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A REVIEW ON THE
LATTICE DESIGN OF LARGE HADRON COLLIDERS*

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ABSTRACT

The conceptual evolution of the accelerator lattice design is discussed. Indicated are aspects of IR design. We emphasize the cancellation of stop-band width in the cluster design. The case of symmetric vs antisymmetric design is also discussed. The SSC lattice is used as an example.

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1 INTRODUCTION

The introduction of the collider concept into accelerator design has increased dramatically the center-of-mass energy available for particle collisions. The design of the collider has also taken advantage of the concept of modular construction.

A traditional fixed higher energy target circular accelerator uses the FODO cell structure or its variants as the basic building block of the accelerator. A machine usually behaves better if it has a large number of identical building blocks. The number of identical building blocks is called the superperiod, P , of the machine. When the superperiod of the machine is large, the number of systematic resonances becomes smaller. Thus the machine has a larger free tune space to work with. Many of the low to medium energy machines such as the AGS and CERN PS have a superperiod of 10 or higher.

A collider usually requires a region of free space for the experimentalists to set up their instruments; this is called the interaction region (IR). To retain large superperiodicity of the machine, the IR also forms a module. The machine is then composed of the IR modules, separated by identical arcs which transport the beam particles in a nearly circular path onto the next IR for the collision. Many colliders have been built of arc-IR basic modules, for example the ISR, Sp $\bar{p}$ S, and LEP at CERN; Tevatron at FNAL; PEP at SLAC; TRISTAN at KEK; etc. Table I lists some of their lattice parameters. The machine has a superperiodicity equal to the number of basic building blocks. The superperiodicity is, however, usually violated because of the demand for different insertion configurations for different types of physics research and machine functions. With different types of IR the collider ends up with a superperiodicity of 1, and therefore the number of systematic resonances is increased. Operation of the machine requires careful correction of these systematic resonances. It is, however, important to be able to minimize these systematic resonances in the design.

The important systematic resonances are the half-integer stop-band and the third-order systematic resonances. The third-order systematic resonances can be minimized by properly adjusted sextupoles in the machine, and they are not of much concern in the IR design except to minimize the chromaticity contribution. The half-integer resonance width will directly affect the beam performance for the off-momentum particles. Minimizing the half-integer stop-band is therefore an important consideration in the IR design.

In this chapter, we discuss some aspects of IR designs: in Section 2 we review briefly the important issues, and in Section 3 we review the conceptual

Table I. Lattice Parameters of Some Colliders

	SSC† pp	RHIC pp, Heavy ions	SPES PP	TEV PP	LED e ⁺ e ⁻	TRISTAN e ⁺ e ⁻
C (m)	82944	3833.8	7911	6283.2	26658.8	3018.08
E (GeV)	20000	100/amu	270	1000	50-100	30
μ	60°	90°	90°	70°	60°	56°
L (m)	192	29.2	64	59.5	7.9	16.12
Q _x	78.27	28.826	26.69	19.39	70.35	32.75
Q _y	78.28	28.822	26.71	19.43	78.2	36.60
S/A*	A	A	S	A	S	S
β_x^* (m)	0.5	3	2	1	1.75	4.5-2
β_y^* (m)	0.5	3	1	1	0.07	4.5-2
$\hat{\beta}$ (m)	8000	400	1600	1000	300	170

†There is also a reference design parameter for 90° phase advance lattice.

*S/A stands for symmetric or antisymmetric insertion layout.

evolution. In Section 4 we analyze the present SSC IR design, and in Section 5 we give the conclusion.

2 ISSUES IN THE LATTICE DESIGN

The collider lattice is normally composed of two parts, the interaction region (IR) and the arc. The interaction region (IR) is designed to match the requirements of experimentalists. The arc acts as a beam transport system guiding the beam particles from IR to IR. The overall consideration is cost vs versatility.

2.1 Arc Considerations

The basic structure of the arc is the FODO cell. A FODO cell is made of a pair of focusing and defocusing quadrupoles separated by spaces for the dipoles. Let L , ϕ , μ , and q be the length of the cell, its total bending angle, its betatron phase advance, and the quadrupole strength, respectively. The lattice function can then be determined as¹

$$\beta_x(F) = \frac{L}{\sin \mu} \left(1 + \sin \frac{\mu}{2} \right), \quad \beta_y(F) = \frac{L}{\sin \mu} \left(1 - \sin \frac{\mu}{2} \right),$$

$$Lq = 4 \sin \frac{\mu}{2},$$

$$\eta_x(F) = \frac{L\phi}{4} \frac{1 + \frac{1}{2} \sin \frac{\mu}{2}}{\sin^2 \frac{\mu}{2}}, \quad \eta_x(D) = \frac{L\phi}{4} \frac{1 - \frac{1}{2} \sin \frac{\mu}{2}}{\sin^2 \frac{\mu}{2}}.$$

Since the beam size is given by $\sigma = \sqrt{\beta\epsilon}$, it is easy to see that the cell length cannot be infinite. Use of longer cells would increase the aperture requirement of the magnet and therefore reduce the dynamical aperture. On the other hand, with shorter cells, the number of magnets would increase, and the machine would cost more. Optimization is indeed important in the choice of cell length and phase advance. The design of every collider has gone through this optimization procedure. Figure 1 shows cell length vs circumference for some accelerators.

