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**TITLE: SUPPRESSION OF ADJACENT-CHANNEL AND COCHANNEL
FM INTERFERENCE VIA EXTENDED KALMAN FILTERING**

AUTHOR(S): Jonathan N. Bradley

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SUPPRESSION OF ADJACENT-CHANNEL AND COCHANNEL FM INTERFERENCE VIA EXTENDED KALMAN FILTERING

by

Jonathan N. Bradley
Los Alamos National Laboratory
C-3, MS B-265
Los Alamos, New Mexico 87545
U.S.A.
Telephone: 505-665-2722
Electronic Mail: bradley@lanl.gov

ABSTRACT

A method is discussed for the problem of demodulating an FM signal in the presence of an adjacent-channel or cochannel interferer. State-space descriptions of the underlying message processes and the measurement process are presented which are then incorporated in an extended Kalman filter framework. The extended Kalman filter was necessary due to the nonlinearity of the measurement equation.

The results of computer simulations are presented which demonstrate the system performance for various selections of the modulation index and signal separation. The derived system is contrasted to the previous state of the art in this area which involved maximum likelihood estimation.

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1 Introduction

This work addresses the design of an FM receiver which is to suppress an unwanted adjacent-channel or cochannel interferer. The problem is stated as follows: the signal $r(t)$ is available which consists of the sum of two FM signals in the presence of additive noise

$$r(t) = \sqrt{2}a_1 \sin(\omega_{c1}t + d_1 \int_0^t m_1(\xi)d\xi) + \sqrt{2}a_2 \sin(\omega_{c2}t + d_2 \int_0^t m_2(\xi)d\xi) + n(t) \quad (1)$$

where it is desired to demodulate the message signal $m_1(t)$. The quantities a_1 and a_2 are assumed to be constant but unknown and the two carrier frequencies, ω_{c1} and ω_{c2} , are “close” or perhaps even equal. The typical approach to the problem is to design a system which jointly estimates all of the unknown FM parameters in (1).

In [5] a continuous-time maximum *a posteriori* estimator was derived for the simultaneous estimation of $m_1(t)$ and $m_2(t)$ in the case where a_1 and a_2 are assumed known. The resulting system was shown to consist of two phase-locked loops (PLLs): one PLL for tracking the desired signal and the other for tracking the unwanted signal. Empirical results were presented in [2] which revealed the regions of stability for the various loop parameters. In a later work [1], a method for estimating the unknown signal amplitudes was developed using adaptive filtering techniques. This amplitude estimator was combined with the maximum *a posteriori* estimator to yield an overall system for demodulating $m_1(t)$ and $m_2(t)$ in the more realistic case where a_1 and a_2 are unknown. The performance of this system is limited by assumptions used in the development of the amplitude estimator which lead to oscillatory behavior in the amplitude estimates and cause the system to become chaotic in the case where the two carrier frequencies are equal.

In this work the interference suppression problem is approached by sampling $r(t)$ and applying discrete-time extended Kalman filtering. The resulting system does not treat the phase and amplitude estimation problems separately—all of the unknown quantities are contained in the filter state vector and are sequentially updated in a joint fashion. The estimator structure is shown to consist of two PLLs cross-coupled as those in the maximum *a posteriori* estimator. However, the system contains amplitude estimators which are significantly different from those developed in [1]. The system can perform successfully in the case of equal carrier frequencies.

2 System and Measurement Models

The application of the extended Kalman filter requires a state-space description of the underlying physical process which generates the desired and interfering FM signals and a model by which the composite of these two signals is measured. In this section, continuous-time versions of these models are presented which are then used to derive the requisite discrete-time model. The development here parallels that in [4] which applied extended Kalman filtering to the problem of FM demodulation.

2.1 The Continuous-Time Model

The system model is represented by the first order matrix differential equation

$$\dot{\mathbf{x}}(t) = F\mathbf{x}(t) + G\mathbf{w}(t) \quad (2)$$

where $\mathbf{w}(t)$ is a stationary zero-mean white Gaussian process with covariance equal to the identity matrix. The scalar measurement model is

$$z(t) = h(\mathbf{x}(t), t) + \nu(t). \quad (3)$$

where $\nu(t)$ is zero-mean Gaussian with variance τ . The phases of the desired and interfering FM signals are, respectively

$$\theta_1(t) = d_1 \int_0^t m_1(\xi) d\xi \quad (4)$$

$$\theta_2(t) = d_2 \int_0^t m_2(\xi) d\xi. \quad (5)$$

Modeling the message signals $m_1(t)$ and $m_2(t)$ as first-order lowpass unit-energy random processes with respective half-power points α_1 and α_2

$$\dot{m}_1(t) + \alpha_1 m_1(t) = \sqrt{2} w_1(t) \quad (6)$$

$$\dot{m}_2(t) + \alpha_2 m_2(t) = \sqrt{2} w_2(t) \quad (7)$$

and defining the state vector $\mathbf{x}$ as

$$\mathbf{x} = \begin{bmatrix} \theta_1 & m_1 & \theta_2 & m_2 & a_1 & a_2 \end{bmatrix}^T \quad (8)$$

the matrices F and G are given by

$$F = \begin{bmatrix} 0 & d_1 & 0 & 0 & 0 & 0 \\ 0 & -\alpha_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_2 & 0 & 0 \\ 0 & 0 & 0 & -\alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

$$G = \text{diag} \left\{ 0 \quad \sqrt{2}\alpha_1 \quad 0 \quad \sqrt{2}\alpha_2 \quad 0 \quad 0 \right\}. \quad (10)$$

