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**Classical Theory
of Beam-Induced
Plasma Currents**

S. P. Hirshman

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DEPARTMENT OF ENERGY**

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CLASSICAL THEORY OF BEAM-INDUCED PLASMA CURRENTS

S. P. Hirshman

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ABSTRACT

A variational calculation of the current induced in a plasma by fast beam particles is presented. A new energy polynomial expansion of the classical Spitzer function is obtained and used to derive an accurate analytic expression for the beam-induced current $j_{\parallel}$, which includes the effects of electron-electron collisions and is valid for all values of $\bar{v}_b = v_b/v_e$ and effective charge $\bar{Z}$. This analytic result is in excellent agreement with previous numerical calculations. An accurate rational form for the beam-induced current as a function of beam velocity is obtained by patching the large and small $\bar{v}_b$ asymptotic expansions of $j_{\parallel}$.

1. INTRODUCTION

Cordey et al.¹ have recently computed a numerical solution of the steady-state electron Fokker-Planck equation in the presence of a beam-injected momentum source. Their computation extended previous results to include the effects of electron-electron collisions, which result in distortions of the electron energy distribution function, as well as arbitrary beam velocities v_b relative to the electron thermal speed $v_e = (2T_e/m_e)^{1/2}$. The remarkable result of their analysis was that for $v_e > v_b$ and $Z = 1$, the net plasma current could be in the direction opposite to the beam current (neglecting trapping effects due to toroidal geometry).

The purpose of the present analysis is to obtain an accurate analytic expression for the classical beam-induced current from a suitable variational principle involving the classical Spitzer function.² Indeed, it is shown that the Spitzer function, which is the electron response to a parallel electric field applied to a plasma, is the Green's function for calculating the beam-induced current. Thus, accurate determination of the Spitzer function (which is generally easier to obtain than the beam-induced electron distribution) is sufficient to compute the beam current from the variational principle.

In Sect. 2, the electron kinetic equation in the presence of a beam of energetic particles is briefly reviewed. The variational principle for the plasma current is established in Sect. 3, and asymptotic forms for the current are computed. The Spitzer function is determined from the classical variational principle³ in Sect. 4, and it is used in Sect. 5 to obtain the desired expression for the beam-induced current.

2. THE ELECTRON FOKKER-PLANCK EQUATION

In a uniformly magnetized plasma, the Fokker-Planck equation describing the electron response f_e along the magnetic field due to a beam distribution f_b is:

$$C_{ee}(f_{e1}, f_{Me}) + C_{ee}(f_{Me}, f_{e1}) + \sum_i C_{ei}(f_{e1}, f_{Mi}) = -C_{eb}(f_{Me}, f_b) \equiv S_b(\underline{v}) . \quad (1)$$

Here, $f_{e1} = f_e - f_{Me} \sim n_b$ is the first order (in beam density n_b) linearized electron response, f_{Mj} are Maxwellian distributions for $j = e, i$, and C_{ee} is the Coulomb collision operator for electron-electron collisions (cf. Appendix A). In the main (thermal) ion rest frame,

$$\sum_i C_{ei}(f_{e1}, f_{Mi}) = \frac{v_{eo}}{x^3} \bar{Z} L f_{e1} , \quad (2a)$$

where $\bar{Z} = \sum_i n_i e_i^2 / n_e e^2$ is the effective charge, $x = v/v_e$, $v_{eo} = 4\pi n_e e^4 \times \ln \Lambda / (m_e^2 v_e^3)$ is the electron-electron collision frequency, and

$$L = \frac{1}{2 \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} \quad (2b)$$

is the pitch angle scattering operator, where $v_{\parallel}/v = \cos \theta$ and $v_{\parallel} = \underline{B} \cdot \underline{v}/B$.

The beam distribution function f_b is assumed to be known from the solution of the beam-slowing-down equation.^{4,5} For computing the current along the magnetic field,

$$j_{\parallel} = e_b \int v_{\parallel} f_b d\underline{v} - |e| \int v_{\parallel} f_{e1} d\underline{v} , \quad (3)$$

it is sufficient to consider only the $\ell = 1$ spherical harmonic of the beam distribution function in Eq. (1) (this follows from the spherical symmetry of the linearized collision operators):

$$f_b^{(1)} \equiv \frac{3}{2} P_1(\xi) \int_{-1}^1 P_1(\xi') f_b(\underline{v}') d\xi' = \frac{3}{2} a^{(1)}(v) P_1(\xi), \quad (4a)$$

where $\xi = \cos \theta$ is the pitch angle variable and $P_1(\xi) = (3/2)\xi^2 - 1/2$. For $v_b/v_e \ll 1$, which is typical of present-day beam injection into tokamaks, the detailed velocity dependence of $a^{(1)}(v)$ is unimportant [cf. Eq. (5)]. To treat more general values of v_b/v_e and arbitrary beam distributions, we shall compute the electron (Green's function) response to a monoenergetic source of beam particles:

$$a^{(1)}(v) = n_b u_{\parallel b} \frac{\delta(v - v_b)}{2\pi v_b^3}, \quad (4b)$$

where $n_b u_{\parallel b} = \int v_{\parallel} \bar{f}_b d\underline{v}$. Once the electron distribution function in response to this source has been obtained from Eq. (1), the beam-induced current for arbitrary beam energy distributions may be computed by integrating Eq. (3), with the appropriate energy weighting factor, over the energy parameter v_b .

