CHAPTER 2

THRESHOLD FOR RAMAN CONVERSION OF FEL LASER
PULSES IN THE ATMOSPHERE

FINAL REPORT

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THRESHOLD FOR RAMAN CONVERSION OF FEL LASER PULSES IN THE ATMOSPHERE

1.0 INTRODUCTION

Transmission of a high intensity laser beam through the atmosphere can be affected by a variety of stimulated and parametric nonlinear processes. As a result, gain at other wavelengths will be generated along the propagation path of the beam. But the conversion of the original laser energy into other wavelengths is not the only result. Unless special measures (such as "seeding") are taken, the wavefront of the Raman shifted radiation will be determined by the turbulence in the atmosphere and by the aspect ratio of the gain medium. These processes will lead to beam divergences which can be significantly greater than the diffraction limit. As a result of these nonlinear processes, it will not be possible to efficiently propagate the beam to distant space targets unless proper measures are taken.

The dominant nonlinear processes in atmospheric propagation at wavelengths of interest are rotational and vibrational stimulated Raman scattering by molecular atmospheric constituents. Of these processes, stimulated rotational Raman scattering (SRRS) by nitrogen dominates because (1) the rotational Raman gain for nitrogen is larger than the vibrational cross section at pressures of one atmosphere and below and (2) the length of pressure broadened gain region (i.e., constant gain region) is several atmospheric scale heights. Because the atmospheric density decreases with increasing altitude, a high

power propagating laser beam generates stimulated Raman gain which is pressure broadened at low altitudes and Doppler broadened at high altitudes. The Doppler width in a scattering process is proportional to the fundamental excitation quantum of the scattering medium. This width is a function of the scattering angle and is narrowest in the forward direction. The Doppler width is especially narrow for rotational Raman scattering because of the small energy spacing between rotational levels. Therefore, the length of the pressure broadened gain region in the atmosphere is especially long in this case.

For laser pulse lengths shorter than the characteristic relaxation time for Raman polarization of the medium, the stimulated scattering as well as the spontaneous scattering become time dependent. In this transient regime, the intensity gain for the stimulated Raman process as well as the spontaneous scattering cross section for a given wavelength are dependent on the temporal history of the pulse. However, the stimulated scattering rate in the transient regime can be shown to be lower than that for the steady state regime. Since the characteristic relaxation time for the Raman process is dependent on the atmospheric density, it is clear that for a given laser pulse length, the stimulated Raman process might be in steady state at low altitudes and in the transient regime at high altitudes.

Developing a quantitative model for the transient regime is especially important for determining the propagation characteristics of FEL laser pulses through the atmosphere. The objective of this

research report is to present such a time dependent model for stimulated Raman conversion of short laser pulse propagating in the atmosphere and to calculate the threshold laser pulse energy and intensity for stimulated rotational Raman scattering as a function of wavelength, pulse width and beam diameter.

Throughout the analysis and the calculations we will assume that the laser field is linearly polarized. Since in that case the Stokes field with the same polarization has the highest gain for SRRS,⁽¹⁾ we will assume that the Stokes field is also linearly polarized and with the same polarization as that of the laser field. However, it should be pointed out, that since the gain for SRRS in nitrogen for circularly polarized light is larger than that for linearly polarized light by a factor of 1.5,⁽²⁾ the threshold intensity and threshold energy for a circularly polarized laser will be considerably lower than those presented in this report.

The classical coupled field equations for the stimulated Raman process have been given by Bloembergen.⁽³⁾ Carman et al,⁽⁴⁾ analyzed the general features of the solutions of these equations for the transient case. In their analysis the scattering molecules are treated as classical oscillating dipoles. In the next section the quantum mechanical equivalent of the Carman equations is derived by treating the scattering molecules as quantized entities. However, the radiation fields will be treated as classical fields, as is appropriate for macroscopic fields participating in a stimulated scattering process. In the third section the relationship between the parameters which appear in the equations, and measurable

quantities such as the steady state cross section for spontaneous Raman scattering, and the steady state gain will be discussed. In the fourth section the linewidth problem, i.e., pressure broadening versus doppler broadening and its relevance to the atmospheric SRRS will be discussed. In the fifth section geometry considerations and the relevant spontaneous source will be discussed. In the sixth section methods of obtaining a solution for the transient case will be discussed and the results of threshold intensities and threshold energies will be presented.

2.0 QUANTUM MECHANICAL DERIVATION OF THE STOKES - LASER FIELD COUPLED EQUATIONS

The classical coupled field equations for the transient stimulated Raman process can be written as:^(3,4)

$$\frac{\partial Q^*}{\partial t} + v_{ph} \frac{\partial Q^*}{\partial z} + \Gamma Q^* = iK_1 E_s E_L^* \quad (2.1)$$

$$\frac{1}{v_s} \frac{\partial E_s}{\partial t} + \frac{E_s}{\partial z} = -K_2 Q^* E_L \quad (2.2)$$

where E_L and E_s are the laser and Stokes time and space dependent field amplitudes, Q is the normal mode coordinate for the molecular degree of freedom, (vibration or rotation) taking part in the interaction, v_{ph} and v_s are the group velocities of the phonons and Stokes field respectively, Γ^{-1} is the characteristic relaxation time of the Raman polarization and K_1 and K_2 are given by:

$$K_1 = \frac{N}{2\omega_{ph}} \left(\frac{\partial \alpha}{\partial Q} \right) \quad (2.3)$$

$$K_2 = \frac{2\pi\omega_s^2}{c^2 K_s^2} \left(\frac{\partial \alpha}{\partial Q} \right) \quad (2.4)$$

where α is the polarizability of the medium, N is the molecular density and ω_{ph} is the phonon frequency. In general, because of the smallness of v_{ph} compared to v_s , the second term in Eq. (2.1) can be neglected.⁽²⁾

The purpose of this chapter is to derive the quantum mechanical equivalent to Eqs. (2.1) and (2.2), i.e. derive similar coupled field equations in which the scattering atomic or molecular system are treated quantum mechanically.

Starting with Maxwell equations one can relate the Stokes field to the macroscopic polarization of the medium at the Stokes frequency. Then the macroscopic polarization is expressed in terms of the polarization expectation value which is calculated by perturbation theory. The nonlinear feature of the coupled field equations is caused by the dependence of this expectation value on both the Stokes field and the laser field.

