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WAVES WITH THE INTEGRAL
EIGENMODE EQUATION

PLASMA PHYSICS LABORATORY

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On the Stability of Drift Waves with the Integral
Eigenmode Equation

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ABSTRACT

An analytical theory on the stability properties of drift-wave eigenmodes in a slab plasma with finite magnetic shear is presented. The corresponding eigenmode equation is the integral equation first given by Coppi, Rosenbluth, and Sagdeev (1967) and rederived here, in a relatively simpler fashion, via the gyrokinetic equation. It is then proved that the universal drift-wave eigenmodes remain absolutely stable and finite electron temperature gradients do not alter the stability.

I. INTRODUCTION

In recent years, there has been a significant progress in the understanding of stability properties of drift-wave eigenmodes in a plasma slab with finite magnetic shear. Most of the work, however, is limited to the second-order differential equation approximation (Ross and Mahajan, 1978; Tsang et al., 1978; Antonsen, 1978; Chen et al., 1978, 1979; and Lee and Chel, 1979a) with a somewhat restricted regime of validity; i.e., $|\rho_i^2 d^2/dx^2| \ll 1$. Later on, stability investigations have been extended analytically (Lee, Chen, and Nevins, 1980, hereafter called Paper LCN) and numerically (Tang, Rewoldt, and Frieman, 1980) to arbitrary $|\rho_i^2 d^2/dx^2|$ via an integral equation formulation. Recently, however, it has been pointed out independently by Chen et al., (1981) and Linsker (1981) that the integral equation employed in previous studies is improper due to the breakdown of WKB assumption, which is generally the case for ion drift modes as well as electron drift modes when the magnetic shear is strong. The proper equation to be used, in fact, was derived some years ago by Coppi, Rosenbluth, and Sagdeev (1967) using orbital integrations. Similar results have also been obtained by Bellew and Bakshi (1977), Drummond and Gentle (1967) as well as Antonsen and Manheimer (1978).

In the present work, we first demonstrate that the proper integral eigenmode equation can be derived in a relatively simpler way via the gyrokinetic formulation first developed by Taylor and Hastie (1968) as well as Rutherford and Frieman (1968). We then show that the electrostatic universal drift-wave eigenmodes remain stable with this proper equation and that finite electron temperature gradients do not affect the stability nature.

The derivation of the integral equation is given in Sec. II. Section III contains the stability proof. Final conclusions and discussion are given in Sec. IV.

II. DERIVATION OF THE INTEGRAL EIGENMODE EQUATION

In the following derivations, we adopt Catto's approach to the linear gyrokinetic formalism (Catto, 1978). Thus, we first make a transformation from the particle phase space (x, v) to the guiding-center phase space $(\tilde{x}, \tilde{v})$ given by

$$\tilde{x} = x + v \times e_{\parallel} / \Omega, \quad (1)$$

$$\tilde{v} = \tilde{v}(\epsilon, \mu, \xi) \quad (2)$$

where $\epsilon = v^2/2$, $\mu = v_{\perp}^2/B$, $\xi_{\parallel} = v_{\parallel}/B$, and ξ is the gyrophase angle. Now the j th-species perturbed distribution function, δf_j , can be written as

$$\delta f_j = -(q/T)_j \delta \phi f_{oj} + \delta h_j. \quad (3)$$

Here, f_{oj} is the equilibrium distribution function and is taken to be a Maxwellian. Assuming perturbations of the form

$$\begin{aligned} \delta h_j(x, v, t) &= h_j(x, v) \exp[i(k_y y - \omega t)] \\ &= H_j(\tilde{x}, \tilde{v}) \exp[i(k_y Y - \omega t)] \quad (4) \end{aligned}$$

It can then be shown (see Catto, 1978) that $\partial H_j / \partial \xi = 0$ and

$$H_j = -(q/T)_j \langle \Phi \rangle_\alpha (\omega + \omega_{*j}) f_{0j} / (k_{\parallel} v_{\parallel} - \omega) \quad (5)$$

