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Issues and Prospects

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Abstract

The capability of a power plant to operate at a wide range of output power is essential for initial commissioning and normal maintenance. In this work, we explore critical physics issues related to operating a tokamak fusion reactor at fractions of its rated power and identify methods for power control. Analysis is carried out with a steady-state, profile-dependent, zero-dimensional power balance model of the plasma in which several empirical transport scalings appropriate to tokamaks are used. It is found that reactor operation depends strongly on the confinement model, plasma β limit, and the effect of alpha power on transport. Parametric calculations indicate that density, auxiliary heating power and an effective external confinement control mechanism are the key control elements, and burn control is required in most cases. Transition between power plateaus is facilitated by operating in the hybrid transformer mode. In general, the impact of fractional power operation on full power reactor designs appears to be small.

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1. Introduction

Conceptual designs of fusion reactors have been based primarily on achieving rated power output by maintaining the plasma parameters at their optimum operating levels. However, as with fossil and fission power plants, the commissioning of fusion power reactors will entail a series of plasma and system component performance checks at low power (<5% of rated power) and subsequent testing at progressive power stages. In addition, we can envision normal start-up/shutdowns of the plant for scheduled maintenance and when system components malfunction. In these procedures, the fusion power can vary from below 5% to 100% of the plant rated power, and the rate of change must be controlled to satisfy physics and engineering constraints on the plasma and its support systems. What this implies is that, in addition to operating at full load, the plant should have the capability of operating at fractions of its full power and of making controlled transitions between power levels.

In this paper, we explore methods to operate a tokamak fusion reactor at fractions of its rated power, identify the more effective control "knobs" and assess the impact of the requirements of fractional power operation on full power reactor design. As a prerequisite to this task, we carefully examine the critical physics issues relevant to reactor performance predictions and offer some insight into their impact on fractional power operation. The basic tool of analysis consists of a zero-dimensional (0-D) plasma power balance code which incorporates the most recent advances in transport and burn plasma

physics relevant to tokamaks.

With respect to confinement scaling, a large physics data base^[1] has been accumulated, both from theory and experiments. Empirical scaling laws, derived by parameter fitting with experimental data,^[2] are used in our analyses. For global control of reactor output power, it is found that programmed density changes by gas or pellet fueling, auxiliary heating power, and external means to control confinement are some of the more effective "knobs" for output power control. Frequently these control elements must be applied simultaneously during power transients and for steady state burn control. Scenarios in which the reactor is brought from one power level to another have been identified, requiring a minimum amount of energy input while satisfying the relevant engineering constraints.

Recent tokamak experiments involving high supplementary heating power consistently indicate degradation of the energy confinement time τ_E with auxiliary power, P_{aux} .^[1] Within the limited range of power available ($P_{aux} < 5-6$ MW), two empirical scalings of τ_E with P_{aux} have been derived. The first, given by $\tau_E \propto a_1 + a_2/P_{aux}$ ^[3], a_1 , a_2 being constants, leads to optimistic predictions for reactor performance, while the second, with $\tau_E \propto P_{aux}^{-1/2}$ ^[3,4], gives pessimistic extrapolations. Another critical consideration is whether the presence of energetic alphas and their thermalization will adversely affect the overall confinement properties. Inclusion of the alpha heating power in P_{aux} for τ_E scaling results in unacceptable performance of a conventional tokamak reactor. The absence of experimental data in this crucial area underlines the urgent need for a burning plasma experiment in the near future.

The paper is organized in the following manner. Section II gives a brief description of the physics basis, both theory and experiment, on which the

analyses in this work depend. In addition, a number of critical issues are identified and an assessment of how they might impact reactor operation in general is given. In Section III we describe the 0-D plasma power balance code used to analyze reactor operating scenarios and some of the techniques involved in the search for power control elements. This is followed by Section IV where fractional power operating scenarios are deduced under varying physics assumptions, and the appropriate control elements are identified. Section V contains some preliminary insight into dynamic analyses of power transients towards various power plateaus, including a qualitative assessment of additional engineering requirements on burn control. Conclusions and discussions are given in Section VI.

2. Physics Basis

The level of thermonuclear power output from a fusion reactor depends on the operating plasma density $\langle n \rangle$ and temperature $\langle T \rangle$. ($\langle \rangle$ denotes volume average.) At thermal equilibrium, the plasma heating power, including ohmic, auxiliary and alpha, is balanced against the plasma losses, which consist of transport and radiation. The properties of each of these power components are, in addition to $\langle n \rangle$ and $\langle T \rangle$, functions of the plasma discharge parameters, namely: the plasma current I_p , size (R = major radius, a = minor radius), shape (K = elongation, etc.) and the toroidal magnetic field B_0 . The profiles of density, temperature and auxiliary power deposition are also important factors in determining the power balance. Externally induced plasma losses, such as transport due to magnetic field ripples and radiation arising from impurity injection can also play an important role.

In the following subsections, we will focus on the more important physics

issues related to reactor-grade plasmas and briefly review their experimental and theoretical status. We will also describe the various physics models to be employed in our analyses and assess the uncertainties and implications that come with these models.

2.a. Energy Confinement

The current status of understanding of energy transport in tokamak plasmas was well documented in a recent review article by S.M. Kaye.^[1] It has long been established that transport in a tokamak is anomalous and governed primarily by plasma turbulence^[5] as a result of the existence of free energy sources within the plasma. These sources can be in the form of current and pressure gradients, or a distorted or nonisotropic particle distribution function, a likely consequence of externally applied auxiliary heating. Experiments to date also confirmed that the main energy loss channel is through the electrons. Because of the complexity of the problem, there are relatively few reliable theoretical models that can adequately describe the parameter dependence of energy confinement obtained from these experiments.

Under these circumstances, tokamak transport has been studied through the so-called empirical scalings, which express the global energy confinement time τ_E as a function of plasma discharge parameters such as I_p , $\langle n_e \rangle$, R , a , etc. These empirical scalings are obtained by curve-fitting to a large collection of data over a wide range of parameters using standard multiple linear regression techniques.^[2] In low-density ohmic discharges, it was found that^[4]

$$\tau_{E,NA}(\text{sec}) = 7.1 \times 10^{-22} \langle n_e \rangle (\text{cm}^{-3}) a^{1.04} (\text{cm}) R^{2.04} (\text{cm}) q^{0.5} \quad (1)$$

where q is the safety factor at the plasma boundary. This is the well-known

neo-Alcator scaling. For auxiliary-heated tokamaks, particularly neutral beam injection heating, the empirical scaling is given by^[4]

$$\tau_{E,aux}(\text{sec}) = 6.4 \times 10^{-8} K^{0.5} I_p(A) P_{tot}^{-0.5}(W) a^{-0.37}(\text{cm}) R^{1.75}(\text{cm}) \quad (2)$$

where P_{tot} is the sum of ohmic power P_{OH} and auxiliary heating power P_{aux} . Comparing Eqs. (1) and (2), we note that for ohmic discharges, τ_E is strongly dependent on the average density $\langle n_e \rangle$ while for $P_{aux} > P_{OH}$, this density dependence disappears. Instead, for these discharges, τ_E increases linearly with I_p and decreases monotonically with P_{aux} . Noting that τ_E levels off above a critical density for ohmic discharges on Alcator-C^[6] and D-III,^[7] Goldston proposed an inverse quadrature relation of τ_E to $\tau_{E,NA}$ and $\tau_{E,aux}$ to model the confinement^[4]:

$$\tau_{E,G} = (\tau_{E,NA}^{-2} + \tau_{E,aux}^{-2})^{-1/2} \quad (3)$$

and was able to fit the experimental data very well. Equations (1), (2), (3) represent what we call Goldston scaling. Clearly, above a critical density, given by the equality $\tau_{E,NA} = \tau_{E,aux}$, τ_E is dominated by $\tau_{E,aux}$. Conversely, for constant density and auxiliary power, τ_E becomes neo-Alcator like above a critical current. The latter has been observed on D-III^[8] and is well modelled by Eq. (3).

