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Numerics for the Simulation of Electromagnetic-Field Propagation in Nonionized and Plasma Media

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NUMERICS FOR THE SIMULATION OF ELECTROMAGNETIC-FIELD PROPAGATION IN NONIONIZED AND PLASMA MEDIA

by

Joseph E. Borovsky

ABSTRACT

The numerical techniques used to simulate the propagation of low-frequency electromagnetic disturbances in Ohmic media and in collisional and collisionless plasmas are discussed. The scheme is based upon a multimedia propagation equation that is derived from Maxwell's equations.

I. INTRODUCTION

In order to follow the propagation of ultra-low-frequency electromagnetic disturbances produced by high-altitude nuclear explosions, a two-dimensional computer code has been developed and is described in this report.

The electromagnetic disturbances of interest are thought to be generated by the expansion of nuclear-explosion debris into the magnetized plasma of the magnetosphere or ionosphere. Some fraction of the electromagnetic disturbance propagates down through the ionosphere producing an electromagnetic signal on the ground, denoted as an mhd-emp (magnetohydrodynamic-electromagnetic pulse). Even though the signal at the ground may be of small amplitude, it is coherent over a very large horizontal distance and therefore can produce large electrical potentials along communication lines and within power networks that lead to the threat of system failures. The purpose of building the numerical

code described in this report is to obtain reflection, transmission, and attenuation coefficients for such electromagnetic disturbances propagating down onto the ionosphere. From the transmission coefficient, the amplitude of the signal on the ground can be predicted from a knowledge of the amplitude of a weapon-generated signal at high altitudes.

Because the signal of interest originates in the collisionless magnetospheric plasma and propagates through the collisional ionospheric plasma and through the atmosphere into the ground or ocean (see Fig. 1), standard mhd codes will not suffice. Rather, in the code described herein, the propagation

Fig. 1. The geometry of the two-dimensional simulation system (with the medium labeled according to the terrestrial magnetosphere-ionosphere-atmosphere-ground system).

of low-frequency electromagnetic signals is followed via Maxwell's equations, with constitutive relations used to describe the currents and charges within any media that are present (as in Ref. 1). For nonionized media, the constitutive relations are obtained from a simple Ohm's law. For ionized media, these constitutive relations are constructed from the collisionally averaged orbit equations that describe the response of the plasma particles to the electromagnetic fields of the propagating disturbance. By combining Maxwell's equations with the constitutive relations, a single wave-like equation describing propagation throughout the entire system is obtained for each problem of interest. This approach eliminates the inaccuracy of matching solutions across artificially constructed transition regions between differing types of media. This single partial differential equation is then numerically solved implicitly by finite differencing the temporal and spatial derivatives. Adjustable relaxation coefficients are introduced into the spatial and temporal differencing. These coefficients can be chosen to maximize the accuracy of the code or to increase the computational speed of the code by making it unconditionally stable.

For the mhd-emp problem, the major restrictions on the numerical simulations are that the properties of the media are only functions of the altitude and that the electromagnetic fields of the disturbance are of small amplitude.

II. FORMULATION OF THE PROPAGATION

To follow the propagation of low-frequency electromagnetic disturbances valid in the magnetosphere, ionosphere, atmosphere, and ground, Maxwell's equations are used:

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t} \quad (1a)$$

$$\nabla \times \vec{E} = - \frac{1}{c} \frac{\partial \vec{B}}{\partial t} \quad (1b)$$

$$\nabla \cdot \vec{B} = 0 \quad (1c)$$

$$\nabla \cdot \vec{E} = 4\pi\rho_q \quad . \quad (1d)$$

To generate a set of equations for the fields $\vec{E}$ and $\vec{B}$ only, the source terms $\vec{J}$ and ρ_q must be expressed in terms of the fields.