With f_b given by Eq. (4), the source of beam momentum in Eq. (1), $S_b^{(1)}(\underline{v}) \equiv \frac{3}{2} P_1(\xi) \int_{-1}^1 P_1(\xi') S_b(\underline{v}') d\xi'$, may be evaluated as follows:

$$S_b^{(1)}(\underline{v}) = - \frac{2v_{\parallel} u_{\parallel b}}{v_e^2} \left(\frac{n_b e_b^2}{n_e e^2} \right) v_{eo} f_{Me}^s(x), \quad (5a)$$

where

$$s(x) = \begin{cases} x^{-3} \left(1 + \frac{6}{5} \bar{v}_b^2\right) & \bar{v}_b < x \\ \bar{v}_b^{-3} \left(\frac{6}{5} x^2 - 2\right) & \bar{v}_b > x \end{cases} \quad (5b)$$

and $\bar{v}_b = v_b/v_e$. Note the discontinuity in $s(x)$ at $v_b = v$.

Since we are interested in calculating the beam current, defined in Eq. (3), it is convenient to introduce the electron distribution in the beam frame:

$$f_{eb} \equiv f_{e1} - \frac{2v_{\parallel} u_{\parallel b}}{v_e^2} \frac{n_b e_b}{n_e |e|} f_{eo} . \quad (6)$$

In terms of f_{eb} , the current is $j_{\parallel} = -|e| \int v_{\parallel} f_{eb} dv$. Furthermore, since electron-electron collisions conserve momentum, $C_{ee}(f_{eb}, f_{Me}) + C_{ee}(f_{Me}, f_{eb}) \equiv C_{ee}(f_{eb}) = C_{ee}(f_{e1})$, f_{eb} satisfies the following modified kinetic equation:

$$C_{ee}(f_{eb}) + \frac{v_{eo}}{x^3} \bar{Z} L f_{eb} \equiv C_e(f_{eb}) = \bar{S}_b(\underline{v}) , \quad (7a)$$

where

$$\bar{S}_b(\underline{v}) = -2(v_{\parallel} u_{\parallel b} / v_e^2) (n_b e_b^2 / n_e e^2) v_{eo} \bar{s}(x) f_{Me}$$

and

$$\bar{s}(x) = \begin{cases} x^{-3} \left[\left(1 - \frac{\bar{Z}}{Z_b}\right) + \frac{6}{5} \bar{v}_b^2 \right] & \bar{v}_b < x \\ -x^{-3} \frac{\bar{Z}}{Z_b} - \bar{v}_b^{-3} \left(2 - \frac{6}{5} x^2\right) & \bar{v}_b > x \end{cases} \quad (7b)$$

Note that for $\bar{Z} \approx Z_b \equiv e_b/|e|$ and $\bar{v}_b < 1$, typical of tokamak parameters, the discontinuity in $\bar{s}(x)$ is especially severe. Thus, the solution of Eq. (7a) must be obtained with extreme precision in order to compute the beam-induced current accurately in this relevant case. To improve the accuracy in the calculation of $j_{||}$, a variational expression for the beam-induced current is now derived.

3. VARIATIONAL EXPRESSION FOR THE BEAM-INDUCED CURRENT

The classical variational principle³ for Eq. (7a) is formed from the entropy production subject to the constraint of nonzero momentum production:

$$\dot{S}_1 = \int \phi_{eb} C_e(f_{eb}) d\underline{v} - 2 \int \phi_{eb} \bar{S}_b(\underline{v}) d\underline{v}, \quad (8)$$

where $\phi_{eb} = f_{eb}/f_{Me}$. Although $\dot{S}_1$ is stationary with respect to variations in ϕ_{eb} when Eq. (7a) is satisfied, its extremum value is $-\int \phi_{eb} \bar{S}_b(\underline{v}) d\underline{v}$, which is not the plasma current for $\bar{S}_b(\underline{v})$ given by Eq. (7b).