The relevant Maxwell's equations in this case are (in M.K.S. units)

$$\nabla \times H = \frac{\partial}{\partial t} (\epsilon E + P) \quad (2.5)$$

$$\nabla \times E = - \frac{\partial}{\partial t} (\mu H) \quad (2.6)$$

$$\nabla \cdot E = 0 \quad (2.7)$$

where E is the total field at the medium given by:

$$E = \left[E_L(z,t) e^{i(K_L z - \omega_L t)} + E_s(z,t) e^{i(K_s z - \omega_s t)} \right] + c.c. \quad (2.8)$$

$P(E)$ is the macroscopic polarization density of the medium, E_L and E_s are the space and time dependent amplitudes of the laser field and the Stokes field respectively and as stated before it is assumed that both

have the same linear polarization.

From Maxwell's equations we obtain by the general procedure for deriving the nonlinear wave equation:

$$\Delta \mathbf{E} = \mu (\epsilon \ddot{\mathbf{E}} + \ddot{\mathbf{P}}) \quad (2.9)$$

Expressing $\mathbf{P}$ as:

$$\mathbf{P} = \left[P_L(z,t) e^{i(K_L z - \omega_L t)} + P_S(z,t) e^{i(K_S z - \omega_S t)} \right] + c.c. , \quad (2.10)$$

Substituting $\mathbf{E}$ from Eq. (2.8) and assuming slowly varying amplitudes for both fields and polarizations one gets for the field and the polarization density at the Stokes frequency ω_S :

$$\frac{\partial E_S}{\partial z} + \frac{1}{c} \frac{\partial E_S}{\partial t} = \frac{i\omega_S}{2\epsilon c} P_S \quad (2.11)$$

where we have substituted $(\mu\epsilon) = 1/c^2$. The macroscopic polarization density amplitude P_S can be written in terms of the expectation value of the polarization at the Stokes frequency by:

$$P_S = \Delta N \langle p_S \rangle \quad (2.12)$$

where ΔN is the difference in the population density of the initial and final states of the molecules taking part in the Raman process and $\langle p_S \rangle$ is the expectation value of the polarization of the molecule at ω_S .

For the derivation of $\langle p_S \rangle$ we use perturbation theory. We

assume that without fields the molecular system is described by the Hamiltonian H_0 , eigenstates ϕ_k and energies ϵ_k so that

$$H_0 \phi_k = \epsilon_k \phi_k \quad (2.13)$$

The interaction Hamiltonian is given by:

$$H = - \hat{p} \cdot E \quad (2.14)$$

where $\hat{p}$ is the polarization operator and E is given by (2.8).

Let $\psi(t)$ be the function describing the state of the molecule after the interaction with the fields is turned on $\psi(t)$ can be expanded in terms of the eigenfunctions of the unperturbed system in the following way:

$$\psi(t) = \psi^{(0)} + \psi^{(1)}(t) + \psi^{(2)}(t) + \dots \quad (2.15)$$

where $\psi^{(0)} = \phi_i$ the initial state of the system and

$$\psi^{(n)}(t) = \sum_k A_k^{(n)}(t) \phi_k \exp(-i \epsilon_k t / \hbar) \quad (2.16)$$

The expansion coefficients are given by standard perturbation theory⁽⁵⁾:

$$\frac{d}{dt} A_k^{(n)}(t) = - \frac{i}{\hbar} \sum_j H_{kj}' e^{i \omega_{kj} t} A_j^{(n-1)} \quad (2.17)$$

where H'_{kj} is the matrix element of the perturbation Hamiltonian (2.16) between the states k and j :

$$H'_{kj} = \langle \phi_k | H' | \phi_j \rangle \quad (2.18)$$

and

$$\omega_{kj} = \frac{E_k - E_j}{\hbar} \quad (2.19)$$

We are interested in the expectation value of the polarization operator given by:

$$\langle p \rangle = \langle \psi(t) | \hat{p} | \psi(t) \rangle \quad (2.20)$$

$\langle p \rangle$ describes a variety of interactions of the molecules with the fields. The component of the expectation value relevant to the Raman process is third order in the interaction and is given by:

$$\langle p_r \rangle = \left[\langle \psi^{(1)} | \hat{p} | \psi^{(2)} \rangle + \langle \psi^{(0)} | \hat{p} | \psi^{(3)} \rangle \right] + \text{c.c} = \quad (2.21)$$

$$\left[\langle {}^1 p^2 \rangle + \langle {}^0 p^3 \rangle \right] + \text{c.c}$$

$\langle p_r \rangle$ also contains terms with different frequencies. For substitution in Eq. (2.11), one is interested in the components of $\langle p_r \rangle$ at frequency $\pm \omega_k$.

Substituting (2.8) in (2.14), one sees that the interaction Hamiltonian has four components and we can write:

$$H' = \sum_{\alpha} H'_{\alpha} \quad (2.22)$$

where α can be L, S, L* and S* according to the different components of E. Similarly the components of the expansion coefficients (see (2.17)) can be defined by:

$$\dot{A}_{k,\alpha}^{(n)} = - \frac{i}{\hbar} \sum_j (H'_{\alpha})_{kj} e^{i\omega_{kj}t} A_j^{(n-1)} \quad (2.23)$$

and also:

$$\dot{A}_{k,\beta}^{(n)} = - \frac{i}{\hbar} \sum_j (H'_{\beta})_{kj} e^{i\omega_{kj}t} A_j^{(n-1)} \quad (2.24)$$

clearly:

$$A_k^{(n)} = \sum_{\alpha} A_{k,\alpha}^{(n)} = \sum_{\alpha,\beta} A_{k,\alpha,\beta}^{(n)} \quad (2.25)$$

Substituting (2.16) in (2.21), and assuming the molecule is initially at ϕ_i and finally at ϕ_f , we find that the only relevant contributions to $\langle^1p^2\rangle$ in (2.21) at frequencies $\pm \omega_e$ are:

$$\langle^1p^2\rangle = \sum_k \left(A_{kL}^{(1)} \right)^* \left[A_{fLkL}^{(2)} + A_{fLkS}^{(2)} \right] p_{kf} e^{i\omega_{kf}t} \quad (2.26)$$

From (2.23) we have:

$$\dot{A}_{kL}^{(1)} = - \frac{i}{\hbar} (H'_L)_{ki} e^{i\omega_{ki}t} = - \frac{i}{\hbar} p_{ki} E_L e^{i(k_L z - \omega_L t)} e^{i\omega_{ki}t} \quad (2.27)$$

Assuming nonresonant Raman scattering, i.e., $|\omega_{k_1} - \omega_L| \gg \Gamma$ for all k , where Γ is the line width, Eq. (2.27) can be integrated between $-\infty$ and t by the so called "fast oscillation approximation" to yield:

$$A_{k_L}^{(1)} = \frac{1}{\hbar} \frac{p_{ki} e^{ik_L z}}{\omega_{ki} - \omega_L} E_L e^{i(\omega_{ki} - \omega_L)t} \quad (2.28)$$

In doing so, we have assumed $E_L(t = -\infty) = 0$.