In Eq. (5), $\langle \dots \rangle_\xi \equiv (1/2\pi) \oint d\xi (\dots)$ corresponds to gyrophase averaging, $\omega_{*j} = (k_y c T / q R L_n)_j$ is the diamagnetic drift frequency, $L_n^{-1} = -d(\ln N_{0j})/dx$ and, for the sheared slab model considered here,

$$\vec{r} = R_0 (\vec{e}_\phi + x \vec{e}_y / L_s) \quad (6)$$

and

$$k_{\parallel} = \langle \vec{v}_\parallel \cdot \vec{\nabla}_X \rangle_\xi = k_y X / L_s \quad (7)$$

We have, furthermore, ignored corrections of $O(x^2/L_s^2)$ and temperature gradients. Defining the Fourier transforms as

$$f(\vec{r}, G) = \int d\vec{k} [\bar{f}, \bar{G}](\vec{k}) \exp[i\vec{k}(\vec{r}, X)] \quad (8)$$

we then have, noting Eqs. (1) and (4),

$$\bar{f}(\vec{k}) = \bar{G}(\vec{k}) \exp[i\vec{k}_\perp \cdot (\vec{y} \times \vec{e}_\parallel) / \omega_c] \quad (9)$$

with $\vec{k}_\perp = k \vec{e}_x + k_y \vec{e}_y$. Using Eqs. (3), (4), and (9), the perturbed density $\bar{n}_j(\vec{k})$ is given by

$$\bar{n}_j(\vec{k}) = -(q/T)_j \bar{\Phi}(\vec{k}) + \bar{n}_{2j}(\vec{k}) \quad (10)$$

and

$$\bar{n}_{2j}(k) = \int d^3v \bar{h}_j(k) = 2\pi \int v_{\perp} dv_{\perp} \int dv_{\parallel} \bar{H}_j(k) J_0(k_{\perp} v_{\perp} / \omega_{cj}) \quad (11)$$

Meanwhile, from Eqs. (5) and (9), we have

$$\bar{H}_j(k) = -(q/T)_j (1 + \omega_{*j}) f_{0j} \int dk' \overline{(k_{\parallel} v_{\parallel} - \omega)^{-1} (k - k')} \bar{\phi}(k') J_0(k'_{\perp} v_{\perp} / \omega_{cj}) \quad (12)$$

and $k'_{\perp} = k'_{\perp x} + k'_{\perp y}$. Substituting Eq. (12) into Eq. (11), we find

$$\bar{n}_{2j}(k) = -(q/T)_j (1 + \omega_{*j} / \omega) \int dk' \bar{\phi}(k') \tilde{F}_{0j}(k, k') \overline{\xi_j Z_j}(k - k') \quad (13)$$

where

$$\tilde{F}_{0j}(k, k') = I_0(B_j) \exp(-\hat{b}_j) \quad (14)$$

$B_j = \rho_j^2 k_{\perp} k'_{\perp} / 2$, $\hat{b}_j = \rho_j^2 (k_{\perp}^2 + k'_{\perp}^2) / 4$, $\xi_j = \omega / |k_{\parallel} v_{\perp j}|$, $Z_j = Z(\xi_j)$ and Z is the plasma dispersion function. The quasi-neutrality condition $\bar{n}_e(k) = \bar{n}_i(k)$ along with Eqs. (10) and (13) then yields the desired integral eigenmode equation

$$\int dk' \bar{\phi}(k') \{ (\tau + 1/\Omega) \tilde{F}_{01}(k, k') \overline{\xi_1 Z_1}(k - k') + (1 - 1/\Omega) \overline{\xi_e Z_e}(k - k') \} + (\tau + 1) \bar{\phi}(k) = 0 \quad (15)$$

In Eq. (15), $\tau \equiv T_e/T_i$, $\Omega = \omega/\omega_{*e}$, and electron Larmor-radius effects have been ignored. We remark that, if k in $\tilde{\Gamma}_{01}$ is replaced by k' , Eq. (15) reduces to the improper integral equation used in the previous analyses. It can be readily recognized that the replacement of k and k' is equivalent to a WKB approximation for the ion responses and becomes invalid when the scale length of $\phi(x)$ becomes comparable to the ion Landau-damping distance, $(1/k) \sim v_{Ti}/\omega$. Thus for ion drift modes which are generally localized by the ion Landau damping, the WKB approximation cannot usually be justified. Similarly, the WKB approximation also breaks down for electron drift modes if the magnetic shear is sufficiently strong; i.e., $L_n/L_s \sim O(1)$.