With regard to beam-heated tokamaks, experimental data on D-III with $P_{tot} \approx 8 \text{ MW}$ reveal that τ_E may equally well be approximated by^[3]

$$\tau_{E,DIII}(\text{sec}) = I_p(\text{MA}) K^{1/2} [0.022 + 0.061/P_{tot}(\text{MW})] \quad (4)$$

for limiter discharges. Here we note that τ_E decreases but eventually levels

off with increasing P_{tot} . With respect to scaling with P_{aux} , $\tau_{E,DIII}$ is much more optimistic than $\tau_{E,aux}$. It is clear that data with high auxiliary power (>15 MW) are needed to clarify this point.

Based on the phenomenological free energy argument, Ohkawa^[5] derived a scaling law for τ_E given by

$$\tau_{E,OK}(\text{sec}) = 6.32 \times 10^{-22} \langle n_e \rangle (m^{-3}) R^2(m) a(m) q \langle T_e \rangle^{-1/2} (\text{keV}) \quad (5)$$

$$\times (1 + P_{aux}(\text{MW})/P_{OH}(\text{MW}))^{-1/2}$$

which has the same functional dependence on P_{aux} as Goldston scaling for $P_{aux} \gg P_{OH}$. For a pure ohmic discharge, we note that $\tau_{E,OK}$ reduces to a Merezghin-like scaling^[9] with the characteristic $\langle T_e \rangle^{-1/2}$ -dependence.

There are several other theoretical transport scalings which appear to fit the experimental data from present-day tokamaks over a wide range of parameters. One of these is due to Perkins,^[10] who postulates that the electron thermal conduction is governed mainly by the strong drift wave turbulence between the $q=1$ and $q=2$ surfaces. He arrived at an empirical scaling of the form^[10]

$$\tau_{E,P} \propto I_p^{0.9} B_o^{-0.1} n_e^{-0.6} P_{tot}^{-0.6} a^{0.4} R^{1.7} \quad (6)$$

which is remarkably close to the form of $\tau_{E,aux}$. Thus it can be said that much progress has been made recently in the theoretical understanding of the physical processes governing transport in toroidal plasmas.

Although a large data base has been accumulated in neutral beam heated discharges, there are as yet very little systematic studies on the effect of RF heating on confinement. Ion cyclotron radio frequency (ICRF) heating experiments on PLT^[11] and TEXTOR^[12] have recorded τ_E degradation with RF

power in a manner very close to that found for beam heated experiments. Very similar behavior of τ_E with P_{aux} was also observed on D-III for electron cyclotron heating.^[3] Definitive scaling laws are not available because of the limited RF power levels involved. Thus, for the time being, we will assume that RF-heated plasmas follow the same confinement scalings as for beam-heated plasmas. This removes the need for us to specify a particular supplementary heating method.

The empirical scalings in Eqs. (2)-(5) are derived for the so-called L-mode neutral beam injection (NBI) discharges, in which a substantial drop in τ_E from that in ohmic discharges is observed. However, "good" confinement in these NBI discharges can be restored by essentially reducing the neutral recycling near the edge of the plasma. This so-called H-mode operation can be obtained by a variety of means, for example, by a "closed" divertor configuration. The H-mode plasma is characterized by a radial T_e profile with a pedestal of several hundred eV and a relatively flat density profile. However, detailed processes involved in the transition from the L- to the H-mode are not well understood, except that H-mode confinement seems to exhibit the same parameter dependence as for L-mode plasmas, and the confinement time for H-mode is roughly twice that of L-mode. There are other methods to improve energy confinement, such as pellet fueling, trace impurity injection and ungettered operation. But these techniques do not yet have sufficiently large data bases to warrant serious consideration for reactors. In the ensuing analyses, we will adopt the conservative approach in assuming L-mode operation only.

In summary, we point out that there are no experimental data on confinement properties for either reactor-sized or D-T burning plasmas. On the other hand, a large collection of data has accumulated over the years from

a variety of tokamak experiments in which r_E appears to exhibit a consistent functional dependence on machine and discharge parameters. These empirical scalings seem to be supported by recent experimental results on TFTR,^[13] JET,^[14] and by transport theory^[10] on tokamaks. Extrapolation to near-term large non-burning tokamaks using the empirical transport scaling is justified. However, for burning tokamaks, the issue of alpha particle effects on confinement needs to be addressed.

2.b. Alpha Particle Effects

From a phenomenological point of view, Ohkawa^[5] argued that the beam ions heat the background plasma, creating a nonisotropic electron distribution function. Part of the beam power then ends up as a source of free energy for plasma turbulence or fluctuations, which can lead to enhanced energy transport. Using an argument along this line, an approximate transport scaling is derived that yields a $P_{aux}^{-1/2}$ dependence for r_E .

It is tempting to evoke the same reasoning and infer that $r_E \propto P_\alpha^{-1/2}$, where P_α is the alpha heating power. We note that a variety of physical processes exist through which alpha particles can give rise to plasma turbulence. For example, energetic trapped alpha particles may interact through their precessional drift with high-mode-number ballooning modes in tokamaks.^[15] The alpha particles may also destabilize the shear Alfvén mode and the fast magnetosonic wave via their anisotropic distribution, or the drift Alfvén spectrum by virtue of plasma inhomogeneities.^[16] Indeed, that the influence of alpha particles on confinement is analogous to that of neutral beam ions may be a very reasonable assumption, which we will consider in our work. However, there exists the possibility that the alpha particles may transfer most of their energy to the background plasma via anomalous absorption through the kinetic shear Alfvén instabilities.^[17] In that case,

the deleterious effect on confinement may be alleviated.

2.c. Plasma Beta Limit

The fusion power P_{fus} from a tokamak reactor has the following parameter dependence:

$$P_{fus} \propto \beta_c^2 B_o^4 a^2 R K \langle \sigma v \rangle_{DT} / T^2 \quad (7)$$

where β_c is the toroidal beta limit for mhd plasma stability. Small β_c can severely limit the range of fusion power output for a reactor of given size. Indeed, if additional constraints such as passive thermal stability and ignited operation are imposed, the range of power output may be further reduced. Thus, reactor concepts that raise the beta limit are a subject of vital interest to the fusion community.

