THERMOVISCOPLASTIC RESPONSE OF CYLINDRICAL
STRUCTURES USING A STATE VARIABLE MODEL

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SUMMARY

A number of combined creep-plasticity constitutive models with state variables have been recently proposed to describe time-dependent inelastic deformation of metals at elevated temperatures. Some of these models seem capable of more faithfully predicting material behavior under complex mechanical and thermal loading histories than is currently possible with traditional models which use separation of time-independent plasticity and time-dependent creep. The author and his coworkers have been interested for some time in the efficient numerical solution of boundary value problems using such constitutive models. A particularly promising model due to Hart has been used in the calculations. This model has been used to predict the uniaxial responses of several metals and alloys to various types of loading histories. The correlation between model predictions and experiments has been found to be very good. Stresses and deformation in cylindrical structures subjected to time-dependent axisymmetric mechanical and thermal loads is the subject of this paper. Several physically observed phenomena that cannot be simulated by traditional models are discussed. The computational scheme used is very efficient.

NOTATION

$\epsilon_{ij}, \epsilon_{ij}^e, \epsilon_{ij}^n, \epsilon_{ij}^T$	total, elastic, nonelastic and thermal strain tensors
σ_{ij}	stress tensor
q_{ij}	state variables
T	temperature
a, b	internal and external radii of cylinder
p, q, P	internal pressure, external pressure and axial force

T_a, T_b	internal and external temperatures
r, θ, z, t	radial, tangential and axial coordinates; time
u_r, u_z	radial and axial displacements
$\epsilon_r, \epsilon_\theta, \epsilon_z$	radial, tangential and axial strains
$\sigma_r, \sigma_\theta, \sigma_z$	radial, tangential and axial stresses
E, ν	Young's modulus and Poisson's ratio
α	coefficient of linear thermal expansion
$\epsilon_r^n, \epsilon_\theta^n, \epsilon_z^n$	radial, tangential and axial nonelastic strains
ϵ_z, ϵ_z^e	total and elastic axial strains
F, y	function, independent variable (equation 11)
Δt	time increment
∇F	backward difference of F
$e, e_{\max}, e_{\min}$	error parameters
$\epsilon_{ij}^a, \epsilon_{ij}^p$	anelastic strain (state variable) and permanent strain tensors
σ^*	hardness (state variable)
$s_{ij}^a, s_{ij}^f, s_{ij}^f$	deviatoric stress and auxilliary stresses
$\sigma, \sigma^a, \sigma^f$	stress invariants
$\epsilon^a, \dot{\epsilon}^n, \dot{\epsilon}^p$	strain and strain rate invariants
$M, m, \lambda, M, \dot{a}^*, \dot{\epsilon}_{sT}^*$	flow parameters
Γ	work hardening function
β, δ	work hardening parameters
σ_s^*, T_B	reference hardness and temperature
R	gas constant
Q_d	activation energy for atomic self diffusion
$(\dot{})$	$\frac{d}{dt} ()$

INTRODUCTION

Several combined creep-plasticity constitutive models using state variables (e.g. ¹⁻⁵) have been proposed in recent years for time-dependent inelastic deformation of metals and alloys. These models attempt to overcome the shortcomings of 'classical' creep models such as power law, time hardening and strain hardening. The author and his co-workers have been interested for some time in developing efficient numerical methods for the solution of uniaxial⁶ and multiaxial structural analysis problems using such constitutive models. Finite difference⁷⁻⁸, boundary-integral⁹⁻¹⁰ and finite element¹¹ methods have been used, together with the constitutive model due to Hart².

The solution technique, in all the papers quoted above, consists of formulating the problem in terms of real time rates which leads to an inhomogeneous linear boundary value problem for stress and displacement rates. The linear boundary value problem thus obtained is solved to find the rates of the variables of interest throughout the body at any time (using finite difference, boundary-integral or finite element methods), and then these rates are integrated forward in time to obtain the time histories of the variables of interest.

