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SUBAPERTURE OPTICAL TESTING TECHNIQUES

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**SPHERICITY MEASUREMENTS OF FULL SPHERES
USING SHAPED-TURE OPTICAL TESTING TECHNIQUES**

by

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ABSTRACT

This paper discusses the progress being made on the development of applying subaperture interferometric testing techniques to making sphericity measurements on the entire surface of a full sphere.

1. INTRODUCTION

Measurement of lack of roundness on full spheres are generally made by using a stylus instrument, as shown in Fig. 1, to take three roundness traces on mutually orthogonal planes. This instrument compares the roundness of the sphere to the circle described by the stylus, as it is rotated about a well defined axis. Accuracies of 25 nm can be routinely achieved with this technique.

To very accurately determine the sphericity of precision gyro rotors, Lipi and Siddall¹ made a series of 16 longitudinal roundness traces and presented the error data on stereo plots. More roundness traces certainly mean that one can define the true shape of the sphere better, however a series of these measurements is quite time consuming (three to four hours for the case of Lipi and Siddall) and requires much contact with the part surface. Surface contact can damage spheres which have fragile surfaces (which is the case for many optical quality parts).

Figure 1. Stylus instrument for making roundness traces.

It is desired to obtain high density sphericity data to an accuracy of 15 nm with very little mechanical contact. These requirements have caused us to investigate optical test approaches requiring the least. Currently, Los Alamos National Laboratory and Applied Optics Research is continuing an effort to adapt the use of interferometric testing techniques for making sphericity measurements on spheres. In principle, the interferometric method may be very fast. Interferometric testing offers the potential for sphere relatively uniform examinations of the surface equally with a minimum of contact with the optical surface. The approach is to take a series of interferograms over the surface of the sphere and mathematically combine together to describe the sphere's surface topography. The principal difficulty is to extend the interferometric technique to the measurement of full spheres.

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With a field lens of number 10, $2\theta_0$ as shown in Fig. 2, about one eighth of the surface can be measured with each interferogram. About six to eight interferograms will be required to approximately cover the surface of the sphere.

Figure 2. Area of sphere illuminated. Interferogram area is constant, changing $2\theta_0$ does.

III. EXPERIMENTAL RESULTS

A first experiment was designed that is shown in Fig. 3 for obtaining the data to compare the techniques for the sphere technique. When taking the data, the sphere is initially rotated by the turntable so that three interferograms are obtained at 120° intervals with the camera located along a latitude 30° above the equator, i.e., the northern hemisphere. The sphere is then rotated by 180° so that north and south sides are examined. Thus, interferograms are then obtained at 120° along the equator, i.e., in the southern hemisphere. The patterns of the southern hemisphere are shifted 180° with respect to the patterns of the northern hemisphere. The patterns are then shifted 120° with respect to the patterns of the northern hemisphere and are shifted 120° with respect to the patterns of the southern hemisphere.

Table 1. Spherical coordinates for centers of six interferograms.

θ	ϕ
0	60
60	120
120	60
180	120
240	60
300	120

This pattern allows coverage over approximately 75% of the surface. The coverage pattern is illustrated in Fig. 4.

A Zygo Interferometer was used to take the data. Processing of the interferometric data to Zernike polynomial coefficients was done with the WAP program.²

Figure 4. Coverage pattern for six interferogram pattern.

III. THEORETICAL APPROACH

The use of interferometric information from subsapertures has been developed over the last several years.^{1,11} This theory addressed determination of a set of Zernike polynomials describing the aberrations of the full aperture from the interferograms over parts of the aperture. Three difficulties arise from working the subsaperture data:

- 1) subsaperture data may not fully cover the surface
- 2) subsaperture data may overlap
- 3) alignment errors are different in each subsaperture.

