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Fields From A Vertical Magnetic Dipole Beneath A Layered Earth

Thurlow W. H. Caffey

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FIELDS FROM A VERTICAL MAGNETIC
DIPOLE BENEATH A LAYERED EARTH

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ABSTRACT

Both the E and H fields from a vertical magnetic dipole placed in the third layer of a three-layer earth are derived. The results are applied to transmission beneath two Wyoming coal seams to see the effects of the overburden upon the voltage induced in a horizontal loop which is coplanar with the dipole. Beneath each seam, the effects of the overburden can be neglected when both the dipole and loop are at least one meter below the seam and when the operating frequency is less than 15 kHz.

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FOREWORD

The study of transmission between coplanar, horizontal loops buried within a layered earth arises from an underground coal gasification experiment.¹ It is necessary, in the experiment, to transmit data from beneath the floor of a burning coal seam to a receiver outside the burning region. To reduce drilling and emplacement costs it is desirable to place the ends of the transmission link just beneath the floor of the seam, but it must first be shown that the close proximity of the burning coal will not adversely affect the transmission. This report, beginning with a simple outline of the telemetry system, shows that the loops can be placed only a short distance beneath the seam. Formulas for the electromagnetic fields in any of the layers and on the surface are provided for use in other situations.

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FIELDS FROM A VERTICAL MAGNETIC DIPOLE BENEATH A LAYERED EARTH

The Telemetry System

Very large coal deposits underlie portions of the state of Wyoming. The deposits consist of several extensive seams, 2- to 10-m thick, separated by layers of earth with seam depths ranging from 47 to 460 m. As an alternative to both strip mining, with its pollution problems, and to conventional mining with its safety hazards, an experimental program to burn coal in place is now underway.

When a coal seam is burned in place, it is necessary to monitor the combustion process. Many vertical strings of thermocouples (TCs) are placed in the seam before ignition and along the expected burn path. Each string is grouted in place and connected to the surface data processing equipment by a messenger cable (Figure 1). Ideally, the burn spreads along the floor of the seam and consumes the seam from the bottom up. In this case the TCs in a string are eventually destroyed, beginning with the lowest TC in the string. However, it is not uncommon for the burn to move to the top of the seam and destroy the cable connecting all the TCs to the surface. All temperature data in the burned-through TC string is thereby lost. This problem is overcome by using a second TC string which is connected to a telemetry package beneath the seam. The telemetry package samples and stores twelve TC outputs every hour, and transmits the data every 6 hours to a receiver in a vertical shaft outside the burning seam. The receiver output is cabled to the surface, demodulated, and sent to a data processor. The use of dual TC-strings guarantees temperature data after the burn has severed the TC-strings until the time that the seam thickness is consumed. The buried system has a range of 50 m and battery capacity for 6 months operation.

The Electromagnetic Fields

The EM fields are determined from the solution to a boundary-value problem which has been discussed for the homogeneous earth by Wait² and for the two-layer earth by Wait and Spies.³

For the present problem, the earth will be modelled as a vertical stack of three homogeneous layers (Figure 2). The third layer is assumed to be infinitely deep. The origin of a cylindrical (ρ, ϕ, z) coordinate system is located at the air-earth interface. A source loop is located at $(0, 0, -h)$ and a receiving loop is centered at $P(\rho, 0, z)$. Both loops lie parallel to

the $z = 0$ plane and are much smaller than either the wavelength or pathlength in the underburden. The permeability of each layer is assumed to be that of free space, μ_0 .

Figure 1. Lateral Transmission Between Vertical Shafts

Figure 2. 3-Layer Earth Model

Time-dependence as $\exp(j\omega t)$ will be assumed where ω is the angular frequency. It is clear that the solutions will possess azimuthal symmetry with respect to the source loop. The fields will be derived in terms of the magnetic Hertz vector, π^* ,⁴ using

$$\vec{H}_i = \nabla \left(\nabla \cdot \pi_i^* \right) - \mu_0 \left(\sigma_i \frac{\partial \pi_i^*}{\partial t} + \epsilon_i \frac{\partial^2 \pi_i^*}{\partial t^2} \right), \quad (1a)$$

$$\vec{E}_i = -\mu_0 \nabla \times \frac{\partial \pi_i^*}{\partial t} \quad (1b)$$

