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EQUILIBRIUM PLASMA CORONA SURFACES

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Equilibrium Plasma Corona Surfaces

The distribution of charge of one sign when the opposite charge density is given is determined. Poisson's equation is solved in plane geometry for a simple specified ion density. This automatically gives the inverse solution for a given electron density, by reversing the sign of the potential. Some solutions can approximate a microwave confined corona, for very over dense cases.

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I. Poisson's Equation

$$\frac{d^2\phi}{dz^2} = 4\pi e N_0 [f_e - f_i]$$

---- 1)

choose for convenience $\eta = \frac{e\phi}{KT}$, $\mathcal{L} = z/\lambda_D$

where $\lambda_D^{-2} = \left(\frac{4\pi N_0 e^2}{KT} \right)$ $N_0 =$ interior plasma particle density at $\mathcal{L} \rightarrow -\infty$

Then Poisson's equation becomes $\frac{d^2\eta}{d\mathcal{L}^2} = [f_e - f_i]$

The specified ion density is

$$\begin{cases} f_i = 1 & \mathcal{L} < 0 \\ f_i = 0 & \mathcal{L} > 0 \end{cases}$$

This choice corresponds to ignoring the ion motion due to their large inertia for times $\omega_{pe}^{-1} < \tau \leq \omega_{pi}^{-1}$ or for indefinite times with infinitely heavy ions. We seek solutions with $\eta < 0$ everywhere, so write $\eta \rightarrow -|\eta|$.

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II. Solution Regions

Region $-\infty < \varphi < 0$

$$\frac{d^2|\eta|}{d\varphi^2} = -[e^{-|\eta|} - 1] \quad \text{multiply by } d|\eta|d\varphi$$

and obtain a first integral

$$\frac{1}{2} (d|\eta|/d\varphi)^2 = e^{-|\eta|} + |\eta| + C_1$$

$C_1 = -1$ in order that $d|\eta|/d\varphi \rightarrow 0$ as $|\eta| \rightarrow 0$ (For convenience, drop the absolute value sign) a second integral is then

$$\int \frac{d\eta}{\sqrt{e^{-\eta} + \eta - 1}} = \sqrt{2}\varphi + C_2$$

Reprint the integral by

$$\begin{aligned} e^{-\eta} + \eta - 1 &\approx \left(1 - \eta + \frac{\eta^2}{2!} - \frac{\eta^3}{3!} + \frac{\eta^4}{4!} + \dots\right) + \eta - 1 \\ &\approx \eta^2 \left[\frac{1}{2} - \frac{\eta}{6} + \frac{\eta^2}{24}\right] \end{aligned}$$

The integral then approximates to:

$$\int \frac{d\eta}{\sqrt{e^{-\eta} + \eta - 1}} \approx \sqrt{24} \int \frac{d\eta}{\eta \sqrt{12 - 4\eta + \eta^2}}$$

$$= -\sqrt{2} \ln \left\{ \frac{\sqrt{12 - 4\eta + \eta^2} + 2\sqrt{3}}{\eta} - \frac{1}{\sqrt{3}} \right\}$$

and hence an implicit solution for $|\eta(\mathcal{L})| : (\eta \equiv |\eta|)$

$$\boxed{\mathcal{L} = \frac{-\sqrt{2} \ln \left\{ \frac{\sqrt{12 - 4\eta + \eta^2} + 2\sqrt{3}}{\eta} - \frac{1}{\sqrt{3}} \right\} - C_2}{\sqrt{2}}} \quad \text{---- 2)}$$

Region $0 < \mathcal{L} < \infty$:

Now 1) becomes $\frac{d^2|\eta|}{d\mathcal{L}^2} = -e^{-|\eta|}$

and a first integral $\frac{1}{2} \left(\frac{d|\eta|}{d\mathcal{L}} \right)^2 = e^{-|\eta|} + K_1$ and $K_1 = 0$ in order that $d|\eta|/d\mathcal{L} \rightarrow 0$ as $|\eta| \rightarrow \infty$ (Note that $|\eta| \rightarrow \infty$ is necessary with the given ion distribution for some $\mathcal{L} > 0$)

The second integral is simple and given by

$$\int \frac{d|\eta|}{e^{-|\eta|/2}} = 2e^{|\eta|/2} = \sqrt{2} \mathcal{L} + K_2$$

or

$$\boxed{|\eta| = \ln \left\{ \frac{2}{\sqrt{2}} \mathcal{L} + K_2 \right\}^2} \quad \text{---- 3)}$$

