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IDENTIFICATION, DETECTION, AND VALIDATION
OF VIBRATING STRUCTURES: A SIGNAL PROCESSING APPROACH

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CONTENTS

Abstract	iii
Introduction	1
On-Line Estimation/Identifier Design	3
Structural Applications	14
Summary	32
Acknowledgements	33
References	34
Appendix A	
Appendix B	
Appendix C	

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LIST OF ILLUSTRATIONS

1. Estimator/Identifier Evaluation	4
2. Simulation of Noisy Accelerometer Measurements of Structural Displacement	6
3. State Estimates for Top and Base of Eight Node Structure (Normal Response)	7
4. Innovations Sequence and Statistics for Normal Structural Response	9
5. Reciprocal Mass (1/m) Parameter Estimates for Eight Node Structure	11
6. Damping Parameter Estimates for Eight Node Structure	12
7. Stiffness Parameter Estimates for Eight Node Structure	13
8. Structural Failure Detection	16
9. Simulated Structural Displacement Subject to Earthquake Force with Structural Failure	18
10. Noisy Accelerometer Displacement Measurements with Structural Failure	19
11. States Estimates for Top and Base of Eight Node Structure	21
12. Innovations Sequence and Statistics for Failure Structural Response	22
13. Structural Failure Detector Response for Simulated Failure	23
14. Structural Design Validation	27
15. Noisy Accelerometer Displacement Measurements for 10% Structural Change	28
16. Innovations Sequence and Statistics for 10% Parametric Difference	29
17. Design Validation Estimation and Erros for 10% Parametric Difference	30

ABSTRACT

This report discusses the application of modern signal processing techniques to characterize parameters governing the vibrational response of a structure. Simulated response data is used to explore the feasibility of applying these techniques to various structural problems. On-line estimator/identifiers are used to estimate structural parameters, validate designed structures, and detect structural failure when used with a detector.

I. INTRODUCTION

One of the major responsibilities of the Nuclear Regulatory Commission (NRC) is the assessment of the structural integrity of Nuclear Power Plant (NPP) structures and equipment. Assessment is accomplished typically in three cases: (1) before the plant is constructed (design model verification); (2) after construction (design specification verification); and (3) after severe shaking (damage verification). The NRC requires a means to acquire structural information in the form of model parameters and measured responses in order to characterize the integrity of a structure before, during, and after the time it is subjected to strong vibration.

We address three fundamental issues in this report: (1) the characterization of structural parameters; (2) the on-line detection of structural failures as well as quantifying structural changes due to failure; and (3) the validation of the constructed structure with the intended design.

We use modern signal processing techniques to develop the proposed methodology. The basic purpose of this work is to assess the feasibility of these techniques for use in such proposed methodologies. It should be noted that the techniques presented can also be useful in experimental design for structural response. The implementation of the algorithms could eventually lead to the development of in-plant instrumentation for on-line structural failure detection as

well as the development of a general methodology for the requalification of components and structures. Also, a general technique for the validation of plant design/construction would evolve.

We apply modern signal processing techniques (both on- and off-line) to characterize a simulated structure from noisy measurements. The problem of characterizing the parameters of a structure from noisy vibration test data is well documented [1,2,3]. System identification or parameter estimation techniques have been applied with some success and can be used with confidence to characterize modes [4]. These techniques can be employed by the NRC to help scrutinize the structural performance of a plant. Besides estimating structural parameters we apply an on-line estimator/identifier to enhance the response signal to a hypothetical structural model. The state estimator---an extended Kalman filter (EKF) is then coupled to an on-line detector to detect structural failure. The estimator is also applied to response data generated from a "constructed" structure and we show how it could be used to decide if the structure meets design specifications.

In Section II we discuss the design of an estimator/identifier. In Section III, we examine the application of the estimator/identifier to two structural problems: (1) failure detection and quantification; and (2) design validation.

II. ON-LINE ESTIMATOR/IDENTIFIER DESIGN

In this section we consider the design and evaluation of an on-line estimator/identifier. In order to evaluate the performance of the estimator, a structural response simulator was developed using a state space description of the n -degree-of-freedom (DOF) vibration model,

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t) \quad (2.1)$$

The resulting state space representation is of the general form

$$\dot{s} = Fs + Gu \quad (2.2)$$

where s and u are the r -state and m -input vectors. Details of this model and simulation are discussed in Appendix A.