Fig. 1. Cell length vs circumference for some hadron colliders.

2.2 Important Considerations for the IR Design

The interaction region (IR) design of a large collider has to meet several important requirements:

(1) The luminosity (see Appendix A) has to meet a certain minimum requirement.

To obtain high luminosity, (a) the beta-function at the interaction point (IP), β^* , should be tunable to a small value without much impact on the optical matching, and (b) the dispersion functions should satisfy

$$\eta_x^* = \eta_y^* = 0 \quad (1)$$

The latter condition is important so that the beam size does not depend on the momentum error of the beam.

The beam size at the IP is then given by $\sigma^* = \sqrt{\beta^*\epsilon}$, where β^* and ϵ are the β -function at IP and the beam emittance respectively. Besides the requirement for small beam size at the interaction point, the luminosity depends on the

crossing angle (see Appendix A) between two beams. To achieve a nearly zero-crossing-angle collision between two proton beams, a pair of beam crossing dipoles is needed. These beam crossing dipoles would induce an antisymmetric contribution to the dispersion function. If, however, the dispersion function is not matched, the closed-orbit behavior of the off-momentum particles will be difficult to predict inside the accelerator. This in turn will affect the chromatic properties as well as higher-order resonance contributions. It is therefore important to have good dispersion function matching in the accelerator. This points to the requirement for the antisymmetric insertion quadrupole configuration (see Appendix B). In other words, the polarity of the quadrupoles is antisymmetric with respect to the IP for the $p\bar{p}$ collider.

In the $p\bar{p}$ collider, the proton and antiproton circulate in the same beam-pipe, and beam crossing dipoles are not needed. The dispersion function can be reduced to zero by the dispersion suppressor in the entire matching and IP region. Therefore a symmetric (refer to the quadrupole polarity relative to the IP) insertion design can be used. The symmetric insertion gives rise naturally to symmetry at IP. The tuning of the quadrupoles in the insertion region gains an extra two degrees of freedom. Therefore it is easier to tune the lattice. This concept was in the CBA symmetric lattice design, which has no beam crossing dipoles for the coasting beam pp collision at a natural crossing angle. Besides the advantage in the tuning of the insertion optics, the symmetric lattice design may have the highest possible superperiodicity. This will lead to smaller half-integer stop-band width. The advantage was exploited fully in the CBA lattice design.

On the other hand, an antisymmetric IR design can also be used for $p\bar{p}$ collision as well. In the antisymmetric IR configuration, the beam optics observed by the p and $\bar{p}$ are identical before and after the collision. Thus these two beams have identical optical properties; however, the superperiodicity may be reduced by a factor of 2.

In this chapter we limit our discussion to the pp option. We therefore discuss mainly the antisymmetric insertion optics.

(2) The magnets in the accelerator have to provide enough focusing strength and aperture for the beam size under consideration. Thus the length of the FODO cell in the arc is determined by the field strengths of dipoles and quadrupoles, the beam size requirement, and the cost effectiveness of the design. The quadrupoles in the IR depend on the achievable gradient. The beta-function at the IP depends on the aperture of the IR quadrupoles, where the maximum beta-function is about 10 times that in the arc.

(3) The region of the nearby resonances should be carefully avoided to increase the dynamical and the linear aperture. Once the lengths of the arc cell and the IR modules have been chosen, the total tune of the machine must be adjusted properly to avoid low-order systematic resonances. Systematic and random resonances become more important in the large superconducting collider because of relatively large higher magnetic multipoles and beam-beam interac-

tions. The tune adjustment of the machine can be obtained by changing the phase advance of the FODO cell in the arc or by tuning the phase-trombone.

One important issue regarding the resonance for the collider is the beam-beam interaction. Because of the Coulomb interaction, the particle would experience a kick due to the crossing of the other bunch. The beam-beam interaction is highly nonlinear; however, it can be characterized by a beam-beam strength parameter ξ .²

$$\xi = \frac{3N_b r_0}{2\pi\epsilon_N} \quad (2)$$

where r_0 is the classical radius of the particle, N_b is number of particles per bunch, and ϵ_N is the normalized emittance. Experience from CERN SpS experiments indicates that ξ should be < 0.005 per crossing. Since the beam-beam parameter is independent of the beta-value at the crossing point, the luminosity can be enhanced by mini- or micro- β insertions without changing the beam-beam strength parameter.