in which the dielectric constant of the i -th layer is

$$\epsilon_i = \epsilon_{r,i} \epsilon_0 \quad (1c)$$

When the source is a loop of area NA carrying an rms current I , the only component of π_i^* is the z -component,

$$\pi_{z,i}^* = \frac{NAI}{4\pi} F_i \quad (2)$$

where F_i is a scalar function of the geometry and the soil parameters. The fields in each region can be found in terms of the F_i as

$$H_{\phi,i} = C \frac{\partial^2 F_i}{\partial \rho \partial z}, \quad (3a)$$

$$H_{z,i} = C \left(\frac{\partial^2 F_i}{\partial z^2} - k_i^2 F_i \right), \quad (3b)$$

and

$$E_{\phi,i} = -j\mu_0 \omega C \frac{\partial F_i}{\partial \rho}, \quad (3c)$$

where

$$k_i^2 = \mu_0 \epsilon_i \omega^2 \left(1 - j \frac{\sigma_i}{\epsilon_i \omega} \right) \quad (3d)$$

and

$$C = \frac{NAI}{4\pi}. \quad (3e)$$

The tangential field components are continuous across the interfaces in the absence of any surface current along the interface, hence

$$\frac{\partial F_i}{\partial \rho} = \frac{\partial F_j}{\partial \rho} \quad (4a)$$

and

$$\frac{\partial^2 F_i}{\partial \rho \partial z} = \frac{\partial^2 F_j}{\partial \rho \partial z} \quad (4b)$$

The F_i and their derivatives must vanish as ρ becomes infinite, so when Eq (4) is integrated with respect to ρ the constant of integration must be zero. The boundary conditions become

$$F_i = F_j \quad (5a)$$

and

$$\frac{\partial F_i}{\partial z} = \frac{\partial F_j}{\partial z} \quad (5b)$$

It only remains to obtain the F_i and apply the boundary conditions.

The F_i are scalars and must be solutions of the wave equation,

$$\nabla^2 F_i - k_i^2 F_i = 0 \quad (6a)$$

or

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial F_i}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 F_i}{\partial \phi^2} + \frac{\partial^2 F_i}{\partial z^2} - k_i^2 F_i = 0. \quad (6b)$$

Because of azimuthal symmetry,

$$\frac{\partial^2 F_i}{\partial \phi^2} = 0, \quad (7)$$

and the F_i can be sought by means of separation of variables,

$$F_i = R(\rho) Z(z). \quad (8)$$

Using Eq (8) in Eq (6) leads to

$$\frac{1}{R\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial R}{\partial \rho} \right) = \frac{-1}{z} \frac{\partial^2 Z}{\partial z^2} + k_i^2. \quad (9)$$

Both sides of Eq (9) can be set equal to the same real constant because they are independent of each other. Beginning with the right-hand side, the equation

$$- \frac{1}{z} \frac{\partial^2 Z}{\partial z^2} + k_i^2 = -\lambda^2 \quad (10a)$$

has this solution for $Z(z)$:

$$Z(z) = Z_a \exp \left[\left(\lambda^2 + k_i^2 \right)^{1/2} z \right] + Z_b \exp \left[- \left(\lambda^2 + k_i^2 \right)^{1/2} z \right]. \quad (10b)$$

Z_a and Z_b are found from boundary conditions, and hereafter it is convenient to write

$$u_i = \left(\lambda^2 + k_i^2 \right)^{1/2}. \quad (11)$$

It is necessary that the real part of $u_i > 0$ in order that $Z(z)$ diminish as $|z|$ becomes large. The left-hand side of Eq (9) becomes

$$\frac{\lambda^2 R}{\rho^2} + \frac{1}{\rho} \frac{\partial R}{\partial \rho} + R \lambda^2 = 0 \quad (12a)$$

which is Bessel's equation of order zero. The solution which is finite at the origin is

$$R(\rho) = R_o J_o(\lambda \rho) \quad (12b)$$

where R_o is to be determined from boundary conditions. The use of Eq (10) and Eq (12) in Eq (8) provides one solution for the F_i as a function of λ . The complete solution requires integration over all λ ,

$$F_i = \int_0^\infty G_j(\lambda) \exp \left(\mp u_i z \right) J_o(\lambda \rho) d\lambda \quad (13)$$

and the $G_j(\lambda)$ are eigenfunctions to be found. Equation (13) is valid throughout all layers, but in the third layer the source function must be added:

$$F_3 = \frac{\exp(-k_3 r)}{r} + \int_0^\infty G_5(\lambda) \exp(u_3 z) J_0(\lambda \rho) d\lambda \quad (14a)$$

where

$$r^2 = \rho^2 + (z + h)^2. \quad (14b)$$

In order to use Eq (14) in Eq (4), it is necessary to represent the source function in an integral form due to Sommerfeld:⁵

$$\frac{\exp(-k_3 r)}{r} = \int_0^\infty \frac{\lambda}{u_3} \exp(-u_3 |z + h|) J_0(\lambda \rho) d\lambda. \quad (15)$$

