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MULTIDIPOLE MAGNETIC FIELDS FOR PLASMA CONFINEMENT

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Multidipole Magnetic Fields for Plasma Confinement*

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Abstract

Plasma confinement by permanent magnets has been studied. An analytic formula for the field of a single bar magnet has been obtained. Generalization to various configurations of multidipole fields has also been found. Any two-dimensional field may now be completely described by a simple function of complex variables in closed form. Vector potential has also been obtained by integrating over a prescribed Riemann surface. The confinement of plasma by multidipole fields then becomes obvious through conservation principles.

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I. Introduction

Confinement of plasma by permanent magnets is a relatively easy and inexpensive task. For a given power level of the d.c. discharge which sustains the plasma, permanent magnets are often used to increase the plasma densities in the laboratory*. The multidipole fields can also be arranged for the purpose of reducing the inhomogeneity in the plasma density distribution.

Even for the large mirror machine in nuclear fusion, small permanent magnets can also be used to confine the secondary electrons generated by the particles escaped from the center cell. As the escaping ions hit the wall, a large number of the secondary thermal electrons may be released from the cool boundary. These cold electrons will thermalize with the main plasma, thus may drastically reduce the temperature in the center cell. If a multitude of small magnets are placed properly on the wall, the sputtered secondary electrons may be confined to the boundary layer at the wall, the fusion plasma will hence not be seriously affected**.

In producing high power neutral beams in Lawrence Berkeley Laboratory, for example, various configurations of permanent magnets arrangements have been tested in the lab. One typical set-up is to place long bar magnets around the wall of the cylindrical bucket of the ion source. The poles of the neighboring magnets face alternately toward and away from the center axis.

* K. N. Ehlers and K. N. Leung, "Characteristics of the Berkeley Multicusp Ion Source", Rev. Sci. Instrum. 50 (11), 1979, p. 1353.

** First proposed by D. Baldwin.

Experimental results have shown that such arrangements are very effective and produce probably the best ion beam transport efficiency*.

Detailed calculations for the plasma confinement by the permanent magnets will involve complicated tracing of the particles trajectories in the multi-line-cusp fields, which is an extremely tedious task, particularly if the magnets are not arranged carefully with any symmetry in mind to allow for some mathematical simplification.

In the present paper, we would like to discuss briefly first how the confinement of plasma can be described by the vector potential arguments through the conservation principles, then proceed to find the magnetic fields and vector potential profiles for various bucket designs. A sample calculation is also included.

11. Vector Potential and Plasma Confinement

For the case that the magnetic field can be described by a vector potential along one fixed direction, say $\vec{A} = A_z \hat{e}_z$ only, the canonical momentum along $\hat{e}_z$ is always conserved, i.e.,

$$P_z = m v_z + q A_z = \text{const.} \quad (1)$$

Where m is the mass, q charge, v_z , the z -component of the velocity of the charged particle.

Put it in another way, from $\vec{B} = \nabla \times \vec{A}$, we have, in Cartesian coordinates,

$$B_x = \frac{\partial A_z}{\partial y}; \quad B_y = -\frac{\partial A_z}{\partial x} \quad (2)$$

* K. N. Leung, K. W. Ehlers, and E. B. Hooper, Jr., "Transport of Low Energy Positive and Negative Ion Beam by Permanent Magnets", Report LBL-11636.

because $A_x = A_y = 0$ here.

The equation of motion in z-direction for a charged particle thus becomes

$$\begin{aligned} m \dot{v}_z &= q (\vec{v} \times \vec{B})_z = q (\dot{x}B_y - \dot{y}B_x) \\ &= -q \left(\dot{x} \frac{\partial A_z}{\partial x} + \dot{y} \frac{\partial A_z}{\partial y} \right) \end{aligned}$$

Hence $m \dot{v}_z + q \dot{A}_z = 0$

or $mv_z + qA_z = P_z = \text{const.}$

which is exactly the statement of equation (1).