3 SOME CONCEPTUAL EVOLUTIONS OF THE COLLIDER DESIGN

We have discussed the need for a large number of repetitive building blocks to minimize the effect of the important systematic resonances, especially the half-integer stop-band (see Appendix C). This is especially important for machines expected to have a large momentum width. For the SSC, since the expected rms momentum spread is no more than 0.000175, the issue is indeed not important at all. The machine would be better, however, if it could be designed with smaller half-integer resonances.

When the machine superperiodicity, P , is small, the half-integer stop-band width can be reduced only via cancellation between various modules of the machine. It was demonstrated earlier in the CBA 2-in-1 lattice³ and the RHIC lattice design⁴ that neighboring IRs separated by an arc can be made to cancel each other for the important half-integer resonance. The effect is important there because of large momentum spread. The linear machine chromatic properties become very good with the cancellation.

More recently, Garren proposed cluster IR modules with special phase advance. The half-integer stop-bands of two identical adjacent IR modules can be made to cancel each other by adjusting the betatron phase advance in each module, i.e.

$$I_2 + (-)^{i2\pi\mu_1} I_2 = 0 \quad (3)$$

where I_2 is the half-integer stop-band integral of the insertion module, and μ_1 is the phase advance across the insertion. Thus the clustering of the IR module pairs with properly chosen phase advance,

$$\mu_1 = n \pm \frac{1}{4}; \quad n = \text{integer}, \quad (4)$$

gives rise to equivalent large superperiodicity of the collider. The total contribution of the half-integer stop-band would come essentially from the arc, where the total contribution can be adjusted to zero by appropriate choice of the phase advance of each cell.

The total contribution of the half-integer stop-band from the arc with M FODO cells and μ phase advance per cell becomes

$$J_{2p} = \left| \int \beta(k - k'\eta)e^{-2ip\phi(s)} ds \right| \frac{\Delta p}{p}$$

$$= \left| \frac{\sin 2\pi M\mu}{\sin 2\pi\mu} \left[\beta(F)k_f - \beta(D)k_D e^{-i\frac{2p}{Q}\mu\pi} - \beta(F)k'_F\eta(F) + \beta(D)k'_D\eta(D)e^{-i\frac{2p}{Q}\mu\pi} \right] \right| \frac{\Delta p}{p} \quad (5)$$

where k and k' are respectively the quadrupole and sextupole strengths. Therefore the stop-band width from the arc will be zero if the phase advance and the number of cells are chosen properly, i.e.

$$M\mu = \text{integer} \quad (6)$$

The choice of Eq. (6) also has the important consequence that the arc is also the first-order achromatic module.⁵ The conceptual evolution of the local cancellation of the half-integer stop-band provides the advantage of the cluster IR.⁶ The beam dynamics of the cluster IR should be equivalent to those of a collider with high superperiodicity. If the phase advance of the arc cell is used to vary the tune of the machine, then $M\mu = \text{integer} + \text{fraction part of tune}$, and the total contribution of the arc cell is not zero but a small number. The accelerator should perform very well.

Besides the half-integer resonances, there are third-order resonances due to the large sextupoles needed for chromatic corrections.^{7,8} The distribution of sextupoles is important in reducing the third-order resonance. This problem is discussed in Appendix D.

4 SSC LATTICE EXAMPLE

Figure 2 shows the conceptual cluster design of the SSC collider. There are two utility modules, two low- β modules, two medium- β modules, and two utility modules for future upgrade. Appendix E (taken from the SSC Conceptual Design)⁹ lists the relevant parameters of the lattice design by Garren et al. Note that the cell length in the arc is 192 m and the length of the IR module is 2400 m. The IR module is composed of DS, straight, DS-, and FD components, where the half-cell FD is added to make the IR module transparent to the optical matching. The modular construction becomes an independent unit. There are three types of straight sections, utility (U), low- β crossing (XL), and medium- β crossing (XM).

Fig. 2. The conceptual SSC cluster design.⁹ There are 8 insertion modules, with symbols U for utility; XL for low- β interaction, and XM for medium- β interaction.

Figure 3 shows the betatron amplitude function of the arc cell. Figure 4 shows the low- β crossing lattice functions. Note here that $\eta_y \neq 0$ in the Q1, Q2, Q3 triplet region discussed in Appendix B. Figure 5 shows an example of integrating the integral J_{2p} of Eq. (C.1) with $2p = 156$ along the two utility modules and two low- β interaction modules XL. We observe that the cumulative effect of the integral is very small in each utility module. The cumulative effect of a low- β module, however, is very important. It is dominated by one of the triplet focusing quadrupoles (Q_1 to Q_3). The effect is fairly well cancelled by the neighboring module. Similar cancellation can be observed also for $2p = 157$ (see Figure 6).

Fig. 3. The betatron and dispersion functions of a SSC cell.⁹

Fig. 4. The low- β interaction module⁹ is shown to demonstrate the dispersion matching at the IP. Note that η_y is antisymmetric with respect to IP. Because of the low- β insertion at IP, the condition in Eq. (B.5) is not satisfied: η_y is not zero at the Q_1 to Q_3 region. Antisymmetric insertion is essential to match dispersion functions.