For low-frequency fields in a magnetized plasma, the current density $\vec{J}$ and charge density ρ_q may be written in terms of the electric field as

$$\vec{J} = \sigma_{ped} \vec{E}_\perp + \sigma_{\parallel} \vec{E}_\parallel + \sigma_{pol} \frac{\partial \vec{E}_\perp}{\partial t} + \sigma_H \hat{B}_0 \times \vec{E} + \sigma_{cpol} \hat{B}_0 \times \frac{\partial \vec{E}}{\partial t} \quad , \quad (2)$$

and

$$\frac{\partial \rho_q}{\partial t} = -\nabla \cdot \vec{J} \quad , \quad (3)$$

where $\perp$ and $\parallel$ refer to directions with respect to the ambient magnetic-field direction $\hat{B}_0$. The various conductivities σ for low-frequency phenomena in magnetized, collisional plasmas are derived from mean-free-path theory as follows. The current density $\vec{J}$ is given by the difference between the time-averaged drift velocity of a typical ion and a typical electron,

$$\vec{J} = n_0 e (\langle \vec{v}_i \rangle - \langle \vec{v}_e \rangle) \quad , \quad (4)$$

where $\langle \rangle$ denotes averaging over time. Between collisions, the orbits of particles of mass m and charge q within a plasma with magnetic induction $\vec{B}_0$ are determined by the equation of motion

$$\frac{\partial \vec{v}}{\partial t} = \frac{q}{m} \vec{E} + \frac{q}{mc} \vec{v} \times \vec{B}_0 \quad , \quad (5)$$

where the Lorentz force arising from the perturbation magnetic induction $\vec{B}$ is

ignored. Taking $\vec{B}_0 = (0,0,B_0)$, to first order in $\frac{1}{\omega_c} \frac{\partial}{\partial t}$, Eq. (5) leads to the orbit equations

$$\begin{aligned}
 v_x = & v_{x0} \cos(Q\omega_c t) + v_{y0} \sin(Q\omega_c t) \\
 & + \left[\frac{c}{B_0} E_y + \frac{mc^2}{qB_0^2} \frac{\partial E_x}{\partial t} \right] [1 - \cos(Q\omega_c t)] \\
 & + \left[\frac{c}{B_0} E_x - \frac{mc^2}{qB_0^2} \frac{\partial E_y}{\partial t} \right] \sin(Q\omega_c t)
 \end{aligned} \tag{6a}$$

$$\begin{aligned}
 v_y = & v_{y0} \cos(Q\omega_c t) - v_{x0} \sin(Q\omega_c t) \\
 & + \left[-\frac{c}{B_0} E_x + \frac{mc^2}{qB_0^2} \frac{\partial E_y}{\partial t} \right] [1 - \cos(Q\omega_c t)] \\
 & + \left[\frac{c}{B_0} E_y + \frac{mc^2}{qB_0^2} \frac{\partial E_x}{\partial t} \right] \sin(Q\omega_c t)
 \end{aligned} \tag{6b}$$

$$v_z = \frac{q}{m} E_z t + v_{z0}, \tag{6c}$$

where $Q = (q)/(|q|)$. Perpendicular to $\vec{B}_0$, the orbit Eqs. (6a)-(6c) describe gyromotion, $\vec{E} \times \vec{B}_0$ drifting, and polarization drifting, and parallel to $\vec{B}_0$, they describe accelerated motion. Following Townsend,^{2,3} a momentum-transfer collision frequency ν is defined and it is assumed that momentum-transferring collisions are distributed in time according to

$$f = \nu e^{-\nu t}, \tag{7}$$

with the particle orbit being described by Eqs. (6a)-(6c) between collisions. This being the case, the time-averaged velocity of a charged particle is given by

$$\langle \vec{v} \rangle = \int_0^\infty \vec{v}_{\text{orbit}} v e^{-vt} dt, \quad (8)$$

where $\vec{v}_{\text{orbit}}$ is the trajectory given by Eqs. (6a)-(6c). Assuming that the momentum-transfer collisions that occur with frequency v leave a particle with an average velocity of zero, $v_{x0} = v_{y0} = v_{z0}$ is taken in (6a)-(6c). Inserting these orbits into Eq. (8) and making use of