The simulated structural response as determined from the solution of (2.2) was input to a measurement system (see Figure 1) simulation to depict typical noisy accelerometer measurements at various nodes of the structure. In the simulation we assume that instrument dynamics are negligible compared to that of the structure. We model the instrument by

$$z_k = Hs_k + v_k \quad (2.3)$$

where $H^T = [I_n | 0_{pn}]$ and $v_k \sim N(0, R_k)^{\dagger}$, I_n and 0_{pn} are the respective $n \times n$ and $p \times n$ identity and null matrices.

[†] Here $x \sim N(\bar{x}, \sigma^2)$ means $\bar{x}$ has a Gaussian distribution with mean $\bar{x}$ and variance σ^2 .

Figure 1 Estimator/Identifier Evaluation

The additive Gaussian noise source is used to represent instrument uncertainties and inaccuracies. The measurements are assumed independent with covariance given by

$$R = \sigma_1^2 I_n \quad i=1, \dots, n$$

With these models in mind we can now precisely state the structural (state) estimation problem as

Given a sequence of noisy accelerometer measurements

$\{z_k\}$, find the "best" (minimum variance) estimates $\{\hat{s}\}$ of the nodal displacements and velocities.

We use the EKF algorithm which degenerates to a standard linear Kalman filter (LKF) for this problem (see DYNES[†] [7] for details). The algorithm and simplified flow diagram are shown in Appendix B.

Simulated noisy accelerometer outputs for the 1st and 8th node displacements for an eight node building model are shown in Figure 2, denoted $\hat{s}_{11}$ and $\hat{s}_{81}$ respectively, where $\hat{s}_{i1}$ is the estimated displacement at the i th node.[†]

Typical simulation outputs for the estimator using the normal noisy accelerometer data (Figure 2) is shown in Figure 3 with the associated estimation error curves.^{††}

[†] The variance was selected to maintain a signal-to-noise (SNR) ratio of 100 where $SNR_i = (\text{maximum positive peak}) / (\text{noise deviation})$.

^{††} The estimator is considered "on track" when the error curve remains within the predicted bounds 95% of the time.

Figure 2 Simulation of Noisy Accelerometer Measurements of Structural Displacement

Figure 3 State Estimates for Top and Base of Eight Node Structure (Normal Response)

Here the error is defined as $\tilde{S}_i = S_{TRUE} - \hat{S}_i$ and the two sigma bounds are predicted by the estimator, i.e., we are 95% confident that the error lies within $\pm 2\sqrt{\hat{S}_i^2}$. Note that in this case the primary purpose of the estimator is to "reconstruct" the respective displacement and velocity states at the i th node $\{S_{i1}, S_{i2}\}$ from noisy measurements and enhance (increase) the SNR. In most cases the signal levels were doubled (+3 db) relative to the noise (see Appendix B for details).

When an estimator is properly tuned and its model is adequate, the estimator innovations or residual sequence[†] is zero mean and white. We see that this is (approximately) the case for the above example as shown by the innovations sequence in Figure 4 for the two nodes of interest. Here note that 95% of the variances must lie within the bounds (2σ) for the innovations to be considered white (see [7] for details).

Now we turn our attention estimating or identifying parameters of the structure using the model of (2.1). We define the structural parameter identification problem as

Given a sequence of noisy measurements $\{z_k\}$, find
(identify) the structural parameters M, C, K .

[†] The innovations are the difference between the actual and predicted (estimated) measurements.

Figure 4 Innovations Sequence and Statistics for Normal Structural Response

To solve this problem, we "augment" the state vector with the unknown model parameters and solve the basic estimation problem with an EKF algorithm. Details of the identifier are given in Appendix B.

The EKF identifier was run over an ensemble of five noisy accelerometer simulations with identical statistics. A typical set of parameter runs is shown in Figures 5 through 7. Note that for these runs the M , C , and K matrices were constrained simple forms $M=mI$, $C=cM$, and $K=kA$ with A a constant. Justification of this structure is given in Appendix A. Here we see that both m_R (reciprocal mass, $1/m$) and c are estimated very well while the estimate for k is slightly biased; however, the maximum relative error is less than 2% (for k) showing excellent performance over this ensemble. The simulation results are depicted in Table I. This completes the section on the estimator/identifier design. In the next section we discuss applying the estimator/identifier to some NRC structural problems.