In order to form the appropriate variational quantity, consider the "adjoint" equation

$$C_e(f_{es}) = v_{eo} \frac{v_{\parallel}}{v_e} f_{eo}. \quad (9)$$

Equation (9) is recognized as the normalized Spitzer equation,² with v_{eo} replacing the usual acceleration term $2|e|E_{\parallel}/(m_e v_e)$. Multiplying Eq. (9) by ϕ_{eb} and using the self-adjoint property of C_e , i.e., $\int \phi_{eb} C_e(f_{es}) d\underline{v} = \int \phi_{es} C_e(f_{eb}) d\underline{v}$, yields

$$j_{\parallel} = - \frac{|e|v_e}{v_{eo}} \int (f_{es}/f_{eo}) \bar{S}_b(\underline{v}) d\underline{v} = j_{\parallel b} Z_b \int \frac{2v_{\parallel}}{v_e} \frac{f_{es}}{n_e} \bar{s}(x), \quad (10)$$

where $j_{\parallel b} = n_b e_b u_{\parallel b}$ is the unshielded current carried by the fast beam ions. Note that the Spitzer function f_{es} is the Green's function for the beam-induced current.

Consider the hybrid entropy production rate:

$$\begin{aligned} \dot{S}_2 = & \int \frac{1}{2} [\phi_{es} C_e(f_{eb}) + \phi_{eb} C_e(f_{es})] d\underline{v} \\ & - v_{eo} \int \frac{v_{\parallel}}{v_e} f_{eb} d\underline{v} - \int \phi_{es} \bar{S}_b(\underline{v}) d\underline{v} . \end{aligned} \quad (11)$$

Variation of $\dot{S}_2$ with respect to ϕ_{es} and ϕ_{eb} yields Eqs. (7a) and (9), respectively. The current can then be evaluated variationally in terms of the extremal value of $\dot{S}_2$, denoted by S^* :

$$j_{\parallel} = - \frac{e_b v_e}{v_{eo}} S^* . \quad (12)$$

For practical computations, it is convenient to expand ϕ_{es} and ϕ_{eb} in a series of energy polynomials $L_n(x)$:

$$\phi_{es} = \frac{2v_{\parallel}}{v} \sum a_n L_n(x) , \quad (13a)$$

$$\phi_{eb} = \frac{2v_{\parallel}}{v} \sum b_n L_n(x) . \quad (13b)$$

For the special case when the same indices of L_n are chosen for the trial functions ϕ_{es} and ϕ_{eb} in the variational principle Eq. (11), the current may be obtained variationally from Eq. (10), with ϕ_{es} determined by the classical variational principle for the electrical conductivity $\delta \dot{S}_3 = 0$, where³

$$\dot{S}_3 = \int \phi_{es} C_e(f_{es}) d\underline{v} - 2v_{eo} \int \frac{v_{||}}{v_e} f_{es} d\underline{v} . \quad (14)$$

As shown in Sect. 5, the beam-induced current can be adequately computed using the restricted variational principle denoted by Eqs. (14) and (10). It should be noted that because the source term in the Spitzer Eq. (9) is a smooth function of velocity, it is easier to obtain an accurate analytic approximation for f_{es} than for f_{eb} .

It is possible to use Eq. (10), together with limited knowledge about f_{es} , to obtain asymptotic expressions for $j_{||}$ in the limits $\bar{v}_b \rightarrow 0$, ∞ , without first solving Eq. (9). Consider the limit $\bar{v}_b \ll 1$, which is of interest for tokamaks. Then:

$$\begin{aligned} \frac{j_{||}}{j_{||b}} &= Z_b \int \frac{2v_{||}}{v_e} f_{es} x^{-3} \left(1 - \frac{\bar{Z}}{Z_b} + \frac{6}{5} \bar{v}_b^2 \right) d\underline{v} \\ &- \int_{x < \bar{v}_b} \frac{2v_{||}}{v_e} f_{es} \left[x^{-3} \left(1 + \frac{6}{5} \bar{v}_b^2 \right) + \frac{1}{\bar{v}_b^3} \left(2 - \frac{6}{5} x^2 \right) \right] d\underline{v} . \quad (15a) \end{aligned}$$

Now, it can be shown^{1,2} that as $x \rightarrow 0$, $f_{es}/x \rightarrow 0$. Thus, the last term in Eq. (15a) is smaller than $O(\bar{v}_b^2)$. Using conservation of momentum in Eq. (9) and recalling $\bar{Z} x^{-3} f_{es} = -C_{ei}(f_{es})/v_{eo}$ yields

$$j_{||}/j_{||b} = 1 - \frac{Z_b}{\bar{Z}} \left(1 + \frac{6}{5} \bar{v}_b^2 \right) , \quad \bar{v}_b \ll 1 , \quad (15b)$$

in agreement with the result obtained in Ref. 1 by an entirely different method.