III. STABILITY ANALYSIS

In the present work, our approach to the stability analysis consists of the following two steps.

(i) We first show that in the adiabatic-electron limit (i.e., without the $\nabla_{\perp}^2$ term) the eigenmodes described by Eq. (15) are always damped.

(ii) We then show that as we gradually turn on the full electron dynamics, the stability properties remain the same.

To achieve (ii), we introduce an artificial parameter $\lambda (0 \leq \lambda \leq 1)$ and construct the following artificial eigenmode equation from Eq. (15)

$$\int dk' \tilde{\phi}(k') [(\tau + 1/\Omega) \tilde{\Gamma}_{01} \overline{\tilde{\Gamma}_{01}}(k - k') + \lambda(1 - 1/\Omega) \overline{\tilde{\Gamma}_{01}}(k - k')] + (\tau + 1) \tilde{\phi}(k) = 0. \quad (16)$$

Thus, in Eq. (16), λ smoothly connects the limits of adiabatic electrons

($\lambda = 0$) and full electron dynamics ($\lambda = 1$). The corresponding equation in the x space can readily be obtained by expanding $k = k' + k - k'$ in $\tilde{\Gamma}_{01}$ about k' and then performing the inverse Fourier transform, and is given by

$$[(\tau + 1/\Omega)\sigma(-id/dx, x) + \rho(x, \lambda)]\phi(x) = 0, \quad (17)$$

where

$$\sigma(s, x) = \sum_{n=0}^{\infty} (-1)^n (\xi_1 Z_1)^{(n)} F_n(s)/n! , \quad (18)$$

$$F_n(s) = [d^n \tilde{\Gamma}_{01}(k, s)/dk^n]_{k=s} , \quad (19)$$

$$\rho(x, \lambda) = \lambda(1 - 1/\Omega)\xi_e Z_e(x) + (\tau + 1) , \quad (20)$$

and $(\xi_1 Z_1)^{(n)} = d^n \xi_1 Z_1 / dx^n$. We note that Eq. (17) will be used later to derive the asymptotic solutions of $\phi(x)$ as $|x| \rightarrow \infty$.

(III.1) Stability with adiabatic electrons ($\lambda = 0$). Let us first assume that unstable eigenmodes with $\Omega = \Omega_r + i\Omega_i$ and $\Omega_i > 0$ exist. Multiplying Eq. (16) with $\lambda = 0$ by $\int dk \bar{\phi}^*(k)$ and restoring the velocity integrations, we obtain

$$|[(C + 1/\tau)/(C - \Omega)]| + R_1 = 0, \quad (21)$$

where

$$|p| \equiv \int dk \int dk' \int d^3 v \bar{\phi}^*(k) \phi(k') f_{01} J_0(k_1 v_1 / \omega_{ci})$$

$$\times J_0(k'_1 v_1 / \omega_{ce}) \bar{P}(k - k') , \quad (22)$$

$$R_1 = (1 + 1/\tau) \int dk |\bar{\Phi}(k)|^2 - ||1|| , \quad (23)$$

and

$$C = v_A k_y^2 / L_s \omega_{*e} . \quad (24)$$

Noting that $C_1 > 0$, we find from the imaginary part of Eq. (21)

$$||1/D|| = -\tau ||C/D|| , \quad (25)$$

where

$$D = (C - \Omega_r)^2 + \Omega_1^2 . \quad (26)$$

Meanwhile, from the real part of Eq. (21), we find

$$R_1 + R_2 = 0 , \quad (27)$$

where

$$R_2 = ||C^2/D|| + (1/\tau) i ||C/D|| . \quad (28)$$

Using Eq. (25), Eq. (28) can be rewritten as

$$R_2 = [||1/n||]^{-1} [||c^2/n|| ||1/D|| - ||c/n||^2] . \quad (29)$$

From the properties of Fourier transform, it can readily be shown that $||1/n|| \geq 0$. Furthermore, using Schwarz inequality, we have

$$||c^2/n|| ||1/D|| - ||c/n||^2 \geq 0 ;$$

and hence,

$$R_2 \geq 0 . \quad (30)$$

Now since $|J_0| \leq 1$, it is clear that

$$R_1 \geq 0 . \quad (31)$$

With both R_1 and R_2 being non-negative, Eq. (27) allows only trivial solutions, and we have, therefore, proved that no unstable eigenmode exists.