Figure 5 Reciprocal Mass ($1/m$) Parameter Estimates for Eight Node Structure

Figure 6 Damping Parameter Estimates for Eight Node Structure

Figure 7 Stiffness Parameter Estimates for Eight Node Structure

III. STRUCTURAL APPLICATIONS

In this section we discuss the application of the on-line estimator/identifier to some structural problems. In the previous section we designed an on-line estimator to enhance the signal-to-noise ratio and an identifier to determine structural parameters from noisy data. We will incorporate both designs to: (1) detect and quantify structural failures; and, (2) validate the structural design and construction of nuclear containment vessels.

1. Structural Failure Detection

One of the current problems plaguing the Nuclear Regulatory Commission (NRC) is the need for a scheme to requalify components, systems, and structure subject to severe environmental or accidental events. To be more precise we define the structural failure detection problem as

Given a structure subjected to earthquake type forces, determine: (1) if the structure has changed, (2) how much it has changed, and (3) if the change is significant enough to present failure.

Assume an unperturbed structure is modeled by

$$M\ddot{x} + C\dot{x} + Kx = f(t)^\dagger \quad (2.1)$$

with M , C , K determined from the design parameters.

† It should be noted that the estimation/identification algorithms employed are not restricted to the linear time invariant problem as (2.1), but can be applied to nonlinear stochastic ordinary and partial differential equation structural models as well.

Let (2.1) be excited by an earthquake type force, f_0 , or a severe vibration caused by mechanical failure, such that the structural properties are altered by amounts $(\Delta M, \Delta K, \Delta C)$. Under these conditions the perturbed structure can be modeled by,

$$(M + \Delta M)\ddot{x} + (C + \Delta C)\dot{x} + (K + \Delta K)x = f_0 \quad (3.1)$$

The problem is to

- (1) Detect (on-line) from noisy measurement data when a significant change occurs;
- (2) Quantify, ΔM , ΔC , and ΔK with 95% statistical confidence; and
- (3) Statistically, test if these changes are within allowable limits (M^*, C^*, K^*) , i.e.,

$$\begin{aligned} (M + \Delta M) &\cdot M^* \\ (C + \Delta C) &\cdot C^* \\ (K + \Delta K) &\cdot K^* \end{aligned} \quad (3.2)$$

A possible solution to the structural failure detection problem is depicted in Figure 8 along with a simulator. Basically a state estimator is used to enhance the SNR in the response and a detector is used to "decide" whether or not the structure has changed. If a change is detected than an identifier (as in Section III) can be used to quantify $(\Delta M, \Delta C, \Delta K)$. In this feasibility

Figure 8. STRUCTURAL FAILURE DETECTION

study we merely developed the detection algorithm and applied it to a simulated structural change. The identification algorithm was not exercised.

The estimator innovations sequence is a zero mean, white (independent) process, only when the estimation model reflects the true structural dynamics and the Kalman filter is properly tuned. However, when the estimator is improperly tuned or the models employed are inadequate, the innovations are biased and correlated. The effect of using estimator models which do not include the perturbed structural changes (ΔM , ΔC , ΔK) as in (3.1) results in a model mismatch reflected by variations in the innovations statistics. Statistical changes in $\{\epsilon_k\}$, reflect changes from the normal or expected operation; therefore, we propose an on-line detector which monitors the innovations for these changes.

To test the feasibility of this concept, we used the structure simulator to generate a perturbed response simulating the building vibrating from an earthquake when a structural failure occurs (see Figure 8). The simulated outputs are shown in Figures 9 and 10. The change has caused a frequency (pole) and damping alteration in the response compared to the normal response. Large SNR's were chosen to demonstrate the

Figure 9 Simulated Structural Displacement Subject to Earthquake Force with Structural Failure ($\Delta C = 0.32$, $\Delta k = 600$)

Figure 10 Noisy Accelerometer Displacement Measurements with Structural Failure ($\Delta C = 0.32$, $\Delta k = 600$)

concepts---the estimator and detector are capable of operating with much lower SNR's, however.

The estimator was first tuned to the normal response as shown previously in Figure 3 and 4 using the noisy data of Figure 2. This scenario would represent actual in-plant, on-line implementation of the algorithms.

The perturbed responses (Figures 9 and 10) were then applied to the estimator. Estimator outputs are shown in Figure 11. As shown by the estimator error curves in Figure 11 the estimator does not track the failure since well over 5% of the error lies beyond the $\pm 2\sigma$ bounds. This is also confirmed by the biased and correlated innovation sequences of Figure 12. Inability of the estimator to track the change permits failure detection using an innovation-based detector.