There are basically two regimes of stability for tokamaks in terms of the plasma beta. The first stability regime is characterized by plasmas with elongated Dee shapes, and $\langle \beta_c \rangle \lesssim 10\%$. Stability against external kinks without wall stabilization imposes a beta limit given by Troyon et al.:^[18]

$$\langle \beta_c \rangle (\%) = C_o I_p (MA) / a(m) B_o (T) \quad (8)$$

where C_o is a constant ($C_o=2.7$). Sykes et al.^[19] found a similar beta limit for ideal ballooning modes with $C_o=4.4$. Consideration of resistive tearing modes will somewhat lower this beta limit. Recently neutral beam experiments on D-III^[19] indicated the same parameter dependence of $\langle \beta_c \rangle$ but with $C_o=3.5$. In our analyses, we will adopt the D-III beta limit scaling.

In recent years, numerical and theoretical studies^[21,22] showed that a marked improvement to the stability of pressure-driven ideal mhd modes can be

achieved by creating an indentation on the inboard side of a conventional Dee-shaped plasma. Access to the so-called second stability regime, characterized by beta limits in excess of 20%, is further facilitated by the presence of a nearby conducting wall. However, stability in this regime requires precise control of the local plasma current profile which may impose severe requirements on the current drive system. The design of the pusher coil, together with its shielding, used for indenting the plasma could also pose a major technological challenge when extrapolating to the reactor regime. Due to the uncertainty of this mode of operation and the lack of an extensive experimental data base, we will limit our analysis to first stability discharges.

An equally important consideration in fractional power operation concerns the plasma transport as $\langle \beta \rangle$ approaches the mhd limit. In the D-III experiments, [20] it was found that the β -limit is generally disruptive, caused by destabilization of the $m=2$, $n=1$ external kink mode. In contrast, "soft" beta limits were observed on ASDEX [23] as the neutral beam power was increased except for $q < 3$ and/or high power discharges. This saturated beta is a direct consequence of enhanced transport caused by fluctuations as the plasma pressure approaches the mhd limit. Very similar confinement properties were also observed on ISX-B experiments [24] a few years ago, in which the plasma transport is well modelled by a thermal diffusivity X_e proportional to $(\epsilon \beta_p)^{3/2}$, [25] where β_p is the poloidal beta and ϵ is the inverse aspect ratio. It is at present not precisely known as to what leads to a "soft" or "disruptive" beta limit. If the "disruptive" beta limit is assumed, a maximum beta of perhaps $0.9 \langle \beta_c \rangle$ will be the practical limit to avoid disruption, thus further reducing the region of operation in the $\langle n \rangle$ - $\langle T \rangle$ space. On the other hand, a "soft" beta limit permits full exploitation of the plasma pressure and

can potentially lead to an excellent scheme for power control with inherent thermal stability.

2.d. Thermal Stability

From a design point of view, it is desirable to operate the reactor over a region of thermal equilibria in $\langle n \rangle$ - $\langle T \rangle$ space in which the system is passively stable. Operation outside of this region probably requires a feedback burn control scheme that may add to the complexity of the discharge scenario and the related engineering subsystems. The issue of reactor thermal stability may best be understood by examining the plasma energy balance near ignition:

$$\frac{dW}{dt} = P_{\alpha} + P_{aux} - P_L' \equiv F(\langle n \rangle, \langle T \rangle) \quad (9)$$

where W , P_{α} , P_{aux} and P_L' are the energy, alpha power, auxiliary power and loss power (including transport and radiation) respectively. Thermal equilibrium is defined by

$$F(\langle n \rangle, \langle T \rangle) = 0 \quad (10)$$

and a necessary and sufficient criterion for thermal stability is

$$\frac{\partial}{\partial \langle T \rangle} F(\langle n \rangle, \langle T \rangle) < 0 \quad (11)$$

For a constant P_{aux} , Eq. (10) defines a contour in the $\langle n \rangle$ - $\langle T \rangle$ space, along which

$$\frac{d\langle n \rangle}{d\langle T \rangle} = \frac{\partial F(\langle n \rangle, \langle T \rangle) / \partial \langle T \rangle}{\partial F(\langle n \rangle, \langle T \rangle) / \partial \langle n \rangle} \quad (12)$$

The part of the contour that satisfies

$$\frac{\partial F(\langle n \rangle, \langle T \rangle)}{\partial \langle n \rangle} \cdot \frac{d\langle n \rangle}{d\langle T \rangle} > 0 \quad (13)$$

is then thermally stable, and will depend on the empirical scaling governing P_L' . As an example, for neo-Alcator scaling (Eq. (1)) and a 50-50 DT plasma, it is found that $\partial F / \partial \langle n \rangle > 0$ for $\langle T \rangle \gtrsim 5$ keV such that in that range of temperatures, the part of the contour with $d\langle n \rangle / d\langle T \rangle > 0$ is thermally stable.

The ideal reactor would produce the range of desirable power output over a region in $\langle n \rangle$ - $\langle T \rangle$ space in an ignited, passively stable mode. However, if the first stability beta limit is imposed, such a region will be severely narrowed and, for some range of power output, driven operation and/or burn control will be required. Externally we may induce enhanced power loss that scales with T^N , with $N > 1$ such that the thermally stable equilibrium contour can be varied over a wide range of $\langle n \rangle$ and $\langle T \rangle$. Mechanisms that may achieve this include enhanced transport due to toroidal field ripples and radiation loss via introduction of a high-Z impurity into the plasma.

In the case of driven operation and/or burn control, there are a variety of control elements that can be used. Burn control requires "knobs" that induce a fast response from the plasma, particularly if it is inside the ignited, thermal runaway regime. Direct transition between power plateaus, if needed, can be achieved in a highly controlled manner and considered as a series of quasi-equilibrium points in $\langle n \rangle$ - $\langle T \rangle$ space. The control elements then comprise all parameters that can affect the energy balance, such as density, auxiliary power, fuel mixture ratio, current, plasma shape and size, the level of impurity and the relevant radial profiles. The viability of some

of these parameters as power control knobs will be assessed in the following section.

2.e. Operating Mode

It has long been established that steady state operation is critical to the future of the tokamak configuration as an economically viable reactor concept. Noninductive current drive by lower hybrid waves has been shown to be effective in recent experiments,^[26] although only at relatively low densities ($\bar{n}_e \lesssim 5 \times 10^{13} \text{ cm}^{-3}$). Theory predicts^[27] that the current drive efficiency scales as $I_p/P_{\text{rf}} \propto n_e^{-1}$ such that within the reactor range of parameters, the efficiency will extrapolate to unattractively low values. To exploit the relatively good lower hybrid current drive efficiency at low densities, the "hybrid transformer" mode of operation was proposed.^[28] In this mode, the plasma current is alternatively driven by the OH coil during the burn phase and by lower hybrid waves during the low-density, non-burning and coil re-charging phase. What emerges is a plasma discharge scenario in which the burn pulse is long ($\gg 1000\text{s}$), depending on the transformer size, while the current can be kept constant, thus minimizing the occurrence of major disruption.