We now show that no marginal solutions ($\Omega_j = 0$) exist, again, by contradiction; that is, we assume eigenmodes with $\Omega = \Omega_r$ do exist, and then demonstrate that, to be physically acceptable, they must be trivial. To do this, we first analytically continue Eq. (17) into the complex plane and examine the asymptotic behavior of $\phi(x)$ as $|x| \rightarrow \infty$ via the WKB approximation $\phi(x) = \phi_0 \exp[i \int^x k_x(x') dx']$. It is then easy to show that k_x satisfies

$$F_0 = i\alpha|x| , \quad (32)$$

where

$$\alpha = (1 + \tau)/[\pi^{1/2} x_1(2\tau + 1)] \quad , \quad (33)$$

$$b_1 = (v_1^2/2)(k_x^2 + k_y^2), \quad F_0 = I_0(b_1) \exp(-b_1), \quad \text{and} \quad x_1 = |v_{\text{ex}} L_S / k_y v_{t1}|.$$

Equation (32) has the following solutions

$$v_x^2 - \frac{v_y^2}{2} = \ln|x| \pm i0(\pi) \quad , \quad (34)$$

where the $\pm i0(\pi)$ term depends on the branch of the logarithmic function. Below v_y , we can determine the asymptotic solutions of $\psi(x)$ and hence, asymptotic Stokes structures. Since Eq. (17) is symmetric about $x = 0$, only $\text{Re}x \geq 0$ needs to be considered. Thus, the asymptotic anti-Stokes lines are given by $\arg(x) = \pm i\pi/2 \pm O(\pi/\ln|x|)$. That is, the subdominant region for $\text{Re}x \geq 0$ is bounded by rays with $\arg(x) = \pm i\pi/2 + O(\pi/\ln|x|)$. The presence of $O(\pi/\ln|x|)$ term indicates that the imaginary axis does not coincide with the anti-Stokes line and thus the flux-conservation technique used in Paper LCN via the $x = -iS_\alpha \eta$ (Antonsen, 1978) transformation is no longer applicable. Here $S_\alpha \equiv \text{sign}(\alpha) = \text{sign}(2\tau + 1)$. Therefore, we introduce a new transformation defined as

$$x = -iS_\alpha \int_0^\eta d\eta' \exp[iS_\alpha \theta(\eta')] \quad , \quad (35)$$

where $\theta(\eta) \geq 0$ is any smoothly varying function in η such that $\theta(\eta) \rightarrow 0$ as $|\eta| \rightarrow 0$ and $\theta(\eta) \rightarrow \theta_0$ as $|\eta| \rightarrow \infty$ with θ_0 being a small but finite positive number. Two examples of $\theta(\eta)$ are

$$\theta(\eta) = \frac{1}{2} \left\{ \begin{array}{l} 1 - \operatorname{sech}^2(\eta/R) \\ 1 - \exp(-\eta^2/R^2) \end{array} \right\} \quad (36)$$

where R is taken to be sufficiently large that

$$R|\alpha| \sim O(R/x_1) \gg 1 \quad (37)$$

We emphasize that the most important property of the transformation given by Eq. (35) is that, since $\theta(\eta) + \theta_0 > O(\eta/\ln|\eta|) \sim O^+$ as $|\eta| \rightarrow \infty$, the η -axis ends inside the subdominant regions.