From Eq. (2.24), one can write for $A_{fs*L}^{(2)}$:

$$\dot{A}_{fs*L}^{(2)} = -\frac{i}{\hbar} \sum_j A_{jL}^{(1)} (H_{s*}^I)_{fj} e^{i\omega_{fj}t} \quad (2.29)$$

Substituting $A_{jL}^{(1)}$ from (2.28), H_{s*}^I from (2.14) and defining $\delta = (\omega_{fj} + \omega_s - \omega_L)$ we get:

$$\dot{A}_{fs*L}^{(2)} = \frac{i}{\hbar^2} \sum_j \frac{p_{fj} p_{ji}}{\omega_{ji} - \omega_L} e^{i(k_L - k_s)z} E_L E_s^* e^{i\delta t} \quad (2.30)$$

In a similar manner one can write for the second coefficient in the square brackets of (2.26):

$$\dot{A}_{LS*}^{(2)} = \frac{i}{\hbar^2} \sum_j \frac{p_{fj} p_{ji}}{\omega_{ji} + \omega_s} e^{i(k_L - k_s)z} E_L E_s^* e^{i\delta t} \quad (2.31)$$

Defining: $b_f = A_{fs*L}^{(2)} + A_{LS*}^{(2)} \quad (2.32)$

we get:
$$\dot{b}_f = \frac{1}{\hbar^2} U e^{i(\omega_L - \omega_s)t} E_L E_s^* e^{i\delta t} \quad (2.33)$$

where U is given by:

$$U = \sum_j P_{fj} P_{ji} \left\{ \frac{1}{\omega_{ji} - \omega_L} + \frac{1}{\omega_{ji} + \omega_s} \right\} \quad (2.34)$$

From Eq. (2.33) it is clear that b_f has a resonance when $\delta = 0$, i.e., when $\omega_{fj} = \omega_L - \omega_s$ as expected. Because of the smallness of δ , we cannot use the "fast oscillations approximation" to integrate (2.33), and we will have to use (2.33) as it is in conjunction with (2.26) which in terms of b_f can be written as:

$$\langle^1 p^2 \rangle = \frac{1}{\hbar} \sum_k \frac{P_{ik} P_{kf}}{\omega_{ki} - \omega_L} e^{-ik_L z} E_L^* b_f e^{i(\omega_L - \omega_{fi})t} \quad (2.35)$$

In deriving (2.35) we have also substituted for $A_{kL}^{(1)}$ from (2.28).

A similar analysis shows that the only relevant contributions to the second term of the polarization expectation value in (2.21) can be written as:

$$\langle^0 p^3 \rangle = \frac{1}{\hbar} \sum_k \frac{P_{ik} P_{kf}}{\omega_L + \omega_{kf}} e^{-ik_L z} E_L^* b_f \quad (2.36)$$

substituting (2.35) and (2.36) in (2.21) we get:

$$\langle p_r \rangle = \frac{1}{\hbar} \sum_k p_{if} p_{kf} \frac{1}{\omega_{ki} + \omega_s} + \frac{1}{\omega_{ki} - \omega_L} e^{-ik_L z} b_f E_L^* e^{i(\omega_L - \omega_{fi})t} + c.c. \quad (2.37)$$

$$= \frac{U^*}{\hbar} e^{-ik_L z} b_f E_L^* e^{i(\omega_L - \omega_{fi})t} + c.c.$$

where U is given by (2.34) and we have taken advantage of the fact that

$$\omega_{ki} + \omega_s = \omega_{kf} + \omega_L.$$

Defining:

$$C_f = b_f e^{i(k_s - k_L)z} \quad (2.38)$$

we can write:

$$\langle p_r \rangle = \frac{U^*}{\hbar} (C_f e^{-i\delta t}) E_L^* e^{i(k_s z - \omega_s t)} + c.c. \quad (2.39)$$

$\langle p_r \rangle$ describes the expectation value of the Raman polarization at frequencies ω_s , comparing (2.39) with (2.10) and (2.12) we find:

$$P_s^*(z, t) = \Delta N \frac{U^*}{\hbar} E_L^* C_f e^{-i\delta t} \quad (2.40)$$

defining:

$$Q = \Delta N C_f e^{-i\delta t} \quad (2.41)$$

we can write:

$$\dot{Q} = \Delta N \dot{C}_f e^{-i\delta t} - i\delta Q \quad (2.42)$$

Substituting for $\dot{C}_r$ from (2.38) and (2.33) and adding phenomenologically a relaxation term which will describe the homogeneous broadening of the transition we get:

$$\frac{\partial Q}{\partial t} + (\gamma + i\delta) Q = \frac{i\Delta N}{\hbar} U E_L E_s^* \quad (2.43)$$

Substituting (2.41) in (2.40) and (2.40) into (2.11) we also get:

$$\frac{\partial E_s}{\partial z} + \frac{1}{c} \frac{\partial E_s}{\partial t} = \frac{i\omega_s}{2\epsilon\hbar c} U \cdot Q^* E_L \quad (2.44)$$

Defining:

$$\eta_1 = -\Delta N \frac{U^*}{\hbar} \quad \text{and} \quad \eta_2 = -\frac{\omega_s U}{2\epsilon\hbar c} \quad (2.45)$$

we can write:

$$\frac{\partial Q^*}{\partial t} + (\gamma - i\delta) Q^* = i\eta_1 E_L^* E_s \quad (2.46)$$

$$\frac{\partial E_s}{\partial z} + \frac{1}{c} \frac{\partial E_s}{\partial t} = -i\eta_2 Q^* E_L \quad (2.47)$$

Equations (2.46), (2.47) are the quantum mechanical equivalent of the coupled field equations (2.1) and (2.2), where the classical coefficients K_1 and K_2 are replaced by quantum mechanical coefficients η_1 and η_2 .

