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PLASMA CONFINEMENT CALCULATIONS FOR TIBER II

FINAL REPORT

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INTRODUCTION

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The project under LLNL Contract 1413307 is to compare the empirically based TIBER II design parameters against the reactor parameters of a tokamak based on a theoretical model of heat transport.

The motivation behind this project is the following. The present TIBER-II design is based on an empirical scaling of confinement time τ_E known as Kaye-Goldston scaling. This empirical scaling is based on data from tokamak plasmas whose temperatures did not exceed a few keV, while the TIBER-II requires electron temperatures of 25 keV. The question, then, is whether the Kaye Goldston scaling is valid at temperatures above the range of empirical data. For this we must turn to theory. If a theoretical transport model is available that shows favorable correlation with the Kaye-Goldston scaling in the range of a few keV, then this scaling could be extrapolated to the 25 keV range required for TIBER-II. Hence the motivation behind this project.

The project is divided into two parts. Part one of the project concerns the development of a transport model from plasma theory that has empirical support. Part two of the project consists of applying the transport model to the TIBER-II design.

PART 1

The transport model that has been developed is a two fluid model based on thermal transport due to microinstabilities and neoclassical diffusion.

In particular the two fluid transport model is based on the following premises:

1. The electron and ion thermal loss channel are treated separately.
2. The electron thermal diffusivity is due to trapped electron modes.
3. The ion thermal diffusivity is due to neoclassical thermal diffusion, in the banana regime, and additionally due to toroidal η_i modes when $\eta_i > \eta_{ic}$

$$\text{where } \eta_i = \frac{d \ln T_i}{d \ln n_i} = \left| \frac{\frac{1}{T_i} \frac{dT_i}{dr}}{\frac{1}{n_i} \frac{dn_i}{dr}} \right| \quad (1)$$

$$\eta_{ic} = 1.5 \quad (2)$$

T_i is the ion temperature

and n_i is the ion density

4. The electron and ion thermal loss channels are coupled by Coulomb collision power transfer H_{ie} .
5. The electrons will be heated by ohmic heating and H_{ie} .
6. The ions are heated by auxiliary heating with a profile given by:

$$H_{aux} = \left(\frac{r}{a}\right)^{\mathcal{L}} H_0 \quad (3)$$

where a is the plasma minor radius, H_0 is a constant, and $\mathcal{L}$ is a measure of the peakedness of the heating profile.

7. We neglect direct effects of η_i mode fluctuations on electron transport and trapped electron mode fluctuation on ion transport.
- The steady state heat balance equation for ions and electrons is written using premises (1), (4), (5), and (6) as follows:

$$H_0 + H_{ie} = -\frac{1}{r} \frac{d}{dr} \left(r n_e \chi_e \frac{dT_e}{dr} \right) \quad (4)$$

$$H_{aux} - H_{ie} = -\frac{1}{r} \frac{d}{dr} \left(r n_i \chi_i \frac{dT_i}{dr} \right) \quad (5)$$

where $H_0 = \eta_{ie} J_{||}^2$

$$= \frac{3.567 \times 10^8 Z J_{||0}^2}{T_e^{3/2}} \exp^{-2(0.5 + q_a) \frac{r^2}{a^2}} \frac{\text{keV}}{\text{m}^3 \text{s}} \quad (6)$$

$$T_e = T_{e0} \exp -\frac{2}{3}(0.5 + q_a) \frac{r^2}{a^2} \quad (7)$$

$$T_i = T_{i0} \exp -\frac{2}{3}(0.5 + q_a) \frac{r^2}{a^2} \quad (8)$$

$$J_{||} = J_{||0} \exp -(0.5 + q_a) \frac{r^2}{a^2} \quad (9)$$

$$n_e = n_i = n = n_0 \left(1 - \left(\frac{r}{a}\right)^2\right)^{q_n} \quad (10)$$

$$H_{ie} = \frac{3}{2} n \frac{(T_i - T_e)}{\tau_{eq}} \quad \frac{\text{keV}}{\text{m}^3 \text{s}} \quad (11)$$

T_i is the ion temperature, T_e is the electron temperature, n_i is the ion density, n_e is the electron density, $H_{||}$ is the ohmic heating power, χ_e is the electron thermal diffusivity, χ_i is the ion thermal diffusivity, $J_{||}$ is the parallel current density, and $\eta_{||e}$ is the electron parallel resistivity.