The mathematical structure of many of the newer constitutive models is especially amenable to the integration technique described above. This structure can be summarized by the following equations

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^n + \dot{\epsilon}_{ij}^T \quad (1)$$

$$\dot{\epsilon}_{ij}^n = f_{ij}(\sigma_{ij}, q_{ij}^{(k)}, T) \quad (2)$$

$$\dot{q}_{ij}^{(k)} = g_{ij}(\sigma_{ij}, q_{ij}^{(k)}, T) \quad (3)$$

Here $\dot{\epsilon}_{ij}^e$, $\dot{\epsilon}_{ij}^n$ and $\dot{\epsilon}_{ij}^T$ are the elastic, nonelastic and thermal strain rates respectively, σ_{ij} is the stress tensor, T the temperature and $q_{ij}^{(k)}$ are state variables. The number of state variables vary in the different models and they can be scalars or tensors. These state variables are assumed to completely characterize the present deformation state of the material and the history dependence of the rate of nonelastic strain upto the current time is assumed to be completely taken into account by their current values. It is important to note that the rates of the nonelastic strain and state variables at any time depend only on the current values of the stress, state variables and temperature. This fact plays a key role in the computational technique using a rate formulation.

The present paper discusses numerical solutions of axisymmetric boundary value problems for cylinders subjected to time-varying mechanical (internal pressure and axial force) and thermal (radial temperature gradient) loads. Plane stress and plane strain problems are also considered. The governing differential equations for the problems are first developed using the rate formulation. Analytical expressions for the rates of stress, displacement and temperature are obtained for any point inside the cylinder. These equations are integrated in time using an Euler type scheme with automatic time-step control discussed later in the paper. Numerical solutions for the case of pure mechanical, pure thermal and combined loads are presented using Hart's constitutive model. The case of combined mechanical and thermal loading chosen is that on a cylindrical nozzle due to an internal fluid (sodium) thermal transient¹².

FORMULATION OF THE PROBLEM AND SOLUTION STRATEGY

The formulation of the problem follows that discussed in an ERDA report from Cornell University¹³. The equations for stress and temperature rates are given next for various end conditions on the cylinder and the technique for obtaining the numerical solution is presented.

Thick Cylinder Subjected to Axisymmetric Mechanical and Thermal Loads

A long thick-walled cylinder of uniform circular cross section with internal and external radii a and b is subjected to internal and external pressures $p(t)$ and $q(t)$ and axial force $P(t)$. The internal temperature is $T_a(t)$ and the external temperature is $T_b(t)$. Neglecting end effects, the only nonzero displacements are the radial and axial ones.

These have the forms

$$u_r = u_r(r, t), \quad u_z = f(t)z \quad (4)$$

These displacements give rise to nonvanishing strains ϵ_r , ϵ_θ and ϵ_z and corresponding stresses σ_r , σ_θ and σ_z . The radial, tangential and axial directions are r , θ and z respectively and t is time.

Using the appropriate kinematic, equilibrium and constitutive equations and the proper boundary conditions, the displacement rate formulation yields the displacement rates and the stress rates. The stress rates are

$$\begin{aligned} \dot{\sigma}_r = & \frac{E}{2(1-\nu^2)} \left(\frac{r}{a} \frac{\dot{\epsilon}_r^n - \dot{\epsilon}_\theta^n}{n} d\eta - \frac{(r^2 - a^2)}{(b^2 - a^2)} \frac{b^2}{r^2} \frac{b}{a} \frac{\dot{\epsilon}_r^n - \dot{\epsilon}_\theta^n}{n} d\eta \right) \\ & + \frac{E(1-2\nu)}{2(1-\nu^2)} \frac{1}{r^2} \left(\frac{r}{a} \int_a^r \eta \dot{\epsilon}_z^n d\eta - \frac{(r^2 - a^2)}{(b^2 - a^2)} \frac{b}{a} \int_a^b \eta \dot{\epsilon}_z^n d\eta \right) - \dot{p} \frac{(b^2 - r^2)}{(b^2 - a^2)} \frac{a^2}{r^2} \\ & - \dot{q} \frac{(r^2 - a^2)}{(b^2 - a^2)} \frac{b^2}{r^2} + \frac{\alpha E (\dot{T}_a - \dot{T}_b)}{2(1-\nu) \ln(b/a)} \left(-\ln \frac{b}{r} + \frac{(b^2 - r^2)}{(b^2 - a^2)} \frac{a^2}{r^2} \ln \frac{b}{a} \right) \end{aligned} \quad (5)$$