In the opposite (cold electron) limit $\bar{v}_b \gg 1$,

$$\begin{aligned}
 j_{\parallel} / j_{\parallel b} &= -Z_b \int \frac{2v_{\parallel}}{v_e} \left[x^{-3} \frac{\bar{Z}}{Z_b} + \bar{v}_b^{-3} \left(2 - \frac{6}{5} x^2 \right) \right] f_{es} dv + 0 \left(e^{-\bar{v}_b^2} \right) \\
 &= 1 - \frac{Z_b}{\bar{v}_b^3} [\Lambda_0(\bar{Z}) + 3\Lambda_1(\bar{Z})] \left(\frac{3\sqrt{\pi}}{4} \right). \quad (15c)
 \end{aligned}$$

Here, Λ_0 is the normalized Spitzer conductivity and Λ_1 is the classical thermoelectric coefficient:⁶

$$\Lambda_0 = - \frac{4}{3\sqrt{\pi}} \int \frac{2v_{\parallel}}{v_e} f_{es} dv = - \frac{1}{Z} \left(\frac{1 + 0.374Z}{1 + 0.292Z} \right) \left(\frac{1.085 + 2.783Z + 0.424Z^2}{1 + 1.161Z + 0.16Z^2} \right), \quad (16a)$$

$$\Lambda_1 = - \frac{8}{15\sqrt{\pi}} \int \frac{2v_{\parallel}}{v_e} \left(x^2 - \frac{5}{2} \right) f_{es} dv = \frac{0.877 + 0.695Z + 0.095Z^2}{(1 + 0.292Z)(1 + 1.161Z + 0.16Z^2)}. \quad (16b)$$

4. ANALYTIC DETERMINATION OF THE SPITZER FUNCTION

An analytic expression for f_{es} will now be obtained using the variational principle, Eq. (14). Expressions for f_{es} have been previously obtained^{6,7} in terms of an expansion in Sonine polynomials with energy argument x^2 . The coefficients in this expansion were determined by taking appropriate moments of the kinetic Eq. (9). [Identical results would have been obtained had the variational principle Eq. (14) been used with the Sonine expansion considered as a trial function.]

The two lowest order terms in such an expansion yield an accurate result for the classical parallel conductivity, which is a variational quantity. However, this truncated expansion does not give an adequate representation of the local energy dependence of the Spitzer function. In view of the discontinuity in the beam source term appearing in Eq. (10), a more accurate expression for f_{es} is required. One possible way to improve the analytic approximation for f_{es} is to increase the number of terms in the Sonine expansion. However, because the energy dependence of the collision operator is benign, $C/v_{eo} \sim x^{-2}$ or x^{-3} , and the source term in the Spitzer Eq. (9) also has a weak energy dependence $\sim x$, a rapid increase in the accuracy of this representation is not to be expected by introducing higher powers of x in the Sonine expansion.

To obtain a more rapidly convergent representation for f_{es} , consider the Lorentz limit $\bar{Z} \rightarrow \infty$, in which electron-electron collisions are ignorable. In this limit, $f_{es} = -\frac{v_{||}}{v_e} \frac{x^3}{\bar{Z}} f_{eo}$; thus, the Sonine expansion in even powers of x attempts to approximate the odd power x^3 dependence of f_{es} . The expansion of f_{es} can be effectively renormalized by

introducing a trial function for f_{es} which includes both even and odd, relatively low powers of x . Therefore, assume $f_{es} = -\frac{v_{\parallel}}{v} D(x) f_{eo}$, where

$$D(x) = x(d_1x + d_2x^2 + d_3x^3 + d_4x^4) . \quad (17)$$

The lowest power of x in Eq. (17) was chosen so that $f_{es}/x \rightarrow 0$, which was required to obtain the low energy asymptotic expansion in Eq. (15b). The highest power of x was chosen to obtain an accurate representation of the Spitzer current (Λ_0) and heat conduction (Λ_1), which are required to obtain the high energy asymptotic behavior in Eq. (15c).

If Eq. (17) is inserted as a trial function in the variational principle Eq. (14), the momentum conservation condition implied by the $v_{\parallel}$ moment of Eq. (9) will not be automatically satisfied. [Had a d_0 term in Eq. (17) been retained, momentum conservation would have been one of the variational constraint equations. However, such a term is not compatible with the small x behavior of f_{es} .] To guarantee momentum conservation for the trial function of Eq. (17), an additional term $-2(\lambda/v_e) \int [v_{\parallel} C_e(f_{es}) - v_{eo} (v_{\parallel}^2/v_e) f_{eo}]$ is appended to the variational quantity $\dot{S}_3$ in Eq. (14). The Lagrange multiplier λ is then determined by the constraint that its coefficient in $\dot{S}_3$ vanish, i.e., momentum conservation.

Define the matrix elements of the collision operator:

$$m_{ij} \equiv -\int \frac{v_{\parallel}}{v_e} x^i C_e \left(\frac{2v_{\parallel}}{v_e} x^j f_{eo} \right) dv = m_{ji} . \quad (18)$$

Inserting $D(x)$ from Eq. (17) into the appended variational principle yields

$$\dot{S}_3 = -\frac{1}{2} \left[\sum_{i=1}^4 \sum_{j=1}^4 m_{ij} d_i d_j - \frac{4n_e}{\tau_{ee}} \sum_{i=1}^4 \langle x^i \rangle d_i + 2\lambda \left(\sum_{i=1}^4 m_{oi} d_i - v_{eo} n_e \right) \right]. \quad (19)$$

Here, $\tau_{ee}^{-1} = (4/3\sqrt{\pi})v_{eo}$ and

$$\langle x^j \rangle = \int_0^\infty x^{j+4} e^{-x^2} dx = \frac{1}{2} \Gamma\left(\frac{1}{2}j + \frac{5}{2}\right). \quad (20)$$

The computation of the matrix elements is considered in the Appendix.

