 4. The result of a 91 MeV/n ¹²C nucleus colliding with a AgBr emulsion (B. Jacobsson, priv. comm.).