Method for Calculating Strong Synchrotron Tune Modulation of Depolarizing Resonances in Storage Rings

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It is well-known that the spins of electrons and positrons circulating in storage rings become spontaneously polarized by the emission of synchrotron radiation; this is now known as the Sokolov-Ternov effect [1]. They predicted that the polarization would build up exponentially to an asymptotic value of $8/(5\sqrt{3}) \approx 92.4\%$. In practice, the actual value of the polarization can be strongly reduced by so-called "depolarizing spin resonances." The spin resonances occur whenever the condition

$$\nu = m_0 + m_1\nu_x + m_2\nu_y + m_3\nu_z \quad (1)$$

is satisfied. Here ν is the spin tune, the ν_i are the orbital tunes, and the m_i are integers, including zero. A formalism for calculating the strengths of these resonances was derived by Mane [2]. It expands a suitably defined perturbation series in powers of the orbital actions, followed by an average over the orbital action-angle variables. Effectively, this yields a series in powers of the orbital emittances.

However, the perturbation series can be summed in other ways, in particular for the so-called "synchrotron sideband resonances" [3] - [7], and all the results have been shown to be mathematically equivalent [7]. The expression for the spin resonance strengths is written, not as a power series, but as a series of the form

$$\sum_{m=-\infty}^{\infty} C_m e^{-\alpha} I_m(\alpha) \quad (2)$$

where the I_m are modified Bessel functions, and the C_m are coefficients whose values are not important here. The perturbation expansion parameter α is not merely $\sigma_c^2 = (\sigma_E/E_0)^2$, but rather $\alpha = (\gamma a)^2 \sigma_c^2 / \nu_s^2$, where ν_s is the synchrotron tune, γ is the particle energy in units of rest energy, and $a = (g - 2)/2 \approx 0.00116$ for an electron. In a ring like LEP, $\gamma a \approx 100$ and $\nu_s \approx 0.1$, so $\alpha \approx 10^6 \sigma_c^2 \gg \sigma_c^2$. Numerical calculations, given in Ref. [7], indicate that $\alpha \approx 1.4$ in HERA at 30 GeV and $\alpha \approx 0.8$ in LEP at 50 GeV,

so a power series expansion in α might not converge suitably rapidly for these rings. In this paper, we shall be concerned with the problem of treating cases where α is large, i.e. $O(1)$ or more. We shall call this "strong synchrotron tune modulation."

Let us define the problem more precisely. There would be no problem if synchrotron spin resonances were the *only* type present in storage rings, because formulas, treating only synchrotron spin resonances, have been given in Refs. [3] - [7], for all values of α . There would also be no problem if there were no synchrotron sideband spin resonances, because it is known empirically (there is no rigorous proof) that a power series expansion for the other resonances, in powers of the orbital emittances, converges rapidly. There is also no problem if there are both "parent" spin resonances and synchrotron sidebands, for weak synchrotron tune modulation, i.e. $\alpha \ll 1$, because a power series expansion, for all the resonances, will converge rapidly. The problem is: there is no known formalism which deals with both parent spin resonances plus *strong* synchrotron tune modulation, $\alpha \geq O(1)$. This paper proposes to offer an *approximate* method for dealing with the problem.

The polarization is given by the Derbenev-Kondratenko formula [8]

$$P = \frac{8}{5\sqrt{3}} \frac{\langle |\rho|^{-3} [\hat{b} \cdot \hat{n} - \hat{b} \cdot \vec{d}] \rangle}{\langle |\rho|^{-3} \left[1 - \frac{2}{9} (\hat{n} \cdot \hat{v})^2 + \frac{11}{18} |\vec{d}|^2 \right] \rangle} \quad (3)$$

where $\hat{n}$ is the quantization axis of the spin eigenstates and $\vec{d} = \gamma(\partial \hat{n} / \partial \gamma)$ is the spin orbit coupling vector. The definitions of $\hat{b}$, $\hat{v}$, ρ and the angular brackets do not matter here. The term in $|\vec{d}|^2$ has the greatest relevance when calculating the strengths of spin resonances. Suppose the parent resonance is a first order betatron spin resonance, $\nu_p = m_0 \pm \nu_{x,y}$, and $|\vec{d}|^2 = |A_p|^2 / (\nu - \nu_p)^2$, i.e. $|\vec{d}|^2 \rightarrow \infty$ as $\nu - \nu_p \rightarrow 0$. Here A_p is a complex number, which determines the strength of the resonance. Then in Ref. [7] it was shown that the synchrotron tune modulation yields