This transport model is based on the assumption that the electron and ion temperature profiles are Gaussian in shape given by Eqs. (7) and (8) respectively, and $J_{||}$ is also Gaussian in shape as shown in Eq. (9). The Gaussian shape profile are known as consistent profiles and there is empirical support for their existence in Ohmic and auxiliary heated toroidal discharges [1].

In Eqs. (4) and (5) the alpha power is omitted which is appropriate for comparison with present experiments in hydrogen or deuterium plasmas.

Equations (4) and (5) can be rearranged to solve for χ_e and χ_i as follows:

$$\chi_e = \frac{\int_0^r r' (H_{||} + H_{ie}) dr'}{r n_e \frac{dT_e}{dr}} \quad (12)$$

$$\chi_i = \frac{\int_0^r r' (H_{aux} - H_{ie}) dr'}{r n_i \frac{dT_i}{dr}} \quad (13)$$

Substituting Eqs. (6), (7), (8), (9), (10), and (11) into Eqs. (4) and (5) gives after integrating:

$$\begin{aligned}
 X_e = & \frac{1.338 \times 10^8 j_{10}^2 a^{4+2a_n} z}{n_{e0} T_{e0}^{5/2} (0.5 + q_a)^2} \frac{\exp \left[\frac{2}{3} (0.5 + q_a) \frac{r^2}{a^2} \right]}{(a^2 - r^2)^{a_n} r^2} \\
 & \times \left(\frac{\exp \left[(0.5 + q_a) / q_a^2 \right]}{2} + \frac{\exp \left[-(0.5 + q_a) / q_a^2 \right]}{2} - \exp \left[-(0.5 + q_a) \frac{r^2}{a^2} \right] \right) \\
 & + \frac{7.883 \times 10^{-19} n_{e0} a^{2+2a_n} (T_{i0} - T_{e0}) \exp \left[\frac{2}{3} (0.5 + q_a) \frac{r^2}{a^2} \right]}{T_{e0}^{5/2} (0.5 + q_a) (a^2 - r^2)^{a_n} r^2} \\
 & \times \left[\frac{a^2}{4 a_n + 2} \left(\exp \left[\frac{1}{3} (0.5 + q_a) / q_a^2 \right] - \exp \left[\frac{1}{3} (0.5 + q_a) / q_a^2 \right] \right) \right. \\
 & \times \left(\frac{q_a^2 - 1}{q_a^2} \right)^{2a_n + 1} \left. + \frac{3 a^{2-4a_n}}{2} \right. \\
 & \times \left. \left(\frac{\exp \left[\frac{1}{3} (0.5 + q_a) / q_a^2 \right] (1 - 1/q_a^2)^{2a_n + 1} a^{4a_n + 2} - \exp \left[\frac{1}{3} (0.5 + q_a) \frac{r^2}{a^2} \right] (a^2 - r^2)^{2a_n + 1}}{3a^2 (2a_n + 1) - (a^2 - r^2) (0.5 + q_a)} \right) \right] \quad (14)
 \end{aligned}$$

$$\begin{aligned}
 X_i = & \frac{0.094 a^{2a_n - 2} p_T}{n_{e0} T_{i0} (0.5 + q_a) n_{ex}^2 R} \frac{r^{2+2} \exp \left[\frac{2}{3} (0.5 + q_a) \frac{r^2}{a^2} \right]}{(a^2 - r^2)^{a_n} r^2} \\
 & - \frac{7.883 \times 10^{-19} n_{e0} a^{2+2a_n} (T_{i0} - T_{e0}) \exp \left[\frac{2}{3} (0.5 + q_a) \frac{r^2}{a^2} \right]}{T_{i0} T_{e0}^{3/2} (0.5 + q_a) (a^2 - r^2)^{a_n} r^2}
 \end{aligned}$$

