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RAYLEIGH-TAYLOR INSTABILITY AND RESULTING FAILURE MODES OF ABLATIVELY IMPILODED INERTIAL-FUSION TARGETS

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I. INTRODUCTION

The ablation process is thought to be useful for driving the spherical implosion of inertial confinement fusion (ICF) targets¹. By "ablation" is meant the process of applying pressure continuously in time to a surface by heating the surface. The pressure applied may be thought of as the reaction to acceleration of heated material away from the surface. In fact, an implosion is only "ablative" if the time scale required for establishment of the outward flow from the surface is short compared to the implosion time; i.e., if the ablative flow is quasistationary. If the spherical target pellet is a shell of initial thickness Δr_p and radius r_p , which is hollow or contains a much lower density fuel, then this shell, whose initial aspect ratio is defined as $A_p = r_p / \Delta r_p$, will accelerate inwardly. This acceleration causes the outside surface of the shell to experience instability of the Rayleigh-Taylor type, which is thought to be potentially troublesome for $A_p \gtrsim 5^2$. Subsequently, as the implosion of the shell is decelerated by compression of lower density fuel material inside, the inside surface becomes unstable³. This article presents a theory of these instabilities and potential modes of failure caused by them. The remaining sections discuss the following:

- II. Small Amplitude Growth of the Outside Surface Instability
- III. Modes of Failure Resulting from Nonlinear Development of the Inside Surface Instability
- IV. Conclusions and Conjecture

II. SMALL AMPLITUDE GROWTH OF THE OUTSIDE SURFACE INSTABILITY

That the outside surface instability could be troublesome when $A_p \approx 5$ was suggested by early linearized, i.e., small amplitude, analytic estimates and then shown more clearly by time dependent simulations. The latter included in the linearized treatment the complicating effects of convection and thermal conduction through a linearized treatment employing an expansion of perturbations of all fluid flow and thermodynamic variables in spherical harmonics, $Y_l^m(\theta)$, and were done "piggyback" with a spherically symmetric time dependent implosion simulation³⁻⁴, i.e., using the symmetric simulation as the zero order solution. These calculations showed that amplification of shell surface perturbations by factors of 10^6 or more for λ values of interest are plausible if $A_p \approx 20$.

More recently, use has been made of the quasistationary character of ablative flows to perform more extensive and systematic parameter studies, and to obtain scalings of growth rates and resulting unstable amplifications of outside surface perturbations. These numerical calculations employ zero order spherically symmetric ablative flows of the type described in References 5 and 6, with a non zero artificial gravity $g(=a)$, representing the instantaneous inward acceleration, a , of the shell. The approximate implosion time at this rate of inward acceleration is taken to be $\tau_I = (2r_p/g)^{1/2}$. The 2 in this expression for τ_I , which comes simply from $r_p = 1/2 g \tau_I^2$, is a bit arbitrary and might be replaced by a slightly-smaller number so as to view the inward accelerating phase of the implosion as being complete before the shell reaches the spherical origin. Such a change in τ_I could be made in the application of the results obtained below in a straightforward way that will become clear. The fraction of the shell "burned off" ablatively during implosion is approximately $B = \Delta M/M = 4\pi \rho_s r_s v_s \tau_I/M$, where quantities with the subscripts have values found at the sonic point in the quasistationary ablative flow^{5,6} and M is the total shell mass. If the dimensionless parameter $\tilde{g} \equiv g r_s / v_s^2$, then

$$B \equiv \frac{\Delta M}{M} = \left(\frac{2r_p}{r_s} \right)^{1/2} \frac{1}{M/4\pi r_s^2 \rho_s} \frac{1}{\tilde{g}} \quad (1)$$

As shown in References 5 and 6, and Figure 1, r_p/r_s is near 1. References 5 and 6 show that $\tilde{g}$ is a measure of the relative importance of spherical divergence and inward acceleration in determining properties of the ablative flow, particularly near the sonic point. In larger $\tilde{g}$ solutions acceleration is more important.

Figure 1 Profiles of the pellet obtained from the stationary flow model for $\bar{\rho}p = 50$ and $g = 10^{1/2}$.