The total energy of the particle is naturally, also conserved, thus,

$$\epsilon = \frac{1}{2} mv^2 + qV = \text{const.} \quad (3)$$

where V is the electrical potential. For the neutral beam experiment at LBL*, the potential of the plasma ion source stays as constant throughout almost the whole bucket, hence (2) may be modified as,

$$\epsilon = \frac{1}{2} mv^2 = \text{const.}$$

For a charged particle starting from negligible B field with $A_z = 0$, and no z-component velocity $v_z = 0$, (1) gives $P_z = 0$.

Since the maximum value v_z can get is $v_z = v$, the total velocity, the maximum value of A_z is $\frac{mv}{q}$, from eq. (1). Hence charged particles can only reach regions where

* K. N. Leung, Private Communication.

$$A_z < (2mc)^{1/2}/q \quad (4)$$

Even if particles start with the largest possible $v_z = v$, rather than $v_z = 0$, then $P_z = mv$ where $A = 0$. The accessible region will be

$$A_z < 2mv = (8mc)^{1/2}/q \quad (5)$$

Therefore, if A_z is known, regions where plasma particles can get to can be determined. Though crude, this will provide us the first picture of confinement. More refined view will require detailed analyses concerning electrical potential profile, diffusion, sheath formation, and collision, etc.

In the present study, we will focus only on the magnetic field and vector potential formulation.

III. Magnetic Field for an Infinite Current Source

In M.K.S. unit, the magnetic field intensity $\vec{B}$ is related to the applying field $\vec{H}$ by

$$\vec{B} = \mu \vec{H} \quad (6)$$

where μ is the permeability. In vacuum, $\mu = \mu_0$.

Field $\vec{H}$ is related to the current density $\vec{j}$ by Maxwell's equation

$$\nabla \times \vec{H} = \vec{j}. \quad (7)$$

For an infinite long current source, we can integrate (7) and get the following equation through Stoke's theorem.

$$\oint \vec{H} \cdot d\vec{l} = \int \vec{j} \cdot d\vec{a} = I \quad (8)$$

where l is the closed cycle integration element enclosing the area σ . The total current is specified by I .

By symmetry $\vec{H} = H_\theta \hat{e}_\theta$, we thus have

$$2\pi r H_\theta = I. \quad (9)$$

This is simply the Biot-Savart's law. H_θ may also be written as,

$$H_\theta = \frac{I}{2\pi i z l} \quad (10)$$

where the complex variable $z = x+iy$ has been introduced.

Since $\vec{H}$ points to $\hat{e}_\theta$ direction, and $\hat{e}_\theta = ie^{i\theta}$, in complex notation, thus

$$\vec{H} = H(z) = \frac{I}{2\pi i z l} ie^{i\theta} \quad (11)$$

The field intensity $\vec{B}$ is then

$$\vec{B} = \mu_0 \vec{H}, \text{ or}$$

$$B(z) = B_x + iB_y = \frac{\mu_0 I}{2\pi i z l} \left(\frac{i}{e^{-i\theta}} \right)$$

If we write $B^* = B_x - iB_y$ as the complex conjugate of $B(z)$, then

$$B^* = B_x - iB_y = \frac{-i\mu_0 I}{2\pi i z l e^{+i\theta}}$$

Since $z = |z|e^{i\theta}$, we finally obtain

$$B^* = \frac{\mu_0 I}{2\pi i z} \quad (12)$$

Equation (12) is a simple expression for the field generated by a current source.

IV. Magnetic Field of a Bar Magnet

If the current density $\vec{j}$ in equation (7) or (8) consists of two sheet currents coming into and out of the paper at $x = a$ and $x = -a$, respectively, as shown

Then

$$\vec{j} = j \hat{e}_z [\delta(x+a) - \delta(x-a)] \theta(b^2 - y^2) \quad (13)$$

where j is the amount of current density, δ is the Dirac delta function, θ the unit step function, $\hat{e}_z$ the unit vector along z .