$$\begin{aligned}
& \times \left[\frac{a^2}{4 a_n + 2} \left(\exp \left[\frac{1}{3} (0.5 + q_a) / q_a^2 \right] - \exp \left[\frac{1}{3} (0.5 + q_a) / q_a^2 \right] \right) \right. \\
& \times \left. \left(\frac{q_a^2 - 1}{q_a^2} \right)^{2a_n + 1} \right) + \frac{3 a^{2-4a_n}}{2} \\
& \times \left(\frac{\exp \left[\frac{1}{3} (0.5 + q_a) / q_a^2 \right] (1 - 1/q_a^2)^{2a_n + 1} a^{4a_n + 2} - \exp \left[\frac{1}{3} (0.5 + q_a) / q_a^2 \right] (a^2 - r^2)^{2a_n + 1}}{3a^2 (2a_n + 1) - (a^2 - r^2) (0.5 + q_a)} \right) \Big] \quad (15)
\end{aligned}$$

where units are a (m), R (m), T_{eo} (keV), T_{fo} (keV), n_o (m⁻³), P_T (keV/s), $J_{||o}$ (A/m²), χ_i (m²/s), R is the major radius, q_a is the edge safety factor, P_T is the total heating power and κ_* is the effective plasma elongation given by:

$$\kappa_*^2 = \kappa^2 (1 + 2\delta^2) \quad (16)$$

In Eq. (15) P_T is the total heating power, which is the total auxiliary heating power since ohmic and alpha heating is neglected. P_T is given by integrating Eq. (3) over the plasma volume as:

$$\begin{aligned}
P_T &= \int H_{aux} dV \\
&= \frac{4\pi^2 R \kappa_* a^2 H_o}{L + 2} \quad (17)
\end{aligned}$$

In Eq. (16) κ is the plasma elongation and δ is the plasma triangularity.

The electron and ion thermal diffusivities are also expressed using premises (2) and (3) as follows. The trapped electron χ_e^H is given as [2]:

$$\chi_e^H = \frac{\epsilon^{3/2} (\omega_{*e}^2 \eta_e + \omega_{*e}^p \omega_{pe})}{\nu_{ei} \left[1 + \frac{\epsilon}{\nu_{*e}} \right] k_{\perp}^2} \quad (18)$$

where $\epsilon = r/R$ is the inverse aspect ratio and

$$\omega_{*e} = \frac{k_{\perp} T_e}{e B L_{Ne}} \quad (19)$$

$$\eta_e = (L_N/L_T) e \quad (20)$$

$$\omega_{*e}^p = \omega_{*e} (1 + \eta_e) \quad (21)$$

$$\omega_{De} = \frac{\omega_{*e} L_N}{R} \quad (22)$$

$$\nu_{*e} = \frac{\nu_{eff}}{\omega_{be}} \quad (23)$$

$$\bar{c} \sim 0.1 - 0.2 \quad (24)$$

$$L_N = \left| \frac{1}{n} \frac{dn}{dr} \right|^{-1} \quad (25)$$

$$L_T = \left| \frac{1}{T} \frac{dT}{dr} \right|^{-1} \quad (26)$$

$$\nu_{eff} = \frac{\nu_{ei}}{\epsilon} \quad (27)$$

$$\omega_{be} = \frac{\epsilon^{1/2} v_{th}}{qR} \quad (28)$$

$k_{\perp}$ is the perpendicular wave number of the trapped electron mode and v_{th} is the thermal speed. The constant $\bar{c}$ simulates a transition to the collisionless regime [2]. ν_{ei} is the electron ion collision frequency and is given as [3]:

$$\nu_{ei} = \frac{4 \sqrt{2} \pi e^4 Z_1^2 n_1}{\sqrt{3} e m_e T^{3/2}} \quad (29)$$

where m_e is the electron mass in g, T_e is the electron temperature in electron volts (eV), L is the Coulomb logarithm approximately equal to 17.5, Z_i is the atomic number of the ion, n_i is the ion density in cm^{-3} , e is the electronic charge in c.g.s. units and ν_{ei} is in s^{-1} .