3.0 THE RELATIONSHIP BETWEEN THE PARAMETERS OF THE COUPLED FIELD EQUATIONS AND MEASURABLE QUANTITIES

To solve the coupled field Eqs. (2.46) and (2.47), one has to know the parameters η_1 and γ in terms of measurable quantities. The linewidth problem will be dealt with in the next section. Here the relationship between η_1 and measurable quantities such as steady state gain per unit intensity and steady state Raman scattering cross section will be discussed.

As it is obvious from Eqs. (2.46) and (2.47), to solve these equations, it is sufficient to know the product of η_1 and η_2 (see also Section 6). To express this product in terms of measurable quantities, let us compare the steady state gain for the Stokes intensity obtained from Eqs. (2.46) and (2.47) with that obtained by conventional steady state calculation.

Under steady state conditions:

$$\frac{\partial E_s}{\partial t} = \frac{\partial Q^*}{\partial t} = 0 \quad (3.1)$$

and it is easy to get from Eqs. (2.46) and (2.47) an exponentiation equation for the Stokes intensity under steady state:

$$\frac{dI_s}{dt} = \frac{2\eta_1\eta_2\gamma}{(\gamma^2 + \delta^2)} |E_L|^2 I_s \quad (3.2)$$

From Eq. (3.2) the steady state gain for the Stokes intensity at line center can be written in terms of the laser intensity I_L (in MKS units)

$$g_{ss} = \frac{\eta_1 \eta_2}{c \epsilon \gamma} I_L \quad (3.3)$$

On the other hand the steady state gain g_{ss} can be expressed by conventional steady state analysis in terms of the cross section for spontaneous Raman scattering:⁽⁶⁾

$$g_{ss} = \frac{2\Delta N}{hc\pi} \frac{\lambda_s^3}{\Delta\nu_R} \left(\frac{\partial\sigma}{\partial\Omega} \right)_{90} \frac{I_L}{(1+\beta)} \quad (3.4)$$

where λ_s is the Stokes wavelength, $(\partial\sigma/\partial\Omega)_{90}$ is the differential cross section for spontaneous Raman scattering at 90° to the polarization of the incident beam, $\Delta\nu_R$ is the F.W.H.M. of the Raman transition in frequency units* and β is the depolarization ratio. Using Eq. (3.3) and (3.4) with the proper units one can express $(\partial\sigma/\partial\Omega)$ in terms of the matrix sum $|U|^2$. We should point out here that the expression obtained thus far $(\partial\sigma/\partial\Omega)$, coincides exactly with that obtained by direct quantum electrodynamic calculation of the cross section.

From Eq. (3.4) one realizes that for a pressure broadened transition, g_{ss} is independent of gas density because both ΔN and $\Delta\nu_R$ are proportional to the gas density. Rokni et al⁽⁷⁾ have shown that at steady state, the stimulated rotational Raman scattering in the atmosphere, is pressure dominated at low altitudes. They have also

*Note that $\gamma = \pi\Delta\nu_R$

calculated the steady state gain per unit incident intensity for both nitrogen and oxygen at atmospheric conditions as a function of wavelength⁽⁷⁾. Their results show that the S.R.R.S. by nitrogen is the dominant process for wavelength longer than 2000 Å (see Fig. 1). This is because the oxygen density is lower by about a factor of 3-5 than that of nitrogen, while its cross section for rotational Raman scattering is of the same order of magnitude as that of nitrogen.⁽⁷⁾ For the sake of completeness, a short outline of the calculation of g_{ss} for atmospheric nitrogen will be presented.

For a rotational Raman transition the differential cross section can be written as^(1,8)

$$\left(\frac{d\sigma}{d\Omega} \right) = \frac{3}{8\pi} \sigma_o(j) \left\{ \frac{6 + \sin^2\theta}{10} \right\} \quad (3.5)$$

where:

$$\sigma_o(j) = \frac{128\pi^5}{9\lambda_s^4} \frac{(j+1)(j+2)}{(2j+1)(2j+3)} \gamma_{oo}^2 \quad (3.6)$$

γ_{oo} is the anisotropic molecular polarizability of the ground state and θ is the scattering angle as measured from the direction of polarization of the incident beam.

γ_{oo} has been measured for nitrogen at 633 nm by Bridge and Buckingham as 0.693 Å^3 .⁽⁸⁾ However its value at other wavelengths can be estimated using the approximate relationship for the cross section

Fig. 1: Gain per unit intensity as a function of wavelength for the S(8) transition of nitrogen and the S(11) transition of oxygen.

at wavelengths far from resonance.⁽¹⁾ Therefore,

$$\left(\frac{d\sigma}{d\Omega}\right) \propto \frac{\nu_0^6}{(\nu_i^2 - \nu_l^2)^2} \quad (3.7)$$

where ν_l is the incident laser frequency and $h\nu_l$ is a characteristic excitation energy (average energy of intermediate levels) for the Raman process. Bischel and Black have measured ν_l for vibrational Raman transition in nitrogen⁽¹⁰⁾ as $\nu_l \sim 9 \times 10^4 \text{ cm}^{-1}$. To calculate σ_{ls} for the rotational Raman transition in nitrogen one can use the same value of ν_l .

To obtain the gain, one has to evaluate ΔN_j the population difference. Clearly only $j = +2$ transitions contribute to stimulated rotational Raman scattering. The population of the rotational level j is given by

$$\frac{N_j}{N} = \sum_{T_j} \frac{(2T_j + 1)(2j + 1) \exp \left(\frac{-B}{KT} j(j + 1) \right)}{Q_R} \quad (3.8)$$

where T_j is the molecular nuclear spin, and Q_R is the rotational partition function:

$$Q_R = \sum_{j, T_j} (2T_j + 1)(2j + 1) \exp \left\{ \frac{-B}{KT} j(j + 1) \right\} \quad (3.9)$$

An analysis of Eq. (3.8) shows that for nitrogen at room temperature ΔN_j is a maximum for the S (8) transition ($j = 8 \rightarrow j = 10$)

for which $\Delta N_j/N \approx 3.5\%$. Also, since the variation of $\Delta N_j/N$ for a temperature range of 200 - 300° K, which is a typical temperature range for altitudes from 0 - 70 km, is less than 25%,⁽⁷⁾ one can neglect this temperature variation.

Figure (1) shows the steady state gain per unit incident intensity for S.R.R.S. at atmospheric conditions as a function of wavelength, both for the S(8) transition of nitrogen and the S(11) transition of oxygen as calculated by Rokni et al.⁽⁷⁾ The results for nitrogen can be approximated by

$$g_{s,s} \approx \left\{ \frac{250}{\lambda} \right\} 1.5 \times 10^{-5} \text{ (cm MW)}^{-1} \quad (3.10)$$

where λ is in nanometers.