Physically, this will produce exactly the same magnetic field as that of an infinitely long bar magnet of uniform magnetization

$$\vec{M} = M_y \hat{e}_y = j \theta(a^2 - x^2) \theta(b^2 - y^2) \hat{e}_y \quad (14)$$

Because the magnetization is related to current density by

$$\vec{j} = \nabla \times \vec{M}, \quad (15)$$

and the curl of eq. (14) is simply

$$\begin{aligned} \nabla \times \vec{M} &= j \frac{\partial}{\partial x} \theta(a^2 - x^2) \theta(b^2 - y^2) \hat{e}_z \\ &= j [\delta(x+a) - \delta(x-a)] \theta(b^2 - y^2) \hat{e}_z \end{aligned}$$

which is exactly prescribed by eq. (13). The arrow in the magnet shows that its magnetization direction is along positive y direction.

The complex field at any point $z = x + iy$ can then be derived from integrating eq. (12) over source element (x_0, y_0)

$$B^*(z) = \frac{\mu_0}{2\pi i} \iint dx_0 dy_0 \frac{j(x_0, y_0)}{z - z_0}$$

where

$$j(x_0, y_0) = j [\delta(x_0 + a) - \delta(x_0 - a)] \theta(b^2 - y_0^2) \quad (16)$$

$$\text{Or, } B^*(z) = \frac{\mu_0 j}{2\pi} \left\{ \text{Ln}(z+a-ib) - \text{Ln}(z+a+ib) + \text{Ln}(z-a+ib) - \text{Ln}(z-a-ib) \right\} \quad (17)$$

This form of expression is much easier to use than the reduced logarithmic form of eq. (17):

$$B^*(z) = \frac{\mu_0 j}{2\pi} \text{Ln} \left\{ \frac{z^2 - (a-ib)^2}{z^2 - (a+ib)^2} \right\} \quad (18)$$

By choosing proper branch lines, the multivalued functions of Ln can readily be replaced by their principal values of ln.

V. Translation and Rotation of Magnets

Eq. (17) gives a general formula for the field produced by a bar magnet located in the origin of the coordinate system, facing y direction. Let's define it as $B_0^*(z)$.

Now, if the magnet is rotated by an angle α with respect to the y axis as shown

Then, the previous formula should still be valid in the rotating system $z' = x' + iy' = ze^{-i\alpha}$. And, the new B^* should also be modified by $B_0^* e^{-i\alpha}$.

Furthermore, if the magnet is moved to a new center $z_0 = x_0 + iy_0$, then B^* should be

$$B^* = e^{-i\alpha} B_0^* (e^{-i\alpha} (z - z_0)) \quad (19)$$

When $z_0 = 0$, and $\alpha = 0$, $B^* = B_0^*$ equation (19) will reduce to eq. (17).

If the magnet is located at z_0 , but facing x direction, then $\alpha = -\frac{\pi}{2}$

$$B^* = i B_0^* (i (z - z_0)) \quad (20)$$

One immediate interesting result of eq. (19) is that when the magnetization of $\vec{M}$ has its orientation increased by an angle α , the orientation of B^* on the complex surface will actually decrease by α . Physically, it is because as M turns, the relative position of the observing point w.r.t. the magnet will change. The field it will now observe is equivalent to another observing point shift by $-\alpha$ w.r.t. the original

coordinate system. This is also described as the Easy Axis Rotation Theorem*.

VI. Field Reversal and Current Reversal

Eq. (17) and (19) both show that $B^*(z)$ will simply change sign when the current j is reversed to $-j$. That is when the direction of magnetization flips, the field will flip with it. This simple fact is, however, not quite so obvious, because when reversing the magnetization, instead of switching j , it can also be interpreted as α has been increased or decreased by π .