In layers 1 and 2, where there are two boundaries, cylindrical waves can propagate in both axial directions, and it will be helpful to write the F_i explicitly.

$$F_0 = \int_0^\infty G_0 \exp(-u_0 z) J_0(\lambda \rho) d\lambda, \quad (16a)$$

$$F_1 = \int_0^\infty \left[G_1 \exp(-u_1 z) + G_2 \exp(u_1 z) \right] J_0(\lambda \rho) d\lambda, \quad (16b)$$

$$F_2 = \int_0^\infty \left[G_3 \exp(-u_2 z) + G_4 \exp(u_2 z) \right] J_0(\lambda \rho) d\lambda, \quad (16c)$$

and

$$F_3 = \int_0^\infty \frac{\lambda}{u_3} \left\{ \exp[-u_3 |z + h|] + G_5 \exp(u_3 z) \right\} J_0(\lambda \rho) d\lambda. \quad (16d)$$

These results are used in Eq (5) to form the following system of equations:

$$\begin{vmatrix}
1 & -1 & -1 & 0 & 0 & 0 \\
u_0 & -u_1 & u_1 & 0 & 0 & 0 \\
0 & e^\alpha & e^{-\alpha} & -e^\beta & -e^{-\beta} & 0 \\
0 & u_1 e^\alpha & -u_1 e^{-\alpha} & -u_2 e^\beta & u_2 e^{-\beta} & 0 \\
0 & 0 & 0 & u_3 e^\gamma & u_3 e^{-\gamma} & -\lambda e^\delta \\
0 & 0 & 0 & u_2 e^\gamma & -u_2 e^{-\gamma} & \lambda e^\delta
\end{vmatrix}
X
\begin{vmatrix}
G_0 \\
G_1 \\
G_2 \\
G_3 \\
G_4 \\
G_5
\end{vmatrix}
= \lambda e^{-\xi}
\begin{vmatrix}
0 \\
0 \\
0 \\
0 \\
1 \\
1
\end{vmatrix}
\quad (17)$$

In Eq (17) the following notation was introduced:

$$\alpha = u_1 d_1, \quad \beta = u_2 d_1, \quad \gamma = u_2 d_2, \quad \delta = u_3 d_2$$

and

$$\xi = u_3 |h - d_2| \text{ or } \xi = u_3 (h - d_2) \text{ because } h > d_2 \text{ always.}$$

The solution of Eq (17) for the eigenfunctions and the formulas for the field components are listed in the appendices. We now turn to the study of communication between horizontal coplanar loops in the third layer.

Communication Within the Third Layer

The formula for the magnitude of the voltage induced in the receiver loop with a turns-area product $(NA)_r$ by the coplanar source loop is

$$V = u_0 \omega (NA)_r |H_{z,3}| \quad \text{volts, rms.} \quad (18)$$

The formula for the magnetic intensity, from Appendix A, consists of two principal terms:

$$\begin{aligned}
H_{z,3} = \frac{NAI}{4\pi} & \left\{ \left[2(z+h)^2 (1+k_3 r) - \rho^2 \left(1+k_3 r + k_3^2 r^2 \right) \right] \frac{e^{-k_3 r}}{r^5} \right. \\
& \left. + \int_0^\infty u_3^2 G_5 e^{u_3 z} J_0(\lambda \rho) d\lambda \right\} \quad \text{amp/meter.} \quad (19)
\end{aligned}$$

The first term is due to a magnetic dipole in an infinite conducting medium,⁶ and the integral is the effect of the layered earth. The method of evaluating the integral is described in Appendix D. The effect of the overburden upon transmission within the third layer can be studied by forming the ratio of the second term to the first. When the loops are coplanar ($z = -h$), the ratio is given by

$$H\text{-ratio} = \frac{\left| \int_0^\infty u_3^2 G_5 e^{-u_3 h} J_0(\lambda \rho) d\lambda \right|}{\left| \left(1 + k_3 \rho + k_3^2 \rho^2 \right) \frac{e^{-k_3 \rho}}{\rho^3} \right|}. \quad (20)$$

The H-ratio is of interest here over the range $|z + d_2| < 5$ m and $10 \leq \rho \leq 70$ m. The earth structure at two Wyoming coal gasification sites, Hanna and Hoe Creek, are listed below.