$$\begin{aligned}
 \dot{\sigma}_{\theta} = & \frac{E}{2(1-\nu^2)} \left(r \frac{\dot{\epsilon}_r^n - \dot{\epsilon}_{\theta}^n}{\eta} d\eta - \frac{(r^2 + a^2)}{(b^2 - a^2)^2} \frac{b^2}{r^2} \frac{b}{a} \frac{\dot{\epsilon}_r^n - \dot{\epsilon}_{\theta}^n}{\eta} d\eta \right) \\
 & - \frac{E(1-2\nu)}{2(1-\nu^2)} \frac{1}{r^2} \left(\int_a^r \eta \dot{\epsilon}_z^n d\eta + \frac{(r^2 + a^2)}{(b^2 - a^2)^2} \frac{b}{a} \int_a^r \eta \dot{\epsilon}_z^n d\eta \right) \\
 & - \frac{E}{1-\nu^2} (\dot{\epsilon}_{\theta}^n + \nu \dot{\epsilon}_z^n) + \dot{p} \frac{(b^2 + r^2)}{(b^2 - a^2)^2} \frac{a^2}{r^2} - \dot{q} \frac{(r^2 + a^2)}{(b^2 - a^2)^2} \frac{b^2}{r^2} \\
 & + \frac{\alpha E (\dot{T}_a - \dot{T}_b)}{2(1-\nu) \ln(b/a)} \left(1 - \ln \frac{b}{r} - \frac{(b^2 + r^2)}{(b^2 - a^2)^2} \frac{a^2}{r^2} \ln \frac{b}{a} \right) \quad (6)
 \end{aligned}$$

$$\begin{aligned}
 \dot{\sigma}_z = & \frac{E\nu}{1-\nu^2} \left(r \frac{\dot{\epsilon}_r^n - \dot{\epsilon}_{\theta}^n}{\eta} d\eta - \frac{b^2}{b^2 - a^2} \frac{b}{a} \frac{\dot{\epsilon}_r^n - \dot{\epsilon}_{\theta}^n}{\eta} d\eta \right) \\
 & + \frac{E(2-\nu)}{(1-\nu^2)} \frac{1}{(b^2 - a^2)^2} \int_a^b \eta \dot{\epsilon}_z^n d\eta + \frac{E}{1-\nu^2} (\nu \dot{\epsilon}_r^n - (1-\nu) \dot{\epsilon}_z^n) + \frac{\dot{P}}{\pi(b^2 - a^2)} \\
 & + \frac{\alpha E (\dot{T}_a - \dot{T}_b)}{2(1-\nu) \ln(b/a)} \left(1 - 2 \ln \frac{b}{r} - \frac{2a^2}{b^2 - a^2} \ln \frac{b}{a} \right) \quad (7)
 \end{aligned}$$

The temperature rate is

$$\dot{T} = \frac{1}{\ln(b/a)} (\dot{T}_b \ln(r/a) + \dot{T}_a \ln(b/r)) \quad (8)$$

In the above, α is the coefficient of linear thermal expansion and $\dot{\epsilon}_r^n$, $\dot{\epsilon}_{\theta}^n$ and $\dot{\epsilon}_z^n$ are nonelastic strain rates. Nonelastic deformation is assumed incompressible, so that

$$\dot{\epsilon}_r^n + \dot{\epsilon}_{\theta}^n + \dot{\epsilon}_z^n = 0 \quad (9)$$

Note that the stress rate equations contain terms involving the nonelastic strain rates together with time derivatives of the thermoelastic solution of the problem.

Thermomechanical coupling is ignored. The temperature on the surface of the body is assumed to change slowly in time and thermal steady state conditions are assumed to exist throughout the body at each time. Thus, the temperature field is assumed to be given by the solution of the appropriate steady state diffusion equation subject to slowly varying surface temperatures $T_a(t)$ and $T_b(t)$.

Plane strain. The case of plane strain requires that ϵ_z be equal to zero. In this case, equations (5) and (6) remain unaltered and equation (7) must be replaced by an alternative equation obtained by using

$$\dot{\epsilon}_z = \dot{\epsilon}_z^e + \dot{\epsilon}_z^n = 0. \text{ This is}$$

$$\dot{\sigma}_z = \nu(\dot{\sigma}_r + \dot{\sigma}_\theta) - E\dot{\epsilon}_z^n - E\alpha\dot{T} \quad (10)$$

Equation (8) for the temperature rate remains the same as before.