The innovations were input to the structural change detector for both the normal and failure run. In Figure 13 we see that during normal operation the threshold is not exceeded but when a failure occurs it is detected dramatically due to the high SNR. Thus, the estimator coupled with a detector can be used to detect and quantify (using an identifier) structural changes. This completes the structural failure detection and quantification problem. In the next sub-

Figure 11 State Estimates for Top and Base of Eight Node Structure (Failure, $\Delta C = 0.32$, $\Delta k = 600$)

Figure 12 Innovations Sequence and Statistics for Failure ($\Delta C = 0.32, \Delta k = 600$) Structural Response

Figure 13 Structural Failure Detector Response for Simulated Failure

section we discuss how the estimator can be used to validate the construction of a designed structure.

2. Structural Design Validation

Another current problem of NRC concern is to develop a methodology to compare analytical and experimental parameters enabling the identification of construction errors. Before a plant is constructed, a model is used to first design the structure according to certain specifications. In this subsection we develop a technique using an on-line estimator to validate from measurement data how well the as-built structure "matches" the design model. To be more precise, we define the structural design validation problem as

Given a structural design model, determine the structural integrity, i.e., validate that the structure meets and continues to meet design specifications. If specifications are not met, then determine the new parameters.

Assume the design model is given by

$$M_d \ddot{x} + C_d \dot{x} + K_d x = f(t) \quad (3.3)$$

and the response data model (unknown) is given by

$$M\ddot{x} + C\dot{x} + Kx = f(t) \quad (2.1)$$

Suppose the structural response is z_k , then an estimator can be constructed using (M_d, C_d, K_d) . The error between the measured response z_k and the response predicted by the estimator $\hat{z}_k$, i.e.,

$$\epsilon_k = z_k - \hat{z}_k \quad (3.4)$$

is a measure of the difference between the actual structure and the "design" model for the structure.[†] The standard deviation of the error, σ_ϵ ^{††} is used to determine with 95% confidence that the structure has met design specifications. If not, new response data model parameters are determined from the data for use in further analysis and investigation.

The problem is to:

- (1) Obtain the design/ construction error, ϵ_k ;
- (2) Specify the statistics of the design ϵ_k^d and construct the 95% confidence limits ($\pm 2\sigma_\epsilon$);
- (3) Statistically test if the design and construction errors are significant; and
- (4) If so, obtain the new structural model (M, C, K) .

[†] More specifically, a measure of model mismatch between (3.3) and 2.1).

^{††} Here $\sigma_\epsilon = R^{1/2}_\epsilon$ of Table II, Appendix B.

A possible solution to this problem is shown in Figure 14. Here we see that the structural design model is incorporated into the estimator so that the innovations provide an estimate of "how well" the structural response meets the design. The filter was tuned as in Figures 3 and 4 for "normal design data" and then a 10% change in the structural parameters was simulated in Figure 15. The innovations sequences for the 1st and 8th nodes are shown in Figure 16 with the design variances ($\pm 2\sigma_c^d$) indicated. The variances can be used to provide a confidence interval test for 'construction' i.e., over 95% of the innovations must fall within the design (variance) interval. In this case both sequences fail the test indicating the errors are significant and a new (M, C, K) should be identified as in Section II. Note also the mean/whiteness test is shown to indicate that indeed there is a model mismatch.

An alternate method of testing the structure would be to look at the design state estimation errors as

$$\tilde{s}_k = s_k^{\text{design}} - \hat{s}_k^{\text{construction}} \quad (3.4)$$

and use $\sigma_{\tilde{s}}$ to test the error significance. A typical state estimator error curve is shown in Figure 17 along with the $\hat{s}_k^{\text{constr.}}$. Again we see that the 10% construction error does not lie within the 95% confidence

Figure 14. STRUCTURAL DESIGN VALIDATION

Figure 15 Noisy Accelerometer Displacement Measurements
for 10% Structural Change
($\Delta C = 0.704$, $\Delta m = 1.1$, $\Delta k = 1320$)

Figure 16 Innovations Sequence and Statistics
 for 10% Parametric Difference
 ($\Delta m = 1.1, \Delta C = 0.704, \Delta k = 1320$)

Figure 17 Design Validation Estimation and Errors
for 10% Parametric Difference
($\Delta m = 1.1$, $\Delta C = 0.704$, $\Delta k = 1320$)

limits ($\pm 2\sigma_g$). Of course, another alternative is to identify the parameters and place bounds on them to decide if design specifications are met. In any case an estimator can be employed to validate the design of a structure.