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sidebands given by

$$|\vec{d}|^2 \rightarrow |A_p|^2 e^{-\alpha} \left\{ \sum_{m=-\infty}^{\infty} \frac{1}{(\nu - \nu_p + m\nu_s)^2} \times \right. \\ \left. \times \left[I_m + \frac{J_c}{J_{x,y}} \frac{\alpha}{2} (I_{m-1} + I_{m+1}) \right] \right. \\ \left. + \sum_{m=-\infty}^{\infty} \frac{1}{\nu - \nu_p + m\nu_s} \frac{mJ_c}{\nu_s J_{x,y}} I_m \right\} \quad (4)$$

where I_m means $I_m(\alpha)$ and the J 's are damping partition numbers.

Now, in general, $\vec{d}$ contains several resonances, and is of the form

$$|\vec{d}|^2 = \left| \sum_p \frac{A_p}{\nu - \nu_p} \right|^2 \\ = \sum_p \frac{|A_p|^2}{(\nu - \nu_p)^2} + \text{cross-terms.} \quad (5)$$

We neglect the cross-terms, because there is no formula to obtain synchrotron sidebands for the cross-terms. We introduce synchrotron sideband spin resonances for the diagonal terms via

$$|\vec{d}|^2 \rightarrow \sum_p \frac{|A_p|^2}{(\nu - \nu_p)^2} \quad (6)$$

followed by

$$|\vec{d}|^2 \rightarrow \sum_{p,m} \frac{|A_p|^2 e^{-\alpha}}{(\nu - \nu_p + m\nu_s)^2} \times \\ \times \left[I_m + \frac{J_c}{J_{x,y}} \frac{\alpha}{2} (I_{m-1} + I_{m+1}) \right] \\ + \sum_{p,m} \frac{|A_p|^2 e^{-\alpha}}{\nu - \nu_p + m\nu_s} \frac{mJ_c}{\nu_s J_{x,y}} I_m. \quad (7)$$

If the parent resonance is not a first-order betatron spin resonance, e.g. if it is a higher order betatron spin resonance, then the factors of $J_c/J_{x,y}$ become more complicated, but the basic idea of starting from $|A_p|^2/(\nu - \nu_p)^2$ and generalizing it remains the same. Thus the basic procedure is: (1) use a power series expansion to obtain an expression for $\vec{d}$, hence $|\vec{d}|^2$, (2) neglect the cross-terms, (3) generalize the diagonal terms to include synchrotron sidebands, using modified Bessel functions. There are several obvious approximations in such a procedure. It is not necessarily true that the cross-terms are negligible. Furthermore, a resonance of the form $\nu = \nu_x + \nu_s$ can be visualized as (i) a synchrotron sideband of the parent resonance $\nu = \nu_x$, or (ii) as a betatron sideband of the parent resonance $\nu = \nu_x$. In the above scheme, we neglect case (ii) and do not include the contribution

to the resonance strength coming from the betatron sideband of the parent resonance $\nu = \nu_x$. This is not a priori justified. In addition, the formalisms of Refs. [3] - [7] assume the ring is flat, with all non-flatness due to imperfections. This is not true in a ring with spin rotators, or Siberian Snakes, etc., so the above Bessel function formula is not a priori valid for a ring such as HERA.

Further work is clearly required to see if the above approximate scheme works satisfactorily in practice. Note that, since the power series expansion of a Bessel function converges absolutely for *all* values of the argument, a power series expansion for the synchrotron sidebands should always converge. The problem is that the convergence may be slow. The above idea may at least provide a starting point for the eventual development of a suitably fast algorithm to deal with the problem of strong synchrotron tune modulation.

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