In the analyses here, we will treat the "hybrid transformer" as the preferred mode of operation at all levels of fusion power output. The study will be focused on the OH-driven burn pulse, and the problem of plasma transients between the low density dwell phase and the fusion operation phase will be the subject of another report. We also note that by adopting this hybrid mode of operation, transition between power plateaus is naturally achieved while starting up from a dwell phase. In the case of genuine steady-state operation with RF current drive, direct transition from one power stage to another may have to be considered. Even then, the choice between direct

transition and fresh start-up remains to be determined.

3. Steady State Analysis

We use a relatively simple, zero-dimensional plasma model to analyze tokamak operation at various power levels. There are two reasons for this simple approach: first, detailed local transport in a tokamak is not well understood, making it difficult to model, especially in reactors where auxiliary and alpha heating power dominate ohmic heating; secondly, a more sophisticated one-dimensional plasma model leads to a much more time-consuming computer code which is unnecessary for reactor parametric analyses. At steady state, the global power balance equation is given by

$$P_{\alpha} + P_{aux} + P_{OH} = P_L + P_{rad} \quad (14)$$

where P_L and P_{rad} are the transport and radiation power respectively. Equation (14) is essentially zero-dimensional, assumes $T_i = T_e = T$ and profiles of the form

$$T(r) = (1 + \alpha_T) \langle T \rangle (1 - \epsilon_T r^2/a^2)^{\alpha_T} \quad (15a)$$

$$n(r) = (1 + \alpha_N) \langle n \rangle (1 - \epsilon_N r^2/a^2)^{\alpha_N} \quad (15b)$$

where $\langle T \rangle$ and $\langle n \rangle$ are spatial averages of $T(r)$ and $n(r)$. In Eq. (15), $\epsilon_T = 1 - [T(a)/T(0)]^{1/\alpha_T}$ and $\epsilon_N = 1 - [n(a)/n(0)]^{1/\alpha_N}$. In MKS units, the alpha heating power P_{α} is given as:

$$P_{\alpha} \text{ (MW)} = 2.214 \times 10^{-17} R K \gamma \int_0^a r dr n^2(r) \langle \sigma v \rangle_{DT} \quad (16)$$

where $\gamma = (n_D/n_T)(1+n_D/n_T)^{-2}$ with n_D , n_T being deuterium and tritium densities and $\langle \sigma v \rangle_{DT}$ is the D-T fusion cross-section. In this model, the alpha particles are assumed to thermalize and be lost from the plasma instantaneously. The ohmic heating power P_{OH} is expressed as

$$P_{OH} \text{ (MW)} = 0.132(1.5\alpha_T + 1)^2 I_p^2 \text{ (MA)} \frac{R}{a^4 K} \int_0^a r dr \frac{(1 - \frac{r^2}{a^2})^{1.5\alpha_T} f_{NC}(r)}{T(r)^{3/2}} \quad (17)$$

where $T(r)$ is in keV and $f_{NC}(r)$ is the neo-classical correction factor to Spitzer resistivity, given by $f_{NC}(r) = [1 - 1.95(\frac{r}{R})^{1/2} + 0.95(\frac{r}{R})]^{-1}$ and $Z_{in} = 20$ is assumed. In arriving at Eq. (17), the Ohmic current density profile $j(r)$ is assumed to be $j(r) = j_0(1 - \frac{r^2}{a^2})^{1.5\alpha_T}$.

In the absence of impurities, the radiated power P_{rad} is mainly Bremsstrahlung in the regime of interest, and is simply calculated by

$$P_{rad} \text{ (MW)} = 1.9 \times 10^{-41} R K \int_0^a r dr n^2(r) T^{1/2}(r) \quad (18)$$

$$= 1.9 \times 10^{-41} R a^2 K \frac{(1 + \alpha_N)^2 (1 + \alpha_T)^{1/2} \langle n \rangle^2 \langle T \rangle^{1/2}}{(1 + 2\alpha_N + 0.5\alpha_T)}$$

In terms of the global confinement time τ_E , the transport loss term P_L is written as

$$P_L \text{ (MW)} = 1.90 \times 10^{-20} R K \int_0^a r dr n(r) T(r) / \tau_E \quad (19)$$

$$\approx 1.90 \times 10^{-20} R a^2 K \frac{(1 + \alpha_N)(1 + \alpha_T)}{(1 + \alpha_N + \alpha_T)} \cdot \frac{\langle n \rangle \langle T \rangle}{\tau_E}$$

We note that τ_E as measured in experiments is given by the expression

$$\tau_E = \frac{\frac{3}{2} \langle n_i T_i \rangle V_p + \frac{3}{2} \langle n_e T_e \rangle V_p}{P_{OH} + P_{aux}} \quad (20)$$

for the flat-top of the discharge, with V_p denoting the plasma volume. When we write $P_L = 3\langle nT \rangle / \tau_E$, we are implying either $\tau_E^i = \tau_E^e = \tau_E$ or $\tau_E^i = \tau_E^e/2$, $\tau_E^i > \tau_E^e$.

In our analysis, the auxiliary power P_{aux} is obtained from the global power balance equation given $\langle n \rangle$ and $\langle T \rangle$ together with their profile information. The results are displayed in the form of contours of constant P_{aux} as a function of $\langle n \rangle$ and $\langle T \rangle$, the so-called POPCON plots.^[29] By overlaying contours of plasma $\langle \beta \rangle$, fusion power and, possibly, $dF(\langle n \rangle, \langle T \rangle)/d\langle T \rangle$, we can easily locate desirable and accessible regions of reactor operation for the prescribed range of fusion power output.

For purposes of illustration, we use INTOR, the International Tokamak Reactor,^[30] as a reference device, which is designed to meet steady-state performance criteria based on its objectives. Since the main objectives of INTOR are to demonstrate net power production beyond the present generation of tokamaks and to assess critical physics and engineering issues related to the tokamak reactor, the device design is not optimized. As such, the parameters of INTOR are listed in Table I. We emphasize here that one of the aims of this work is to assess the impact of startup and fractional power operation requirements on a fusion reactor design based on delivering its rated full-power performance parameters. The results of our analyses may unfold critical physics issues that may imply modifications and further optimizations of conventional reactor designs. However, quantification of these design improvements is beyond the scope of this work.

4. Results of POPCON Analysis

Due to the uncertainty in tokamak confinement scaling and how it extrapolates to reactors, we have studied the problem of fractional power operation using four plausible scaling laws, namely: (i) neo-Alcator, (ii) Goldston, (iii) Ohkawa and (iv) D-III. For each scaling law, we present the POPCON plot using the standard parameters of INTOR (Table I) and discuss the list of possibilities for varying the fusion power output.