IV. SUMMARY

In this report we have indicated the feasibility of using on-line estimator/identifiers to estimate parameters of a vibrating structure. We have also shown that the estimator can be used to validate the design and construction process, quantify the construction errors and determine if they are significant. We also showed how an estimator/identifier when coupled with a detector can be used to detect and quantify structural failures caused by earthquake type forces.

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A-1

APPENDIX A

STATE SPACE DESCRIPTION OF MULTI-DEGREE-OF-FREEDOM STRUCTURES

In this section we develop a state space model of a structure. We consider a linear, n -degree-of-freedom (DOF) system characterized by the vibration system model [5]

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t) \quad (A.1)$$

where

x, f are n -vector displacement and forcing functions,

M is the $n \times n$ mass matrix;

C is the $n \times n$ damping matrix; and

K is the $n \times n$ stiffness matrix.

Recall that the general state space representation of a linear system [6] is given by

$$\begin{aligned} \dot{s} &= Fs + Gu \\ y &= Hs \end{aligned} \quad (A.2)$$

where s, u, y are the r -state, m -input, and p -output vectors; and F, G, H are appropriately dimensioned matrices.

The vibration model can be placed in state space form by rewriting (A.1) as

$$\ddot{x}(t) = M^{-1}f(t) - M^{-1}C\dot{x}(t) - M^{-1}Kx(t)$$

and expressing these n -second order differential equations into $2n$ first order equations. By defining the state vector as

$$\mathbf{s}^T = [x_1 \dots x_n | \dot{x}_1 \dots \dot{x}_n]^T = [s_{11} \dots s_{n1} | s_{12} \dots s_{n2}]^T \quad (\text{A.3})$$

we obtain a state representation of the vibration system as

$$\begin{aligned} \dot{s}_{i1} &= s_{i2} \\ \dot{s}_{i2} &= M^{-1}f(t) - M^{-1}Cs_{i2} - M^{-1}Ks_{i1}, \quad i \in \underline{n}^\dagger \end{aligned} \quad (\text{A.4})$$

If we use the vector-matrix notation of (2.2), we see that

$$\mathbf{F} = \begin{bmatrix} \mathbf{O}_n & \mathbf{I}_n \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \quad \text{and} \quad \mathbf{G} = \begin{bmatrix} \mathbf{O}_n \\ \mathbf{M}^{-1} \end{bmatrix} \quad (\text{A.5})$$

where $\mathbf{O}_n$ and $\mathbf{I}_n$ are the $n \times n$ null and identity matrices.

For the problem considered in this report we place constraints on the entries of $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ such that

$$\begin{aligned} \mathbf{M} &= \text{diag} (m_i), \quad i \in \underline{n}; \\ \mathbf{C} &= \text{diag} (c_i), \quad i \in \underline{n}; \quad \text{and} \\ \mathbf{K} &= \text{tri-diag} (k_{ii-1}, k_{ii}, k_{ii+1}), \quad i \in \underline{n}. \end{aligned} \quad (\text{A.6})$$

[†] This notation means $i=1, \dots, n$

Thus, from (A.4) we have

$$\dot{s}_{i1} = s_{i2}$$

$$\begin{aligned} \dot{s}_{i2} = & m_r(i)f_i(t) - m_r(i)c_i s_{i2} - m_r(i)k_{ii-1}s_{i-11} - m_r(i)k_{ii}s_{i1} \\ & - m_r(i)k_{ii+1}s_{i+11} \end{aligned} \quad (A.7)$$

where

$$m_r(i) = 1/m_i.$$

If we further constrain these entries to be constant, we have

$$\begin{aligned} m_r(1) &= m_r(2) = \dots = m_r \\ c_1 &= c_2 = \dots = c \\ k_{ii} &= k_1; k_{ii-1} = k_{ii+1} = k_2 \quad \forall i, \end{aligned} \quad (A.8)$$

simplifying the equations of (A.7).