The measurement equation is simply

$$h(\mathbf{x}(t), t) = \sqrt{2}a_1 \sin(\omega_{c1}t + x_1) + \sqrt{2}a_2 \sin(\omega_{c2}t + x_2) \quad (11)$$

2.2 The Discrete-Time Model

Sampling the received signal $r(t)$ at times $t_k = k\Delta T$ defines an equivalent discrete-time model. The equivalent state-space system model is

$$\mathbf{x}_k = \Phi_k \mathbf{x}_{k-1} + \mathbf{w}_{k-1} \quad (12)$$

and the measurement equation is

$$z_k = h_k(\mathbf{x}_k) + \nu_k. \quad (13)$$

Here, Φ_k is given by

$$\Phi_k = \Phi(t_{k+1}, t_k) \quad (14)$$

where Φ is the state transition matrix

$$\Phi(t, t_0) = \Phi(t - t_0) = e^{F(t-t_0)}. \quad (15)$$

Φ_k reduces to

$$\Phi_k = \begin{bmatrix} 1 & \phi_{12} & 0 & 0 & 0 & 0 \\ 0 & \phi_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \phi_{34} & 0 & 0 \\ 0 & 0 & 0 & \phi_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

where

$$\phi_{12} = \frac{d_1}{\alpha_1} \left[1 - e^{-\alpha_1 \Delta T} \right] \quad (17)$$

$$\phi_{22} = e^{-\alpha_1 \Delta T} \quad (18)$$

$$\phi_{34} = \frac{d_2}{\alpha_2} \left[1 - e^{-\alpha_2 \Delta T} \right] \quad (19)$$

$$\phi_{44} = e^{-\alpha_2 \Delta T}. \quad (20)$$

The random vector $\mathbf{w}_k$ is given by the matrix superposition integral

$$\mathbf{w}_k = \int_{t_{k-1}}^{t_k} \Phi(t_k, \xi) G \mathbf{w}(\xi) d\xi \quad (21)$$

and has covariance

$$Q = \int_{t_{k-1}}^{t_k} \Phi(t_k, \xi) G G^T \Phi(t_k, \xi) d\xi \quad (22)$$

$$= \begin{bmatrix} q_{11} & q_{12} & 0 & 0 & 0 & 0 \\ q_{21} & q_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & q_{33} & q_{34} & 0 & 0 \\ 0 & 0 & q_{43} & q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (23)$$

where

$$q_{11} = \frac{d_1^2}{\alpha_1^2} [2\alpha_1 T + 4e^{-\alpha_1 T} - e^{-2\alpha_1 T} - 3] \quad (24)$$

$$q_{12} = q_{21} = \frac{d_1}{\alpha_1} [1 - e^{-\alpha_1 T}]^2 \quad (25)$$

$$q_{22} = 1 - e^{-2\alpha_1 T} \quad (26)$$

$$q_{33} = \frac{d_2^2}{\alpha_2^2} [2\alpha_2 T + 4e^{-\alpha_2 T} - e^{-2\alpha_2 T} - 3] \quad (27)$$

$$q_{34} = q_{43} = \frac{d_2}{\alpha_2} [1 - e^{-\alpha_2 T}]^2 \quad (28)$$

$$q_{44} = 1 - e^{-2\alpha_2 T}. \quad (29)$$

The discrete time measurement equation is obtained by sampling (11)

$$h_k(\mathbf{x}) = \sqrt{2}x_3 \sin(\omega_1 t_k + x_1) + \sqrt{2}x_6 \sin(\omega_2 t_k + x_4). \quad (30)$$

3 Estimator Derivation

In this section, the discrete-time system and measurement models derived in the previous section are incorporated in an extended Kalman filter. The pertinent filter equations are [3]

$$\hat{\mathbf{x}}_k(-) = \Phi_{k-1} \hat{\mathbf{x}}_{k-1}(+) \quad (31)$$

$$P_k(-) = \Phi_{k-1} P_{k-1}(+) \Phi_{k-1}^T + Q \quad (32)$$

$$\hat{\mathbf{x}}_k(+) = \hat{\mathbf{x}}_k(-) + K_k \left[z_k - h_k(\hat{\mathbf{x}}_k(-)) \right] \quad (33)$$

$$P_k(+) = \left[I - K_k H_k(\hat{\mathbf{x}}_k(-)) \right] P_k(-) \quad (34)$$

$$K_k = P_k(+) H_k^T(\hat{\mathbf{x}}_k(-)) r^{-1} \quad (35)$$

where

$$H_k(\mathbf{x}) = \frac{\partial h_k(\mathbf{x})}{\partial \mathbf{x}}. \quad (36)$$