III. Determination of C_2 , K_2

The continuity condition between the regions $\mathcal{L} < 0$ ($\eta_<$) and $\mathcal{L} > 0$ ($\eta_>$) is:

$$\left\{ \begin{array}{l} \eta_< = \eta_> \\ \frac{d\eta_<}{d\mathcal{L}} = \frac{d\eta_>}{d\mathcal{L}} \end{array} \right\} \quad \mathcal{L} = 0$$

From the derivative condition, with $\eta(0) \equiv \eta_0$,

$$\left(\frac{d\eta_<}{d\mathcal{L}} \right)_{\mathcal{L}=0} = \sqrt{2} \cdot \sqrt{e^{-\eta_0} + \eta_0 - 1} = \left(\frac{d\eta_>}{d\mathcal{L}} \right)_{\mathcal{L}=0} = \frac{\sqrt{2}}{K_2}$$

Also from the first condition we require that $\eta_< \rightarrow \eta_0$ as $\mathcal{L} \rightarrow 0$ as well as $\eta_> \rightarrow \eta_0$ as $\mathcal{L} \rightarrow 0$. Hence from 3)

$$(\eta_>)_{\mathcal{L} \rightarrow 0} = \eta_0 = \ln K_2^2 \quad \text{or} \quad K_2 = e^{\eta_0/2}$$

using this in the expression above for the derivative results in the eigenvalue for η_0 ;

$$\frac{\sqrt{2}}{e^{\eta_0/2}} = \sqrt{2} \sqrt{e^{-\eta_0} + \eta_0 - 1} \cdot \boxed{\eta_0 = 1}$$

and $K_2 = \sqrt{e}$

Also from 2)

$$C_2 = -\sqrt{2} \ln \left(3 + 2\sqrt{3} - \frac{1}{\sqrt{3}} \right)$$

To summarize:

$$\boxed{\begin{aligned} \eta_0 &= 1 \\ K_2 &= \sqrt{e} \\ C_2 &= -\sqrt{2} \ln \left(3 + 2\sqrt{3} - \frac{1}{\sqrt{3}} \right) \end{aligned}}$$

IV. Summary of Solutions for η , Particle Density, Charge Density and Electric Field (See Figs. I and II)

$$-\infty < \varphi < 0 :$$

$$\varphi = \ln \left\{ \frac{\left(3 + 2\sqrt{3} - \frac{1}{\sqrt{3}} \right)}{\frac{\sqrt{12 - 4|\eta| + |\eta|^2 + 2\sqrt{3}}}{|\eta|} - \frac{1}{\sqrt{3}}} \right\}$$

$$N_e(\mathcal{L}) = e^{-|\eta|} ; \quad E = -\text{grad}|\eta| = \sqrt{2} \sqrt{e^{-|\eta|} + |\eta| - 1}$$

$$\rho_< = (N_i - N_e) = (1 - e^{-|\eta|})$$

numerical integration of this charge density shows that the total charge $Q_<$ from $-\infty$ to 0 is

$$Q_< = \int_{-\infty}^0 \rho_< d\mathcal{L} \approx \sqrt{\frac{2}{e}} \text{ to within } < 1\%$$

$0 < \mathcal{L} < \infty :$

$$|\eta| = \ln \left\{ \frac{\sqrt{2}}{2} \mathcal{L} + \sqrt{e} \right\}^2$$

$$N_e = e^{-|\eta|} = \frac{1}{\left[\frac{\sqrt{2}}{2} \mathcal{L} + \sqrt{e} \right]^2} ; \quad E = \frac{\sqrt{2}}{\left[\frac{\sqrt{2}}{2} \mathcal{L} + \sqrt{e} \right]}$$

$$\rho_> = -e^{-|\eta|} = -\frac{1}{\left[\frac{\sqrt{2}}{2} \mathcal{L} + \sqrt{e} \right]^2}$$

and the electron density can be integrated analytically to provide the total electron charge from 0 to $+\infty$:

$$Q_> = \int_0^{\infty} \rho_> d\mathcal{L} = -\sqrt{\frac{2}{e}}$$

which just cancels that from Q_c and provides a charge neutral solution

$$\int_{-\infty}^{+\infty} \rho d\mathcal{L} = 0$$

Discussion of Results

The accuracy of the potential arrived at by approximating $e^{-|\eta|}$ with a five term expansion in the integrand of 2) can be checked as follows: We know the exact eigenvalue, $|\eta|_0 = 1$. By using $|\eta|_c$ as given by the integrated expression, we can form $d|\eta|/d\mathcal{L}$ and ask for the predicted eigenvalue:

$$(\text{with } \eta = |\eta|)$$

$$\left(\frac{d\eta}{d\mathcal{L}}\right)_{\mathcal{L}=0} = \frac{\sqrt{2}}{e^{\eta/2}} \approx - \left\{ \frac{\eta(\eta - 2)}{\sqrt{12 - 4\eta + \eta^2}} - \left(\sqrt{12 - 4\eta + \eta^2} + 2\sqrt{3} \right) \right\} \\ \eta^2 \left\{ \frac{\sqrt{12 - 4\eta + \eta^2} + 2\sqrt{3}}{\eta} - \frac{1}{\sqrt{3}} \right\}$$

yields $.993 < |\eta|_0 < .998$ and consequently the potential is accurate to $< 1\%$. (See Fig. III for table of numerical values.)

The potential falls off much more rapidly going into the plasma, $\mathcal{L} < 0$, than for $\mathcal{L} > 0$. We note that only $\sim 46\%$ of the total electron charge from $0 \rightarrow +\infty$ is contained out to $\mathcal{L} = +2$. While $\sim 87\%$ of the net

charge from $0 \rightarrow -\infty$ is contained within $\mathcal{V}^5 - 2$.

The solutions obtained correspond to a fully ionized plasma with $z \approx 1$. Extension to other geometry and other z values is straightforward.

To obtain solutions for a stationary electron front, where some force field confines the electrons (for instance a magnetic field intensity or the momentum of reflecting light off the electrons, etc.) replace $|\eta| \rightarrow \eta$ and solutions with $\eta > 0$ everywhere result for the equilibrium ion distribution, only now $\omega_{pi}^{-1} < \tau$, where τ is the length of time the electron force field is on.

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FIG. I

FIG. II

η eta	$l = \frac{3}{\lambda_D}$ L	-12- E	q Q	Ne n
-1	0	.85776389	-.36767944	.36787944
-1.3885677	.5	.70630348	-.24943231	.24943231
-1.7137846	1	.60030424	-.18018258	.18018258
-1.9934407	1.5	.52196916	-.13623591	.13623591
-2.2387471	2	.46171846	-.10659197	.10659197
-2.4577264	2.5	.41393778	-.08562942	.08562942
-2.6541721	3	.37511882	-.07035706	.07035706
-2.8334507	3.5	.34295646	-.05880957	.05880957
-2.9979726	4	.31587373	-.04988811	.04988811
-3.1499835	4.5	.2927553	-.04285283	.04285283
-3.2912524	5	.27279011	-.03720722	.03720722
-3.4231977	5.5	.25537422	-.032608	.032608
-3.5469744	6	.24004866	-.02881168	.02881168
-3.6635352	6.5	.2264584	-.0256417	.0256417
-3.7736752	7	.21432451	-.0229675	.0229675
-3.8780651	7.5	.20342478	-.02069082	.02069082
-3.9772757	8	.19358003	-.01873661	.01873661
-4.0717965	8.5	.18464418	-.01704674	.01704674
-4.1620511	9	.1764969	-.01557558	.01557558
-4.2484081	9.5	.16903822	-.01428696	.01428696
-4.3811901	10	.16218438	-.01315189	.01315189
-1	0	.85776389	.63212056	.3678944
-.9	-.12095718	.78303213	.59343034	.40656966
-.8	-.25456856	.70615716	.55067104	.44932896
-.7	-.40412858	.62703318	.5034147	.4965853
-.6	-.57447781	.5455486	.45118836	.54881164
-.5	-.7731389	.46158566	.39340934	.60653066
-.4	-1.0127	.37502012	.32967995	.67032005
-.3	-1.3169579	.28572091	.25918178	.74081822
-.2	-1.7390206	.19354975	.18126925	.81873075
-.1	-2.4488178	.09836075	.09516258	.90483742
-.05	-3.1503369	.0495868	.04877057	.95122943
-.01	-4.7663998	.00998343	.00995017	.99084983
0	infinitely -	0	0	1

FIG. III