In Eq. (28) q is the safety factor given as:

$$q = \frac{r B_T}{R B_\theta}$$

$$= \left(\frac{r}{a}\right)^2 q_a \frac{(1 - \exp^{-(0.5 + q_a)})}{1 - \exp^{-(0.5 + q_a)} \frac{r^2}{a^2}} \quad (30)$$

where the consistent current density profile given by Eq. (9) has been used in the definition of $q[2]$. B_T is the toroidal magnetic field strength and B_θ is the poloidal magnetic field strength.

Substituting Eqs. (19) - (30) into Eq. (18) gives after simplifying:

$$\chi_e^* = \chi_{\text{trapped electron}}$$

$$= \frac{0.018 T_{eo}^{3/2} q_a (1 - \exp^{-(0.5 + q_a)})}{B_r^2 \tau a^4} \left(\frac{r^3}{\exp^{(0.5 + q_a)} \frac{r^2}{a^2} - 1} \right)$$

$$\times \left(\frac{6a^2 a_n + 4(0.5 + q_a) (a^2 - r^2) + 8r R(0.5 + q_a) q_n}{a^2 - r^2} \right) \quad (31)$$

where units are $a(\text{m})$, $R(\text{m})$, $B_T(\text{T})$, $T_{eo}(\text{KeV})$, and $\chi_e^* (\text{m}^2/\text{s})$.

In Eq. (31) the density profile is given by Eq. (10), the temperature profile is given by Eq. (7) and for simplicity $\bar{C}/\nu_{*e} \gg 1$ is assumed appropriate for the regimes of interest, and $B \sim B_T$.

The ion thermal diffusivity is given as:

$$\chi_i^* = \chi_{\eta i} + \chi_{i \text{ ban}}^{\text{neo}} \quad (32)$$

$$\text{where } \chi_{\eta 1} = 0.4 \left(\frac{\pi}{4} \right) \ln^2 (1 + \eta_1) \left(\frac{1 + \eta_1}{\tau} \right)^2 \frac{\rho_s^2 c_s}{L_s} \quad (33)$$

is the G.S. Lee P.H. Diamond expression for χ_1 due to η_1 modes in Eq. (33):

$$\eta_1 = \left| \frac{1}{\bar{T}_1} \frac{dT_1}{dr} \right| \left| \frac{1}{\bar{n}_1} \frac{dn_1}{dr} \right| \quad (34)$$

$$\rho_s = \frac{m_i}{eB} \left(\frac{2 T_e}{m_i} \right)^{1/2} \quad (35)$$

$$c_s = \left(\frac{5}{3} \frac{T_e}{m_i} \right)^{1/2} \quad (36)$$

$$L_s = qR \quad (37)$$

$$\tau = T_e/T_i \quad (38)$$

Substituting Eqs. (7), (8), (10), (30), (34), (35), (36), (37), and (38) into Eq. (33) and simplifying gives:

$$\chi_{\eta 1} = \frac{13.030 a^2 T_{i0}^2 \exp [-(0.5 + q_a) \frac{r^2}{a^2}] \{1 - \exp [-(0.5 + q_a) \frac{r^2}{a^2}]\}}{r^2 T_{e0}^{1/2} B_T^2 q_a R \{1 - \exp [-(0.5 + q_a)]\}} f^* \times \left(1 + \frac{2 (0.5 + q_a) (a^2 - r^2)}{3a^2 a_n} \right)^2 \ln^2 \left(1 + \frac{2 (0.5 + q_a) (a^2 - r^2)}{3a^2 a_n} \right) \quad (39)$$

where it has been assumed that $B \sim B_T$ for the sake of simplicity and units are $a(m)$, $R(m)$, $B_T(T)$, $T_{i0}(keV)$, $T_{e0}(keV)$, and $\chi_{\eta 1}(m^2/s)$.

In Eq. (39) $f^* = 1$ if $\eta > \eta_{1c}$ and 0 if $\eta < \eta_{1c}$ where $\eta_{1c} = 1.5$.