Figure 1 shows profiles of $\tilde{\rho} \equiv \rho/\rho_s$, $\tilde{T} = T/T_s$, and $\tilde{V} \equiv V/V_s$, as a function of $\tilde{r} \equiv r/r_s$, obtained from the stationary flow model with the programmatically interesting parameters values $\tilde{g} = 10^{1/2}$ and $\rho_p/\rho_s = \tilde{\rho}_p = 50$. These and all other 0th and 1st order calculations presented here were done with a $\gamma = 5/3$ equation of state electron thermal conductivity of the form $\kappa = \kappa_0(Z)(T/T_0)^{5/2}$. The parameters $\tilde{g}$ and $\tilde{\rho}_p$, may be taken to be the two parameters of the family of stationary flow imploding shell solutions⁶.

Alternatively, the inflight aspect ratio, A , and the burnthrough parameter B , which are perhaps more physical than $\tilde{g}$ and $\tilde{\rho}_p$, may be taken to be the two parameters of the family of solutions. The mapping of one pair of these parameters into the other is seen from the contours of constant $\tilde{g}$ and $\tilde{\rho}_p$ versus A and B in Figure 5 or 6. Energy efficient implusions are obtained near $B = 1$, where the mass of the shell has been reduced from its initial value by a factor of about $1/e$. As will be discussed below, greater instability growth occurs when A is larger, while unperturbed shells with smaller A are believed to have less desirable implusion properties. From consideration of these limitations, the ranges of A and B for which solutions are shown in Figures 5 and 6 are those thought to be programmatically interesting. It may be important to note that shells with a given B , and with A in the interesting range between 10 and 10^2 , have qualitatively different properties than those with much larger values of A . The $\tilde{g}$ and $\tilde{\rho}_p$ values increase with increasing A . Consequently, theoretical and experimental studies of ablation-driven Taylor instability in plane geometry, i.e., with $A \rightarrow \infty$, may be misleading when applied to spherical systems.

By using these spherically symmetric stationary flow solutions as the zero order solutions, and by expanding all necessary angle, $\tilde{\Omega}$, and time dependent first order quantities in spherical harmonics, $Y_l^m(\tilde{\Omega})$, as in the "piggyback" method described above, a set of equations for first order, r and t dependent coefficient $V_{lr}(\rho, t)$, $\rho_l(r, t)$, etc., is obtained. The spatial forms that these coefficients take for the unstable modes and the growth rates of these modes have now been obtained by two very different methods with essentially identical results.

The first method integrates the time dependent equations for the first order quantities for a given l (the equations are degenerate with respect to m), starting with some arbitrary initial perturbation. When the mode growth comes sufficiently close to being purely exponential, and the spatial mode form becomes essentially stationary, the exponentiation rate, $\gamma(l)$ is taken to be that of the fastest growing mode and the spatial forms are taken to be those of that mode. Some results obtained with this method have been published before⁷.

More recently, a method has been developed which assumes exponential growth and solves for the growth rates, $\gamma(\ell)$, as eigenvalues, and the mode forms as eigenfunctions. Slower growing unstable modes, as well as the fastest, can be obtained. This method, which is more effective than the first method in dealing with large $\tilde{\rho}_p$ cases with their larger spatial gradients, is described in Reference 8. The agreement between the two methods has given confidence in the parameter study shown below. Figures 2a and 2b show the form of perturbed radial velocity and radial displacement, V_{r1} and ξ_r , for the fastest growing $\ell = 400$ mode of the 0th order case shown in Figure 1. Note the similarity of $\xi_r(\tilde{r})$ to the higher ℓ modes shown in Reference 3. A peak in ξ_r is seen to occur near $\tilde{r} = 0.92$ (the sonic point by definition is at $\tilde{r} = 1$) at approximately the point of largest ablation front gradients in Figure 1. To the left of this point, where thermal conduction and convection are not very important, $\xi_r(\tilde{r})$ has the decaying exponential form of the classical Taylor mode. To the right structure, including the second much smoother peak, is quite different from the classical mode form because of the effects of thermal conduction and convection.

Figure 3 shows the dimensionless growth rate, $\tilde{\gamma}(\ell) \equiv \gamma(\ell) (\tau_s/V_s)$ for $\tilde{\rho}_p = 50$ and various values of $\tilde{g}$. For each $\tilde{g}$ the corresponding in flight values of A and values of $\gamma_{\max} \tau_1$ are given. The value of $\tilde{\gamma}$ for the $\ell = 400$ mode shown in Figure 2 is seen to lie near the maximum of the $\tilde{\gamma}(\ell)$ curve for $\tilde{g} = 10^{1/2}$.