4.a. Neo-Alcator Scaling

Shown in Fig. 1 is the POPCON plot for INTOR with neo-Alcator confinement scaling, given by Eq. (1). Contours of P_{aux} , P_{fus} and $\langle \beta_c \rangle$ are plotted. We observe that full power (500 MW) operation brings the plasma way above ignition and is limited by $\langle \beta_c \rangle = 3.4\%$, spanning a parameter range of $\langle T \rangle = 5-15$ keV and $\langle n \rangle = 0.65-2. \times 10^{20} m^{-3}$, between points A and B in Fig. 1. It is also evident that a burn control mechanism is necessary to keep the plasma in equilibrium along the full power contour, particularly if the $\langle \beta \rangle$ limit is disruptive. Operation at $\langle \beta \rangle < 0.9 \langle \beta_c \rangle$ may be necessary. By introducing a toroidal field ripple or a controlled amount of impurity into the plasma, the ignition boundary ($P_{aux}=0$) above $\langle T \rangle > 10$ keV can be brought to intersect the $P_{aux}=500$ MW contour between A and B, with a slope of $d\langle n \rangle / d\langle T \rangle > 0$, thus ensuring thermal stability. It must be pointed out that injection of impurity will dilute the plasma fuel and cause a reduction in fusion power output, while enhancing transport via field ripples may increase the heat load on the edge particle removal system. On the other hand, if the $\langle \beta \rangle$ limit is soft, τ_E is expected to deteriorate as $\langle \beta \rangle$ approaches $\langle \beta_c \rangle$ until a stable equilibrium is reached. It is speculated, and not entirely impossible, that the soft $\langle \beta \rangle$ limit may be used as a passive thermal stability control mechanism, and with programmed density and auxiliary power, may serve as a

"knob" for fusion power control.

Referring to Fig. 1, reactor operation at 50% and 25% of rated power can be achieved at points C and D respectively, along the ignition boundary. We note that point D is unstable while point C is only marginally stable. To ensure thermal stability, points C' and D' are suggested, requiring 5 and 15 MW of auxiliary driving power respectively. Apparently there is a tradeoff between driven operation (finite $Q = P_{fus}/P_{aux}$) and the need for burn control. However, if full power operation requires burn control anyway, the obvious choice will be points D and C for 25% and 50% power operation. Likewise, if passive control via "soft" beta limit is available for full power operation, points D' and C' will be the preferred equilibria for maintaining 25% and 50% fractional power.

Based on the hybrid transformer mode of operation, we can envision a likely scenario for transiting between power plateaus. For example, let us take the case of transiting from 25% to 50% of rated power and we are interested in going from point D to point C. From point D, we may turn off the fueling to bring the plasma down from ignition and both $\langle n \rangle$ and $\langle T \rangle$ will drop with a characteristic time of τ_E . At the same time, the current I_p will decrease with a resistive skin time of $\tau_s \approx L/R_p$, where L is the plasma inductance, R_p is the plasma resistance and $\tau_E \ll \tau_s$ usually. When $\langle n \rangle$ decays to below $3 \times 10^{19} \text{ m}^{-3}$, 10 MW of lower hybrid power for current drive is switched on. The rate of density drop will be controlled to slowly bring the plasma to point E, when the plasma current will be totally driven by RF. The point E is determined by the empirical lower hybrid current drive efficiency formula:[31]

$$\frac{I_p}{P_{RF}} (\text{A/W}) = \frac{0.15}{R(\text{m}) \langle n_{20} \rangle} \quad (21)$$

where $\langle n_{20} \rangle$ is in units of 10^{20} m^{-3} , and by global power balance is given by Eq. (14), assuming all the RF power is dissipated in the plasma. Substituting $I_p = 6.4 \text{ MA}$ and $P_{RF} = 10 \text{ MW}$ into Eq. (21), we obtain $\langle n \rangle = 4.5 \times 10^{18} \text{ m}^{-3}$ and $\langle T \rangle = 3.2 \text{ keV}$ at point E from power balance. To reach point C after recharging the transformer primary, we increase the ion fueling rate and turn on the auxiliary heating power while removing the RF current drive power. In a highly programmed fashion, the plasma can be brought to the equilibrium point C by increasing the density to and maintaining it at $5 \times 10^{19} \text{ m}^{-3}$. The trajectory for the whole procedure is shown by the dashed curve DEC in Fig. 1 with the arrows pointing in the appropriate direction. Clearly, a more detailed, time-dependent analysis is required to determine the exact path of this operation. Note also that during this process, at most only 10 MW of P_{aux} for heating and 10 MW of P_{RF} for current drive are involved, which are well within the power requirements for the full power steady state design.

4.b. Goldston Scaling

The main feature of Goldston empirical confinement scaling, as given by Eqs. (1)-(3), is that it describes correctly two experimentally observed phenomena: (i) for an ohmically heated tokamak, τ_E rises with density but saturates above a critical value; (ii) for tokamaks where $P_{aux}/P_{OH} > 1$, τ_E drops with P_{aux} . When extrapolating to a reactor plasma, the issue of how the energetic alphas may affect global confinement becomes critical. Using a phenomenological argument, it can be inferred that the effect of alpha power may be similar to that of auxiliary heating power, in which case very pessimistic predictions may result. On the other hand, anomalous absorption by the background ions may reduce the turbulence level to a minimum with the result that overall confinement is unaffected. In the absence of any experimental data in this regard, we will examine two extreme cases: (1)

confinement without alpha degradation, and (ii) confinement with alpha degradation.

(i) Confinement without alpha degradation:

Using Eqs. (2) and (3), we can rewrite the plasma power balance equation as

$$\frac{W}{\tau_{NA}} (1 + \nu P_{tot})^{1/2} = P_{tot} + P_{\alpha}' \quad (22)$$

where $W = 3\langle nT \rangle V_p$ is the total plasma energy, $P_{tot} = P_{aux} + P_{OH}$, $\nu = (\tau_{NA}/\tau_{aux})^2/P_{tot}$ and $P_{\alpha}' = P_{\alpha} - P_{rad}$. We note here that since τ_E is dependent on P_{aux} , it opens up an avenue for controlling τ_E via an external means. To illustrate this notion, we first point out that, with Goldston scaling, the ignition curve is obtained by substituting $P_{aux}=0$ in Eq. (22), given explicitly by

$$P_{\alpha}' + P_{OH} = \frac{W}{\tau_{NA}} \quad (23)$$

with $\tau_{NA}' = \tau_{NA}(1 + \nu P_{OH})^{-1/2}$. For nonzero P_{aux} , the solution to Eq. (22) given $\langle n \rangle$ and $\langle T \rangle$ is divided into two regions: subignition and ignition, as illustrated in Figure 2. Below ignition, the plasma equilibria are given solely by the solution $P_{aux}=P_2$, as shown in Fig. 2(A). The corresponding POPCON plot is depicted in Fig. 3, using the standard INTOR parameters in Table I. Comparison with Fig. 1 indicates that the ignition boundary is only slightly modified from that for neo-Alcator transport, since $\tau_{NA} = \tau_{NA}'$ near ignition. However, due to the deleterious effect on confinement by auxiliary heating, the power required for driven equilibrium is increased by approximately tenfold. Practically, all these equilibria are inherently

thermally stable, since P_{aux} is comparable to P_α and dominates the overall heating power. Figure 2(a) also shows that $P_{aux} > P_2$ has to be satisfied at all points along the path to ignition. This implies that within this plasma model, at least 100 MW of auxiliary power is required to ignite an INTOR-like device, as shown in Fig. 3. It is speculated that the finite thermalization time τ_α of the alpha particles can be utilized to somewhat reduce this power requirement. The scenario is as follows: as the plasma approaches ignition, P_{aux} is turned off and τ_E relaxes from $\tau_{E,G}$ to τ_{NA}' in a characteristic time τ_r . If $\tau_r \ll \tau_\alpha$, the reduction in input power ($P_\alpha + P_{tot}$) is less than the drop in P_L such that $d\langle T \rangle / dt > 0$ and ignition can be attained.