Earth Parameter	Location	
	Hanna	Hoe Creek
d_1	100 m	49 m
d_2	110 m	56 m
σ_1	0.02 S/m	0.05 S/m
σ_2 , (unburned)	0.005 S/m	0.02 S/m
σ_2 , (burning)	0.2 S/m	0.8 (est) S/m
σ_3	0.02 S/m	0.07 S/m

When the conductivity of a layer is at least 10^{-3} S/m and the operating frequency is no greater than 15 kHz, the effect of ϵ_r on the propagation constant of the layer is less than 1% as long as $\epsilon_r \leq 1050$. In coal at low frequencies the principal cause of polarization is the water molecule for which $\epsilon_r = 82$. This value is used in all computations.

The H-ratio for each site is presented in Figures 3 and 4 as a function of ρ with the coal state and the distance beneath the coal seam as parameters. A frequency of 10500 Hz was used at Hanna, but 14500 Hz was used at Hoe Creek to increase the induced voltage. The H-ratio is less than -38 dB in all cases so that the integral term in Eq (19) can be ignored. In other words, the state of the coal seam, whether unburned or burning, is insignificant and the spacing of the coplanar loops from the seam is inconsequential as far as transmission is concerned.

Figure 3
Hanna H-Ratio vs ρ with Coal State and $\Delta = |z + d_2| = h - d_2$ as Parameters

Figure 4
Hoe Creek H-Ratio vs ρ with Coal State and $\Delta = |z + d_2| = h - d_2$ as Parameters

References

1. Thurlow W. H. Caffey and David E. Barnes, "A Telemetry System Beneath a Burning Coal Seam," Proceedings 25th International Convention of the ISA, Anaheim, CA, May 1979.
2. James R. Wait, "The Magnetic Dipole over the Horizontally Stratified Earth," Can. J. Physics, Vol. 29 (Nov. 1951) pp. 577-592.
3. James R. Wait and K. P. Spies, "Evaluation of the Surface Electromagnetic Fields for a Buried Magnetic Dipole Source," Air Force Cambridge Research Laboratories, Report No. 52, February 1971.
4. J. A. Stratton, Electromagnetic Theory, McGraw-Hill, New York, 1941, p. 29.
5. Arnold Sommerfeld, Partial Differential Equations in Physics, Academic Press, New York, 5th Printing 1967, pp. 240-242.
6. James R. Wait, "The Magnetic Dipole Antenna Immersed in a Conducting Medium," Proceedings IRE, Oct. 1952, pp. 1244-1245.

APPENDIX A

Field Formulas in a Three-Layer Earth

The formulas for the three field components in each layer are listed. The semi-infinite region above the earth is denoted by a zero subscript, and the propagation coefficient in each layer is given by

$$u_i = \left(\lambda^2 + k_i^2 \right)^{1/2} . \quad (A1)$$

A reflection function at the interface between two layers is given by

$$U_{ij} = \frac{u_i - u_j}{u_i + u_j} . \quad (A2)$$

The determinant of coefficients is a common denominator term in the solution for the eigenfunctions, G_j , and is given by the product of two major terms:

$$\begin{aligned} D = & \left\{ \lambda (u_0 + u_1)(u_1 + u_2)(u_2 + u_3) \exp \left[u_1 d_1 + u_2 (d_2 - d_1) - u_3 d_2 \right] \right\} \\ & \times \left\{ 1 + U_{01} U_{12} \exp (-2u_1 d_1) + U_{12} U_{23} \exp \left[-2u_2 (d_2 - d_1) \right] \right. \\ & \left. + U_{01} U_{23} \exp \left[-2u_1 d_1 - 2u_2 (d_2 - d_1) \right] \right\} . \end{aligned} \quad (A3)$$

The first term will be divided into numerator terms of the G_j , but the second term always appears intact and is given the separate designation "DENOM". The field components and layer eigenfunctions are listed below for each layer starting from the top.