For the present case of interest, the required normalized coefficients

$\bar{m}_{ij} = m_{ij}(n_e/\tau_{ee})^{-1}$ are given explicitly as:

$$\bar{m}_{01} = (\sqrt{\pi}/2)\bar{Z} \quad (21a)$$

$$\bar{m}_{02} = \bar{Z} \quad (21b)$$

$$\bar{m}_{03} = (3\sqrt{\pi}/4)\bar{Z} \quad (21c)$$

$$\bar{m}_{04} = 2\bar{Z} \quad (21d)$$

$$\bar{m}_{11} = \left(\frac{151}{30}\sqrt{2} - \frac{104}{15}\right) + \bar{Z} \quad (21e)$$

$$\bar{m}_{12} = \frac{4}{\sqrt{\pi}} - \sqrt{\pi} + (3\sqrt{\pi}/4)\bar{Z} \quad (21f)$$

$$\bar{m}_{13} = (607/40)\sqrt{2} - 102/5 + 2\bar{Z} \quad (21g)$$

$$\bar{m}_{14} = (26/\sqrt{\pi}) - 7\sqrt{\pi} + (15\sqrt{\pi}/8)\bar{Z} \quad (21h)$$

$$\bar{m}_{22} = \sqrt{2} + 2\bar{Z} \quad (21i)$$

$$\bar{m}_{23} = (6/\sqrt{\pi}) + (15\sqrt{\pi}/8)\bar{Z} \quad (21j)$$

$$\bar{m}_{24} = (11/2) \sqrt{2} + 6\bar{Z} \quad (21k)$$

$$\bar{m}_{33} = (6147/160) \sqrt{2} - 228/5 + 6\bar{Z} \quad (21l)$$

$$\bar{m}_{34} = (45/\sqrt{\pi}) - (9\sqrt{\pi}/4) + (105\sqrt{\pi}/16)\bar{Z} \quad (21m)$$

$$\bar{m}_{44} = (157/4) \sqrt{2} + 24\bar{Z} . \quad (21n)$$

The variation of $\dot{S}_3$ in Eq. (19) with respect to d_j , for $j = (1,2,3,4)$, yields the matrix equation

$$\underline{\bar{m}} \cdot \underline{d} + \lambda \underline{\bar{m}}_0 = \underline{\Gamma} , \quad (22a)$$

where $(\underline{\bar{m}})_{ij} = \bar{m}_{ij}$, $(\underline{d})_i = d_i$, $(\underline{\bar{m}}_0)_i = \bar{m}_{0i}$, and $(\underline{\Gamma})_i = \Gamma\left(\frac{1}{2}i + \frac{5}{2}\right)$. The Lagrange multiplier λ is determined by momentum conservation:

$$\underline{\bar{m}}_0 \cdot \underline{d} = (3\sqrt{\pi}/4) . \quad (22b)$$

Solving Eqs. (22a,b) simultaneously yields:

$$d_1 = (4.397 - 2.32\bar{Z} - 0.283\bar{Z}^2)/G(\bar{Z}) \quad (23a)$$

$$d_2 = (0.793\bar{Z}^2 + 8.053\bar{Z} - 4.627)/G(\bar{Z}) \quad (23b)$$

$$d_3 = (0.0467\bar{Z}^3 + 0.108\bar{Z}^2 - 4.136\bar{Z} + 2.006)/G(\bar{Z}) \quad (23c)$$

$$d_4 = (-0.011\bar{Z}^2 + 0.716\bar{Z} - 0.304)/G(\bar{Z}) , \quad (23d)$$

where $G(\bar{Z}) = \bar{Z}(1 + 0.292\bar{Z})(1 + 1.161\bar{Z} + 0.16\bar{Z}^2)$. Note that for $\bar{Z} \rightarrow \infty$, $d_j = \delta_{j3}/\bar{Z}$ and the Lorentz result is recovered. For $\bar{Z} = 1$ and 2, the function $\bar{Z} D(x, \bar{Z})$ computed variationally here is compared in Table I with the numerical results.² Even though the analytic distribution function has been computed from a variational principle for the current (a moment of f_{es}), there is good point-wise agreement in velocity space between the present result and the numerical one.