The conditions and requirements for full and fractional power operation with Goldston scaling are similar to those with neo-Alcator confinement, except that the auxiliary power required for driven equilibrium are much higher. Therefore, there is much incentive in locating the operating points on the ignition boundary, even at the expense of a costly feedback stabilization system.

With Goldston scaling, it may be possible to use a modest amount of injection power to control the burn well above ignition. Shown in Fig. 2(3) are the solutions P_1 and P_2 ($P_2 > P_1$) for P_{aux} in Eq. (22) for $\langle n \rangle$, $\langle T \rangle$ which satisfies the ignition condition: $P_\alpha' + P_{OH} > W / \tau_{NA}'$. Contours of constant $P_{aux} = P_1$ are shown as dashed curves in the POPCON plot of Fig. 4. It is evident that full power operation can be maintained with auxiliary power not more than 15 MW. For $\langle T \rangle > 12.5$ keV above ignition, these "driven" high Q equilibrium points may actually be thermally stable. Unfortunately this stable region is limited by the mhd critical beta. However, an important prerequisite for this scheme to work is that the effect of the alpha power on confinement is small compared to that of the driving power which is usually

less than 10% of P_α . Concrete experimental data is needed to determine the validity of the scheme.

(ii) Confinement with alpha degradation:

Full account of alpha degradation of confinement can be taken by solving the equation

$$\frac{W}{\tau_{NA}} [1 + v(l_{tot} + P_\alpha)]^{1/2} = P_{tot} + P_\alpha \quad (24)$$

for P_{aux} given $\langle n \rangle$ and $\langle T \rangle$. The result is given by the POPCON plot in Fig. 5, where we note that (i) ignited operation is not available, and (ii) even for driven reactor mode, the auxiliary power is considered too high to make the INTOR-like machine economically viable. This highly pessimistic extrapolation to reactors underscores the importance of the need to understand the behavior of alpha particles in a burning plasma and, in particular, how they affect confinement.

4.c. Ohkawa Scaling

Based on the argument that anomalous energy confinement results from turbulence destabilized by the existence of free energy in the plasma, Ohkawa^[5] derived an empirical scaling law given in Eq. (5). We can rewrite the expression for $\tau_{E,OK}$ as

$$\tau_{E,OK} = \tau_{E,OH} (1 + P_{aux}/P_{OH})^{-1/2} \quad (25)$$

where $\tau_{E,OH}(s.) = 6.32 \times 10^{-22} \langle n_e \rangle (m^{-3}) R^2(m) a(m) q / \langle T_e \rangle^{-1/2} (keV)$. We note that $\tau_{E,OK}$ and $\tau_{E,G}$ have roughly the same form for $P_{aux} \gg P_{OH}$. However, for an ohmically heated plasma, $\tau_{E,OK} = \tau_{E,OH}$ which is neo-Alcator like except for a $\langle T_e \rangle^{-1/2}$ - dependence. As a result, the ignition boundary is markedly

different from that for Goldston scaling, as depicted in Fig. 6, where β confinement degradation due to alphas is assumed. Because of the $\langle T_e \rangle^{-1/2}$ variation, unacceptably high levels of auxiliary power are necessary to drive the reactor near or below ignition, particularly for electron temperatures above 5 keV. Above ignition, as in the case of Goldston scaling, thermal equilibrium, whether or not stable, can be maintained by low levels of auxiliary power of the order of a few megawatts. Because of the nature of the confinement scaling, marginally viable regimes of operation are limited to the low-temperature, high-density portion of the $\langle n \rangle$ - $\langle T \rangle$ space. Full power (500 MW) can be maintained at the point $\langle n \rangle = 1 \times 10^{14} \text{ cm}^{-3}$ and $\langle T \rangle = 8 \text{ keV}$ without any auxiliary power. To minimize the auxiliary power requirement, access to this point has to be achieved via a high-density path, e.g. for $\langle n \rangle = 3 \times 10^{14} \text{ cm}^{-3}$, a minimum of 200 MW is required (see Fig. 6(b)). It is evident by now that the basic problem with reactor operation under this scaling law is that the access to ignited scenario requires a very large amount of auxiliary power and an active feedback control scheme, which may make this concept economically unattractive. Even if the above difficulty is overcome, ignited operation regimes are also extremely limited, while the alternative will be the low-Q driven mode. By the same argument, fractional power operation, particularly at lower levels (<50%), appears to be problematical.

Inclusion of the fact that alpha power could degrade confinement much like auxiliary heating leads to extremely unattractive reactor power balance for the INTOR-like machine. However, substituting for $P_{\text{aux}} \approx P_\alpha$ in Eq. (25), we can define an ignition condition

$$\frac{P_\alpha}{P_L} = \frac{\tau_{E,OH} P_\alpha^{1/2} P_{OH}^{1/2}}{W} > 1 \quad (26)$$

Noting that $P_\alpha/P_L \propto \langle n \rangle \langle T \rangle^{1/2-9/4} B_0^2 K$, where $\langle \alpha v \rangle \propto \langle T \rangle^{1/2}$, we conclude that, even with alpha confinement degradation, we can build an ignition tokamak by choosing parameters cautiously. According to Eq. (10), we need a reactor with high density, high current, high magnetic field and relatively large size to achieve ignition. As an example, we take an unoptimized reactor with the following parameters: $R = 5$ m, $a = 0.8$ m, $K = 1.6$, $B_0 = 14$ T, $I_p = 8$ MA and $q = 2.0$. The corresponding POPCON plot for auxiliary power is shown in Fig. 7. We observe that ignition occurs at a minimum $\langle n \rangle$ of $1.1 \times 10^{21} \text{ cm}^{-3}$ and $\langle T \rangle$ of 5 keV. Above $\langle T \rangle = 4.5$ keV, the ignition curve is thermally stable with fusion power ranging in excess of 5000 MW. The part of the ignition boundary with $d\langle n \rangle/d\langle T \rangle > 0$ provides a stable region of operation over a wide range of fusion power. However, the $\langle \beta \rangle$ limit of 2.5% confines ignition operation to a narrow region, with P_{fus} well below 5000 MW. One of the caveats for this high density tokamak is that large amounts of RF power are needed to drive the plasma current, removing any possibility of an economic steady state reactor. A more detailed parameter study clearly is needed to optimize the reactor design based on this scaling law.