These results were used for the calculation of the product $\eta_1 \eta_2$, which is necessary for the solution of the coupled field equations of the time dependent problem as discussed at the beginning of this section.

4.0 LINEWIDTH CONSIDERATIONS

The coupled field Eqs. (2.46) and (2.47) were derived for a homogeneously broadened Raman transition. Jammu et al⁽¹¹⁾ have measured the pressure broadening of the rotational Raman transitions in N_2 as $\Delta\tilde{\nu}_R = 0.08 \times P \text{ cm}^{-1}$, where P is the pressure in atmospheres.

The Doppler width in Raman scattering, as in any light scattering process, is angle dependent. For a molecule moving at a velocity, $\underline{v}$, the Doppler shift in the Stokes frequency is given by

$$\Delta\omega = \underline{v} \cdot (\underline{K}_L - \underline{K}_S) \quad (4.1)$$

where $\underline{K}_L$ and $\underline{K}_S$ are the incident photon and scattered photon wavevectors. To calculate the Doppler width, the velocity distribution of the molecules must be introduced. It is easy to show that the Doppler width is smallest for forward scattering and is given by

$$\Delta\nu_D = \frac{2\bar{v}}{(\pi)^{1/2}c} \nu_R \quad (4.2)$$

where $\bar{v}$ is the average thermal velocity and ν_R is the frequency of the molecular excitation. For the $j = 8 \rightarrow 10$ transition in N_2 , $\tilde{\nu}_R = 76 \text{ cm}^{-1}$. So the Doppler width for rotational Raman scattering in N_2 in the forward direction can be evaluated to be $\Delta\tilde{\nu}_D = 1.2 \times 10^{-4} \text{ cm}^{-1}$.

For rotational Raman scattering, pressure broadening will dominate down to a pressure of about 1.1 torr. The dependence of the atmospheric pressure on altitude can be expressed as:

$$P(t) = P_0 e^{-bz} \quad (4.3)$$

where z is the altitude and $b = 0.13 \text{ Km}^{-1}$. So η_1 and the linewidth under pressure broadening conditions can be written by:

$$\eta_1 = \eta_0 e^{-bz} \quad \gamma = \gamma_0 e^{-bz} \quad (4.4)$$

Comparison of the pressure broadening to the Doppler broadening shows that for the S(8) rotational Raman transition in nitrogen, the pressure broadening is dominant below an altitude of about 50 km.

Rokni, et al⁽⁷⁾ have shown that under steady state conditions, the contribution of the region beyond 50 km, where Doppler broadening is dominant, to S.R.R.S is negligible. This is because, as we have stated before under pressure dominated conditions the steady state gain g_{ss} is independent of the gas density, while under Doppler dominated conditions g_{ss} is proportional to the gas density. Hence, g_{ss} is constant below 50 km and is an exponentially decreasing function of height above 50 km. Similar arguments can be made in the time dependent case. However, we have also incorporated in the analysis the region above 50 km where Doppler broadening is dominant by introducing phenomenologically into Eq. (2.46) an effective Doppler width γ_D which is independent of z , and whose value is given by Eq. (4.2) ($\gamma_D = \pi \Delta \nu_D$). Thus our modified coupled field equations at line center ($\delta=0$) take the form:

$$\frac{\partial Q^*}{\partial t} + (\gamma_0 e^{-bz} + \gamma_D) Q^* = i\gamma_{10} e^{-bz} E_L^* E_s \quad (4.5)$$

$$\frac{\partial E_s}{\partial z} + \frac{1}{c} \frac{\partial E_s}{\partial t} = -i\gamma_2 Q^* E_L \quad (4.6)$$

It should be pointed out here that for very short pulses, for which the Fourier transform of the pulse is broader than the Doppler width γ_D , the transition from pressure broadening to Doppler broadening at an altitude of 50 km is irrelevant. This is because the gain line width will stop decreasing at height below 50 km when $\gamma_0 e^{-bz} = 1/t_0$, where t_0 is the pulse width. This behavior is fully accounted for in the coupled field Eqs. (4.5) and (4.6).

5.0 GEOMETRY CONSIDERATIONS AND THE SPONTANEOUS EMISSION SOURCE

We are interested in the calculation of the threshold for S.R.R.S. of short laser pulses propagating through the atmosphere. Clearly, to minimize the interference of atmospheric scattering processes of all kinds, the laser beam has to propagate in the zenith direction. In our calculations we will assume that this is the case. We will assume that the incident beam has a large Fresnel number, which is true for the uplink of FELSTAR so that diffraction can be neglected. We will also assume that the beam has a circular cross section with uniform spatial intensity distribution and a Gaussian time dependence.

The coupled field Eqs. (4.5) and (4.6) describe the amplification of the Stokes field E_s , provided we know the value of this field at $z=0$ as a function of time. That is to say, these equations inherently assume a source of radiation with frequency ω_s at $z=0$. Clearly the source of radiation for stimulated Raman scattering is provided by the spontaneous Raman scattering, which is not described by Eqs. (4.5) and 4.6). But in reality this source of radiation is distributed over the entire volume of the beam, because each volume element within the beam will emit spontaneously, and this emission will be amplified along the path of the beam.

For the solution of this problem, an effective source of spontaneous Raman scattering, located at $z=0$ will be defined in such a way that its effect on the calculation result will be equivalent to that of the distributed source of radiation.

Before doing that, the following points should be noted: (a)

clearly the main contribution to the spontaneous source for S.R.R.S in the atmosphere will be from low altitudes. This is because a source located at low altitudes has larger gain path length and also because the density of the atmosphere drops exponentially with altitude. (b) Referring to Fig. 2 we realize that only the fraction of the spontaneous scattering which is emitted within the solid angle $\Delta\Omega = S/z^2$, where S is the cross section of the beam, will be amplified all the way up to z . With increasing z , the gain path length increases but $\Delta\Omega$ decreases. Since beyond the pressure broadening region, the gain per unit length is proportional to the exponentially decreasing gas density,* there is a value of z for which the conversion of laser energy (or power) into Stokes energy (or power) is maximal. We will denote this value of z by $z_{\max}$. For threshold calculation we will be interested in calculating the Stokes energy flux (or intensity) for $z_{\max}$ and the corresponding $\Delta\Omega$. In Section 6 we will discuss the method by which $z_{\max}$ is determined. (c) Each volume element of the atmospheric column confined by the laser beam contributes to the buildup of the Stokes field at z . It is clear that near threshold the contributions of the different volume elements add up incoherently. This is because (1) the spontaneous sources at the different elements are incoherent and (2) near threshold where the depletion of the laser intensity can be neglected, the coupled field Eqs. (4.5) and (4.6) are linear for E_s . Therefore, also the contributions to E_s at $z_{\max}$ are

*As discussed in the previous section the pressure broadened region is the largest for the steady state case, where it is about 50 km, but for short laser pulses it can be much smaller.