Fig. 5. The integral of Eq. (C.1) for $2p = 156$ is evaluated element by element through the west cluster (U, U, XL, XL) (Fig. 2). The cancellation of the two nearby modules (XL) is clearly seen.

Fig. 6. Same as Fig. 5 except $2p = 157$.

5 CONCLUSION

We have reviewed some conceptual evolutions in the design of large hadron colliders. The evolution of the arc-IR module into the clustered IR module while maintaining good cancellation of the half-integer stop-band is an important contribution to accelerator design. This requires careful choice of the phase advance in each IR module. To obtain proper matching for the proton-proton collider with beam-crossing dipoles, antisymmetric configuration is preferred in order to provide the needed flexibility of optical matching. With an even number of clustered IRs, the phase advance of the total insertion can be made as a unit insertion of phase advance integer $\cdot 2\pi$. This gives rise further to the advantage that the machine superperiodicity becomes the number of the arc FODO cells, which is a very large number in the large hadron collider.

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Appendix A. Luminosity

Let us assume that the proton bunches form Gaussian profile functions, i.e.

$$\rho_{\pm}(x, y, z) = \frac{N_b}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} e^{-\left(\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2} + \frac{(z \pm vt)^2}{2\sigma_z^2}\right)}. \quad (\text{A.1})$$

Fig. 7. Simple view of vertical bunch crossing at an angle.

The opposite moving bunches collide at a crossing angle (see Fig. 7). Normally, σ_x, σ_y at the collision point are much smaller than σ_z , the rms bunch length. The luminosity is then defined by

$$L = 2 B f_{\text{rev}} \int f_x(x, y, z) f(x', y', z') dx dy dz dv \quad (\text{A.2})$$

where B is the number of bunches and f_{rev} the revolution frequency; (x', y', z') and (x, y, z) are related by coordinate transformation. Since the crossing angle α is usually small, we obtain then

$$L = f_{\text{rev}} \frac{BN_b^2}{4\pi\sigma_x\sigma_y(1 + q^2)^{1/2}} \quad (\text{A.3})$$

with

$$q = \frac{\alpha\sigma_z}{2\sigma_y} \quad (\text{A.4})$$

where the beam is crossing at an angle in the vertical plane. When the length of the bunch is not small compared with the β -function at the interaction point, one should take into account the β -function variation within the bunch:

$$\beta = \beta^* + \frac{S^2}{\beta^*} \quad (\text{A.5})$$

Therefore the transverse dimension σ_x , σ_y should be replaced by the average $\bar{\sigma}_x$, $\bar{\sigma}_y$ within the bunch as

$$\bar{\sigma}_x = \sigma_x \left(1 + \frac{\sigma_z^2}{\beta_x^*} \right)^{1/2} \quad (\text{A.6})$$

$$\bar{\sigma}_y = \sigma_y \left(1 + \frac{\sigma_z^2}{\beta_y^*} \right)^{1/2} \quad (\text{A.7})$$

Appendix B. Dispersion Matching at the Interaction Point

Let us consider the vertical beam crossing of the SSC. The beam transport can be schematically represented by the following transfer matrix:

$$\begin{pmatrix} \eta_y(S_1) \\ \eta'_y(S_1) \\ 1 \end{pmatrix} = M(S_1|S_0) \begin{pmatrix} \eta_y(S_0) \\ \eta'_y(S_0) \\ 1 \end{pmatrix} \quad (\text{B.1})$$

where

$$M(S_1|S_0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \phi \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{\frac{\beta_1}{\beta_0}} (\cos\phi + \alpha_0 \sin\phi) & \sqrt{\beta_0 \beta_1} \sin\phi & 0 \\ \frac{-1}{\sqrt{\beta_0 \beta_1}} [(1 + \alpha_0 \alpha_1) \sin\phi + (\alpha_1 - \alpha_0) \cos\phi] & \sqrt{\frac{\beta_0}{\beta_1}} (\cos\phi - \alpha_1 \sin\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\phi \\ 0 & 0 & 0 \end{pmatrix} \quad (\text{B.2})$$

where $\phi = \delta\psi$ is the phase advance between the vertical bending magnets, and the transfer matrices between the vertical bending magnets are indicated by a phase advance representation. The condition that

$$\eta_y(S_0) = \eta'_y(S_0) = 0, \quad \eta_y(S_1) = \eta'_y(S_1) = 0 \quad (\text{B.3})$$

with dispersion-free conditions on both sides of the vertical bending unit can be obtained only if the following conditions are met:

$$\sin \Delta\psi = 0 , \quad (\text{B.4})$$

$$\phi \left[1 - \sqrt{\frac{\beta_0}{\beta_1}} (\cos \Delta\psi - \alpha_1 \sin \psi) \right] = 0 . \quad (\text{B.5})$$

Note here that the condition of Eq. (B.5) can not be satisfied in the low- β module. Therefore the dispersion is nonzero (see Appendix D) in the Q1, Q2, Q3 region. In this case antisymmetric arrangement of the quadrupoles becomes essential in dispersion matching.