$$\int_0^\infty v e^{-vt} t dt = \frac{1}{v} \quad (9a)$$

$$\int_0^\infty v e^{-vt} \cos(Q\omega_c t) dt = \frac{v^2}{v^2 + \omega_c^2} \quad (9b)$$

$$\int_0^\infty v e^{-vt} \sin(Q\omega_c t) dt = \frac{Q\omega_c v}{v^2 + \omega_c^2} \quad (9c)$$

yields

$$\begin{aligned} \langle \vec{v}_\perp \rangle = & \frac{Q\omega_c v}{v^2 + \omega_c^2} \frac{c}{B_0} \vec{E}_\perp + \frac{\omega_c^2}{v^2 + \omega_c^2} \frac{c}{B_0^2} \vec{E} \times \vec{B}_0 \\ & + \frac{\omega_c^2}{v^2 + \omega_c^2} \frac{mc^2}{qB_0^2} \frac{\partial \vec{E}_\perp}{\partial t} + \frac{Q\omega_c v}{v^2 + \omega_c^2} \frac{mc^2}{qB_0^2} \frac{\vec{B}_0}{B_0} \times \frac{\partial \vec{E}}{\partial t} \end{aligned} \quad (10)$$

and

$$\langle v_\parallel \rangle = \frac{q}{m} \frac{1}{v} E_\parallel. \quad (11)$$

By inserting relations (10) and (11) for ions and electrons into relation (4), the current density can be written in the form of Eq. (2), where the various conductivities σ are:

$$\sigma_{\text{ped}} = \frac{n_o e c}{B_o} \left[\frac{v_e \omega_{ce}}{v_e^2 + \omega_{ce}^2} + \frac{v_i \omega_{ci}}{v_i^2 + \omega_{ci}^2} \right] \quad (12a)$$

$$\sigma_{\text{pol}} = \frac{n_o e c}{B_o} \left[\frac{\omega_{ce}}{v_e^2 + \omega_{ce}^2} + \frac{\omega_{ci}}{v_i^2 + \omega_{ci}^2} \right] \quad (12b)$$

$$\sigma_H = \frac{n_o e c}{B_o} \left[\frac{\omega_{ce}^2}{v_e^2 + \omega_{ce}^2} - \frac{\omega_{ci}^2}{v_i^2 + \omega_{ci}^2} \right] \quad (12c)$$

$$\sigma_{\text{cpol}} = - \frac{n_o e c}{B_o} \left[\frac{v_e}{v_e^2 + \omega_{ce}^2} - \frac{v_i}{v_i^2 + \omega_{ci}^2} \right] \quad (12d)$$

$$\sigma_{\parallel} = n_o e^2 \left[\frac{1}{m_e v_e} + \frac{1}{m_i v_i} \right] \quad , \quad (12e)$$

where n_o is the electron number density, e is the electronic charge, B_o is the magnetic induction of the medium, m_e and m_i are the electron and ion mass, ω_{ce} and ω_{ci} the electron and ion gyrofrequencies, and v_e and v_i are the electron and ion momentum-exchange collision frequencies. The Pederson σ_{ped} , parallel $\sigma_{\parallel}$, and Hall σ_H conductivities are those commonly derived^{4,5} for a non-isotropic d-c plasma Ohms Law, $\vec{J} = \vec{\sigma} \cdot \vec{E}$. The polarization σ_{pol} and cross-polarization σ_{cpol} conductivities give non-Ohmic terms with $\vec{J} \propto \frac{\partial \vec{E}}{\partial t}$ rather than $\vec{J} \propto \vec{E}$. In a collisionless plasma, only the polarization and parallel conductivities are non-zero ($\sigma_{\parallel}$ becoming infinite according to this formulation).