For instance, by reversing the arrow of a magnet along y-

axis, it means either the outgoing current has been switched to ingoing and vice versa, or the α factor in eq. (19) becomes π or $-\pi$. For the former case, $B^*_{(-y)}$ is simply $-B^*_{(y)}$ which is given by equation (17) or (18) in this case. For the latter, we have to substitute $\alpha = \pi$ into eq. (19), thus

$$B^*_{(-y)} = e^{-i\pi} B^*_0 (e^{-i\pi} z) = -B^*_0 (-z) \quad (21)$$

And $B^*_0(z)$ is given by eq. (17) or (18). Hence,

$$B^*_{(-y)} = -\frac{\mu_0 j}{2\pi} \left\{ \ln(-z+a-ib) - \ln(-z+a+ib) + \ln(-z-a+ib) - \ln(-z-a-ib) \right\} \quad (22)$$

* K. Holbach, Nuclear Instruments and Methods (169) (1980) p. 1-10.

The bracket in the R.H.S. is a multiple value function, because when $z = re^{i\theta}$, $\text{Ln } z = \text{Ln } r + i(\theta + 2\pi k)$. However, if equation (18) is being used, since $z^2 = (-z)^2$, it becomes obvious,

$$B_{(-y)}^* = -B_0^*(-z) = -B_{(y)}^*$$

The reason is that the branch cuts that define the Riemann surface to assure that the logarithmic functions are single-valued have to be connecting all current sheets, i.e.,

Therefore, although eq. (17) is easier to handle, all the singular current sheets should be considered consistently so that they will not be treated on different Riemann surfaces.

VII. Alternating Magnets Arrangements of A Bucket

Typical arrangements of magnets in the neutral beam ion source experiment at LBL can be described as follows.

A multitude of identical magnets are placed around the cylindrical wall of the bucket evenly. Their polarities are flipped alternately. The center of the n th magnet is then located at R_n , where

$$R_n = (R + b)e^{i\theta_n} \quad (23)$$

with $\theta_n = n\Delta\theta$

where $\Delta\theta$ is the angular separation between two neighboring magnets.

Its magnetization can be described either by the alternating current $(+/-) j$, thus $\vec{M}_n = (-1)^n j \hat{e}_n$ where $\hat{e}_n = e^{i\theta_n}$ or by the angular difference α with respect to the y -axis where $M_n = j \hat{e}_n$. The latter gives the following formula of B_n^* of the n th magnet.

$$B_n^* = e^{-i\alpha} \frac{\mu_0 j}{2\pi} \left\{ \ln(z_n + a - ib) - \ln(z_n + a + ib) + \ln(z_n - a + ib) - \ln(z_n - a - ib) \right\} \quad (24)$$

$$\text{with } z_n = e^{-i\alpha}(z - R_n)$$

$$\text{where } \alpha = \theta_n + (-1)^{n+1} \frac{\pi}{2} \quad (25)$$

Note that we now use the principal values $\ln$ in eq. (24).

Now, if we use $(+/-)j$ to indicate the alternating nature of the magnets, then B_n^* will be given by the same formula described by eq. (24), however, with j being replaced by $(-1)^n j$, or

$$B_n^* = e^{-i\alpha} \frac{\mu_0 (-1)^n j}{2\pi} \left\{ \ln(z_n + a - ib) - \ln(z_n + a + ib) + \ln(z_n - a + ib) - \ln(z_n - a - ib) \right\} \quad (26)$$

and α is no longer given by eq. (25), rather

$$\alpha = (\theta_n - \frac{\pi}{2}) \quad (27)$$

$$e^{-i\alpha} = ie^{-i\theta n} \quad (28)$$

$$z_n = ie^{-i\theta n}(z - R_n) \quad (29)$$

As discussed in Section VI, these two descriptions given by eq. (24) and eq. (26) are, in fact, exactly the same.

If there are N magnets placed evenly around the complete circle, then

$$\Delta\theta = \frac{2\pi}{N} \quad (30)$$

If the N magnets are placed evenly only around part of the circle leaving a gateway for other instrumentations, then

$$\Delta\theta = \frac{2\pi - \textcircled{H}}{N} \quad (31)$$

where $\textcircled{H}$ is the angle subtained by the gateway.