In Eq. (32) $\chi_{1 \text{ Ban}}^{\text{neo}}$ is given by [5]:

$$\chi_{1 \text{ Ban}}^{\text{neo}} \approx \frac{q^2}{\epsilon^{3/2}} \frac{r_{L1}^2}{\tau_{i1}} \quad (40)$$

$$\text{where } r_{L1} = \frac{m_i}{eB} \left(\frac{2 T_i}{m_i} \right)^{1/2} \quad (41)$$

$$\text{and } \tau_{i1} = \frac{3 m_i T^{3/2}}{4 \pi L e^4 Z^4 n_i} \quad (42)$$

In Eq. (42) the units are identical to those for ν_{e1} in Eq. (29) and T_i is also in eV. Substituting Eqs. (8), (10), (30), (41) and (42) into Eq. (40) gives after simplifying:

$$\begin{aligned} \chi_{1 \text{ Ban}}^{\text{neo}} \approx & \frac{1.421 \times 10^{-21} n_o Z q_a^2 R^{3/2} (1 - \exp^{-(0.5 + q_a)})^2}{B_T^2 T_{i0}^{1/2} a^4 + 2 q_n (1 - \exp^{-(0.5 + q_a)}) \frac{r_a^2}{a^2}} \\ & \times \left(\frac{r^{5/2} \exp^{1/3(0.5 + q_a)} \frac{r^2}{a^2} (a^2 - r^2)^{q_n}}{1} \right) \end{aligned} \quad (43)$$

where units are $a(\text{m})$, $R(\text{m})$, $n_{eo}(\text{m}^{-3})$, $B_T(\text{T})$, $T_{fo}(\text{keV})$, and $\chi_{1 \text{ Ban}}^{\text{neo}} (\text{m}^2/\text{s})$.

Substituting Eqs. (39) and (43) into Eq. (32) gives:

$$\begin{aligned}
 \chi_1^* = & \frac{13.030 a^2 T_{i0}^2 \exp^{-(0.5 + q_a) \frac{r^2}{a^2}} (1 - \exp^{-(0.5 + q_a) \frac{r^2}{a^2}}) f^*}{r^2 T_{eo}^{1/2} B_T^2 q_a R (1 - \exp^{-(0.5 + q_a)})} \\
 & \times \left(1 + \frac{2 (0.5 + q_a) (a^2 - r^2)}{3 a^2} \right)^2 \ln^2 \left(1 + \frac{2 (0.5 + q_a) (a^2 - r^2)}{3 a^2} \right) \\
 & + \frac{1.421 \times 10^{-21} n_0 Z^2 q_a R^{3/2} (1 - \exp^{-(0.5 + q_a)})^2}{B_T^2 T_{i0}^{1/2} a^4 + 2 a n (1 - \exp^{-(0.5 + q_a) \frac{r^2}{a^2}})} \\
 & \times \left(\frac{r^{5/2} \exp^{1/3 (0.5 + q_a) \frac{r^2}{a^2}} (a^2 - r^2) a_n}{1} \right) \quad (44)
 \end{aligned}$$

In Eqs. (14), (15), (31), and (44) the unknown variables are T_{i0} and T_{eo} . In order to determine T_{i0} and T_{eo} , Tang's procedure of averaging χ over the consistent profile is used [2]. The entropy production equations from Tang's averaging procedure for ions and electrons are:

$$\int_{r_1}^{r_2} r \chi_e \left(\frac{1}{T_e} \frac{dT_e}{dr} \right)^2 dr = \int_{r_1}^{r_2} r \chi_e^* \left(\frac{1}{T_e} \frac{dT_e}{dr} \right)^2 dr \quad (45)$$

$$\int_{r_1}^{r_2} r \chi_i \left(\frac{1}{T_i} \frac{dT_i}{dr} \right)^2 dr = \int_{r_1}^{r_2} r \chi_i^* \left(\frac{1}{T_i} \frac{dT_i}{dr} \right)^2 dr \quad (46)$$

where χ_e , χ_i , χ_e^* , and χ_i^* are given by Eqs. (14), (15), (31), and (44) respectively.