For isotropic nonionized Ohmic media (ground, water, and air), the constitutive relation (2) is replaced by the relation

$$\vec{J} = \sigma \vec{E} = \sigma \vec{E}_{\perp} + \sigma \vec{E}_{\parallel} \quad , \quad (13)$$

which is equivalent to relation (2) with $\sigma_{\text{ped}} = \sigma_{\parallel} = \sigma$ and $\sigma_H = \sigma_{\text{pol}} = \sigma_{\text{cpol}} = 0$.

III. THE NUMERICAL METHOD

For each problem to be numerically simulated, Maxwell's Eqs. (1a)-(1d) and the constitutive relations (2) and (3) or (13) and (3) can be combined into a single partial differential equation describing the propagation of one component of $\vec{E}$ or $\vec{B}$, this one component being a function of y , z , and t (see Fig. 1). Restricting the problems to cases where the media are not functions of y , the single two-spatial-variable partial differential equation is Fourier-transformed in the y direction $e^{ik_y y}$, leaving a z - and t -dependent partial differential equation for each value of k_y . The problems are further restricted to cases where the z - and t -dependent partial differential equations are of the form

$$\begin{aligned}
 C_a \frac{\partial^4 f}{\partial z^2 \partial t^2} + C_b \frac{\partial^3 f}{\partial z^2 \partial t} + C_c \frac{\partial^3 f}{\partial z \partial t^2} + C_d \frac{\partial^2 f}{\partial z^2} \\
 + C_e \frac{\partial^2 f}{\partial z \partial t} + C_f \frac{\partial^2 f}{\partial t^2} + C_g \frac{\partial f}{\partial z} + C_h \frac{\partial f}{\partial t} \\
 + C_i f + C_j = 0 \quad , \quad (14)
 \end{aligned}$$

where $f(k_y, z, t)$ is the field component and where the coefficients $C_a, C_b, \dots, C_j$ are functions of k_y, z , and t . Equation (14) is a fourth-order partial differential equation in z and t with no higher than second derivatives in z or t .

Equation (14) is finite differenced in both z and t and then solved implicitly for the new-time values of f on a z - k_y grid. Thus, $f(k_y, z, t)$ is discretized as $f_I^N(k_y)$, I being an integer representing the z -position on the grid and N being an integer representing the discrete value of the time. For a system finite in y , the k_y values are also discrete. The nine derivative operators in Eq. (14) become nine 3×3 computational molecules. Each molecule is arranged as in Fig. 2, the nine elements of each molecule corresponding to three z gridpoints, $I-1, I$, and $I+1$, and three levels of time, $N-1, N$, and $N+1$, where $z = I\Delta z$ and $t = N\Delta t$, Δz and Δt being the gridspacing and the timestep.

Fig. 2. The arrangement of the elements of a 3×3 computational molecule.

Relaxation factors α and γ are introduced into the z -splitting scheme and t -splitting scheme as follows. Whenever a molecule M contains no z -derivatives, $\alpha \Delta z^2 (\partial^2)/(\partial z^2) M$ is added to M , and whenever a molecule M contains no t -derivatives, $\gamma \Delta t^2 (\partial^2)/(\partial t^2) M$ is added to M . As an example, the standard $\frac{\partial^2}{\partial t^2}$ molecule is transformed according to:

$$\begin{array}{ccc}
 0 & 1 & 0 \\
 \frac{1}{\Delta t^2} & 0 & -2 & 0 \\
 0 & 1 & 0
 \end{array}
 +
 \begin{array}{ccc}
 \alpha & (1-2\alpha) & \alpha \\
 -2\alpha & -2(1-2\alpha) & -2\alpha \\
 \alpha & (1-2\alpha) & \alpha
 \end{array}
 . \quad (15)$$