The classical growth rates $\tilde{\gamma}_c$ for $g = 1$ and Atwood number of one, which scale as $\tilde{\gamma}_c (\tilde{g}^2)^{1/2}$, are shown for comparison. An Atwood number of one is thought to be the best classical approximation to the ablative flows of interest because the ratio of the shell material density to the much lower density of the hot ablated material, whose pressure may be thought of as causing the inward acceleration of the shell, is of the order of 10^2 . The classical lines for other values of $\tilde{g}$ are displaced vertically from the $\tilde{g} = 1$ classical line by the ratios of $(\tilde{g})^{1/2}$ values because of the classical scaling. Figure 4, like Figure 3, shows $\tilde{\gamma}(\ell)$ for different $\tilde{g}$'s but for $\tilde{\rho}_p = 10^2$. As can be seen from Figure 5 the $\tilde{g}$ and $\tilde{\rho}_p$ parameters of Figures 3 and 4 lie in the programmatically interesting region.

The steady flow model of imploding shells gives solutions which are a good approximation to the shell center and ablation front parts of time dependent simulations⁹ but do not represent the inside surface of an imploding shell very well. Consequently the $\tilde{\gamma}(\ell)$ curves in Figures 3 and 4 are terminated at ℓ values below which the penetration of the mode structure through to the inside surface of the shell is large enough to make the results questionable.

Figure 2 Perturbed quantities for the case in Fig. 1 a) the perturbed radial velocity b) the perturbed radial displacement in arbitrary units.

Figure 3 The dimensionless growth rate, $\tilde{\gamma}$, as a function of $\tilde{l}$ for $\beta_p = 50$ and various values of $\tilde{g}$.

Figure 4 Similar results as in Fig. 3 but for $\beta_p = 10^2$.

The question is often asked "by what factor do ablative effects reduce growth rates from their classical value?" From the ($\tilde{g} = 1$) and (classical $\tilde{g} = 1$) curves in Figure 4 it can be seen that the question is ambiguous. At the peak ablative rate the reduction factor is seen to be about 2. Because this, 2 is essentially a factor in an exponent of an exponential function which determines perturbation amplification during implosion it can be important. What may be more important though is that ablative effects reduce $\tilde{\gamma}(\lambda)$ to zero above some λ , instead of just holding $\tilde{\gamma}$ below some constant value, as a finite density gradient alone in the unstable region would do. Consequently sufficiently high λ structure is completely suppressed.

Also note that the amplification during implosion of a perturbation, say of radial displacement, ξ_r , is approximately given by $\xi_{rI}(\lambda) = \xi_{r0}(\lambda) e^{\gamma(\lambda)\tau_I}$ and that when the form and magnitude of $\gamma(\lambda)$ τ_I given in Figures 3 and 4 are substituted, $\xi_{rI}(\lambda)$ has a sharp peak at $\lambda_{\max}$, the λ value at which $\tilde{\gamma}_{\max}$ occurs. Consequently, particularly in cases with larger $\tilde{\gamma}_{\max}$ τ_I , it can be expected that the spectral amplitude of perturbations near $\lambda_{\max}$ will be most amplified and that these modes will dominate, at least as long as linearized theory is applicable.

Figures 5 and 6 show the results of a parameter study over the three dimensional space of $\tilde{g}$, $\tilde{\rho}_p$, and λ . Consider the case $\tilde{g} = 1$, $\tilde{\rho}_p = 10^2$. The $\gamma_{\max}$ τ_I value of 8.08, at $\lambda_{\max} = 150$ (Figure 6), gives an amplification of 3×10^3 which would seem to be at or perhaps a little over the limit of acceptability. If the inflight $A \approx 50$ is 4 times the initial A , a common but not infallible rule of thumb, then initial aspect ratio's of 10 are a little too large to be safe. Of course, more detailed, fully nonlinear, and case specific simulations are necessary to reach more definite conclusions. However, even after allowing for small adjustments to τ_I values and ratio of initial to inflight A , and varying B values, it seems clear from Figures 5 and 6 that shells with initial A_p 's of 5 or less are not likely to have outside surface instability difficulty, and those with initial A over about 25 are likely to fail. Figure 6 indicates that the dangerous outside surface modes should be expected to have λ values of order 10^2 or a little larger.

Figure 5 The product of $\gamma_{\max}$ ξ_{imp} , a measure of the amplification, as a function of either A and B or $\bar{\rho}_p$ and $\bar{q}$.