The total B^* is simply the sum of all B_n^* 's.

$$B^* = \sum_{n=0}^{N-1} B_n^* \quad (32)$$

where $n=0$ describes the starting magnet placed on the x-axis, where the coordinate system is defined.

VIII. Vector Potential and Magnetic Field

All the magnets in the present work are simulated by infinite long currents flowing into and out of the paper. Defining this direction as $\pm \hat{e}_z$, then vector potential will have only the $\hat{e}_z$ - component also. Eq. (2) then provides the proper relationship between $\vec{B}$ and $\vec{A}$.

$\vec{B}$ can, furthermore, be described by a scalar potential ϕ .

where $\vec{B} = -\nabla\phi$ (33)

Thus, $B_x = -\frac{\partial\phi}{\partial x}$, $B_y = -\frac{\partial\phi}{\partial y}$ (34)

Combining eqs. (34) and (2), we find that if we define,

$$\psi = A_z + i\phi$$
 (35)

then $i\psi = iA_z - \phi$ (36)

becomes an analytic function.

The real and imaginary parts of function $i\psi$ satisfy the following Cauchy-Riemann equations:

$$\begin{cases} \frac{\partial(-\phi)}{\partial x} = \frac{\partial A_z}{\partial y} = B_x \\ \frac{\partial(-\phi)}{\partial y} = \frac{\partial A_z}{\partial x} = B_y \end{cases}$$
 (37)

Since $B^* = B_x - iB_y = \frac{\partial(-\phi)}{\partial x} + i \frac{\partial A_z}{\partial x}$

$$B^* = \frac{d}{dz} (i\psi)$$
 (38)

The vector potential, which is simply the imaginary part of $i\psi$ as given by eq. (36) is thus

$$A_z = \text{Im} (i\psi) = \text{Im} \int B^* dz$$
 (39)

Using eqs. (26)-(29) for B_n^* ,

$$B_n^* = ie^{-i\theta n}(-1)^n \frac{\mu_0 j}{2\pi} \left\{ \ln(z_n + a-ib) + \dots \right\}$$
 (40)

with $z_n = ie^{-i\theta n} (z - R_n)$

$$R_n = (R+b)e^{i\theta n}$$

$$\begin{aligned} & \int \ln(z_n + a - ib) dz \\ &= \int \ln[ie^{-i\theta n} (z - R_n) + a - ib] dz \\ &= -ie^{i\theta n} \int \ln z' dz' = -ie^{i\theta n} [z'(\ln z' - 1)] \end{aligned} \quad (41)$$

where $z' = z_n + a - ib$ (42)

Thus,

$$A_z^n = I_m \int B^* dz = I_m (-1)^n \frac{\mu_0 j}{2\pi} \left\{ A_1^n (\ln A_1^n - 1) - A_2^n (\ln A_2^n - 1) + A_3^n (\ln A_3^n - 1) - A_4^n (\ln A_4^n - 1) \right\} \quad (43)$$

Where

$$\left\{ \begin{aligned} A_1^n &= ie^{-i\theta n} (z - R_n) + a - ib \\ &= a - r \sin(\theta - \theta n) - i(R+2b) + ir \cos(\theta - \theta n) \\ A_2^n &= ie^{-i\theta n} (z - R_n) + a + ib \\ &= a - r \sin(\theta - \theta n) - iR + ir \cos(\theta - \theta n) \\ A_3^n &= -a - r \sin(\theta - \theta n) - iR + ir \cos(\theta - \theta n) \\ A_4^n &= -a - r \sin(\theta - \theta n) - i(R+2b) + ir \cos(\theta - \theta n) \end{aligned} \right. \quad (44)$$

Since $A_1^n - A_2^n + A_3^n - A_4^n = 0$, eq. (43) can be further simplified as

$$A_z^n = I_m (-1)^n \frac{\mu_0 j}{2\pi} \left\{ A_1^n \ln A_1^n - A_2^n \ln A_2^n + A_3^n \ln A_3^n - A_4^n \ln A_4^n \right\} \quad (45)$$