4.e Doublet-III Scaling

An empirical scaling can be used to fit the Doublet-III confinement experimental data,^[3] in which τ_E saturates to a constant value if the auxiliary power is increased indefinitely. Unfortunately there is no size scaling corresponding to this set of data. We assume, as an example, an Alcator-like size scaling and, normalizing to a typical D-III parameter set, obtain the following confinement law:

$$\tau_{E,G}(s) \approx 0.156 I_p(\text{MA}) a^2(\text{m}) K^{1/2} \{1 + 2.32/P_{tot}(\text{MW})\} \quad (27)$$

where we conservatively take the coefficient obtained from L-mode data. For this case, we will include the fact that the alpha power is deleterious to confinement the same way auxiliary power is, such that $P_{\text{tot}} = P_{\alpha} + P_{\text{aux}} + P_{\text{OH}}$. The POPCON plot for the INTOR-like device (Table I) under the D-III confinement model is shown in Fig. 8, where the $P_{\text{fus}} = 500$ MW and $\langle \beta_c \rangle = 3.4\%$ contours are also depicted. Full power operation has to be run in the ignited mode with burn control required in the event that the beta-limit is disruptive. However, fractional power operation can be carried out in the thermally stable portion of the $\langle n \rangle$ - $\langle T \rangle$ space with $\langle n \rangle < 6 \times 10^{19} \text{ m}^{-3}$ and $\langle T \rangle > 7.5$ keV, with plasma Q in excess of 10. The minimum auxiliary power requirement for ignition is 15 MW as long as the density is kept below $6 \times 10^{19} \text{ m}^{-3}$ during the heating phase. It should be pointed out that the D-III data can be fitted equally well by a $\tau_E \propto P_{\text{aux}}^{-1/2}$ scaling. Thus, higher power experiments should be carried out to determine the true scaling with P_{aux} .

5. Transient Analysis

In Section 2.e, it was determined that the most plausible mode of operation for tokamak reactors, given the known experimental data base and tradeoffs between economics and technology,^[32] is the hybrid transformer mode. Using this mode as the base discharge scenario, transition from one power state to the next is naturally achieved during a normal dwell phase, when the plasma is taken out of its burn mode. On the next transition to the burn phase, a separate heating path through the $\langle n \rangle$ - $\langle T \rangle$ space is taken to reach the next prescribed power plateau. The implication is that no direct

transition between power plateaus will be needed.

The complete time-dependent analysis including thermal burn control for tokamak reactors is complicated and is dealt with in a separate report.^[33] Instead, we will illustrate through an example the issues involved and possible solutions associated with burn control and fractional power operation. The plasma energy balance equation, as given by Eq. (9), is expanded into three equations governing the time evolution of density, ion and electron temperature, which are then solved as an initial value problem. We take the case of Goldston scaling without alpha degradation (Fig. 4) as an example and study the possible scenarios of heating to ignition with a constant density of $1.5 \times 10^{20} \text{ m}^{-3}$. Again, we emphasize here that the case we studied is not an optimized disc arge. In Fig. 9(a), a scenario is shown in which auxiliary power is ramped up at a programmed rate such that $d\langle T \rangle / dt \approx 1.2 \text{ keV/sec}$ and is turned off at $t = 6.2 \text{ sec}$. The corresponding evolution of the average temperature $\langle T \rangle$ is also depicted. We note that at the instant P_{aux} is turned off, $\langle T \rangle = 10 \text{ keV}$ and, from Fig. 4, the plasma is ignited and thermal runaway to very high temperature occurs, if not limited by the beta limit. In Fig. 9(b), the same programmed auxiliary heating pulse is used except that at $t = 5 \text{ sec.}$, the power is reduced to 32 MW and kept constant subsequently. We observe that, for $t > 5 \text{ sec.}$, the plasma stays close to equilibrium for 5 sec. before the temperature falls back to the ohmic equilibrium level. Obviously at $t = 5 \text{ sec.}$, $P_{\text{aux}} = 32 \text{ MW} > P_1$ in Fig. 2(b), resulting in the temperature drop. If P_{aux} is kept at 31 MW for $t > 5 \text{ sec.}$, as is shown in Fig. 9(c), corresponding to $P_{\text{aux}} < P_1$ in Fig. 2(b), the temperature keeps increasing until it stops at $\langle T \rangle = 40 \text{ keV}$, corresponding to a stable equilibrium point. We note that in all three cases, the $\langle \beta \rangle$ -limit is not taken into account. To model this effect properly, a $\langle \beta \rangle$ -dependent τ_E is

used in the case of soft β -limit while a feedback control scheme will have to be installed to keep the operating β sufficiently far away from β_c , while passive thermal stability is absent. The scenarios in Fig. 9 serve to illustrate one of the techniques of using auxiliary power to control the plasma discharge and the sensitivity of these techniques to the transport model assumed. Since the DC power supplies for the auxiliary heating systems have a response time of the order of microseconds, we do not anticipate any severe requirement on these systems to produce the desired programmed pulses.

6. Conclusions and Discussions

It has been recognized that fractional power operation is an important consideration in the design of power reactors, whose commissioning requires that their power output be brought in stages to their rated level for equipment testing and safety checks. This work attempts to assess the impact of this additional procedural requirement on tokamak fusion reactor design. The critical physics issues related to fractional power operation are carefully reviewed within the framework of experimental data obtained in non-burning tokamaks. Due to the lack of data on burning plasma experiments, both "bold" and conservative extrapolations of the plasma confinement model have been used in evaluating the reactor performance in the fusion regime. In particular, several well-known confinement scalings, such as neo-Alcator, Goldston, Ohkawa and Doublet-III, have been employed, resulting in varying conclusions.

Since the plasma transport loss dominates the power balance in the reactor regime of parameters, the requirements for fractional power operation are sensitive to the empirical transport model used in the analysis. In

general, optimistic performance, characterized by high Q or ignited operation, is predicted for scaling laws for which τ_E does not depend on or degrade indefinitely with heating power, such as neo-Alcator and D-III scalings. However, beta limits in the first stability regime often dictate that the full power operating point be located well above the ignition boundary. This implies that, if the beta limit is disruptive, an elaborate feedback scheme, involving both programmed ion fueling and confinement control, will be needed to stabilize the output power. Programmed ion fueling may include changing the gas puffing or pellet injection rate, and varying the fuel n_0/n_T ratio to modify the power balance. Control of energy confinement, on the other hand, may be achieved by enhanced radiation induced by injecting appropriate amounts of impurities into the plasma, or by simply varying the confinement sensitive parameters, such as plasma size and shape, and current. If the beta limit turns out to be "soft," we may be able to utilize this inherent "confinement control" to maintain a stable operating point at full power, depending on how τ_E degrades with $\langle \beta \rangle$. Operation at fractions of full power can be accomplished in the low-density, high-temperature region, with inherent passive thermal stability, and either ignited or high- Q operation is possible. For driven modes, plasma equilibrium points can be found where the required auxiliary heating power does not exceed the minimum power needed to heat the plasma to ignition. As for transition between power plateaus, a convenient spot at which to achieve this is during the normal dwell phase of the hybrid transformer reactor mode. As a result, only the start-up and heating procedures to reach these various power stages need to be determined individually, once the target operating points are determined. Based on these optimistic transport scalings, the physics impact of fractional power operation on full power reactor designs appears to be minimal.

For transport models where τ_E degrades with P_{aux} , such as Goldston and Ohkawa scalings, ignition can still be reached provided the auxiliary power is turned off at a programmed fashion as the plasma approaches the ignition boundary. Above ignition, only moderate levels of driving power are needed to sustain equilibrium below the critical $\langle \beta \rangle$. However, these equilibrium points are thermally unstable thus requiring burn control with programmed ion fueling and auxiliary power. Generally speaking, fractional power can be maintained in the low-density, high-temperature, thermally stable regime, as is the case for neo-Alcator scaling, except that higher auxiliary power is required to sustain equilibrium. Consequently, there may be stronger incentive to operate in the ignited mode even for lower fractional power output, in which case complex burn control schemes may have to be installed. As with neo-Alcator scaling, the impact of fractional power consideration on reactor designs based on full power does not appear to be significant.