Figure 2: The model geometry

incoherent. Hence rather than adding up the contributions to E_s , we can sum up the contributions of the volume elements to the Stokes intensity I_s at $z_{\max}$.

To determine the effective equivalent spontaneous source let us write the Stokes intensity at $z_{\max}$ in terms of the incoherent contributions

$$I_s(z_{\max}) = \int_0^{z_{\max}} F_s(z) dz \quad (5.1)$$

where $F_s(z)$ is the contribution of a column element with a length of dz (cross section S) located at z , to the Stokes intensity at $z_{\max}$. $F_s(z)$ includes the amplification of the Stokes intensity from z to $z_{\max}$. As mentioned before, near threshold the coupled field Eqs (4.5) and (4.6) are linear for E_s . Also since the different scatters in the volume element contribute incoherently, the interference terms average to zero and one can write:

$$F_s(z) = \phi_{sp}(z) \cdot f(z) \quad (5.2)$$

where $\phi_{sp}(z)$ is the intensity per unit length of the spontaneous scattering emitted at z within the solid angle $\Delta\Omega$, and $f(z)$ is the contribution of the volume element at z to the Stokes intensity at $z_{\max}$, for $\phi_{sp}(z) = 1$. Note that $f(z)$ is independent of the source intensity and is completely determined by the coupled field Eqs. (4.5) and (4.6). We can write

$$f(z) = \exp [g(z)] \quad (5.3)$$

where for the steady state case $g(z)$ is a linear function of $(z_{\max} - z)$, while for the extreme transient case $g(z)$ is proportional to $(z_{\max} - z)^{1/2}$ as shown by Carman et al⁽⁴⁾. Taking into account the dependence of $\phi_{sp}(z)$ on the gas density at z and on the solid angle $\Delta\Omega(z)$ we can write

$$\phi_{sp}(z) = \phi_{sp}(0) \frac{z_{\max}^2}{(z_{\max} - z)^2} e^{-bz} \quad (5.4)$$

Substituting in (5.1) and expanding $g(z)$ we get

$$I_s(z_{\max}) = \phi_{sp}(0) \int_0^{z_{\max}} \frac{z_{\max}^2}{(z_{\max} - z)^2} e^{-bz} \times \exp [g(0) - |g'| z + 1/2 g'' z^2 + \dots] dz \quad (5.5)$$

Note that the linear term in the expansion of $g(z)$ is negative because, as defined here, $g(z)$ decreases with increasing z . From (5.5) it is clear that the main contribution to the integral is from $z \leq 1/(b+|g'|)$. Also, a posteriori one finds that $z_{\max} \gg 1/(b+|g'|)$. Thus neglecting the quadratic and higher order terms in the expansion of $g(z)$ and also the z dependence in the denominator, substituting $z_{\max} \sim \infty$ as the integration limit and noting that $e^{g(0)} = f(0)$ we get

$$I_s(z_{\max}) = h \phi_{sp}(0) f(0) \quad (5.6)$$

where $h = [b + |g|]^{-1}$. The meaning of (5.6) is that rather than summarizing the contributions of the distributed sources within the volume, we can assume the existence of an equivalent effective source located at $z=0$ with an effective volume of $S \times h$. Using this method of equivalent spontaneous source we can calculate a value for $E_s(z=0, t)$, as will be discussed in the following, for substitution in the coupled field Eqs. (4.5) and (4.6).

When a laser pulse enters a medium, spontaneous Raman scattering occurs with a distribution of Stokes frequencies, which is determined either by the Fourier transform of the laser pulse (for very short laser pulses), or/and by the broadening mechanism of the Raman transition. As mentioned before, for a given frequency within the Stokes band, the pulse shape in general does not follow the pulse shape of the incident laser, and the intensity of the scattered light at that frequency cannot be described by the cross section for spontaneous Raman scattering. However, as shown by Rousseau et al,⁽¹²⁾ the pulse shape of the total spontaneous Stokes intensity, (i.e., the intensity integrated over the Stokes frequency distribution), is identical to the incident laser pulse shape. Therefore, the total intensity of the spontaneous Stokes scattering can be expressed in terms of the spontaneous scattering cross section, similarly to the steady state case. Thus, the total intensity of the spontaneous Stokes scattering within the solid angle $\Delta\Omega$, for an effective source with height h can be written as

$$I_{s,p}(0, t) = I_L(0, t) \left(\frac{\partial \sigma}{\partial \Omega} \right)_{\nu_0} \frac{S}{z^2} N \cdot h ; \quad (5.7)$$

where $I_L(0,t)$ is the laser intensity at $z=0$, and N is the density of the scatters.

Equation (5.7) describes the total intensity of the spontaneous Stokes scattering, i.e., the integrated intensity over the spontaneous Stokes bandwidth. But for the determination of the spontaneous source intensity, we are interested only in that portion of the spontaneous scattering, which is within the bandwidth of the gain at z_{max} . Thus, we can write the spontaneous source intensity within the proper bandwidth by

$$I_{sp}(0,t) = I_L(0,t) \left(\frac{\partial \sigma}{\partial \Omega} \right)_{\omega_0} \frac{S}{z^2} N \cdot h \cdot \alpha \quad (5.8)$$

where α is the ratio of the bandwidth of the Stokes gain at z_{max} to the bandwidth of the spontaneous Stokes scattering at $z=0$.

For the determination of α we differentiate between three cases:

(a) When $t_0 > 1/\Gamma$ for all z , where t_0 is the pulse width and $\Gamma = \gamma_0 e^{-bz} + \gamma_D$. This is the steady state case. The bandwidth of the spontaneous Stokes field at $z=0$ is given by γ_0 (see Eq. (4.4)). The gain bandwidth for the Stokes field at z_{max} in this case is given by γ_D . Hence in this case $\alpha = \gamma_D/\gamma_0$.