The fields in the upper semi-infinite layer, $z > 0$, are

$$H_\phi = C \int_0^\infty \lambda u_o G_o \exp(-u_o z) J_1(\lambda \rho) d\lambda \quad (A4)$$

$$H_z = C \int_0^\infty \lambda^2 G_o \exp(-u_o z) J_o(\lambda \rho) d\lambda \quad (A5)$$

$$E_\phi = j\omega\mu_o C \int_0^\infty \lambda G_o \exp(-u_o z) J_1(\lambda \rho) d\lambda \quad (A6)$$

in which

$$G_o = \frac{\left(\frac{2\lambda}{u_o + u_1}\right) \left(\frac{2u_1}{u_1 + u_2}\right) \left(\frac{2u_2}{u_2 + u_3}\right) \exp\left[-u_1 d_1 - u_2 (d_2 - d_1) - u_3 (h - d_2)\right]}{\text{DENOM}} \quad (A7)$$

The fields in the first layer, $-d_1 < z < 0$, are

$$H_\phi = C \int_0^\infty \lambda u_1 \left[G_1 \exp(-u_1 z) - G_2 \exp(u_1 z) \right] J_1(\lambda \rho) d\lambda \quad (A8)$$

$$H_z = C \int_0^\infty \lambda^2 \left[G_1 \exp(-u_1 z) + G_2 \exp(u_1 z) \right] J_o(\lambda \rho) d\lambda \quad (A9)$$

$$E_\phi = j\omega\mu_o C \int_0^\infty \lambda \left[G_1 \exp(-u_1 z) + G_2 \exp(u_1 z) \right] J_1(\lambda \rho) d\lambda \quad (A10)$$

in which

$$G_1 = \frac{\left(\frac{2\lambda}{u_1 + u_2}\right) \left(\frac{2u_2}{u_2 + u_3}\right) \exp\left[-u_1 d_1 - u_2 (d_2 - d_1) - u_3 (h - d_2)\right]}{\text{DENOM}} \quad (A11)$$

or

$$G_1 = \left(\frac{u_o + u_1}{2u_1}\right) G_o \quad (A12)$$

and

$$G_2 = -U_{01} G_1 \quad (A13)$$

or

$$G_2 = -\left(\frac{u_0 - u_1}{2u_1}\right) G_0. \quad (A14)$$

The fields in the second layer, $-d_2 < z < -d_1$, are

$$H_\rho = C \int_0^\infty \lambda u_2 \left[G_3 \exp(-u_2 z) - G_4 \exp(u_2 z) \right] J_1(\lambda \rho) d\lambda \quad (A15)$$

$$H_z = C \int_0^\infty \lambda^2 \left[G_3 \exp(-u_2 z) + G_4 \exp(u_2 z) \right] J_0(\lambda \rho) d\lambda \quad (A16)$$

$$E_\phi = j\omega\mu_0 C \int_0^\infty \lambda \left[G_3 \exp(-u_2 z) + G_4 \exp(u_2 z) \right] J_1(\lambda \rho) d\lambda \quad (A17)$$

in which

$$G_3 = \frac{\left(\frac{2\lambda}{u_2 + u_3}\right) \left(1 + U_{01} U_{12} \exp(-2u_1 d_1)\right) \exp\left[-u_2 d_2 - u_3 (h - d_2)\right]}{\text{DENOM}} \quad (A18)$$

$$G_4 = -\frac{\left[U_{12} + U_{01} \exp(-2u_1 d_1)\right] \exp[2u_2 d_1] G_3}{\left[1 + U_{01} U_{12} \exp(-2u_1 d_1)\right]}. \quad (A19)$$

The fields in the lower semi-infinite layer, $z < -d_2$, are

$$H_\rho = C \left\{ \rho (z + h) \left(3 + 3k_3 r + k_3^2 r^2\right) \frac{e^{-k_3 r}}{r^5} - \int_0^\infty \lambda u_3 G_5 \exp(u_3 z) J_1(\lambda \rho) d\lambda \right\} \quad (A20)$$

$$H_z = C \left\{ \left[2(z + h)^2 (1 + k_3 r) - \rho^2 (1 + k_3 r + k_3^2 r^2) \right] \frac{e^{-k_3 r}}{r^5} + \int_0^\infty u_3^2 G_5 \exp(u_3 z) J_0(\lambda \rho) d\lambda \right\} \quad (A21)$$