Plane stress (thin disc). In this case the axial stress, σ_z , must vanish for all time. Using the appropriate constitutive equations, the rate equations are obtained. The equations for $\dot{\sigma}_r$ and $\dot{\sigma}_\theta$ have exactly the same forms as equation (5) and (6) with the Poisson's ratio ν set to zero. The axial stress rate $\dot{\sigma}_z$ is zero and, once again, equation (8) for the temperature rate remains unaltered.

Numerical Solution Strategy

The initial values of the stresses, strains and temperature are obtained from the solution of the corresponding thermoelastic problem¹⁴ and the initial values of state variables are prescribed. The initial rates of nonelastic strains and state variables are obtained from equations (2) and (3) and the stress and temperature rates from equations like (5-8). The

rates of total strain and displacement are then obtained from Hooke's law, equation (1) and kinematic equations. These rates are numerically integrated to obtain the values of the variables after a small time interval Δt and so on, and in this way the time histories of all the variables are determined.

Many of the state variable constitutive models give rise to stiff systems of differential equations and it is very important that a suitable time integration strategy with automatic time step control be used. An Euler type strategy which is simply yet very efficient is used to obtain the numerical results discussed later in the paper. A brief summary of the method in terms of a single differential equation

$$\frac{dy}{dt} = F(y,t) \quad (11)$$

is presented below. Further details are available in Kumar and others⁶. The value of $y(t+\Delta t)$ in terms of $y(t)$ is

$$y(t+\Delta t) = y(t) + F(y,t)\Delta t \quad (12)$$

and the error at this step used for automatic time step control is defined as

$$e = \Delta t |VF| / |y(t)| \quad (13)$$

where $VF = F(y,t) - F(y,t-\Delta t_{\text{previous}})$, is the first backward difference of F . Two error parameters e_{max} and e_{min} are initially prescribed. The algorithm proceeds as follows:

$e_{\max} < e$: replace Δt by $\Delta t/2$; recompute e .

$e \leq e_{\max}$: accept Δt ; calculate $y(t+\Delta t)$

The time step for the next step, Δt_{next} , is decided according as

$e_{\min} < e \leq e_{\max}$: $\Delta t_{\text{next}} = \Delta t$

$e \leq e_{\min}$: $\Delta t_{\text{next}} = 2\Delta t$

The initial time step is prescribed.

HART'S CONSTITUTIVE MODEL

The constitutive relations proposed by Hart² are used in the numerical calculations for the problems discussed in the previous section. Hart's equations have been used to predict uniaxial deformation behavior of several metals and alloys under various prescribed histories of stress and strain¹¹ and the correlation between theory and experiment has been found to be very good. A brief review of the model is presented next.

According to Hart, the nonelastic strain is decomposed into two (time-dependent) components

$$\dot{\epsilon}_{ij}^n = \dot{\epsilon}_{ij}^a + \dot{\epsilon}_{ij}^p \quad (14)$$

where ϵ_{ij}^a is the anelastic strain, a stored strain which reflects the magnitude and direction of prior deformation history and ϵ_{ij}^p is the completely irrecoverable and path-dependent permanent strain. The two state variables in the model are the anelastic strain and a scalar σ^* , called

hardness, which is similar to an isotropic strain hardening parameter. The constitutive equations prescribe flow rules for the anelastic strain tensor and the permanent and nonelastic strain rates. Two auxiliary tensors s_{ij}^a and s_{ij}^f are used, the sum of which equals s_{ij} , the deviatoric part of the stress tensor. Certain scalar invariants of the relevant tensors are defined (e.g., $\sigma^a = ((3/2)s_{ij}^a \cdot s_{ij}^a)^{1/2}$) and these are related to each other through the uniaxial equations

$$\sigma^a = M\epsilon^a, \quad \dot{\epsilon}^n = \dot{\epsilon}^*(\sigma^f/M)^M \quad (15,16)$$

$$\dot{\epsilon}^p = \dot{\epsilon}^* [\ln(\sigma^*/\sigma^a)]^{-1/\lambda} \quad (17)$$

$$\dot{\epsilon}^* = \dot{\epsilon}_{ST}^* (\sigma^*/\sigma_s^*)^m \exp(Q_d/RT_B) \exp(-Q_d/RT) \quad (18)$$

$$\dot{\sigma}^* = \dot{\epsilon}^p \sigma^* \Gamma(\sigma^*, \sigma^a) \quad (19)$$

$$\Gamma(\sigma^*, \sigma^a) = (\beta/\sigma^*)^\delta (\sigma^a/\sigma^*)^{\beta/\sigma^*} \quad (20)$$