Table I. Spitzer function $ZD(x, Z)$

x	Z = 1		Z = 2	
	Numerical	Analytic	Numerical	Analytic
0.15	0.0053	0.018	0.0012	0.003
0.20	0.017	0.034	0.005	0.009
0.30	0.066	0.087	0.028	0.037
0.40	0.155	0.17	0.084	0.094
0.60	0.45	0.45	0.33	0.33
0.80	0.90	0.88	0.77	0.75
1.20	2.27	2.27	2.35	2.33
1.60	4.38	4.39	5.10	5.06
2.08	8.19	8.15	10.54	10.33
2.40	11.83	11.7	16.00	15.6
2.88	19.53	19.8	27.7	27.6
3.20	26.00	28.1	38.5	39.9
$ZD(x, Z) = \begin{cases} x^2(0.60 + 1.41x - 0.66x^2 + 0.134x^3); & Z = 1 \\ 2x^2(-0.11 + 1.17x - 0.44x^2 + 0.086x^3); & Z = 2 \end{cases}$				

5. EVALUATION OF THE BEAM-INDUCED CURRENT

Using the variational expression Eq. (10) and the results of Sect. 4, the parallel beam-induced current $j_{\parallel}/j_{\parallel b} = F(\bar{v}_b, \bar{z})$ may be evaluated:

$$F = -Z_b \sum_{j=1}^4 d_j \bar{T}_j, \quad (24a)$$

where

$$\bar{T}_j \equiv \frac{8}{3\sqrt{\pi}} \int_0^{\infty} x^{4+j} e^{-x^2} \bar{s}(x) dx. \quad (24b)$$

Using momentum conservation, Eq. (22b), this may be reduced to the following convenient form:

$$F = 1 - \frac{Z_b}{\bar{z}} \left(1 + \frac{6}{5} \bar{v}_b^2 \right) \text{erfc}(\bar{v}_b) + Z_b \sum_{j=1}^4 d_j(\bar{z}) \bar{T}_j(\bar{v}_b). \quad (24c)$$

Here,

$$T_1 = \frac{32}{15} \frac{1}{\bar{v}_b^3} \left[\left(1 + \bar{v}_b^2 + \frac{3}{16} \bar{v}_b^4 \right) \text{erf}'(\bar{v}_b) - 2/\sqrt{\pi} \right] \quad (25a)$$

$$T_2 = \frac{4}{3\sqrt{\pi}} \left(1 + \frac{6}{5} \bar{v}_b^2 \right) \text{erfc}(\bar{v}_b) - \frac{11}{2} \frac{1}{\bar{v}_b^3} \left[\text{erf}(\bar{v}_b) - \bar{v}_b \left(1 + \frac{6}{11} \bar{v}_b^2 \right) \text{erf}'(\bar{v}_b) \right] \quad (25b)$$

$$T_3 = \frac{56}{5} \frac{1}{\bar{v}_b^3} \left[\left(1 + \bar{v}_b^2 + \frac{23}{56} \bar{v}_b^4 \right) \text{erf}'(\bar{v}_b) - 2/\sqrt{\pi} \right] \quad (25c)$$

$$T_4 = \frac{8}{3\sqrt{\pi}} \left(1 + \frac{6}{5} \bar{v}_b^2 \right) \text{erfc}(\bar{v}_b) - \frac{119}{4\bar{v}_b^3} \left[\text{erf}(\bar{v}_b) - \bar{v}_b \left(1 + \frac{74}{119} \bar{v}_b^2 + \frac{20}{119} \bar{v}_b^4 \right) \text{erf}'(\bar{v}_b) \right], \quad (25d)$$

where $\text{erf}(x) = (2/\sqrt{\pi}) \int_0^x \exp(-t^2) dt$, $\text{erfc}(x) = 1 - \text{erf}(x)$, and $\text{erf}'(x) = (2/\sqrt{\pi}) \exp(-x^2)$. The small and large $\bar{v}_b$ limits of this expression confirm the asymptotic results obtained in Eq. (15).

Table II gives values for the beam-induced current ratio F for various values of $\bar{Z}$ ($Z_b = 1$ was assumed) and $\bar{v}_b$. Comparison with Table IV of Ref. 1 shows that the analytic result is in very close agreement, over a wide range of effective charges and beam energies, with the calculated numerical values for F . Figures 1 and 2 show the comparison between the analytic and numerical results for F , for $\bar{Z} = 1$ and 2, respectively. Also shown are the asymptotes calculated in Eq. (15).

Finally, recall that the expression for F in Eq. (24c) is a Green's function (of argument $\bar{v}_b$) for the actual beam current [cf. Eq. (46)]. It is therefore useful to obtain a simple analytic approximation for the quantity F . Such an expression is suggested by patching the asymptotic limits for F computed in Eq. (15) to obtain:

$$F \cong 1 - \frac{Z_b}{\bar{Z}} \frac{\left(1 + \frac{6}{5} \frac{\bar{v}_b^2}{n}\right)^n}{\left[1 + \left(\frac{\bar{v}_b}{\alpha}\right)^m\right]^{(3+2n)/m}} \quad (26a)$$

where $m > 2$ and

$$\alpha = \left[\frac{3\sqrt{\pi}}{4} \bar{Z} (\Lambda_0 + 3\Lambda_1) \right]^{1/(3+2n)} \left(\frac{5n}{6} \right)^{n/(3+2n)} \quad (26b)$$