- 1) $Y_{00}(\theta, \phi) = \sqrt{\frac{1}{4\pi}}$ radius change
- 2) $Y_{11}(\theta, \phi) = \sqrt{\frac{3}{8\pi}} \sin\theta \cos\phi$ x-translation
- 3) $Y_{10}(\theta, \phi) = \sqrt{\frac{3}{4\pi}} \cos\theta$ z-translation
- 4) $Y_{1-1}(\theta, \phi) = \sqrt{\frac{3}{8\pi}} \sin\theta \sin\phi$ y-translation
- 5) $Y_{22}(\theta, \phi) = \sqrt{\frac{15}{96\pi}} 3\sin^2\theta \cos^2\phi$ astigmatism
- 6) $Y_{21}(\theta, \phi) = \sqrt{\frac{15}{24\pi}} 3\sin\theta \cos\phi$ astigmatism
- 7) $Y_{20}(\theta, \phi) = \sqrt{\frac{5}{4\pi}} \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$ astigmatism
- 8) $Y_{2-1}(\theta, \phi) = \sqrt{\frac{15}{24\pi}} 3\sin\theta \sin\phi$ astigmatism
- 9) $Y_{2-2}(\theta, \phi) = \sqrt{\frac{15}{96\pi}} 3\sin^2\theta \sin^2\phi$ astigmatism

Spherical harmonic functions are a complete, orthogonal set over a sphere. These functions are convenient to represent the defects which are slowly varying across the surface. Slowly varying defects may be represented by a modest number of polynomials. Defects which are very localized require large numbers of polynomials.

Incomplete coverage is the most difficult problem. Clearly we have no information in areas not covered by at least one subaperture, so defects which are limited to the areas not observed can not be fit. However since these small defects require large numbers of polynomials for representation, they would not be included in a low order polynomial fit in any case. In general, greater coverage leads to better fits, but the lower order polynomials may be fit with modest gaps in the data. With a least-squares approach, overlap of subapertures is actually helpful, since random errors are reduced by the use of redundant information.

Different misalignments in the subapertures may be treated by augmenting the spherical harmonic functions with subaperture misalignment functions. By simultaneously fitting the augmented set of functions to the subaperture data, the subaperture misalignments may be ignored and only the higher order subaperture information contributes to the spherical harmonic terms.

IV. THEORY

In this section we outline the theory needed to calculate the deformations of a sphere as mentioned by a set of spherical harmonic functions. The data from which the calculations are to be made consist of sets of Zernike coefficients describing the wavefront error observed in the subaperture measurements.

Each interferogram is subject to misalignment errors which result from movement of the sphere with respect to the interferometer and which result from changes in the reference mirror position in the interferometer.

To solve the problem, we augment the spherical harmonic over the full sphere with alignment polynomials defined at each subaperture. The most convenient set of polynomials to use for a circular subaperture is the four lowest Zernike polynomials. These functions are orthonormal over a circular region. Strictly speaking they are not orthonormal over a circular patch on a sphere, but the error is very small for the $f/7.5$ optics to be used. Orthonormality is not required for these functions, but it is convenient.

We also delete the four lowest spherical harmonic functions from the spherical harmonic set. The set of augmented functions is

$$W_A(\theta, \phi) = W_L(\theta, \phi) + W_S(\theta, \phi) \quad (1)$$

$W_A(\theta, \phi)$ - augmented polynomial set
 $W_L(\theta, \phi)$ - local alignment polynomials
 $W_S(\theta, \phi)$ - deleted spherical harmonic set

$$W_L(\theta, \phi) = \sum_{k=1}^K \sum_{n=1}^4 b_n Z_n(\theta, \phi) x_k(\theta, \phi) \quad (2)$$

$Z_n(\theta, \phi)$ - Zernike polynomials
 b_{nm} - Zernike coefficients
 $x_k(\theta, \phi)$ - function defining the subaperture
 $x_k(\theta, \phi) = 1$ if (θ, ϕ) is inside the k th subaperture,
 $x_k(\theta, \phi) = 0$ otherwise.
 K - the number of subapertures

$$W_S(\theta, \phi) = \sum_{n=2}^{NS} \sum_{m=-n}^{m=n} a_{nm} Y_{nm}(\theta, \phi) \quad (3)$$

$Y_{nm}(\theta, \phi)$ - spherical harmonic functions
 a_{nm} - spherical harmonic coefficients
 NS - the maximum order of the spherical harmonic functions

To keep the functions strictly real, we have modified the standard set in the following way.

$$\begin{aligned} (-1)^{m+nm} e^{im\phi} &= \cos\theta \text{ for } m > 0 \\ (-1)^{m+nm} e^{im\phi} &= \sin\theta \text{ for } m < 0 \end{aligned}$$

The modified set is orthogonal and complete for real functions.

Our procedure, to be described below, we fit the function, $W_L(\theta, \phi)$, to the subaperture data. The function, $W_S(\theta, \phi)$, describes the aberrations to the sphere. The function, $W_A(\theta, \phi)$, is usually desired.