Appendix C. Superperiodicity and the Half-Integer Resonances

Following Courant and Snyder,¹ the half-integer stop-band due to gradient error $\Delta K(s)$ is defined to be

$$|J_{2p}| = \left| \int \beta \Delta K(s) e^{-i2p\phi(s)} ds \right| \quad (\text{C.1})$$

where

$$\phi(s) = \frac{1}{Q} \int^s ds / \beta(s) . \quad (\text{C.2})$$

The change of β -function of the accelerator due to the gradient error satisfies the following equation:

$$\frac{d^2}{d\phi^2} \left(\frac{\Delta\beta}{\beta} \right) + 4Q^2 \frac{\Delta\beta}{\beta} = -2 Q^2 \beta^2 \Delta K(s) \quad (\text{C.3})$$

where the gradient error due to the momentum spread, $\delta = \Delta p/p$, is given by

$$\Delta K(s) = - [K(s) - K'(s)\eta] (\delta - \delta^2 + \delta^3 - \dots) . \quad (\text{C.4})$$

Using the Fourier analysis of the error,

$$\beta^2 \Delta K(s) = \frac{1}{2\pi Q} \sum_p J_p e^{ip\phi} , \quad (\text{C.5})$$

we obtain

$$\frac{\Delta\beta}{\beta} = -\frac{Q}{4\pi} \sum_p \frac{J_p e^{ip\phi}}{Q^2 - \left(\frac{p}{2}\right)^2} \quad (C.6)$$

This indicates that the amplitude function is most sensitive to the harmonic component of the error field whose order is nearest to $2Q$.

When the accelerator has P superperiods, the Fourier components [Eq. (C.5)] of the systematic errors $\Delta K(s)$ are zero except $p = \text{integer} \cdot P$. Therefore the resonance-free tune space becomes considerably larger. The beat-factor of Eq. (C.6) will be smaller.

The tune shift of the machine is given by

$$\Delta Q = \frac{1}{4\pi} \int_0^c \beta \Delta k ds \quad (C.7)$$

Expanding the stop-band integral and tune shift in power series of $\delta = \frac{\Delta p}{p}$, we obtain

$$J_{2p} = I_{2p}^{(1)} \delta + I_{2p}^{(2)} \delta^2 + I_{2p}^{(3)} \delta^3 + \dots \quad (C.8)$$

$$\Delta Q = X^{(1)} \delta + X^{(2)} \delta^2 + X^{(3)} \delta^3 + \dots \quad (C.9)$$

where

$$I_{2p}^{(1)} = - \int \beta_0(s) [K(s) - k'(s)\eta] e^{-2ip\phi(s)} ds,$$

$$I_{2p}^{(2)} = -I_{2p}^{(1)} - \frac{Q}{4\pi} \sum_m \frac{I_m^{(1)} I_{2p-m}^{(1)}}{Q^2 - \left(\frac{m}{2}\right)^2},$$

$$I_{2p}^{(3)} = I_{2p}^{(1)} + \frac{Q}{4\pi} \sum_m \frac{I_m^{(1)} I_{2p-m}^{(1)}}{Q^2 - \left(\frac{m}{2}\right)^2} + \left(\frac{Q}{4\pi}\right)^2 \sum_{mn} \frac{I_m^{(1)} I_{n-m}^{(1)} I_{2p-n}^{(1)}}{\left[Q^2 - \left(\frac{m}{2}\right)^2\right] \left[Q^2 - \left(\frac{n}{2}\right)^2\right]}$$

and

$$X^{(1)} = \frac{1}{4\pi} I_0^{(1)},$$

$$X^{(2)} = -\frac{1}{4\pi} I_0 - \frac{Q}{4\pi} \sum_m \frac{|I_m^{(1)}|^2}{\left[Q^2 - \left(\frac{m}{2}\right)^2\right]},$$

$$X^{(3)} = \frac{1}{4\pi} I_0 + \frac{Q}{4\pi} \sum_m \frac{|I_m^{(1)}|^2}{Q^2 - \left(\frac{m}{2}\right)^2} - \frac{Q}{4\pi} \sum_m \frac{I_m^{(1)} I_m^{(2)}}{Q^2 - \left(\frac{m}{2}\right)^2}.$$

Eq. (C.6) indicates that the dominant contribution arises from $p \equiv \pm Q$ and $m \equiv \pm 2Q$. Thus a careful elimination of the first-order $I_{2p}^{(1)}$, where $p \equiv \pm Q$, will also give rise to smaller contributions to the higher-order terms, $I_{2p}^{(2)}$, $I_{2p}^{(3)}$, etc. Similar observation is also applicable to the chromaticity.