$$E_{\phi} = j\omega u_o C \left\{ \rho (1 + k_3 r) \frac{e^{-k_3 r}}{r^5} + \int_0^{\infty} \lambda G_5 \exp(u_3 z) J_1(\lambda \rho) d\lambda \right\} \quad (A22)$$

where

$$G_5 = - \left\{ U_{23} + U_{01} U_{12} U_{23} e^{-2u_1 d_1} + U_{12} e^{-2u_2 (d_2 - d_1)} \right. \\ \left. + U_{01} e^{-2[u_1 d_1 + u_2 (d_2 - d_1)]} \right\} \times \frac{e^{u_3 (2d_2 - h)}}{\text{DENOM}}. \quad (A23)$$

APPENDIX B

Field Formulas in a Two-Layer Earth

The formulas for the three-layer earth reduce to those for a two-layer earth when either of two sets of changes are made in the earth model:

1. Set $d_2 = d_1$ and $u_3 = u_2$, or
2. Set $d_1 = 0$, $d_2 = d_1$ and $u_2 = u_1$, $u_3 = u_2$.

Alternatively, the formulas may be derived using the procedure in the text. The denominator term common to all the formulas is given by

$$\text{DENOM} = \left\{ 1 + U_{01} U_{12} \right\} \exp (-2u_1 d) \quad (\text{B-1})$$

where "d" is the depth of the single interface between the earth layers. The fields in the upper semi-infinite layer, $z > 0$, are given by Eqs (A4), (A5), and (A6) in which

$$G_o = \frac{\left(\frac{2\lambda}{u_o + u_1} \right) \left(\frac{2u_1}{u_1 + u_2} \right) \exp \left[-u_1 d_1 - u_2 (h - d) \right]}{\text{DENOM}} \quad (\text{B-2})$$

In the first earth layer, $-d < z < 0$, Eqs (A8) thru (A12) apply with G_o given by Eq (B-2) above. The second layer in this case is the lower semi-infinite layer, $z < -d$, and Eqs. (A20), (A21), and (A22) apply with k_3 replaced by k_2 , u_3 replaced by u_2 , and G_5 replaced by G_3 given by

$$G_3 = \frac{- \left[U_{12} + U_{01} \exp (-2U_1 d) \right] \exp \left[u_2 (2d - h) \right]}{\text{DENOM}} \quad (\text{B-3})$$

APPENDIX C

Field Formulas in a Single-Layer or Homogeneous Earth

In terms of the three-layer model a homogeneous earth results when all of the u_i are set equal u_1 , and both d_2 and d_1 are set to zero. The common denominator term reduces to unity, DENOM = 1. In the upper semi-infinite layer, $z > 0$, Eqs (A4), (A5), and (A6) apply with

$$G_o = \left(\frac{2\lambda}{u_o + u_1} \right) \exp \left(-u_1 h \right) . \quad (C-1)$$

In the homogeneous earth, $z < 0$, Eqs (A20), (A21), and (A22) apply with k_3 replaced by k_1 , u_3 replaced by u_1 , and G_5 replaced by G_1 given by

$$G_1 = - U_{01} \exp \left(-u_1 h \right) . \quad (C-2)$$

APPENDIX D

Computation Notes

The computation of the fields requires the evaluation of the integral of a complex kernel over the positive semi-infinite range of a real variable. The substitution $\lambda = x/h$ makes the variable of integration dimensionless, and the real and imaginary parts of the kernel are integrated separately. In each case the integral is evaluated over an initial range $0 \leq x \leq X_1$, and then over a tail range $X_1 \leq x < \infty$. The first value chosen for X_1 is in excess of the value required to produce the peak magnitude of the kernel aside from the Bessel term. The initial quadrature therefore provides the major contribution, and the tail quadrature provides a correction term for improved accuracy. The initial quadrature divides the interval $0 \leq x \leq X_1$ into several smaller regions depending on X_1 and uses an eight-point Legendre-Gauss algorithm over each subinterval. The error bound in each subinterval is 10^{-7} . The tail quadrature uses a 40-point Gauss-Laguerre method. If the absolute ratio of the tail contribution to the initial contribution exceeds 10^{-5} , the value of X_1 is incremented and the entire process is repeated. The uncertainty in each part of the integral is estimated to be less than 0.1% which is adequate for present purposes.

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