Figure 6 The l number at which maximum growth occurs, $l_{\max}$, as a function of A and B .

II. MODES OF FAILURE RESULTING FROM NON-LINEAR DEVELOPMENT OF THE INSIDE SURFACE INSTABILITY

As shown above in discussion of small amplitude growth of angle development perturbations which have grown unstably on the outside of a shell during inward acceleration, these outside distortions can be expected to penetrate through to the inside surface and to add to whatever fabrication imperfections are already there. Some pellets of interest may consist of multiple concentric shells, which makes more complex the process of transfer and penetration of perturbations from outside to inside, but in any case it is the degree of distortion of the inside of the innermost shell that determines the failure or success of an implosion.

All other things being equal, outside modes with lower spherical harmonic l number can be expected to cause larger inside perturbations because the exponentiation lengths in r of Taylor mode penetration are approximately r/l .

The innermost shell typically contains material which is of much lower density than the shell itself. As the implosion approaches maximum compression, the pressure of this low density material decelerate the colder and more dense shell and causes the inside shell surface to be Taylor unstable. The small amplitude growth of perturbations on this inside surface during deceleration is described in Ref. 3 in the context of cases in which the shell has a much higher density than the material inside, i.e., Atwood number of 1, and the heat flow can be neglected. Exponential growth of small perturbations proceeds about as expected. What is not so clear is just how and when the subsequent large amplitude distortion of the inner surfaces might cause failure of a implosion.

An effort has been made to gain a qualitative understanding of failure modes with idealized calculations of large amplitude development in which, again, the low density material inside the innermost shell has a relatively negligible density but a $\gamma = 5/3$ pressure law, and, as in Ref. 3, heat flow is initially neglected. For initialization, the self similar spherically symmetric flow solution,

$$\begin{aligned}
 v_r &= v_1(t) \frac{r}{r_1}(t) \\
 \rho &= \rho_1(t) f\left(\frac{r}{r_1}\right), \quad f = [1 - C\left(\frac{r^2}{r_1^2} - 1\right)]^{1/(\gamma-1)} \quad (2) \\
 r &\geq r_1
 \end{aligned}$$

is used, where C is a dimensionless parameter of the family of solutions, which determines inflight aspect ratio, A . Here ρ_1, r_1^3 and V_1 and r_1

$$\frac{d}{dt} \left(\frac{V_1}{r_1} \right) + \left(\frac{V_1}{r_1} \right)^2 = \frac{C_{so}^2 r_0^3 C}{r_1^3}, \text{ and } \frac{dr_1}{dt} = V_1 \quad (3)$$

C_{so} and r_0 are the inside shell surface sound speed and radius at initial time $t = t_0$, and $V_1 C_{so}$, as well as C , are initialization parameters. In keeping with the subsonic character of Taylor modes, this system is then given an incompressible initial displacement perturbation of the form $\xi = \xi_0 [r_0^{-2} p_c(0)]$, i.e., 0 is in the r, z plane and $m = 0$, and is then allowed to proceed. Figure 7A shows such an initialized system with parameters $C = 0.8$, which gives an inflight shell aspect ratio $A = r_0 / \Delta r_0 = 6$, and $\Delta r_0 = 16$, which is a value in the range thought likely to be most provoked by outside instability on the inside of a shell which has this A value after some convergence has occurred. The dimensionless initial inward velocity was chosen to be $V_1 / C_{so} = -4$. The sign and magnitude of ξ_0 are such as to give a spike instead of a bubble on the cylindrical axis and to be in the interesting range of magnitude which produces a disruption of fuel compression and heating that is quite noticeable but, as will be seen, does not necessarily cause failure. Figure 8 shows the time histories of the radii of, from bottom to top, 1) the tip of innermost spike, which is the one on the axis; 2) the mean position of the inside surface 3) the center of the deepest bubble protruding back into the surface; and 4) the mean position of the next to outermost zone boundary. It may be interesting to note that at late times the spike tip does not simply go into "free fall", but instead accelerates inward slightly. Figure 7B shows the grid at $t = 0.16$ (arbitrary time and length units). Note the length scale change from Figure 7A and 7B. Figure 9A shows the time history of the temperature of the inside fuel region, which is obtained from this inside fuel volume and the γ law used to estimate fuel pressure as well as temperature. Figure 9B shows the history of the area of the inside surface of the shell. The dashed curves in Figure 9 are from the perturbed case shown in Figure 7 and the solid curves are from the otherwise identical case run with no perturbation, for comparison.