Appendix D. Third-Order Resonances

Because the large accelerator requires careful correction of chromaticity and half-integer stop-band band width with sextupoles, the third-order integer resonance becomes important as well. The single particle in the accelerator will experience the Hamiltonian

$$H = \frac{1}{2} \left[P_x^2 + K_x(s) x^2 \right] + \frac{1}{2} \left[P_y^2 + K_y(s) y^2 \right] + \frac{B''}{6B\rho} (x^3 - 3xy^2) \quad (D.1)$$

where $K_x(s)$ and $K_y(s)$ are gradients of quadrupoles and $B''/B\rho$ is the effective sextupole strength. Particles in the quadrupole field will execute betatron motion,

$$z = \sqrt{2I_z\beta} \cos [\psi_z(\theta) - Q_z\theta + \phi_z],$$

$$P_z = -\sqrt{\frac{2I_z}{\beta}} \left(\sin [\psi_z(\theta) - Q_z\theta + \phi_z] - \frac{\beta'}{2} \cos (\psi_z - Q_z\theta + Q_z) \right) \quad (D.2)$$

where $z = x$, or y , is the Courant-Snyder invariant. Since the circular machine is periodic in θ , the sextupole part of the Hamiltonian can be expanded into Fourier series, i.e.

$$H = Q_x I_x + Q_y I_y + (2I_x)^{3/2} \sum_m (A_{3m} \sin q_{3m} + 3A_{1m} \sin q_{1m}) \\ - (2I_x)^{1/2} (2I_y) \sum_m (2B_{1m} \sin p_{1m} + B_{+m} \sin p_{+m} + B_{-m} \sin p_{-m}), \quad (D.3)$$

$$A_{1m} e^{ia_{1m}} = \frac{i}{24\pi} \sum_k s_k e^{i(\psi_x(\theta_k) + (m-Q_x)\theta_k)}, \quad (D.4.1)$$

$$A_{3m} e^{ia_{3m}} = \frac{i}{24\pi} \sum_k s_k e^{i(3\psi_x(\theta_k) + (m-3Q_x)\theta_k)}, \quad (D.4.2)$$

$$B_{1m} e^{ib_{1m}} = \frac{i}{8\pi} \sum_k \bar{s}_k e^{i(\psi_x(\theta_k) + (m-Q_x)\theta_k)}, \quad (D.4.3)$$

$$B_{\pm m} e^{ib_{\pm m}} = \frac{i}{8\pi} \sum_k \bar{s}_k e^{i(\psi_x(\theta_k) \pm 2\psi_y(\theta_k) + (m-Q_x \mp 2Q_y)\theta_k)} \quad (D.4.4)$$

$$q_{3m} = 3\varphi_x - m\theta + a_{3m}, \quad q_{1m} = \varphi_x - m\theta + a_{1m},$$

$$p_{1m} = \varphi_x - m\theta + a_{1m}, \quad p_{\pm m} = \varphi_x \pm 2\varphi_y - m\theta + b_{\pm m}$$

$$\text{with } s_k = \beta_x^{3/2} \left(\frac{B''L}{2B\rho} \right)_k, \quad \bar{s}_k = (\beta_x \beta_y^2)^{3/2} \left(\frac{B''L}{2B\rho} \right)_k.$$

Note here that the third-order driving terms are obtained from Eqs. (D.4) with $m = Q_x$, $m = 3Q_x$; $m = Q_x \pm 2Q_y$.

On the other hand, when the accelerator has superperiod P , the nonvanishing terms in Eqs. (D.4) corresponds to $m = nP$, or multiple superperiodicity. Alternatively, the summations of the terms in Eq. (D.4) can be made to cancel each other if the same sextupole strengths are located 180° phase advance apart. This latter configuration has been used extensively in the accelerator design.⁸

One can solve the equation of motion from Eq. (D.3) and canonically transform the Hamiltonian. The new Hamiltonian contains the fourth-order effect of sextupoles, which induces tune modulation with amplitude and nonlinear coupling resonance at $2Q_x - 2Q_y = 0$.

Appendix E. SSC Lattice Parameter List

The relevant parameters of the SSC lattice are listed in this Appendix.

