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PRESSURE LIMITS OF AN AXISYMMETRIC TORUS

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ABSTRACT

In order to clarify the pressure limit of a toroidal plasma equilibrium in a three-dimensional geometry, an azimuthally symmetric toroidal plasma was considered. In a tokamak-like plasma, the net toroidal current is necessary for a finite β plasma equilibrium. If external conductors are used to provide the rotational transform, ι , the plasma pressure is limited to $\beta = (\iota/2\pi)^2/2A$, where A is the aspect ratio.

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Berk et al.¹ have recently proposed a general toroidal plasma equilibrium in a three-dimensional geometry. The solution is based on the existence of a sheet current on the magnetic surface, or a finite jump in plasma pressure. While a jump solution can be obtained as a limit of a diffuse boundary equilibrium,² the reverse may not necessarily be true. Here we shall reexamine the two-dimensional, tokamak-like equilibria which have been obtained previously³⁻⁶ and establish the maximum equilibrium β (i.e., maximum plasma pressure divided by the average magnetic pressure).

Using a virial theorem [Eq. (13) of Ref. 6] in an azimuthally symmetric toroidal geometry of a doubly connected system (that is, tokamak-like), it follows that

$$-\int \rho J_{\phi}' B_z^{\text{ext}} d\tau = \int \left(\frac{B_z'^2}{\mu_0} + 2p \right) d\tau \quad (1)$$

Here cylindrical coordinates (ρ, ϕ, z) are used, $\partial/\partial\phi = 0$, J_{ϕ}' and B_z' are, respectively, the current density and magnetic field created by the plasma, p is the plasma pressure, B_z^{ext} is the external magnetic field, and $\int d\tau$ is the volume integral over the whole space.

It is clear from Eq. (1) that in the absence of a toroidal current in the plasma, there will be no equilibrium. Since B_{ϕ} is not involved in Eq. (1), for an arbitrary current J_{ϕ}' , the equilibrium β can equal one.²

Is it possible to construct a toroidal equilibrium where the net toroidal current is zero? It is possible to make two disjointed tokamaks with opposite toroidal currents cancel out each other in one device.

We then ask the question whether there is a closed surface S (on which p is constant), even if the net toroidal current within the volume (bounded by S) is zero. Such an S does not exist. The proof is given below. (This proof is implicit in Ref. 3).

The equation

$$\vec{J} \times \vec{B} = \nabla p \quad (2)$$

is satisfied on S . We require that p be constant on S and $|\vec{\nabla} p|$ not be zero.

Because of the axisymmetry, we must have the poloidal field, $\vec{B}_p$, expressed as

$$\vec{B}_p = \frac{1}{\rho} \nabla \Psi \times \vec{e}_\phi, \quad (3)$$

where

$$\Psi \equiv \rho A_\phi \quad (4)$$

with A_ϕ the ϕ component of the vector potential.

As

$$\vec{B} \cdot \nabla p = 0 \quad (5)$$

leads to

$$\frac{\partial(\Psi, p)}{\partial(\rho, z)} \equiv \frac{\partial \Psi}{\partial \rho} \frac{\partial p}{\partial z} - \frac{\partial \Psi}{\partial z} \frac{\partial p}{\partial \rho} = 0, \quad (6)$$

we get

$$\Psi = f(p) \quad (7)$$

Since $|\vec{v}_p|$ is not zero, on S , $\vec{B}_p$ must be nonzero everywhere or identically zero.

If $\vec{B}_p \neq 0$, we get

$$\int J_\phi dS_p = \int \vec{B}_p \cdot d\vec{s}_p \neq 0 \quad (8)$$

Here S_p is the poloidal cross section of the plasma and $\vec{s}_p$ is the rim of S in a constant ϕ plane. Hence the net toroidal current is finite.

Also,

$$\vec{J} \cdot \vec{v}_p = 0 \quad (9)$$

tends to

$$\rho B_\phi = g(p) \quad (10)$$

If $\vec{B}_p = 0$, Eq. (2), along with Maxwell's equations, leads to

$$\frac{1}{\rho} \frac{\vec{v}}{2} \frac{g^2}{2\mu_0} + \vec{v}_p = \vec{v}_p \left\{ \frac{1}{\rho} \frac{1}{2\mu_0} \frac{dg^2}{dp} + 1 \right\} = 0 \quad (11)$$

This equation cannot be satisfied unless $\rho = \text{const.}$ on S , which is impossible. Thus such an S cannot exist.

Even if there are lines on S where $|\vec{v}_p| = 0$, the relation (8) holds. We assume that the plasma pressure decreases (outwards from) S . This is $\vec{n} \cdot \vec{v}_p \leq 0$, where $\vec{n}$ is the normal vector pointing outwards from S . Then $\vec{B}_p \cdot d\vec{s}_p$ is always the same sign on S . Hence, the relation (8) still holds.