(b) When $t_0 < 1/\gamma_0$. This is the extreme transient case. The gain bandwidth here is constant and given by $1/t_0$ and this is also the bandwidth of the spontaneous scattering in this case. So we have $\alpha = 1$.

(c) When $1/\Gamma > t_0 > 1/\gamma_0$. Here the bandwidth of the spontaneous scattering is again γ_0 , but the gain bandwidth at z_{max} is $1/t_0$. So in

this case $\alpha = 1/(\gamma_0 t_0)$.

Thus α is determined in a manner which is independent of the actual value of z_{max} or h . For the complete determination of the effective source intensity, one has to specify the method by which h is calculated. From Eqs. (5.2-5.5) we realize that we can write

$$F_s(z) = F_s(0) e^{-bz} e^{-|g|z} \quad (5.9)$$

Hence we can write for h :

$$h = \frac{1}{b + |g|} = \frac{F_s(0)}{\left| \frac{dF_s}{dz} (z=0) \right|} \quad (5.10)$$

Since below and near threshold, the coupled field equations are linear for E_s , the ratio given by (5.10) is independent of the magnitude of the effective source field at $z=0$. For the determination of h , Eqs. (4.5) and (4.6) were numerically solved for a given laser pulse once from $z = 0$ to $z = \infty$ and once from $z = \epsilon$ to $z = \infty$, where ϵ is a small increment. The runs were carried out for source fields $E_s(0, t)$ and $E_s(0, t) \cdot e^{-b\epsilon}$ for the two cases respectively. The first run provided $F_s(0)$, while from the difference between the two runs, $dF_s/dz|_{z=0}$ was determined. Substituting in (5.10), h was calculated for a given laser pulse.

From (5.8) we can write for the amplitude of the effective spontaneous source at $z=0$

$$E_{sp}(0,t) = \beta E_l(0,t) \quad (5.11)$$

where

$$\beta = \left[\left(\frac{\partial \sigma}{\partial \Omega} \right)_{\eta_0} \frac{S}{z^2} N.h.a \right]^{1/2} \quad (5.12)$$

6.0 SOLUTION METHOD AND RESULTS

First we will transform the coupled field Eqs. (4.5) and (4.6) in a few steps into a more convenient form. Defining a moving frame by

$$t' = t - \frac{z}{c}, \quad z' = z \quad (6.1)$$

we can write Eqs. (4.5) and (4.6) in the form:

$$\frac{\partial Q^*}{\partial t} + (\gamma_0 e^{-bz} + \gamma_D) Q^* = i\eta_{10} e^{-bz} E_L^* E_s \quad (6.2)$$

$$\frac{\partial E_s}{\partial z} = -i\eta_2 Q^* E_L \quad (6.3)$$

We note that since we assume that the laser field is not depleted (near threshold conditions), then E_L is independent of z and we have

$$E_L(t') = E_0 e_L(t') \quad (6.4)$$

where E_0 is the maximum amplitude of $E_L(t)$ and $e_L(t')$ is the shape function which for a Gaussian is given by:

$$e_L(t') = e^{-\left(t'/t_0\right)^2} \quad (6.5)$$

Substituting in (6.2) and (6.3) we get

$$\frac{\partial Q^*}{\partial t} + (\gamma_0 e^{-bz} + \gamma_D) Q^* = (i\eta_{10} E_0^* e^{-bz}) e_L^* E_s \quad (6.6)$$

$$\frac{\partial E_s}{\partial z} = (-i\eta_2 E_0) e_L Q^* \quad (6.7)$$

Defining dimensionless time and space coordinates by

$$\tau = t/t_0 \quad (\text{time in units of the laser pulse width } t_0), \quad (6.8)$$

$$x = 1 - e^{-bz} \quad (6.9)$$

and $\delta_D = \gamma_D/\gamma_0$, we get

$$\frac{\partial Q^*}{\partial \tau} + (\gamma_0 t_0) (1 + \delta_D - x) Q^* = (i\eta_{10} E_0^* t_0 (1-x)) e_L^* E_s \quad (6.10)$$

and

$$\frac{\partial E_s}{\partial x} = \frac{-i\eta_2 E_0}{b(1-x)} e_L Q^* \quad (6.11)$$

Now we proceed by defining new variables Q' and E'_s given by

$$Q' = Q^* (i\eta_{10} E_0^* t_0 (1-x))^{-1} \quad (6.12)$$

$$\text{and} \quad E'_s(x, \tau) = E_s(x, \tau)/e_L(\tau) \quad (6.13)$$

Substituting in (6.10) and (6.11) one can write

$$\frac{\partial Q'}{\partial \tau} + (\gamma_0 t_0) (1 + \delta_D - x) Q' = (e_L e_L^*) E'_s \quad (6.14)$$

$$\frac{\partial E_s'}{\partial x} = \frac{\gamma_{10} \gamma_2 |E_0|^2 t_0}{b} Q' \quad (6.15)$$

$$\text{Defining} \quad \tau_0 = (\gamma_0 t_0) \quad (6.16)$$

and

$$G = \frac{\gamma_{10} \gamma_2 |E_0|^2 t_0}{b} \quad (6.17)$$

and substituting from (6.5) $e_L(\tau) = e^{-\tau^2}$ we have

$$\frac{\partial Q'}{\partial \tau} + \tau_0 (1 + \delta_D - x) Q' = e^{-2\tau^2} E_s \quad (6.18)$$

and

$$\frac{\partial E_s'}{\partial x} = G Q' \quad (6.19)$$

We have to solve Eqs. (6.18 and (6.19) in the space-time region given by $0 < x < 1$ ($0 < z < \infty$) and $-\infty < \tau < \infty$. Because of the Gaussian shape of the laser pulse it will be sufficient to limit the solution within the time domain $-4 < \tau < 4$.