For the purpose of this investigation, we chose an eight-node stick model for a cantilever beam assuming out of plane displacements, longitudinal displacements and rotation are negligible. These assumptions yield an eight DOF model depicted in Figure A-1. This model was subject to a simulated "earthquake" input or ground acceleration,

$$\ddot{x}_g(t) = \begin{cases} \left(\frac{A}{2T}\right)t & 0 \leq t < T \\ \left(A - \left(\frac{A}{2T}\right)t\right) & T \leq t < 2T \\ 0 & t \geq 2T \end{cases} \quad (A.9)$$

Figure A-1 Lumped Mass Stick Model

where

$$f_g(t) = -M\ddot{x}_g(t) \quad (A.10)$$

The stick model was subjected to the simulated ground motion of (A.10) with the following structure

$$M = mI$$

$$C = cM$$

and

$$k = k \begin{bmatrix} 1 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & & \ddots & \ddots & \ddots & \\ & & & \ddots & \ddots & \\ & & & & -1 & 2 & -1 \\ & & & & & -1 & 2 \end{bmatrix}$$

where

$$k = 12 \frac{EI}{L^3}$$

E is Young's modulus,

I is area moment of inertia,

L is the distance between nodes.

The simulation parameters are shown in Table A-1. A typical simulator output for the 1st and 3th nodes are shown in Figure A-2. This completes the discussion of the structural representation.

TABLE A-1
SIMULATION PARAMETERS

Model

m (kip/ft ²)	1
c (kip/ft/sec)	0.64
E (kip/ft ²)	1
I (ft ⁴)	100
L (ft)	1

Input (Ground Acceleration)

A (kip/ft/sec ²)	1
T (sec)	0.03
t (sec)	0.01

Measurement (Accelerometer)

σ_1^2 (ft ²)	3.6×10^{-9}
σ_2^2 (ft ²)	3.6×10^{-9}
σ_3^2 (ft ²)	2.5×10^{-9}
σ_4^2 (ft ²)	1.6×10^{-9}
σ_5^2 (ft ²)	1.0×10^{-9}
σ_6^2 (ft ²)	6.25×10^{-10}
σ_7^2 (ft ²)	2.89×10^{-10}
σ_8^2 (ft ²)	1.0×10^{-10}

Figure A-2 Simulated Displacement of Structure
Subject to Earthquake Force

B-1

APPENDIX B

ESTIMATOR/IDENTIFIER IMPLEMENTATION

In this section we consider the design of an on-line state estimator and identifier using the EKF algorithm of DYNES [7]. The general state estimation/parameter identification problem is

Given noisy measurements $\{z_k\}$ from a nonlinear dynamical system of the form

$$\dot{s} = f(s, \theta) + g(u, \theta) + w_\theta \quad (B.1)$$

and a discrete measurement system

$$z_k = h(s, \theta) + v_\theta \quad (B.2)$$

where s , u , z , are the r -state, m -input, p -output, and q -parameter vector with associated nonlinear functions $f(\cdot)$, $g(\cdot)$, $h(\cdot)$; and $w \sim N(0, Q_\theta)$, $v \sim N(0, R_\theta)$

find the "best" (minimum variance) estimates $\hat{s}, \hat{\theta}$ of s and θ . [8]

For our state estimation problem the model of (2.1) is linear and therefore,

$$f(s, \theta) = Fs; \quad g(u, \theta) = Gu; \quad h(s, \theta) = Hs;$$

however, as will be seen, the parameter estimation problem is of the general form of (B.1,2).

The solution to the state estimation/parameter identification problem is afforded by the EKF algorithm of Table B-1 and Figure B1. Here the EKF degenerates to a linear Kalman filter (LKF) because the model is linear.

The LKF filter is used to enhance the signal level. To see this we define the filter input SNR as

$$\begin{aligned} \text{SNR}_{\text{IN}}(i): E\{z^T z\} R^{-1} &= E\{H^T S^T S H\} R^{-1} \\ &= \frac{E\{s_{i1}^2\}}{\sigma_i^2} = \frac{\pi_{i1}}{\sigma_i^2} \text{ for } i \in \underline{n} \end{aligned} \quad (\text{B.3})$$

and the output (enhancement) SNR as

$$\text{SNR}_{\text{OUT}}(i) = E\{\hat{z}^T \hat{z}\} \tilde{\pi}_{\text{SS}}^{-1} \approx \frac{E\{\hat{s}_{i1}^2\}}{\tilde{\pi}_{\text{SS}}(i)} \approx \frac{(\tilde{\pi}_i - \tilde{\pi}_{\text{SS}}(i))}{\tilde{\pi}_{\text{SS}}(i)} \text{ for } i \in \underline{n} \quad (\text{B.4})$$

where π is the solution to the steady state Lyapunov equation

$$F\pi + \pi F^T = Q \quad (\text{B.5})$$

If we approximate the numerator of (B.3,4) by s_{ij}^2 and s_{ij}^2 , respectively rather than calculate (B.5) directly then we can approximate the various SNR's. These values are shown in Table B-2. We see that the estimator improves the SNR significantly in most cases.