Equation (15) represents a linear anelastic element, (16) a nonlinear dash-pot, (17) and (18) a "plastic" element and, finally (19) and (20) describe strain hardening. The flow parameters are M , M , m , λ , $\dot{\epsilon}^*$ and $\dot{\epsilon}_{ST}^*$ (at a reference hardness level σ_s^* and reference temperature T_B); β and δ are strain hardening parameters, R is the gas constant and Q_d the activation energy for atomic self diffusion. The material parameters, many of which are temperature dependent, are obtained from load relaxation and constant strain rate tension experiments. They are given in Appendix A of the EPRI report by Kumar and others¹¹.

NUMERICAL RESULTS AND DISCUSSION

A computer program capable of solving inelastic axisymmetric problems for cylinders subjected to mechanical and thermal loads, has been developed. This program can handle prescribed axial load, plane strain and plane stress problems. Numerical results for 304 stainless steel cylinders are presented in this section. Results for pure mechanical loads appear in Figs. 1-4, for pure thermal loads in Figs. 5-8 and those for the nozzle simulation problem with both mechanical and thermal loads in Figs. 9-10.

Material and Geometrical Parameters

The material parameters for 304 stainless steel at 200°C¹¹ are given below. Their values at other temperatures are available in Kumar and others¹¹.

$$\lambda = 0.15 \quad M = 7.8 \quad m = 5$$

$$E = 26.7 \times 10^6 \text{ psi} \quad M = 14.1 \times 10^6 \text{ psi}$$

$$\nu = 0.298$$

$$\dot{\epsilon}_{ST}^* = 9.572 \times 10^{-34} \text{ sec}^{-1} \quad \text{for } \sigma_s^* = 10 \text{ ksi and } T_B = 200^\circ\text{C.}$$

$$\dot{a}^* = 2.41 \times 10^{24} \text{ sec}^{-1}$$

$$\beta = 1.965 \times 10^5 \text{ psi} \quad \delta = 1.33$$

$$Q_d/R = 3.25 \times 10^4 \text{ }^\circ\text{K} \quad \alpha = 1.25 \times 10^{-5} / ^\circ\text{C}$$

The value of $b/a = 1.5$ except in the nozzle simulation problem where $a = 7.625 \text{ in.}$ and $b = 8 \text{ in.}$

Pure Mechanical Loads

Displacements and stresses in a cylinder for the case of internal pressure increasing at a constant rate at a uniform temperature of 200° C are shown in Figs. 1-4. Figure 1 shows pressure-strain plots for various end conditions. The results show the usual features of classical plasticity with linear work hardening. An extra feature, not included in classical plasticity, is the anelastic effect, which causes the slopes of the curves to decrease continually (at low strain levels) from the initial slope due to the elastic solution. For low stresses, the "plastic" element in Hart's model is essentially inactive and the behavior in shear is that of a non-linear viscoelastic Kelvin element. Accumulation of anelastic strains causes decrease of the slope of the stress-strain curve. The anelastic effect has been demonstrated in carefully controlled uniaxial tension tests at low strain rates^{15,16}. The "plastic" element becomes active once the anelastic stress invariant σ^a approaches σ^* . Redistribution of radial, tangential and axial stresses are displayed in Figs. 2-4 respectively. A discontinuity in the slope of the stress is seen to propagate into the cylinder in Figs. 3-4, as in classical plasticity. Hart's model is capable of predicting many other inelastic features like creep, strain rate sensitivity, creep strain recovery and material hardening^{7,11}.

Pure Thermal Loads

Stresses in a cylinder subjected to internal temperature increasing at a constant rate are shown in Figs. 5-8. The thermoanelastic effect, shown in Fig. 5, reduces the stress obtained from a purely thermoelastic solution by about 11% for $\dot{T}_a = 6^\circ\text{C/hr}$. Nonelastic strain rates are typically proportional to high powers of stress and this reduction in thermal stress can

be quite significant in many applications (for example crack growth in ductile materials at elevated temperatures). Rate effects are evident in that the reduction in stress is very much less for a rapid heating rate of $10^{\circ}\text{C}/\text{sec}$. The radial stress (Fig. 6) remains compressive throughout and, of course, is zero at the inside and outside surfaces of the cylinder. Redistribution of tangential and axial thermal stresses (Figs. 7-8) have the usual features of classical plasticity with two separate yield fronts, one starting at the inside and the other at the outside of the cylinder. Similar results for spheres using classical plasticity appear in Johnson and Mellor¹⁷. The tangential and axial stresses are of comparable magnitude.