Least-squares fitting of Eq. (26) with the data in Table II yields, for $\bar{Z} \geq 1$:

$$n(\bar{Z}) = 1.65 + \frac{0.22}{\bar{Z} - 0.55} \quad (27a)$$

$$m(\bar{Z}) = 3.37 - \frac{0.85}{\bar{Z} + 0.48} \quad (27b)$$

As an example, consider $\bar{Z} = Z_b = 1$. Then, $n = 2.14$, $m = 2.80$, $\alpha = 1.47$ and

$$F = 1 - \frac{(1 + 0.56\bar{v}_b^2)^{2.14}}{(1 + 0.34\bar{v}_b^{2.8})^{2.6}} \quad (28)$$

For intermediate values of $\bar{v}_b = (0.8, 1.0, 1.3, 1.5, 2.0)$, for which this approximation is expected to be worst, one obtains $F \cong (-0.247, -0.210, -0.033, 0.124, 0.474)$ in close agreement with the values of F in Table II.

Table II. Ratio of net current to fast ion current, F ,
calculated from Eq. (24c)

v_b/v_e	$Z = 1.0$	1.1	1.2	1.5	2.0	4.0	8.0	16.0
0.1	-0.012	0.080	0.156	0.325	0.493	0.747	0.873	0.937
0.2	-0.041	0.053	0.131	0.304	0.477	0.738	0.869	0.934
0.3	-0.084	0.012	0.093	0.270	0.450	0.723	0.861	0.931
0.5	-0.177	-0.078	0.005	0.191	0.383	0.683	0.840	0.920
0.7	-0.237	-0.140	-0.058	0.129	0.324	0.643	0.816	0.907
0.8	-0.243	-0.149	-0.070	0.113	0.307	0.627	0.806	0.901
1.0	-0.203	-0.118	-0.046	0.121	0.302	0.612	0.793	0.893
1.3	-0.034	0.032	0.088	0.221	0.368	0.633	0.797	0.892
1.5	0.116	0.169	0.215	0.324	0.445	0.669	0.813	0.899
2.0	0.470	0.499	0.524	0.584	0.653	0.784	0.873	0.930
3.0	0.822	0.831	0.839	0.858	0.881	0.924	0.955	0.975
3.8	0.912	0.917	0.921	0.930	0.941	0.963	0.978	0.988
6.0	0.978	0.979	0.980	0.982	0.985	0.991	0.994	0.997
10.0	0.995	0.995	0.996	0.996	0.997	0.998	0.999	0.999

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APPENDIX

The matrix elements of the Coulomb collision operator defined in Eq. (18) will now be computed. There are two contributions to m_{ij} :

$$m_{ij} = m_{ij}^{ei} + m_{ij}^{ee}, \quad (\text{A.1})$$

where $m_{ij}^{ab} = -\int (v_{\parallel}/v_e) x^i C_{ab} \left(\frac{2v_{\parallel}}{v_e} x^i f_{Me} \right) d\underline{v}$. The contribution to m_{ij} from electron-ion collisions is readily evaluated using the Lorentz form of the collision operator:

$$m_{ij}^{ei} = \frac{n_e \bar{Z}}{\tau_{ee}} \Gamma \left[\frac{1}{2} (i + j) + 1 \right]. \quad (\text{A.2})$$

The electron-electron collision operator contribution is most easily computed using the Landau form⁷ for the collision operator. Letting

$$\phi_j(\underline{v}) = \frac{v_{\parallel}}{v_e} x^j \text{ yields}$$

$$m_{ij}^{ee} = \frac{n_e}{\tau_{ee}} (m_{ij}^T + m_{ij}^F), \quad (\text{A.3})$$

where

$$m_{ij}^T = \frac{3\sqrt{\pi}}{4\pi^3 v_e^3} \int d\underline{v} d\underline{v}' \partial_{\ell} \phi_i(\underline{v}) \partial_k \phi_j(\underline{v}) \exp[-(x^2 + x'^2)] U_{\ell k} \quad (\text{A.4a})$$

$$m_{ij}^F = - \frac{3\sqrt{\pi}}{4\pi^3 v_e^3} \int d\underline{v} d\underline{v}' \partial_{\ell} \phi_i(\underline{v}) \partial_{\ell'} \phi_j(\underline{v}') \exp[-(\underline{x}^2 + \underline{x}'^2)] U_{\ell k} . \quad (\text{A.4b})$$