From the above proof, S can be called a magnetic surface with a finite rotational transform.

The limit of equilibrium β is unity if the safety factor q ($\equiv B_\phi/B_p A$) can be made arbitrarily small. Here A is the aspect ratio. We address the next question of the β limit, if $q \geq 1$. The condition that no separatrix exists on a magnetic surface with a constant plasma pressure is

$$|B_z'| \geq |B_{ext}| \quad (12)$$

because of Eq. (11). For a large aspect ratio,⁷

$$\beta \sim \frac{1}{A} \frac{1}{q^2} \quad (13)$$

Special solutions exist apparently for higher β .⁸ In those solutions, the plasma is highly noncircular.

With a small external rotational transform applied to the system, the equilibrium β should become finite. However, the presence of Pfirsch-Schlüter current will limit β such that⁹

$$\beta_c \approx \left(\frac{1}{2\pi}\right)^2 \frac{1}{2A} \quad (14)$$

if the net toroidal plasma current is zero.

A small helical perturbation of an otherwise azimuthally symmetric, constant-plasma-pressure surface would provide a small rotational transform, ι , proportional to the square of the perturbation distance normalized with respect to the plasma radius, δ . Therefore,

$$B_c \sim \delta^4 \frac{1}{A} \quad . \quad (15)$$

Thus a geometrical shape of the bounding magnetic surface (where there is a jump in the plasma pressure) would dictate the pressure limit.

To see this more clearly, we shall calculate exactly a levitron or spherator-like plasma equilibrium. Plasma occupies a volume exterior to a closed surface S which is characterized by

$$F(\rho, z) = 0 \quad . \quad (16)$$

Within S , azimuthally symmetric current-carrying conductors are located (Fig. 1). The toroidal field $\vec{B}_T$ will be

$$B_T = \frac{B_0 R}{\rho} \quad \text{within } S \quad (17)$$

$$B_T = \lambda \frac{B_0 R}{\rho} \quad \text{outside } S \quad , \quad (18)$$

where λ is a constant.

The magnetic surface potential, Ψ , is constant on S . The function Ψ satisfies the simpler version of the Grad-Shafranov equation

$$\frac{\partial^2 \Psi}{\partial \rho^2} + \frac{\partial^2 \Psi}{\partial z^2} - \frac{1}{\rho} \frac{\partial \Psi}{\partial \rho} = 0 \quad (19)$$

within S except for the locations of current-carrying conductors. If Ψ is known within S , $\vec{B}_p$ on S would then be determined.

The jump condition S then requires that

$$B_p^2 = (1 - \lambda^2) B_0^2 \frac{R^2}{\rho^2} + \frac{1}{\rho^2} \left(\frac{\partial \Psi}{\partial n} \right)^2 - 2\mu_0 p \quad (20)$$

Here $\partial \Psi / \partial n$ is the gradient of Ψ normal to S .

Equation (20) defines the poloidal magnetic field except for the arbitrary parameters λ . The range of parameter λ is restricted by the condition that the right-hand side of Eq. (20) is positive everywhere on S .

The solution for B_p of Eq. (20) has the same sign everywhere on S . Let B_p be in the same sign as $(\partial \Psi / \partial n) / \rho$. Then the net current

$$I_p = \int (B_p - \frac{1}{\rho} \frac{\partial \Psi}{\partial n}) ds = 0 \quad (21)$$

is required for the stellarator-like confinement. Equation (21) uniquely defines the free parameter λ .

It is not difficult to see that for a large aspect ratio ($A \gg 1$) plasma

$$(1 - \lambda^2) B_0^2 \approx 2\mu_0 p \quad (22)$$

and $B_p^2 > 0$ everywhere means that

$$\frac{1}{\rho^2} \left(\frac{\partial \Psi}{\partial n} \right)^2 > \frac{4\mu_0 p}{A} \quad (23)$$

We note

$$\frac{1}{2\pi} = \left| \frac{1}{\rho} \frac{\partial \Psi}{\partial n} \right| \frac{A}{B_0} \quad (24)$$

Thus, Eq. (23) reduces to

$$\delta \equiv \frac{2\mu_0 p}{B_0^2} = \frac{1}{2} \frac{1}{A} \left(\frac{1}{2\pi}\right)^2, \quad (25)$$

recovering Eq. (14).

No equilibrium exists in an azimuthally symmetric toroidal device with zero net current. A small helical perturbation should produce a correspondingly small δ limit as defined in Eq. (14). Plasmas with internal conductors will have equilibrium solutions given in Eqs. (20) and (21).

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FIGURE CAPTIONS

FIG. 1. Schematic view of an azimuthally symmetric toroidal plasma.

Fig. 1

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