Combined Mechanical and Thermal Loads

A problem of practical interest, that of a cylindrical nozzle subjected to combined mechanical and thermal loads due to an internal fluid (sodium) thermal transient has been solved. The history of mechanical and thermal loads¹² is shown in Fig. 9 and the corresponding stress strain plot in Fig. 10. The rapid ($16.6^{\circ}\text{C}/\text{sec}$) drop in the inside temperature due to the downshock causes a rapid increase in the circumferential stress to about 15.5 ksi in this case. A small amount of residual strain remains at the end of the cycle. The strains in this problem are calculated with respect to reference configuration at 593°C .

Discussion of Computational Scheme

The computational scheme used for these problems is very efficient. The nozzle simulation problem, for example, requires cpu time of only 26 seconds in an IBM 370/168 computer. No matter how accurately a given constitutive theory models the behavior of a material, it is of little use if it cannot

be applied on a practical scale. The author is greatly encouraged that the rate formulation together with the time-integration strategy proposed in this paper will make it economically feasible to design and analyse structures using a state variable model. While Hart's model is used as an example, other similar constitutive modes with state variables could be easily incorporated into the computer program.

CONCLUSIONS

Several boundary value problems for thermoviscoplastic deformation of cylindrical structures subjected to axisymmetric mechanical and thermal loads have been solved. A state variable constitutive model is used to describe material behavior. Important physical phenomena like the thermoanelastic effect, rate effects etc. are discussed. The computational scheme, based on a rate formulation of the governing differential equations and a time integration method with a newly developed logic for time-step control, is very efficient. It appears that this time-integration scheme will prove very useful for the solution of more complicated inelastic boundary value problems using state variable type constitutive models.

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Figure 1. Internal pressures as functions of circumferential strain at the inside of a cylinder (plane strain) and disc (plane stress). 304 SS 200°C $\sigma_o^* = 17 \text{ ksi}$

Figure 2. Cylinder in plane strain. Redistribution of radial stress. 304 SS 200°C $\dot{p} = 10$ ksi/sec
 $\sigma_0^* = 17$ ksi

Figure 3. Cylinder in plane strain. Redistribution of circumferential stress. 304 SS 200°C
 $\dot{p} = 10$ ksi/sec $\sigma_o^* = 17$ ksi

Figure 4. Cylinder in plane strain. Redistribution of axial stress. 304 SS 200°C $\dot{p} = 10$ ksi/sec $\sigma_o^* = 17$ ksi

Figure 5. Thermo-anelastic effect. Stress invariant at inside of cylinder as functions of rise of inside temperature for different uniform temperature rates. 304 SS $p = q = P = 0$. $T_a(0) = 200^\circ\text{C}$ $T_b = 200^\circ\text{C}$ $\sigma_o^* = 17$ ksi

Figure 6. Cylinder with zero axial force. Redistribution of radial stress. 304 SS $T_a(0) = 100^\circ\text{C}$
 $T_b = 100^\circ\text{C}$ $\dot{T}_a = 6^\circ\text{C/hr}$ $p = q = P = 0$ $\sigma_o^* = 17$ ksi

Figure 7. Cylinder with zero axial force. Redistribution of circumferential stress. 304 SS
 $T_a(0) = 100^\circ\text{C}$ $T_b = 100^\circ\text{C}$ $\dot{T}_a = 6^\circ\text{C/hr}$ $p = \bar{q} = P = 0$ $\sigma_o^* = 17$ ksi

Figure 8. Cylinder with zero axial force. Redistribution of axial stress. 304 SS $T_a(0) = 100^{\circ}\text{C}$
 $T_b = 100^{\circ}\text{C}$ $\dot{T}_a = 6^{\circ}\text{C/hr}$ $p = q = P = 0$ $\sigma_o^* = 17 \text{ ksi}$

Figure 9. Nozzle simulation. Input time histories of temperature, internal pressure and axial force.

Figure 10. Nozzle simulation. Stress-strain plot at the inside of the nozzle. 304 SS
 $\sigma_o^* = 17$ ksi $a = 7.625$ in. $b = 8$ in.