It is seen from the time history of the temperature in the perturbed case that the temperature inside the shell, and therefore the compression of the inside volume, is greatest at approximately $t = 0.15$, slightly before the time at which the grid has the configuration shown in Figure 7B. The peak temperature in the perturbed case is seen to come a little earlier than, and reach a maximum about 10% lower than, in the

Figure 7A The two-dimensional grid showing the initialization of the perturbation on the inside surface.

Figure 7B The same grid but at a later time showing the development of the instability.

Figure 8 The time histories of various shell radii (see text for further explanation).

Figure 9A The fuel temperature for the unperturbed (solid line) and for the perturbed case (dashed line) as a function of time.

unperturbed case. The perturbed surface area, however, is larger than the unperturbed area by about a factor of 3 at peak compression.

Here we will define failure as a failure of the fuel to reach ignition conditions, without regard for the effects of shell distortion on any subsequent burning. Three contributions to such failures from inside surface instability are identified from the example presented here:

- (1) Reduction of the heating by PdV work, ΔT_{pv} , which in this example, (Figure 9A) is about 10%.
- (2) Increase of heat loss into the shell from the fuel by thermal conduction, ΔE_c , because of the increased inner surface area of shell of particles, which would otherwise contribute to further heating and ignition, also because of the increase of the inner surface area of the shell.

These mechanisms, including requirements on PdV work and thermal conduction losses, have been discussed from a different point of view in a paper by Widner¹⁰ on the requirements for reaching fuel ignition conditions, a paper which the reader may find to be a useful complement to the work presented here.

There is little more to be added about mechanism (1), loss of PdV work, beyond what is seen in Figure 9B, except to note that distortions of the magnitude seen in Figure 7B can be expected to reduce the peak temperature through this mechanism by of the order of 10%. To the authors it seemed that more reduction might have been expected and that this was on a balance perhaps a positive result.

To estimate contribution (2), suppose first that the implosion is spherically symmetric, and that instead of the purely isentropic conditions of the simulations presented, there is a bit of thermal conduction from the fuel into the shell. Suppose also that, as is likely, the conductivity of the shell is low enough that conduction into it can be treated as subsonic and therefore in pressure balance, and that the fuel conductivity is enough greater than that of the shell that the fuel may be treated as isothermal. Then, as fuel heat penetrates the shell, spatial profiles of density and temperature normal to the inner surface are as shown in Figure 10.

Figure 9B Time histories of the inside surface area for the case of no initial perturbation and for the perturbed case.

Figure 10 Spatial profiles of density and temperature as thermo conduction penetrates the shell subsonically.

To estimate this heat loss, we assume a thermal conductivity in the shell material of the form $\kappa = \kappa_0(z)(T/T_0)^\alpha (\rho_0/\rho)^\beta$, using the notation of Reference 5. Then the conduction equation is

$$\rho C_V \left[\left(\frac{\partial T}{\partial t} \right)_x + V_x \left(\frac{\partial T}{\partial x} \right)_t \right] = \left[\frac{\partial}{\partial x} \left(\kappa \frac{\partial T}{\partial x} \right) \right]_t, \quad (5)$$

where the partial derivative subscripts are the variable held constant, x is the coordinate normal to the shell surface, and the depth of heat penetration is assumed to be sufficiently small that shell curvature can be neglected. A Lagrangian transformation of spatial variable of the form $y = y(x, t)$, $y(x, t=0) = x$ is useful. The effect of stretching the surface area in the directions parallel to the surface by large amplitude Taylor mode growth is included in this transformation by the mass conservation relation, $A_0 \rho_0 dy = A \rho dx$ where ρ_0 and A_0 are the initial density and cross-sectional area of a small mass element, and $A(t)$ is taken to be independent of x or y . Under this transformation and the assumption of constant pressure, $R \rho_0 T_0 = R \rho T$, the conduction equation becomes

$$\frac{\rho_0 C_V T_0}{T} \left(\frac{dT}{dt} \right)_y = \left[\frac{A(t)}{A_0} \right]^2 \frac{T_0 \kappa_0(z)}{T} \left\{ \frac{\partial}{\partial y} \left[\left(\frac{T}{T_0} \right)^{\alpha + \beta - 1} \frac{\partial T}{\partial y} \right] \right\}.$$