<u>Lattice Parameters</u>		Remarks & References SSC-N-139
configuration	2 rings, over/under, 0.7m vertical separation, clustered IRs	
circumference	82.944 km	
revolution time, frequency	0.27667 ms, 3.614 kHz	
straight sections (8 total)		
2 XL	2 low-beta crossing straight sections ($\theta^* = 0.5\text{m}$; $\pm 20\text{m}$ free space at IP)	
2 XM	2 medium-beta crossing straight sections ($\theta^* = 10\text{m}$; $\pm 101.65\text{m}$ free space at IP)	
2 U	2 utility straight sections (1 for injection and abort, beam 1) (1 for injection and abort, beam 2)	
2 U (future IRs)	2 straight sections for future development as IRs, configured initially as utility straight sections	
arrangement	north arc, east cluster, south arc, west cluster	
arc (north & south)	166 cells, length 31.872 km (each)	
cell lattice	QD/2 ⁺ SP 5 B QF SP 5 B QD/2	Note 1
QF, QD	arc quads	
SP	arc spool (corr. coils; cryo., vac., elec. conn.)	
5 B	5 bend magnets, horiz., 6.60 tesla	
magnetic rigidity, 20 TeV	6.67128×10^6 T-m	
magnetic radius	10.108 km	
arc slot lengths:		
cell	192.00 m	
QF, QD	4.32 m [0.4(end)+3.32(L_{eff})+0.4(end) +0.2(BPM on CW end)]	Note 1
QF/2, QD/2	2.06 m [0.4(end)+1.66($L_{eff}/2$)]	
QF/2 ⁺ , QD/2 ⁺	2.26 m [$L_{eff}/2$ +0.4(end)+0.2(BPM, CW end)]	
B	17.34 m [0.4(end)+16.54(eff)+0.4(end)]	
5 B	86.70 m	
SP	4.98 m	
west cluster	2 (DS, U, DS-, FD), 2 (DS, XL, DS-, FD)	
east cluster	2 (DS, U, DS-, FD), 2 (DS, XM, DS-, FD)	

Note 1: Component sequences are listed in the clockwise (CW) direction as seen from above.

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Clusters (cont.)Remarks &
References
SSC-N-139

DS, DS-	dispersion suppressors					
DS lattice	QD/2 ⁺⁺	SPd	5 B	QFK	SPd	5 B
	QDJ	SPd	5 B	QFI	SPd	5 B
	QDH	SPd	5 B	QFG	SPd	5 B
DS- lattice	5 B	QDG	SPd	5 B	QFH	SPd
	5 B	QDI	SPd	5 B	QFI	SPd
	5 B	QDK	SPd	5 B	QF/2 ⁻	
FD	half cell:	QF/2 ⁺	SP	5 B	QD/2	
QFG, QDG, --, QFK, QDK QD/2 ⁺⁺ , QF/2 ⁻	dispersion-suppressor quads special arc-quad slots at arc-disp.supp. transitions					
SPd	dispersion-suppressor spool					
SP	arc spool					
cluster and dispersion suppressor slot lengths:						
DS, U, DS-, FD	2,400.00 m					
DS, XL, DS-, FD	2,400.00 m					
DS, XM, DS-, FD	2,400.00 m					
U, XL, XM	1,161.30 m (each)					
angle between adj. straights	106.4 mrad					
tangential offset from adj. IP	127.7 m					
DS	573.09 m					
DS-+FD	569.61 + 96.00 = 665.61 m					
QFG--K, QDG--K	5.95 m					
QD/2 ⁺⁺	3.04 m					
QF/2 ⁻	2.91 m					
SPd	3.35 m					
SP	4.98 m					

Note 1

Note 1: Although the slot lengths of QFG, --, QDK are identical, the effective lengths are all different. See Section 4d, Insertion Magnets.

Remarks &
References
SSC-N-139

XL	low-beta crossing straight section						
lattice:							
doublet,	D4.75	QDL6	D47.56	QFL5			
vertical step,	D0.6	BV4	D11	BV3	D0.5		
M=-1 section,	QDV	D18.31	2 QFV	D17.69	2 QDL4		
		D17.69	2 QFV	D18.31	QDV		
vertical step,	D25.5	BVL2	D122	BVL1	D5.08		Note 1
triplet,	QFL3	2 QDL2	QFL1				
IR,	D19.6	IP	D19.6				
triplet,	QDL1	2 QFL2	QDL3				
vertical step,	D5.08	BVL1	D122	BVL2	D25.5		
M=-1 section,	QFV	D18.31	2 QDV	D17.69	2 QFL4		
		D17.69	2 QDV	D18.31	QFV		
vertical step,	D0.5	BV3	D11	BV4	D0.6		
doublet	QDL5	D47.56	QFL6	D8.23			
QFL1-6, QDL1-6	XL insertion quads						
QFV, QDV	vertical dispersion quads						
BVL1, BVL2	strings of 2 SC vertical-bend magnets, 5.212 tesla						
BV3, BV4	strings of 5 SC vertical-bend magnets, 5.212 tesla						
D4.75	drift distance of 4.75 m, etc.						
slot lengths:							
XL straight section	1,161.30 m (betw. end bend slots of DS & DS-)					Note 2	
QFL1, QDL1	15.30 m [0.4(end)+L _{eff} +0.4(end), BPM not incl.]						
QFL2, QDL2	12.59 m "						
QFL3, QDL3	13.84 m "						
QFL4, QDL4	8.965 m "						
QFL5, QDL5	14.40 m "						
QFL6, QDL6	7.60 m "						
QFV, QDV	8.345 m [0.4(end)+L _{eff} +0.4(end), BPM not incl.]						
BVL1	18.00 m [2 magnets, each 0.5(end)+8.0+0.5(end)]						
BVL2	18.00 m "						
BV3	45.00 m [5 magnets "]						
BV4	45.00 m "						
space at IP	±20 m between magnetic quad ends, ±17 m between collimators						

Note 1: Components in bold-face type have single apertures common to the two rings.