Substituting Eq. (35) into Eq. (17) and noting that $O(\eta) \ll 1$, Eq. (17) with $\lambda = 0$ then becomes, to $O[\theta(\eta)]$,

$$[(2\tau + 1)(\sigma_0 - iS_x \sigma_1 + iS_x \eta_1 \sigma_2) + \eta_0] \Phi(\eta) = 0 \quad (38)$$

where

$$\begin{aligned} \sigma_0(d/d\eta, \eta) &= \sigma(S_x d/d\eta, -iS_x \eta) \\ &= - \sum_{n=0}^{\infty} g_1^{(n)} F_n(d/d\eta)/n! \quad (39) \end{aligned}$$

$$\eta_0 = 2(1 + \tau) \quad (40)$$

$$\sigma_1 = i\sigma_0(d/d\eta, \eta)/\partial(d/d\eta)(d/d\eta) \quad (41)$$

$$\sigma_2 = \partial\sigma_0(d/d\eta, \eta)/\partial\eta \quad (42)$$

$$\eta_1 = \int_0^{\eta} \theta(\eta') d\eta' , \quad (43)$$

$$g_1(\eta) = \pi^{1/2} \bar{\xi}_1 \exp(\bar{\xi}_1^2) \operatorname{erfc}(\bar{\xi}_1) , \quad (44)$$

and $\bar{\xi}_1 = Q S_\alpha x_1 / |\eta|$. Since the η -axis ends inside the subdominant regions and hence $\Phi(\eta)$ decays spatially as $|\eta| \rightarrow \infty$, we can multiply Eq. (34) by $\int_{-\infty}^{\infty} d\eta \Phi^*(\eta)$ and obtain

$$\int d\eta \Phi^*(\eta) \{ (\Omega\tau+1)\sigma_0 + \sigma_0 \} - i S_\alpha [(\Omega\tau+1)(\theta\sigma_1 - \eta_1\sigma_2)] \Phi(\eta) = 0 . \quad (45)$$

Noting σ_0 is purely real, we have $\operatorname{Im} \int d\eta \sigma_0 |\Phi(\eta)|^2 = 0$. Furthermore, due to the Hermitian property of the operator, $\hat{H}_{01}$, it is straightforward to demonstrate

$$\operatorname{Im} \int d\eta \Phi^*(\eta) \hat{H}_{01} \Phi(\eta) = 0 . \quad (46)$$

Thus, the imaginary part of Eq. (45) demands

$$\operatorname{Re} \int d\eta \Phi^*(\eta) (\Omega\tau+1)(\theta\sigma_1 - \eta_1\sigma_2) \Phi(\eta) = 0 . \quad (47)$$

From Eqs. (36), (37), and (43), it can be seen that $\theta(\eta)$ and $\eta_1(\eta)$ become non-negligible only for $|\eta| \gtrsim R \gg |\alpha|^{-1}$. Therefore, we may employ the WKB approximation for $\Phi(\eta)$ in Eq. (47); i.e.,

$$\Phi(\eta) = \int_y \Delta_q(\eta) \exp[i \int_{\eta}^{\eta} k_q(\eta') d\eta'] , \quad (48)$$

where k_q is purely real given by

$$(\Omega\tau + 1)\sigma_0(ik_\lambda, \eta) + \Omega(1 + \tau) = 0, \quad (49)$$

and

$$\sigma_0 = -\pi^{1/2} \bar{E}_i \rho_0(ik_\lambda); \quad (50)$$

that is, c.f. Eq. (34),

$$k_\lambda \rho_0 = (\ln|\eta\alpha|)^{1/2} [1 - O(\pi^2/|\ln\eta\alpha|^2)]. \quad (51)$$

Here, the $O(\pi^2/|\ln\eta\alpha|^2)$ term depends on l , i.e., the branch of the logarithmic function. Meanwhile, the slow spatial decay of $\Phi(\eta)$ as $|\eta| \rightarrow \infty$ is contained within $A_\lambda(\eta)$. Substituting Eq. (48) into Eq. (47), we find

$$\int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} |A_\lambda(\eta)|^2 [(2\tau + 1)(\partial k_\lambda \partial \sigma_0 / \partial k_\lambda - \eta_l \partial \sigma_0 / \partial \eta)] = 0. \quad (52)$$

Using Eqs. (49) and (50), we find

$$(\Omega\tau + 1)k_\lambda \partial \sigma_0 / \partial k_\lambda = (\Omega\tau + 1)2k_\lambda^2 \rho_1^2 \sigma_0 = -\Omega(1 + \tau)2k_\lambda^2 \rho_1^2, \quad (53)$$

and

$$(\Omega\tau + 1)\partial \sigma_0 / \partial \eta = \Omega(1 + \tau)/\eta. \quad (54)$$