Simplifying and defining $\Gamma \equiv \alpha + \beta - 1$ gives

$$\rho_0 C_V \frac{\partial T(y, t)}{\partial t} = \left(\frac{A}{A_0} \right)^2 \kappa_0(z) \frac{\partial}{\partial y} \left[\left(\frac{T}{T_0} \right)^\Gamma \frac{\partial T}{\partial y} \right]. \quad (6)$$

For the sake of obtaining an estimate, we will make the assumption that $A(t)/A_0$ is independent of position on the shell surface or is an average over the surface. Then A/A_0 is the ratio of total perturbed to unperturbed surface areas which we have from Figure 9B. The Equation (6) becomes

$$\rho_0 C_V \frac{\partial T}{\partial \tau} = \kappa_0(z) \frac{\partial}{\partial y} \left[\left(\frac{T}{T_0} \right)^\Gamma \frac{\partial T}{\partial y} \right] \quad (7A)$$

where

$$d\tau \equiv \left[\frac{A(t)}{A_0} \right]^2 dt \quad (7B)$$

From Ref. 10, the solution of Equation (7) with a fixed fuel...

temperature at $y = 0$, which we take to be the fuel temperature at maximum compression, T_{fM} , has the typical nonlinear thermal wave appearance with the wave front position, Y_f , satisfying the proportionality

$$Y_f \sim [\kappa_0(z) \left(\frac{T}{T_0}\right)^{\Gamma} \tau]^{1/2}, \quad (8)$$

and the constant of proportionality depending on Γ . Then, if C_v can be taken to be independent of ρ and T , the energy absorbed by conduction, ΔE_c , satisfies

$$\Delta E_c \sim \rho_0 C_v A Y_{fM} T_{fM} \quad (9)$$

where Y_{fM} is the value at maximum compression.

Short of including thermal conduction in these numerical simulations, an estimate of the effect of increased surface area on energy loss can be obtained with approximate analytic expressions for time histories of A and T_f , the surface area and fuel temperatures, obtained with some guidance from Figures 9A and 9B. We take

$$A \approx A_H [1 + \delta_0(t) e^{\gamma_A(t)(t-t_0)}] \quad (10)$$

$$T_f \approx T_{fM} \left[1 - \left(\frac{t-t_M}{t_T} \right)^2 \right] \quad (11)$$

A term quadratic in t could be included in (10) to represent the time dependence of the unperturbed A , as is done in Equation (11) for T_f , but the view taken is that the exponential term dominates when it is present. Significantly better estimates would need to be obtained from simulations including conduction. That the second term in Equation (10) for A is exponential can be learned from plotting the difference between unperturbed and perturbed curves of A in Figure 9B and observing that this difference is very nearly exponential from about $t = 0.10$ until about $t = 0.15$, and with slope $\gamma_A \approx 40$. This could have been guessed from the exponentially growing character of the instability, except that to first order in the surface perturbations, ξ_r , the increase of surface area is zero, and only appears in second order. Consequently, the initial spectral density, $\delta_0(t)$, is of order $\xi_{ro}^2(t)$ and $\gamma_A(t) = 2 \gamma_{\xi}(t)$, where γ_{ξ} is the first order

perturbation growth rate. The characteristic time t_T in Equation (11) could be estimated from Figure 9A or estimated from Equation (2) near maximum compression to be

$$t_T \sim \frac{\tau_H}{C_{SM}} \frac{(2/C)^{1/2}}{3(\gamma-1)} \gg_{\gamma=5/3} \frac{\tau_H}{C_{SM}} (2C)^{1/2} \quad (12)$$

By either method, $t_T \approx .015$ in our example. To estimate conduction we must then obtain the effective conduction time, τ , from Equations (7B) and (10). These give

$$\tau = \left[t + \frac{2\delta}{\gamma_A} e^{\frac{2\gamma_A(t-t_0)}{\delta}} + \frac{\delta^2}{2\gamma_A} e^{\frac{2\gamma_A(t-t_0)}{\delta}} \right] t_H \quad (13)$$

In the absence of a more exact treatment of the time dependence of the surface area, the linear and exponential terms must be treated differently, i.e., with a different t . We choose a bit arbitrarily, to limit the time duration of the t term in Equation (13) to the time before maximum compression when $TF \leq 1/2 T_M$, when the thermal conduction is near maximum. This is a better approximation for larger Γ . From Equations (11) and (12), this condition is satisfied when

$$(t_M - t_1) \approx \Delta t_T = \frac{t_T}{2\Gamma} = \frac{t_T}{2\Gamma} = \frac{1}{2\Gamma(2C)^{1/2}} \cdot \frac{\tau_H}{C_{SM}} \quad (14)$$