Total vector potential is then

$$A_z = \sum_{n=0}^{\infty} \frac{1}{n!} A_z^n \quad (46)$$

IX. Sample Calculation

One typical case for consideration is the following:

There are ten long magnets placed radially away from the center line with alternating polarities around the boundary of a circular cylinder of radius 10 cm. This design is currently being adopted for plasma confinement at Lawrence Berkeley Laboratory. The dimensions of each magnet are $a=0.6$ cm, and $b=1$ cm. At the poles, the magnetic field strength is $B_0 = 3.5$ k Gauss.

By assigning $\Delta\theta = 36^\circ$, eqs. (23), (30), (44), (45) and (46) are just enough for the determination of A_z . The result is given by Fig. 1 where $A = A_z$. A few remarks can easily be observed.

a) At the center line of this cylindrical bucket, where $r = 0$, $A = 0$. Because the symmetry of this arrangement produces just as much $A>0$ as $A<0$.

b) The symmetrical pattern of Fig. 1 is also expected. In fact, not only the value of A repeats itself every 36° as it should, it changes signs every 18° . However, since only the absolute value of A matters in the energy principle shown by eq. (4) or (5), the whole pattern then becomes symmetric every 18° .

c) The results presented by Fig. 1 are in c.g.s. unit. Although it is easy to convert them into M.K.S. or other units, it is not really necessary. Because what is needed here is just

the ratio of quantities reflecting the relative magnitude of A in different zones in space. For instance, at the surface of the poles of a magnet where $z = bi$, equations (17) or (18) gives

$$B_y^* = - \frac{\mu_0 j}{2\pi} \times 2 \tan^{-1} \left(\frac{2b}{a} \right) = - B_y^0$$

$$\therefore B_y^0 = \frac{\mu_0 j}{2\pi} 2 \tan^{-1} \left(\frac{2b}{a} \right)$$

By replacing $\frac{\mu_0 j}{2\pi}$ by

$$\frac{\mu_0 j}{2\pi} = \frac{B_y^0}{2 \tan^{-1} \left(\frac{2b}{a} \right)}$$

all formulas of the magnetic field will have the same unit as B_y^0 . The vector potential has the unit as the product of the field intensity and the distance. The significant part of the A's is thus simply the quantities on the right hand side of eq. (45) without the constant factor of $\left(\frac{\mu_0 j}{2\pi} \right)$. The results of A at various points plotted in Fig. 1 are evaluated by omitting this factor which is numerically equal to $3.5 \text{ k Gauss} / 2 \tan^{-1} \left(\frac{2}{0.6} \right)$ in the present case.

d) The slope changes of A are directly related to the field variations. In cylindrical coordinates, the relationships between B and A are:

$$B_r = \frac{\partial A_z}{r \partial \theta} ; \quad B_\theta = - \frac{\partial A_z}{\partial r}$$

e) If we let $A = 10^{-3}$ in fig. I, then, other than the small cone regions where the magnets are, plasma in the middle can only reach $r = 2.3$ cm in the middle of 2 magnets. While for $A = 10^{-2}$, r could reach 3.6 cm and for $A = 10^{-1}$, r may be 5.9 cm.

f) In regions close to the magnets, there exist "loss cones" where plasma may escape easily. When $A = 10^{-3}$, the cone extends an angle of $\pm 0.5^\circ$, while when $A = 10^{-2}$, it's $\pm 2^\circ$, and when $A = 10^{-1}$, the loss cone has an angle of $\pm 5^\circ$ at the magnet.

g) Note that in all the confinement assessments made above, the plasma is assumed collisionless. Otherwise the estimates of the "loss cones", for example, need to be reconsidered, because plasma particles which are perpendicular to the magnetic field lines originally may very well become parallel to them after one collision. Such calculations are to be discussed in later papers. We thus have here a first order evaluation of the plasma confinement by permanent magnets.

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