Given sufficient numbers of Zernike polynomials, enough subapertures, aperture radii of sufficient size, and good location of the subapertures, we can determine the spherical harmonic coefficients by a least squares calculation.

where a_j are the augmented spherical harmonic coefficients, u_{jk} are the Zernike coefficients for the k th subaperture, and l is the number of Zernike terms. Taking the inner product (or dot product) of both sides of Eq. 4 with $\langle Z_i|$, the transpose of $|Z_i\rangle$, we have

$$h_{ik} = \sum_{j=1}^l \langle Z_i S_j \rangle_k a_j \quad (5)$$

where $\langle Z_i S_j \rangle_k$ is the inner product of Z_i and S_j (evaluated by integration over the k th subaperture) and

$$\langle Z_i Z_j \rangle_k = \delta_{ij}$$

where δ_{ij} is the Kronecker delta function.

This concept can be extended to include all subapertures. We extend the definition of $|b\rangle$ to include all sets of Zernike polynomials for each a total of K subapertures.

$$|b\rangle = G |a\rangle \quad (6)$$

where,

$$G = \begin{bmatrix} \langle Z_1 S_1 \rangle_1 & \langle Z_1 S_2 \rangle_1 & \cdot & \langle Z_1 S_l \rangle_1 \\ \cdot & \cdot & \cdot & \cdot \\ \langle Z_1 S_1 \rangle_2 & \langle Z_1 S_2 \rangle_2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \langle Z_1 S_1 \rangle_K & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \langle Z_1 S_1 \rangle_K & \cdot & \cdot & \langle Z_1 S_l \rangle_K \end{bmatrix}$$

$$|b\rangle = \begin{bmatrix} b_{11} \\ \cdot \\ b_{1l} \\ b_{21} \\ \cdot \\ b_{2l} \\ \cdot \\ b_{1K} \\ \cdot \\ b_{lK} \end{bmatrix}$$

Given the matrix, G , we may calculate its pseudo-inverse, G^{-1} , by

$$G^{-1} = (G^T G)^{-1} G^T \quad (7)$$

where G^T is the matrix transpose.

Creation of the matrix, G , and calculation of the inverse, G^{-1} take considerable computer time. Most of the computer time is consumed in numerical calculation of the dot products. Approximately 80 seconds of Cray 1 time is required to calculate the case of six apertures, Order 4 in spherical harmonics functions, and eleven Zernike functions. This process need be calculated only once for a given pattern of apertures. Eq. 6 must be calculated for each data set but consists of a simple matrix calculation which may be easily calculated on a microcomputer.

V. PRELIMINARY EXPERIMENTAL VALIDATION

Experimental validation of procedure requires measurement and data reduction of a ball which is accurately known. The ideal test ball would have a deformation of in a single low order spherical harmonic mode. The magnitude of aberration should be several wavelengths so that interferogram fitting errors are not significant. The single mode ball could then be used to determine the test the accuracy and orthogonality of method. Repeated tests with the ball in various orientations would help to confirm that the technique works equally well in all directions. Unfortunately the variety of readily available ball is very limited and accurate determination of the shape by roundness traces is difficult. To date the method has been shown to yield comparable values of peak-to-valley errors as the roundness methods. In an equatorial trace of a very complex ball, the measured and calculated peak-to-values were

	<u>Measured</u>	<u>Calculated</u>
Peak-to-valley	825. nm	915. nm

This information suggests that the method is useful for testing balls against a peak-to-valley specification. The ball used for these measurements does not serve well for testing the capability of the technique to match the aberrations in shape because the aberrations are of very high spatial frequency — above those which we can address with the number of polynomials in the current code.

We plan to test the accuracy of the shape of the deformations as well. Our plan is to first validate the technique against a computer simulated ball. Computer generated interferograms for each subaperture will be made and data reduced in the standard way. The Zernike coefficients from this data reduction will be used to reproduce the original spherical harmonic aberrations.

VI. SUMMARY AND CONCLUSION

The theory for analysis of aberrations of full spheres has been developed and implemented in a computer program. Numerical validation checks have been performed. Experimental verification is in a preliminary state but results, so far, are encouraging. Future work will consist of more thorough validation and ultimately an upgrade of the experimental configuration to employ phase measuring interferometry, automated data taking, interferogram averaging, and use of a larger number of polynomial terms.

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