The initial conditions for the solution are

$$Q'(x, \tau = -4) = 0 \quad \text{for all } x \quad (6.20)$$

and

$$E_s'(0, \tau) = \frac{E_{sv}(0, \tau)}{e_L(\tau)} = B \frac{E_L(\tau)}{e_L(\tau)} = B E_0 \quad (6.21)$$

where for the derivation of (6.21) Eqs. (6.13), (5.11) and (6.4) have been used. Defining a new set of variables by:

$$e_s(x, \tau) = E'_s(x, \tau) / (BE_0) \quad (6.22)$$

and

$$q(x, \tau) = Q'(x, \tau) / (BE_0) \quad (6.23)$$

we can write the coupled field equations in the final form:

$$\frac{\partial q}{\partial \tau} + \tau_0 (1 + \delta_D - x) q = e^{-2\tau^2} e_s \quad (6.24)$$

$$\frac{\partial e_s}{\partial \tau} = Gq \quad (6.25)$$

where the initial conditions now take the form:

$$q(x, \tau = -4) = 0 \quad \text{for all } x \quad (6.26)$$

$$\text{and} \quad e_s(0, \tau) = 1 \quad (6.27)$$

The advantage in writing the coupled field equations in the form of (6.24) and 6.25) is twofold: (a) These equations are relations between dimensionless quantities and (b) the equations themselves are independent of the problem parameters h , L and a and the dependence on these parameters is through the initial condition (6.27). So Eqs. (6.24) and (6.25) can be solved numerically in the general case.

Since in the Raman process described here there is no loss mechanism for the Stokes field, the definition of threshold for the process is not unique. For calculations presented here somewhat arbitrary definitions of intensity threshold and energy flux threshold are used. The intensity threshold is defined as the case when the peak intensity of the Stokes field is one percent of the peak intensity of the incident laser. The energy flux threshold is defined as the case when the energy flux of the Stokes field (time integrated intensity) is one percent of the energy flux of the incident laser.* The magnitude of one percent was selected as the value below which conversion into Stokes field is negligible.

From the discussion in the previous section it is clear that $z_{\max}$ is not the same for the two threshold definitions. To find $z_{\max}$ for the intensity threshold one has to find the z value for which $|E_s(\tau = \tau_{\max}, z)|^2$ is maximal, where $\tau_{\max}$ is the time for which the peak Stokes intensity is reached, while for the energy threshold we have to find the z value for which $\int_0^\infty |E_s(\tau, z)|^2 d\tau$ is maximal. Using Eqs. (6.13), (6.22) and (5.12) one finds that to calculate $z_{\max}$ for the intensity threshold, one has to solve the coupled wave Eqs. (6.24) and (6.25) for a given incident laser pulse width, wavelength etc, and to find the z value for which the expression

$$\frac{1}{z^2} \left| e_s(\tau = \tau_{\max}, z) e_L(\tau = \tau_{\max}) \right|^2$$

*A posteriori it was found that for the case considered here, the two threshold definitions were practically identical (see Fig. 3).

Figure 3: Comparison of the incident laser intensity at energy threshold (a) with that at the intensity threshold (b) as a function of laser pulse length, for a wavelength of 5000Å and a beam cross section area of 1 m²

is maximal, where $\tau_{\max}$ is the time for which the product $e_s \cdot e_l$ is maximal for a given z . Similarly $z_{\max}^e$ for the energy threshold is given by the z value for which the expression

$$\frac{1}{z^2} \int_{-\infty}^{\infty} \left| e_s(\tau, z), e_l(\tau) \right|^2 d\tau$$

is maximal. Once $z_{\max}$ was determined, the intensity of the Stokes field at $z_{\max}^i$ and the energy flux of the Stokes field at $z_{\max}^e$ were calculated from Eqs. (6.24) and (6.25) and by changing the value of G by trial and error, G^i and G^e were determined, where G^i is the value of G corresponding to intensity threshold conditions, and G^e is that corresponding to energy threshold conditions. From G^i and G^e the peak incident laser intensity and incident laser energy flux at threshold were determined by using (6.17).

Figure 3 shows a comparison of the incident laser intensity at energy threshold with that at the intensity threshold as a function of laser pulse length, for a wavelength of $5000 \text{ \AA}$ and a beam cross section area of 1 m^2 . As is seen from the figure the two curves practically coincide and the difference between the two definitions is less than 5%. Therefore, we can neglect the difference between the two threshold definitions.

Figure 4 shows plots of threshold laser intensity as a function of pulse length for a variety of wavelengths between $3000 \text{ \AA}$ and $1 \text{ \mu}$ and a beam cross section area of 1 m^2 . Figure 5 shows plots of laser threshold energy as a function of pulse length for the same wavelengths and beam cross section as in Fig. 4.

Figure 4: Threshold laser intensity as a function of pulse length for wavelengths between 3000 Å and 1μ and a beam cross section area of 1 m^2 .

Figure 5: Threshold laser energy flux density as a function of pulse length for wavelengths between 3000 Å and 1μ and a beam cross section area of 1 m^2 .

Figures 6 and 7 show the same data as in Fig. 4, in an expanded scale. Note that the intensity scale in Fig. 7 is linear. Figures 8 and 9 show the same data as in Fig. 5 in an expanded scale. Note that the energy scale in Fig. 8 is linear. The dependence of the threshold intensity on the beam cross section area is shown in Fig. 10 for a wavelength of $5000 \text{ \AA}$. As expected the threshold intensity changes only slightly as a function of this parameter.

From Figs. 4, 6, and 7 one sees that pulse lengths longer than 50 nsec can be considered as steady state. For shorter pulse lengths the threshold intensity increases initially moderately, but a further decrease causes a sharp increase in the threshold intensity. This is a manifestation of the finite relaxation time of the rotational Raman polarization. The results of our calculation for the steady state case are lower by about 20% than those given by Rokni et al.⁽⁷⁾ This is because they took the height of 40 km as the transition point between the pressure broadening and Doppler broadening for the steady state case, while our review of the calculation showed that transition occurs at an altitude of about 50 kms.

From Figs. 5, 8 and 9 one sees that the pulse energy at threshold is the largest for the steady state case, and decreases sharply with decreasing pulse length. On the other hand for the extreme transient case ($t_0 < 0.1 \text{ nsec}$) the threshold energy is practically independent of pulse length indicating that for these short pulse lengths the threshold intensity is about inversely proportional to the pulse length, as observed in Fig. 4.

Figure 6: Threshold intensity as a function of wavelength for pulse lengths between 0.01 nsec and 2 nsec and beam cross section area of 1 m².

Figure 7: Threshold intensity as a function of wavelength for pulse lengths between 1 nsec and steady state and a beam cross section area of 1 m^2 .

Figure 8: Threshold energy flux density as a function of wavelength for pulse lengths between 0.001 nsec and 2 nsec and a beam cross section area of 1 m^2 .

Figure 9: Threshold energy flux density as a function of wavelength for pulse lengths between 2 nsec and 100 nsec and a beam cross section area of 1 m².

Fig. 10: Threshold intensity as a function of laser pulse length for beam cross section areas of 0.1 m², 1 m² and 10 m² .

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