In cases of interest, near failure conditions, $(t_m - t_0) \gamma_A \gg 1$, and the exponential terms in Equation (13) contribute significantly only near $t = t_M$ when $TF = T_M$, and for them we can take $t_A = t_0$. With these two approximations, Equation (13) gives

$$\tau = \frac{t_T}{2\Gamma} + \frac{2\delta}{\gamma_A} [e^{\frac{2\gamma_A(t_m - t_0)}{\delta}} - 1] + \frac{\delta^2}{2\gamma_A} [e^{\frac{2\gamma_A(t_m - t_0)}{\delta}} - 1]. \quad (15)$$

Returning to our example with $t_T \approx 1, 5 \times 10^{-2}$, $\gamma_A \approx 40$, $(t_m - t_0) \approx 0.16$, Spitzer electron conductivity values, $\alpha = 5/2$, $\beta = 0$, $\Gamma \approx \alpha - 1 = 3/2$, gives values for the corresponding terms in Equation (15) of

$$\tau \approx 5 \times 10^{-3} + 1.5 \times 10^{-1} + 1.2 \times 10^{-1}$$

From Equations (7) and (8), the ratio of perturbed to unperturbed conduction is then obtained

$$\frac{\Delta E_c}{\Delta E_{co}} = \left[\frac{\tau}{\tau(\delta=0)} \right]^{1/2} \approx 7.3 \quad (16)$$

This example contains various scaling relations for fuel energy loss by conduction, and shows that the loss into a perturbed shell can be considerably greater than that estimated from shell parameters and dynamics under the assumption of spherical symmetry. Suppose, in this example, that the 10% reduction of T_M caused by the reduction of PdV work had been anticipated and the system parameters had been anticipated and the system parameters had been chosen so that the reduction of T_M caused by conduction, and estimated to be ΔE_{co} , was smaller, but not much smaller than this 10%. Then from Equation (16), the total reduction of T_M would have been much greater than 10% and the fuel would have failed to ignite unless the system had been greatly oversized.

Also, it should be noted that the surface area increases in the example shown here could be thought of as relatively modest. In larger aspect ratio shells in which modes with larger k are expected to be important, γ_A values will be larger and the exponential terms in Equation (15), particularly the $e^{2\gamma_A(t_m - t_0)}$ term, can be expected to be relatively much larger than the $t_T/2$ term from which ΔE_{co} is obtained.

III. CONCLUSIONS AND CONJECTURE

It appears clear that pellets with initial pellet aspect ratio, $A_p > 5$ may have difficulty with Rayleigh-Taylor instability and that shells with $A_p \geq 10$ will probably demand rather stringent smoothness specification in order not to experience failure in the final implosion. These conclusions are based on the observation that the linear amplification of the outside surface instability can easily exceed 10^3 for A_p and resulting A values in the range of programmatic interest. Amplifications of this order, starting from attainable surface finishes, can then penetrate to the inside shell surface, producing perturbations there which approach the nonlinear development amplitude and at the start of the final deceleration and are of the order of that used in the example shown in section II above. In section II it is seen that such inside surface perturbations can be amplified to large amplitude by the inside instability and cause failure through reduction of the maximum fuel temperature achieved. Some insight into the scaling of failure mechanisms has been obtained.

Case specific linearized outside surface calculations of the "piggy back" type are expected to be needed for aspect ratios of order 10 to make more quantitative estimates of the spectral inside surface perturbation amplitude at the time the shell deceleration and inside surface instability begins. To treat

shells of larger aspect, nonlinear calculations (of the type discussed in Ref. 12) of the outside surface, in and continued through to nonlinear treatment of the final fuel compression and heating.

It is our expectation that the strong improvement of pellet performance with increasing aspect ratio that is predicted from spherically symmetric simulations in competition with the increase in sensitivity to instability and resulting failure with increasing aspect ratio will lead to:

- (1) calculating development of Taylor instability carefully to determine more accurately the failure boundary in the space of pellet parameters, and
- (2) a corrected effort to diagnose experiments in such a way as to make possible quantitative comparison of instability calculations with these experiments, and
- (3) use of fusion targets as close as possible to the failure boundary.

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