Finally, we consider the implementation of the estimator/identifier for the problem of (B-1,2). We define the

TABLE B-1: SUMMARY OF CONTINUOUS-DISCRETE EXTENDED KALMAN FILTER

Prediction:

$$\hat{x}_{k+1|k} = \hat{x}_{k|k} + \int_k^{k+1} [(f(\hat{x}_{a|k}) + g(u_a))] da$$

$$\tilde{\pi}_{k+1|k} = \int_k^{k+1} [I(\hat{x}_{k|k})\tilde{\pi}_{a|k} + \tilde{\pi}_{a|k}I^T(\hat{x}_{k|k}) + Q_a] da$$

Innovation:

$$e_{k+1} = z_{k+1} - h(\hat{x}_{k+1|k})$$

$$R_{k+1|k} = H(\hat{x}_{k+1|k})\tilde{\pi}_{k+1|k}H^T(\hat{x}_{k+1|k}) + R_{k+1}$$

Correction:

$$K_{k+1} = \tilde{\pi}_{k+1|k}H^T(\hat{x}_{k+1|k})(R_{k+1|k})^{-1}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}e_{k+1}$$

$$\begin{aligned} \tilde{\pi}_{k+1|k+1} &= [I - K_{k+1}H(\hat{x}_{k+1|k})]\tilde{\pi}_{k+1|k}[I - K_{k+1}H(\hat{x}_{k+1|k})]^T \\ &\quad + K_{k+1}R_{k+1}K_{k+1}^T \end{aligned}$$

where

$$F(\hat{x}_{k|k}) := \left. \frac{\partial f(x)}{\partial x} \right|_{x=\hat{x}_{k|k}}, \quad H(\hat{x}_{k|k}) = \left. \frac{\partial h(x)}{\partial x} \right|_{x=\hat{x}_{k|k}}$$

and

$$\tilde{\pi}_{k+1|k} := \text{Cov}(\tilde{x}_{k+1|k}) \text{ for } \tilde{x}_{k+1|k} := x_{k+1} - \hat{x}_{k+1|k}$$

$$x_0 \sim N(\hat{x}_{0|0}, \tilde{\pi}_{0|0})$$

Figure B-1. Simplified Flow Chart of Dynest Algorithms Including Routing for Least-Squares Estimate

TABLE B-2

SIGNAL ENHANCEMENT OF LKF

<u>Node</u>	<u>SNR</u> _{IN} (db)	<u>SNR</u> _{OUT} (db)
1	33.3 (15.2)	48.6 (16.9)
2	21.9 (13.4)	45.5 (16.6)
3	17.1 (12.3)	20.6 (13.1)
4	17.0 (12.3)	45.8 (16.6)
5	20.08 (13.0)	43.03 (16.3)
6	18.7 (12.7)	34.3 (15.3)
7	26.6 (14.2)	62.5 (18.0)
8	29.6 (14.7)	33.7 (15.3)

"augmented" state vector as

$$s^T = [s_{11} \dots s_{n1} | s_{12} \dots s_{n2} | m_r \ c \ k]$$

and the state equations of (A.7) become

$$s_{i1} = s_{i2}$$

$$s_{i2} = s_{2n+1} [f_i(t) - s_{2n+2}s_{i2} - s_{2n+3}(s_{i-11} - (2 - \delta_{i1})s_{ii} + s_{i+11})]$$

$$s_{2n+j} = 0 \quad \text{for } i \in \underline{n}, \ j \in \underline{3}$$

and

$$\delta_{ij} \text{ the kronecker delta.}$$

To implement the EKF identifier algorithm we must calculate the system and measurement Jacobian matrices. The measurement Jacobian is simply

$$H(s) = [I_3 \ | \ 0_2 \ | \ 0_{g3}].$$

The system Jacobian is characterized by

$$\frac{\partial f_{i1}}{\partial s_{i1}} = \frac{\partial f_{i1}}{\partial s_{2n+j}} = \frac{\partial f_{2n+j}}{\partial s_{2n+j}} = 0 \quad j \in \underline{3}$$