With the relaxation factors, the nine 3×3 molecules appear in Fig. 3 (for $\alpha = \gamma = 0$, the standard molecules are recovered). These nine molecules, times their respective z -dependent C -coefficients, are added together to form one 3×3 molecule at each z -gridpoint I ,

$$\begin{aligned}
M_a &= \frac{\partial^4}{\partial t^2 \partial z^2} = \frac{1}{\Delta t^2 \Delta z^2} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{pmatrix} & M_d &= \frac{\partial^2}{\partial z^2} = \frac{1}{\Delta z^2} \begin{pmatrix} \gamma & -2\gamma & \gamma \\ (1-2\gamma) & 2(1-2\gamma) & (1-2\gamma) \\ \gamma & -2\gamma & \gamma \end{pmatrix} & M_g &= \frac{\partial}{\partial z} = \frac{1}{2\Delta z} \begin{pmatrix} \gamma & 0 & \gamma \\ -(1-2\gamma) & 0 & (1-2\gamma) \\ -\gamma & 0 & \gamma \end{pmatrix} \\
M_b &= \frac{\partial^3}{\partial z^2 \partial t} = \frac{1}{2\Delta t \Delta z^2} \begin{pmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ -1 & 2 & -1 \end{pmatrix} & M_e &= \frac{\partial^2}{\partial z \partial t} = \frac{1}{4\Delta t \Delta z} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{pmatrix} & M_h &= \frac{\partial}{\partial t} = \frac{1}{2\Delta t} \begin{pmatrix} a & (1-2a) & a \\ 0 & 0 & 0 \\ -a & -(1-2a) & -a \end{pmatrix} \\
M_c &= \frac{\partial^3}{\partial z \partial t^2} = \frac{1}{2\Delta t^2 \Delta z} \begin{pmatrix} -1 & 0 & 1 \\ 2 & 0 & -2 \\ -1 & 0 & 1 \end{pmatrix} & M_f &= \frac{\partial^2}{\partial t^2} = \frac{1}{\Delta t^2} \begin{pmatrix} a & (1-2a) & a \\ -2a & 2(1-2a) & -2a \\ a & (1-2a) & a \end{pmatrix} & M_i &= 1 = \begin{pmatrix} \gamma a & \gamma(1-2a) & \gamma a \\ (1-2\gamma)a & (1-2\gamma)(1-2a) & (1-2\gamma)a \\ \gamma a & \gamma(1-2a) & \gamma a \end{pmatrix}
\end{aligned}$$

Fig. 3. The nine 3×3 computational molecules of Eq. (16).

$$\begin{aligned}
M_{\text{split}}(I', N', I, k_y) &= C_a(I, k_y) M_a(I', N') + C_b(I, k_y) M_b(I', N') \\
&+ C_c(I, k_y) M_c(I', N') + C_d(I, k_y) M_d(I', N') \\
&+ C_e(I, k_y) M_e(I', N') + C_f(I, k_y) M_f(I', N') \\
&+ C_g(I, k_y) M_g(I', N') + C_h(I, k_y) M_h(I', N') \\
&+ C_i(I, k_y) M_i(I', N') + C_j(I, k_y) \quad . \quad (16)
\end{aligned}$$

The set of molecules $M_{\text{split}}(I)$ contains all of the information about the numerical splitting scheme. Thus they contain Maxwell's equations plus the properties of the medium in the geometry of the problem to be simulated. Eq. (14) is now written as the set of linear equations

$$\sum_{I'=1}^3 \sum_{N'=1}^3 M_{\text{split}}(I', N', I, k_y) f_{\substack{N-2+N' \\ I-2+I'}} = 0 \quad . \quad (17)$$

To solve for the new-time set $(N+1)$ of f values, Eq. (17) is written in the form

$$a_I f_{I+1}^{N+1} + b_I f_I^{N+1} + c_I f_{I-1}^{N+1} = d_I, \quad (18)$$

where

$$\begin{aligned} a_I &= M_{\text{split}}(3,3,I,k_y) \\ b_I &= M_{\text{split}}(2,3,I,k_y) \\ c_I &= M_{\text{split}}(1,3,I,k_y) \\ d_I &= \sum_{I'=1}^3 \sum_{N'=1}^2 M_{\text{split}}(I',N',I,k_y) f_{I-2+I'}^{N-2+N'}. \end{aligned} \quad (19)$$

Equation (18), with the appropriate boundary conditions, is solved with a tridiagonal algorithm.