Finally, if we consider the possibility that alpha heating power degrades confinement in the same manner as does auxiliary power for either Goldston or Ohkawa scaling, highly pessimistic performance predictions for reactors with conventional size and plasma parameters will result. These reactors will definitely not be viable options and are excluded from further consideration. However, there is a class of tokamak reactors that may exhibit desirable features under these highly conservative transport assumptions. With Ohkawa's scaling, these reactors are characterized by high densities ($\langle n \rangle \sim 10^{21} \text{ m}^{-3}$), moderate temperatures ($\langle T \rangle \sim 5 \text{ keV}$), high magnetic field ($B_0 \sim 15 \text{ T}$), high current ($I_p \sim 10 \text{ MA}$), high fusion output power ($P_{fus} \sim 5000 \text{ MW}$), and ignition is possible in these devices. Parametric studies more extensive than those carried out in this work are clearly needed to reach an optimized version of this class of reactors. Amongst all these analyses, we

should keep in mind the fact that the effect of alpha particles on energy confinement remains a major open question in plasma physics. The answer to this question may significantly impact the direction of fusion reactor design and the methodology to operate the tokamak at different power levels. The sensitivity of reactor operation on alpha particle dynamics also underlines the urgent need for a near-term burning plasma tokamak experiment.

To complete our discussions we point out that there are recently two noteworthy advances in tokamak research. First, an extension of Goldston scaling, the Kaye-Goldston scaling,^[1] into discharges with $P_{aux} \gtrsim 2 P_{OH}$ has resulted in a slightly more optimistic model for $\tau_{E,aux}$, given by

$$\tau_{K-G}(\text{sec}) = 6.02 \times 10^{-8} K^{0.28} B^{-0.09} (T) I_p^{1.24} (MA) \langle n_e \rangle^{0.26} (\text{cm}^{-3}) \\ \times P_{tot}^{-0.58} (MW) a^{-0.49} (\text{cm}) R^{1.65} (\text{cm}) . \quad (28)$$

We note that τ_{K-G} has a stronger dependence on I_p and $\langle n_e \rangle$ than $\tau_{E,aux}$ and, according to Eq. (28), ignition operation tends to favor devices with high current and high toroidal field, a fact that is borne out by recent studies of compact tokamak ignition experiments.^[34] In these studies, it is assumed that $P_{tot} = P_\alpha + P_{aux} + P_{OH}$.

Secondly, recent numerical studies^[35] of mhd stability of advanced-shaped tokamaks indicate that, within the first stability regime, the coefficient C_0 for $\langle \beta_c \rangle$ [Eq. (18)] can be increased substantially beyond that for existing devices by appropriately shaping the plasma cross-section. In particular, high triangularity/indentation with a sharp tip are favorable for achieving higher beta, with the same q-profile, resulting in a crescent-shaped cross-section. This may lead to a relaxed pusher-coil/shield design problem. A higher beta limit will certainly enlarge the region of operation

for tokamak reactors at various power levels and its attainment should be a major goal in tokamak concept improvement studies.

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Table I

Parameters of INTOR Tokamak Reactor

Major Radius R	5.2 m
Minor Radius a	1.2 m
Elongation K	1.6
Magnetic Field on Axis B_0	5.5 T.
Plasma Current I_p	6.4 MA
Safety Factor q	2.1
Critical Beta $\langle \beta_c \rangle$	3.4%
Rated Fusion Power P_{fus}	500 MW
D-T Fuel Mixture Ratio n_D/n_T	1
Effective Charge Number Z_{eff}	1
Temperature Profile Factor α_T	1
Density Profile Factor α_N	1

Figure Captions

Fig. 1. POPCON plot of auxiliary power P_{aux} , fusion power P_{fus} and critical beta $\langle \beta_c \rangle$ over $\langle n \rangle$ - $\langle T \rangle$ space, for an INTOR-like tokamak with parameters listed in Table I. Neo-Alcator scaling is used. Solid lines: P_{aux} (MW), dashed lines: P_{fus} (MW) and dotted line: $\langle \beta_c \rangle = 3.4\%$. Arrowed dash-dotted lines indicate possible trajectory for transition from 25% to 50% rated power.

Fig. 2. Typical diagram of power balance terms as a function of auxiliary power P_{aux} , with a Goldston transport model without alpha degradation for (A) sub-ignition and (B) ignition, for a given $\langle n \rangle$ and $\langle T \rangle$. $P_{aux} = P_1$ and $P_{aux} = P_2$ are solutions to the power balance equation (22).

Fig. 3. Same as in Fig. 1, except that Goldston scaling without alpha degradation is used, i.e., $P_{tot} = P_{aux} + P_{OH}$.

Fig. 4. Same as in Fig. 3, except that contours of modest auxiliary power above the ohmic ignition boundary, given by $P_{aux} = P_1$ in Fig. 2(B). These contours are shown as dashed-dot lines and the labels are in MW.

Fig. 5. Same as in Fig. 1, except that Goldston scaling with alpha degradation is used, i.e. $P_{tot} = P_{aux} + P_{OH} + P_{\alpha}$.

Fig. 6. (a) Same as in Fig. 1, except that Ohkawa scaling without alpha degradation is used.

(b) POPCON plot for parameters used in Fig. 6(a), in the high density: $\langle n \rangle = 1.5-3.5 \times 10^{20} \text{ m}^{-3}$ and low temperature: $\langle T \rangle = 0-10 \text{ keV}$ region.

Fig. 7. POPCON plot of auxiliary power P_{aux} , fusion power P_{fus} and critical beta $\langle \beta_c \rangle$ over $\langle n \rangle - \langle T \rangle$ space, for a reactor with: $R = 5 \text{ m}$, $a = 0.8 \text{ m}$, $K = 1.6$, $B_0 = 14 \text{ T}$, $I_p = 8 \text{ MA}$ and $q = 2.0$. Ohkawa scaling with alpha degradation is used. Solid lines: P_{aux} (MW), dashed lines: P_{fus} (MW) and dotted line: $\langle \beta_c \rangle = 2.5\%$.

Fig. 8. Same as in Fig. 1, except that D-III scaling with alpha degradation is used.

Fig. 9 Time evolution of auxiliary power P_{aux} and average temperature $\langle T \rangle$ for an INTOR-like device with parameters listed in Table I. Goldston scaling without alpha degradation is used. Depending on the heating pulse, three scenarios are shown: (a) thermal runaway, (b) non-ignition, and (c) ignited equilibrium.

Fig.1.

Case A. $P_{\alpha}' + P_{OH} < W / \tau_{NA}'$ Sub-ignition

Case B. $P_{\alpha}' + P_{OH} < W / \tau_{NA}'$ Ignition

Fig.2.

Fig.3.

Fig.4.

Fig.5.

Fig.6(a)

Fig.6(b)

Fig.7.

Fig.8.

Fig.9(a)

Fig.9(b)

Fig.9(c)