The initial error covariance matrix is selected as

$$P_0(+) = \text{diag} \left\{ \sigma_{\theta_1}^2, \sigma_{m_1}^2, \sigma_{\theta_2}^2, \sigma_{m_2}^2, \sigma_{a_1}^2, \sigma_{a_2}^2 \right\} \quad (37)$$

Substituting (30) into (36) gives

$$H_k^T(\mathbf{x}) = \begin{bmatrix} \sqrt{2}x_5 \cos(\omega_{c1}t_k + x_1) \\ 0 \\ \sqrt{2}x_6 \cos(\omega_{c2}t_k + x_3) \\ 0 \\ \sqrt{2} \sin(\omega_{c1}t_k + x_1) \\ \sqrt{2} \sin(\omega_{c2}t_k + x_3) \end{bmatrix}. \quad (38)$$

The overall form of the estimator can be obtained by first substituting the state extrapolation (31) and Kalman gain (35) equations into the state update equation (33)

$$\hat{\mathbf{x}}_k(+) = \Phi_k \hat{\mathbf{x}}_{k-1}(+) + P_k(+) r^{-1} H_k^T(\hat{\mathbf{x}}_k(-)) [z_k - h_k(\hat{\mathbf{x}}_k(-))]. \quad (39)$$

Substituting (38) into (39), simplifying and ignoring double frequency terms yields

$$\dot{\theta}_{1k}(+) = \dot{\theta}_{1,k-1}(+) + \phi_{12} \dot{m}_{1,k-1}(+) + P_{k11}(+) r^{-1} u_{1k} \quad (40)$$

$$\dot{\theta}_{2k}(+) = \dot{\theta}_{2,k-1}(+) + \phi_{34} \dot{m}_{2,k-1}(+) + P_{k33}(+) r^{-1} u_{2k} \quad (41)$$

$$m_{1k}(+) = \phi_{22} m_{1,k-1}(+) + P_{k21}(+) r^{-1} u_{1k} \quad (42)$$

$$m_{2k}(+) = \phi_{44} m_{2,k-1}(+) + P_{k43}(+) r^{-1} u_{2k} \quad (43)$$

$$a_{1k}(+) = (1 - P_{k55} r^{-1}) \hat{a}_{1,k-1}(+) + P_{k55} r^{-1} v_{1k} \quad (44)$$

$$a_{2k}(+) = (1 - P_{k66} r^{-1}) \hat{a}_{2,k-1}(+) + P_{k66} r^{-1} v_{2k} \quad (45)$$

where

$$u_{1k} = \sqrt{2} \dot{\theta}_{1,k-1}(+) \cos(\omega_{c1}t_k + \theta_{1k}(-)) [z_k - \sqrt{2} \dot{a}_{2,k-1}(+) \sin(\omega_{c2}t_k + \theta_{2k}(-))] \quad (46)$$

$$u_{2k} = \sqrt{2} \dot{\theta}_{2,k-1}(+) \cos(\omega_{c2}t_k + \theta_{2k}(-)) [z_k - \sqrt{2} \dot{a}_{1,k-1}(+) \sin(\omega_{c1}t_k + \theta_{1k}(-))] \quad (47)$$

$$v_{1k} = \sqrt{2} \sin(\omega_{c1}t_k + \theta_{1k}(-)) [z_k - \sqrt{2} \dot{a}_{2,k-1}(+) \sin(\omega_{c2}t_k + \theta_{2k}(-))] \quad (48)$$

$$v_{2k} = \sqrt{2} \sin(\omega_{c2}t_k + \theta_{2k}(-)) [z_k - \sqrt{2} \dot{a}_{1,k-1}(+) \sin(\omega_{c1}t_k + \theta_{1k}(-))]. \quad (49)$$

Assuming that $\phi_{12} \approx 1$ and $\phi_{41} \approx 1$, this system of equations can be implemented with the block diagram shown in Figure 1 where the various gains are given by

$$G_{11k} = p_{k11}(+)r^{-1} \quad (50)$$

$$G_{12k} = p_{k21}(+)r^{-1} \quad (51)$$

$$G_{13k} = p_{k55}(+)r^{-1} \quad (52)$$

$$G_{14k} = 1/(1 - G_{13k}) \quad (53)$$

$$G_{21k} = p_{k44}(+)r^{-1} \quad (54)$$

$$G_{22k} = p_{k54}(+)r^{-1} \quad (55)$$

$$G_{23k} = p_{k66}(+)r^{-1} \quad (56)$$

$$G_{24k} = 1/(1 - G_{23k}). \quad (57)$$

The structure consists of two PLLs, each with an associated first order amplitude estimator. The gains of all of the system components are variable. The PLLs are cross-coupled such that the output of each is subtracted from the input of the other, thus allowing each PLL to track one of the two FM signals. This system bears a strong similarity to the continuous-time estimator derived in [1] which consists of two PLLs similarly cross-coupled but with a significantly different amplitude estimation scheme.

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Figure 1
Cross-Coupled Phase-Locked Loop