$$\frac{f_{i1}}{\partial s_{i2}} = 1$$

$$\frac{\partial f_{i2}}{\partial s_{i1}} = -s_{2n+1}s_{2n+3}(2 - \delta_{i1})$$

$$\frac{\partial f_{i2}}{\partial s_{i2}} = -s_{2n+1}s_{2n+3}$$

$$\frac{\partial f_{i2}}{\partial s_{2n+1}} = f_i(t) - s_{2n+2} s_{i2} - s_{2n+3} (s_{i-11} - (2 - \delta_{i1}) s_{i1} + s_{i-11})$$

$$\frac{\partial f_{i2}}{\partial s_{2n+2}} = -s_{2n+1} s_{i2}$$

$$\frac{\partial f_{i2}}{\partial s_{2n+3}} = -s_{2n+1} (s_{i-11} - (2 - \delta_{i1}) s_{i1} + s_{i+11})$$

where

$$F_{ij}(s); = \frac{\partial f_i(s)}{\partial s_j}$$

This completes the discussion of the implementation of the EKF estimator/identifier.

C-1

APPENDIX C

STRUCTURAL CHANGE DETECTOR IMPLEMENTATION

In this appendix we discuss the design of a detector used to "decide" when the structural model employed in the estimator changes due to a structural failure. When the estimator model is not adequate, the innovations sequence becomes biased and correlated. The detector is designed to sense this phenomenon. One of the simplest tests for model mismatch is the Chi-squared or weighted sum squared residual (WSSR) test. The WSSR statistic essentially aggregates all of the information available in the innovation vector over some finite window N. The WSSR defined by $p(l)$ is

$$\begin{aligned} p(l) &= \sum_{k=l-N+1}^l \epsilon^T(k) [R^e(k|k)]^{-1} \epsilon(k) \\ &= \sum_{k=l-N+1}^l \left(\sum_{i=1}^l \epsilon_i^2(k) / r_i^e(k) \right) \quad \text{for } l \geq N \end{aligned} \quad (C.1)$$

The test of simple hypotheses is:

$$\begin{array}{c} H_1 \\ p(l) > \tau \\ < \\ H_0 \end{array} \quad (C.2)$$

where H_0 is the hypothesis that there is normal operation, while H_1 is a structural failure. Under the zero mean assumption, the WSSR test is equivalent to testing that the aggregated innovations sequence is white. To see this, assume we construct the normalized innovations $\{\nu_k\}$ from $\{\epsilon_k\}$ as

$$\nu(k) = T_k^{-1} \epsilon(k) \quad (C.3)$$

where

$$T_k^{\dagger} = [R^e(k|k)]^{1/2} \text{ and}$$

$$\text{Cov}(\nu(k), \nu(j)) = I \delta(k-j)$$

Note that even though the statistics of $\{c(k)\}$ are time dependent, those of $\{\nu(k)\}$ are not.

We test that

$$\begin{aligned} H_0: \quad \{\nu(k)\} &\sim N(0, I) && \text{Normal} \\ H_1: \quad \{\nu(k)\} &\neq N(0, I) && \text{Failure} \end{aligned} \tag{C.4}$$

which is equivalent to testing that $\rho(l)$ is white over the interval N (see C.2). From (C.1) and C.3)

$$\rho(l) = \sum_{k=l-N+1}^l c^T(k) [R^e(k|k)]^{-1} c(k) = \sum_{k=l-N+1}^l \nu^T(k) \nu(k), \quad l \geq N \tag{C.5}$$

which approximates the sample variance of ν multiplied by N .

Under H_0 , $\rho(l)$ is distributed $\chi^2(Np)$; however, since $Np > 30$, we tacitly assume that $\rho(l) \sim N(Np, 2Np)$ and construct a one-sided test due to the transformation of χ^2 to Normal (see Anderson [9] for details).^{††} Thus, we have for a Type I error probability that

[†] $(R_e)^{1/2}$ means the matrix square root of R_e such that $R_e = (R_e)^{1/2} (R_e)^{T/2}$.

^{††} From a practical viewpoint, this assumption enables us to eliminate the need for Chi-squared tables.

$$P_I = P \left\{ \frac{\rho(l) - Np}{\sqrt{2Np}} > \frac{\tau - Np}{\sqrt{2Np}} \right\} = \alpha \quad (C.6)$$

For a level of significant of $\alpha = 0.05$, we have that

$$\tau = Np + 1.96 \sqrt{2Np} \quad (C.7)$$