Note 2: Straight-section length between anti-symmetric optical match points is 1155.32 m.

Clusters (cont.)

Remarks &
References
SSC-N-139

XM

medium-beta crossing straight section

lattice:

doublet,
vertical step,
M=-1 section,

D11.5	QDM6	D36.71	QFM5	
D0.1	BV4	D11	BV3	D0.5
QDV	D18.31	2 QFV	D18.31	2 QDM4
	D18.31	2 QFV	D18.31	QDV

vertical step,
triplet,

D12.27	BVM2	D34	BVM1	SP
QFM3	D4.2	QDM2	D3.7	QDM2

Note 1

IR,
triplet,

D101.25	IP	D101.25		
QDM1	D5.1	QFM2	D3.7	QFM2
	D4.2	QDM3		

vertical step,
M=-1 section,

SP	BVM1	D34	BVM2	D12.27
QFV	D18.31	2 QDV	D18.31	2 QFM4
	D18.31	2 QDV	D18.31	QFV

vertical step,
doublet

D0.5	BV3	D11	BV4	D0.5
QDSM	D36.71	QF6M	D14.98	

QFM1-6, QDM1-6
QFV, QDV
BVM1, BVM2
BV3, BV4
SP
D11.5XM insertion quads
vertical dispersion quads
strings of 4 SC vertical-bend magnets, 5.212 tesla
strings of 5 SC vertical-bend magnets, 5.212 tesla
standard arc spool
drift distance of 11.5 m, etc.

slot lengths:

XM straight sect.

1161.30 m (betw. end bend slots of DS and DS-)

Note 2

QFM1, QDM1	10.0 m	[0.4(end)+L _{eff} +0.4(end), BPM not incl.]
QFM2, QDM2	8.30 m	"
QFM3, QDM3	7.80 m	"
QFM4, QDM4	8.345 m	"
QFM5, QDM5	8.30 m	"
QFM6, QDM6	8.30 m	"
QFV, QDV	8.345 m	"
BVM1, BVM2	36.00 m	[4 magnets each]
BV3, BV4	45.00 m	[5 magnets each]
space at IP	±101.65 m	between magnetic quad ends

Note 1: Components in bold-face type have single apertures common to both rings.

Note 2: Straight-section length between anti-symmetric optical match points is 1155.32 m.

					Remarks & References
U	utility straight section				SSC-N-139
lattice:					
doublet,	D0.85	QDR	D7.57	QFS	Note 1 abort system
long drift, doublet,	D199.2	QDT	D0.2	QFU	
long drift,	D329.61	U-center	D329.61		
doublet, long drift	QDU	D0.2	QFT	D199.2	
doublet	QDS	D7.57	QFR	D4.33	
QFR-U, QDR-U	utility insertion quads				
D0.85	drift distance of 0.85 m, etc.				
slot lengths:					
U straight sect.	1,161.30 m (betw. end bend slots of DS and DS-)				Note 2
QFR, QDR	8.63 m $[0.4(\text{end}) + L_{\text{eff}} + 0.4(\text{end})]$, BPM not incl.]				
QFS, QDS	9.26 m	-	-	-	
QFT, QDT	11.68 m	-	-	-	
QFU, QDU	11.91 m	-	-	-	

Lattice Optics

phase shift per cell, x,y	60°, 60°				
phase shift between adjacent XL IPs	3.75 x 360° (each plane)				
phase shift between adjacent XM IPs	3.25 x 360° "				
phase shift between adjacent U centers	2.25 x 360° "				
betatron tune, x,y	78.265, 78.28, nominal				
tuning range, x,y	± 2				
momentum compaction factor	2.227 x 10 ⁻⁴				
transition gamma	67.00				
natural chromaticity, x,y	-204, -204				
zero-chrom. sext. B ² L/Bρ, F,D	0.00987, -0.01591 m ⁻²				
	$\beta_{\text{max},x,y}$	$\beta_{\text{min},x,y}$	$\eta_{x,\text{max}}$	$\eta_{x,\text{min}}$	$\eta_{y,\text{max}}$
normal arc	332 m	111 m	3.94 m	2.36 m	0 m
dispers. supp. (horiz.)	357	37.7	4.04	0	0
XL (B*=0.5m, η*=0)	7,958	0.50	0	0	±0.38
XM (B*=10m, η*=0)	2,944	5.43	0	0	±0.34
U	1,099	70.9	0	0	0

Note 1: The RF and injection systems are in the first long drift (199 m) reached by each injected beam.

Note 2: Straight-section length between anti-symmetric optical match points is 1155.32 m.