Here, $U_{\ell k} = (u^2 \delta_{\ell k} - u_{\ell} u_k)/u^3$, $\underline{u} = \underline{v} - \underline{v}'$, $\partial_{\ell} \equiv \partial/\partial v_{\ell}$, and repeated indices are summed over. Taking a coordinate system aligned with the local $\underline{B}$ -field, so that $\underline{B}/B = \hat{e}_1$, yields

$$\begin{pmatrix} m_{ij}^T \\ m_{ij}^F \end{pmatrix} = \frac{3\sqrt{\pi}}{4\pi^3 v_e^5} \int d\underline{v} d\underline{v}' x^i \left(\delta_{\ell 1} + \frac{iv_1 v_{\ell}}{v^2} \right) U_{\ell k} \exp[-(\underline{x}^2 + \underline{x}'^2)] \times \left\{ \begin{array}{l} x^j \left(\delta_{k1} + \frac{iv_1 v_k}{v^2} \right) \\ -(x')^j \left[\delta_{k1} + \frac{iv'_1 v'_k}{(v')^2} \right] \end{array} \right\} . \quad (\text{A.5})$$

In order to evaluate these coefficients, it is convenient to write $\underline{v} = v\hat{v}$ in spherical coordinates (v, θ, ϕ) where the azimuthal angle θ is measured with respect to $\hat{e}_1$, i.e., $\hat{v} = \hat{e}_1 \cos \theta + \hat{e}_2 \sin \theta \cos \phi + \hat{e}_3 \sin \theta \sin \phi$. Then, $\hat{v}$ can be taken as a new axis from which $\underline{v}'$ is measured in spherical coordinate (v', θ', ϕ') . Thus, for fixed $\hat{v}$, two orthogonal unit vectors can be defined, $\hat{x} = \sin \theta \hat{e}_1 - \cos \theta \cos \phi \hat{e}_2 - \cos \theta \sin \phi \hat{e}_3$, $\hat{y} = \sin \phi \hat{e}_2 - \cos \phi \hat{e}_3$, such that $\hat{x} \cdot \hat{y} = \hat{x} \cdot \hat{v} = \hat{y} \cdot \hat{v} = 0$. In terms of this $(\hat{v}, \hat{x}, \hat{y})$ coordinate system, $\underline{v}' = v'(\cos \theta' \hat{v} + \sin \theta' \times \cos \phi' \hat{x} + \sin \theta' \sin \phi' \hat{y})$, and $-\underline{u} = (v' \cos \theta' - v) \hat{v} + v' \sin \theta' \times \cos \phi' \hat{x} + v' \sin \theta' \cos \phi' \hat{y}$. After some straightforward integrations, the following results are obtained:

$$m_{ij}^T = \frac{2}{\sqrt{\pi}} \Gamma\left(\frac{i+j+3}{2}\right) \left\{ G(i+j+1, 0) + \left(1 + \frac{ij}{i+j}\right) \left[G(i+j+1, 0) - 2^{-\frac{(i+j+1)}{2}} \right] \right\} \quad (A.6a)$$

$$m_{ij}^F = -\frac{4}{\sqrt{\pi}} \Gamma\left(\frac{i+j+5}{2}\right) \left\{ \left(2 + \frac{19}{15}i + \frac{1}{5}i^2\right) G(j+1, i+2) + \left(2 + \frac{19}{15}j + \frac{1}{5}j^2\right) G(i+1, j+2) - \left[\frac{1}{3} + \frac{1}{10}(i+j)\right] 2^{-\frac{(i+j+1)}{2}} \right\} \quad (A.6b)$$

where

$$G(m, n) \equiv \int_0^{\pi/4} \cos^m \theta \sin^n \theta \, d\theta. \quad (A.7)$$

Useful recursion relations for G are obtained by integrating Eq. (A.7)

by parts:

$$G(m, n) = G(m-2, n) \frac{(m-1)}{m+n} + \frac{2^{-(m+n)/2}}{m+n} \quad (A.8a)$$

$$G(m, n) = G(m, n-2) \frac{(n-1)}{m+n} - \frac{2^{-(m+n)/2}}{m+n} \quad (A.8b)$$

Initial values are easily obtained by direct integration:

$$G(0, 0) = \frac{\pi}{4} \quad (A.9a)$$

$$G(0,1) = 1 - \frac{1}{\sqrt{2}} \quad (\text{A.9b})$$

$$G(1,0) = \frac{1}{\sqrt{2}} \quad (\text{A.9c})$$

$$G(1,1) = \frac{1}{4} . \quad (\text{A.9d})$$

Previously, these matrix elements had been computed for even integer powers $(i,j) = (2m,2n)$ only. The formulas in Eqs. (A.6)-(A.8) are, however, valid for arbitrary (nonintegral) real numbers i and j , and were used to compute the matrix elements required in Eq. (21).

FIGURE CAPTIONS

FIG. 1. Graph of beam current ratio F as a function of $\bar{v}_b$ from Eq. (24c) for $\bar{Z} = Z_b = 1$ (solid curve). Open points are numerical results from Ref. 1. Dashed curves are asymptotic curves from Eqs. (15b-c).

FIG. 2. Graph of beam current ratio F as a function of $\bar{v}_b$ from Eq. (24c) for $\bar{Z} = 2$ and $Z_b = 1$ (solid curve). Open points are numerical results from Ref. 1. Dashed curves are asymptotic curves from Eqs. (15b-c).

FIG. 1.

FIG. 2.

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