Thus, Eq. (52) becomes

$$2(1 + \tau) \int d\eta \sum_{\ell} |\Lambda_{\ell}(\eta)|^2 [\theta(\eta) 2k_{\ell}^2 p_1^2 + \eta_1/\eta] = 0. \quad (55)$$

Since it can readily be shown that no eigenmode with $\theta = 0$ exists and that $\tau_1/\eta + \theta(\eta) > 0$, Eq. (55) can be satisfied only if

$$\Lambda_{\ell}(\eta) = 0, \text{ for all } \ell\text{'s}; \quad (56)$$

that is, $\Lambda(\eta)$ is a trivial solution. Therefore, we have completed our proof that no marginally stable eigenmode exists. This along with the preceding proof that there is no unstable eigenmode show that the eigenmodes, if they exist, must be damped.

III.1b) Stability with full electron dynamics ($\lambda = 1$). Our consideration here consists of the following three subsidiary proofs; i.e., for $0 \leq \lambda \leq 1$,

- (1) no new unstable eigenmode is introduced in the $\lambda \rightarrow 0^+$ limit,
 - (2) no eigenmode exists in the $|\omega| \rightarrow \infty$ limit,
- and finally,
- (3) no marginally stable eigenmode exists.

(1) and (2) can be proved using, as in Paper LCN, the rescaling technique; that is, we rescale for proof (1), Ω , k_x , and x with respect to λ and for proof (2), x and k_y with respect to Ω . The analyses are straightforward and will not be presented here. Meanwhile, it is apparent that our proof on the absence of marginally stable eigenmodes in the adiabatic-electron limit can be readily extended to prove (3).

(1), (2), and (3) along with the proof in III.1) that the eigenmodes

are damped in the ($\lambda = 0$) adiabatic-electron limit clearly demonstrate that for $0 \leq \lambda \leq 1$, the eigenmode frequencies are confined within the lower half of the Ω plane, i.e., $\text{Im}\Omega < 0$, and thus the absolute stability of the integral eigenmode equation, Eq. (15).

We now consider effects of finite electron temperature gradients which modify the electron nonadiabatic ($\xi_e Z_e$) term in Eq. (15) to

$$(1 - 1/\Omega)\xi_e Z_e - (\eta_e/\Omega)\xi_e [\xi_e + (\xi_e^2 - 1/2)]Z_e \quad (57)$$

with $\eta_e \equiv d\ln T_e / d\ln N_e$. Again, introducing the artificial parameter λ in front of both terms, the present analysis can then be extended easily to show that the drift-wave eigenmodes remain damped with finite η_e .

IV. CONCLUSIONS AND DISCUSSION

We have presented an analytical stability theory for electrostatic drift-wave eigenmodes in a sheared slab plasma. The eigenmode equation employed is the proper integral equation first derived by Coppi, Rosenbluth, and Sagdeev (1967) via particle orbital integrations. We first demonstrate a simpler derivation of this integral eigenmode equation using the linear gyrokinetic equation of Taylor and Hastie (1968) as well as Rutherford and Frieman (1968). Stability properties of the integral eigenmode equation are then analyzed via an artificially constructed equation which smoothly connects the limits of adiabatic electrons and full electron dynamics. We find that, even with this proper integral equation, the universal drift-wave eigenmodes are stable in the sheared slab geometry. Further consideration of finite electron temperature gradients shows no change in the stability properties.

While in the present work we have limited our investigations to electrostatic collisionless drift waves, our technique for the stability analysis could, in principle, be extended to cases with fully electromagnetic and collisional effects included. However, as also noted in Paper ICM, our stability analysis is not applicable to drift-wave instabilities driven by finite ion-temperature gradients, i.e., $\eta_i \neq 0$, (Coppi, Rosenbluth, and Sagdeev, 1967; Kadomtsev and Pogutse, 1970). This is because for $\eta_i \gtrsim 0(1)$ it is known, at least in the differential equation approximation, that there are marginally stable eigenmodes due to the existence of singular turning points (Lee and Chen, 1979b). Thus, the eigenmodes can move across the real Ω axis and our technique would fail in ascertaining the stability properties.

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