For simple Dirichlet or Neumann boundary conditions on the field quantity $f(k_y, z, t)$, an $f = \text{constant}$ boundary condition produces a reflection with the sign of f reversed and a $\frac{\partial f}{\partial z} = \text{constant}$ boundary condition produces a reflection with the sign of f staying the same. If f represents the electric field E_x or E_y , then an $E = 0$ boundary condition corresponds to a perfectly conducting boundary and a $\frac{\partial E}{\partial z} = 0$ boundary condition corresponds to a plane of mirror symmetry. Conversely, if f represents the magnetic induction B_x or B_y , then a $B = 0$ boundary corresponds to a plane of symmetry and a $\frac{\partial B}{\partial z} = 0$ boundary corresponds to a perfect conductor. Wherever Neumann boundary conditions are specified, guard-cell (false-boundary) techniques are employed to bring the order of the numerical boundary condition up to the order of the numerical scheme.

To test the stability of the numerical scheme and boundary conditions, a matrix-eigenvalue solver is applied by the computer program to Eq. (18) and it is checked that no eigenvalues of Eq. (18) exceed unity. The matrix-eigenvalue method is used because the media of propagation is inhomogeneous, more standard analyses not being applicable.

The two relaxation factors α and γ are contained in the splitting-scheme molecules M_{split} . These factors allow the accuracy and/or stability of the numerical scheme to be adjusted. Optimal values for α and γ depend on the partial differential equation that is being solved, hence they depend on the

Fig. 4. Eight computer runs of the propagation of truncated-sine-wave pulses according to Eq. (20). The only difference between the runs is the choice of the α and γ relaxation factors.

media and geometry of the physical problem. Examples of the variation in accuracy are displayed in Fig. 4. These are obtained by solving the second-order nondissipative wave equation

$$\frac{\partial^2 E}{\partial t^2} - v_c^2 \frac{\partial^2 E}{\partial z^2} = 0 \quad (20)$$

with the characteristic velocity v_c being a constant. Equation (20) is represented by Eq. (14) when coefficients $C_d = -v_c^2$ and $C_c = 1$, and all other coefficients are zero. With these coefficients, Eq. (17) yields the set of linear equations

$$\begin{aligned}
 & [\alpha - c_o^2 \gamma] E_{I+1}^{N+1} + [(1-2\alpha) + 2c_o^2 \gamma] E_I^{N+1} + [\alpha - c_o^2 \gamma] E_{I-1}^{N+1} \\
 & = - [-2\alpha - c_o^2 (1-2\gamma)] E_{I+1}^N - [-2(1-2\alpha) + 2c_o^2 (1-2\gamma)] E_I^N \\
 & - [-2\alpha - c_o^2 (1-2\gamma)] E_{I-1}^N - [\alpha - c_o^2 \gamma] E_{I+1}^{N-1} \\
 & - [(1-2\alpha) + 2c_o^2 \gamma] E_I^{N-1} - [\alpha - c_o^2 \gamma] E_{I-1}^{N-1}, \quad (21)
 \end{aligned}$$

$c_o = \frac{\Delta t v_c}{\Delta z}$ being the Courant number. When the time relaxation factor $\gamma = 0$, an explicit scheme for the E values results; when $\gamma = \frac{1}{2}$, a time-averaged scheme results; and when $\gamma = \frac{1}{12}$, a cubic spline in time results. When the z-direction relaxation factor $\alpha = 0$, the standard spatial split results, and when $\alpha = \frac{1}{12}$, a cubic spline in the z-direction results. Note that whereas the standard splits are second-order accurate in Δt and Δz , the cubic splines are fourth-order accurate. An analysis of Eq. (21) indicates that the numerical scheme is unconditionally stable for $\gamma > \frac{1}{4}$ and $\alpha < \frac{1}{2}$. The results of numerical schemes employing various combinations of these relaxation factors may be qualitatively compared by examining the propagating truncated-sine-wave pulses in Fig. 4. The boundary conditions used pertain to a perfect conductor at left-hand boundary and reflection symmetry about the right-hand boundary. Each scheme employed has a Courant number $c_o = \frac{1}{4}$. The wave pulses were all started near the $z = 0$ boundary propagating toward the right. After 11 200 time steps, the pulses have made a few passes through the system, have reflected off of the right-hand boundary for their third time, and are each depicted in Fig. 4 propagating to the left. As can be seen, the $\alpha = \gamma = \frac{1}{12}$ choice provides a very good representation of the pulse, with little distortion and little ringing (dispersion). This choice corresponds to cubic splines in both the z-direction and in time. The $\alpha = 0$, $\gamma = \frac{1}{2}$ scheme is the standard, unconditionally stable, time-implicit scheme. The $\alpha = 0$, $\gamma = \frac{1}{4}$ scheme is the unconditionally stable, implicit scheme given by Section 10.3 of Richtmyer and Morton.

Another general scheme for solving Eq. (20), attributed to Von Neumann, replaces Eq. (20) with

$$\frac{\partial^2 E}{\partial t^2} - v_c^2 \frac{\partial^2 E}{\partial z^2} - v_c^2 A \Delta t^2 \frac{\partial^4 E}{\partial z^2 \partial t^2} = 0, \quad (22)$$

the derivatives being split in the standard way. The resulting implicit scheme is unconditionally stable for relaxation factors $A > \frac{1}{4}$. Equating the Von Neumann scheme with the scheme of Eq. (14), it is found that the two schemes are identical whenever

$$\alpha = c_0^2 (\gamma - A), \quad (23)$$

and the critical value $a = \frac{1}{4}$ maps into a set of lines given by

$$\alpha = c_0^2 \gamma - \frac{c_0^2}{4}; \quad (24)$$

these lines are plotted in Fig. 5. For a particular scheme to be unconditionally stable, the point (α, γ) must lie on or below the appropriate line. For $\Delta t \rightarrow \infty$ (hence $c_0 \rightarrow \infty$), any scheme with $\gamma > \frac{1}{4}$ is unconditionally stable.

The structure of the discussed computer code is outlined below.

1. Choose the numerical scheme
(select the α and γ relaxation factors).
2. Set up the ambient medium
(generate the conductivities).
3. Load in the wave equation
(derived from Maxwell's equations and the geometry).
4. Fourier transform in y is performed.
5. The matrix equations are generated.

Fig. 5. For given values of the Courant number c_0 , the α - γ lines are drawn for the Von Neumann scheme from Eq. (24).

6. The numerical stability is tested
(via the matrix-eigenvalue method).
7. The scheme is run.

IV. CODE TESTING AND FUTURE WORK

The numerical code described in this report has been successfully tested for hyperbolic and parabolic equations of the form of Eq. (14). Elliptic equations conforming to Eq. (14) can also be solved via relaxation methods. The code has been used to study the propagation of ultra-low-frequency electromagnetic signals in collisional plasmas, with particular emphasis on the propagation of such signals through linearly varying plasma-density ramps and linearly varying conductivity ramps.

The simulation system pertinent to the mhd-emp propagation problem is depicted in Fig. 1. For these simulations, the International Reference Ionosphere¹⁰ will be used as a model for the densities and collision frequencies within the ionospheric plasma, and a model magnetosphere will be fit onto the topside ionosphere. To ensure that the electromagnetic signal at sea level is properly obtained, several kilometers of ocean water or ground material will be used to isolate the lower boundary of the simulation system from the atmosphere.

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