Equations (45) and (46) are simultaneous equations in T_{eo} and T_{i0} given a , R , q_a , B_T , n_{eo} , Z , F_T , $\tilde{c}$, a_n , and $\mathcal{L}$. These equations have been solved

simultaneously for T_{i0} and T_{e0} using an iterative procedure. The values of T_{i0} and T_{e0} are substituted in Eqs. (7) and (8) to give the magnitude of the ion and electron temperature profiles. The limits r_1 and r_2 designate the location of the $q = 1$ and $q = 2$ surfaces. Approximately $r_1 = a q^{-1}$ [2]. A flat profile is imposed on T_i and T_e inside the $q = 1$ surface such that $T_i(0) = T_i(r_1)$ and $T_e(0) = T_e(r_1)$. In determining T_{i0} and T_{e0} the value of ζ is fixed at 0.15 and $\alpha_n = 1$.

A data base of T_{i0} and T_{e0} vs a , R , q_a , B_T , n_{e0} , Z , and P_T has been generated for three auxiliary heating profiles ($\ell = -1.5$, $\ell = 0$, and $\ell = 2$). The corresponding data base for τ_E , $\langle x_e^* \rangle$, $\langle x_{\eta 1} \rangle$, and $\langle x_{ban} \rangle$ has been generated using the following equations:

$$\tau_E = \frac{3 \int n (T_i + T_e) dV}{2 P_T} \quad (47)$$

$$\langle x_e^* \rangle = \frac{2\pi \int_{r(q=1)}^{r(q=2)} r x_E dr}{2\pi \int_{r(q=1)}^{r(q=2)} r dr} \quad (48)$$

$$\langle x_{\eta 1} \rangle = \frac{2\pi \int_{r(q=1)}^{r(q=2)} r x_{\eta 1} dr}{2\pi \int_{r(q=1)}^{r(q=2)} r dr} \quad (49)$$

$$\langle x_{ban} \rangle = \frac{2\pi \int_{r(q=1)}^{r(q=2)} r x_{i ban}^{neo} dr}{2\pi \int_{r(q=1)}^{r(q=2)} r dr} \quad (50)$$

The data base is generated around a fixed set of reactor parameters given below:

$$a = 0.75 \text{ m} \quad (51)$$

$$R = 3.0 \text{ m} \quad (52)$$

$$q_a = 3 \quad (53)$$

$$B_T = 6 \text{ T} \quad (54)$$

$$n_0 = 5 \times 10^{20} \text{ m}^{-3} \quad (55)$$

$$Z = 1.5 \quad (56)$$

$$P_T = 160 \text{ MW} \quad (57)$$

Statistical regression analysis of the data base gives the following overall scalings:

$$\underline{\ell = -1.5}$$

$$\tau_E = 1.832 a^{2.45} R^{0.59} q_a^{-2.07} B_T^{0.91} \langle n \rangle^{0.46} Z^{-0.09} p_T^{-0.55} \kappa_*^{0.55} \quad (58)$$

$$\langle \chi_e^* \rangle = 2.728 a^{-0.82} R^{0.24} q_a^{-2.60} B_T^{-0.72} \langle n \rangle^{0.57} Z^{-0.11} p_T^{0.56} \kappa_*^{-0.56} \quad (59)$$

$$\langle \chi_{\eta i} \rangle = 109.522 a^{0.22} R^{-1.68} q_a^{-2.35} B_T^{-0.52} \langle n \rangle^{-1.15} Z^{-0.20} p_T^{0.83} \kappa_*^{-0.83} \quad (60)$$

$$\langle \chi_{neo} \rangle = 0.146 a^{-1.64} R^{1.72} q_a^{2.49} B_T^{-2.48} \langle n \rangle^{1.33} Z^{-2.06} p_T^{-0.25} \kappa_*^{0.25} \quad (61)$$

$$T_{e0} = 2.363 a^{0.67} R^{-0.38} q_a^{-1.38} B_T^{0.85} \langle n \rangle^{-0.38} Z^{-0.07} p_T^{0.37} \kappa_*^{-0.37} \quad (62)$$

$$T_{i0} = 2.671 a^{0.28} R^{-0.43} q_a^{-1.89} B_T^{0.95} \langle n \rangle^{-0.67} Z^{-0.12} p_T^{0.51} \kappa_*^{-0.51} \quad (63)$$

$$\underline{\ell = 0}$$

$$\tau_E = 2.718 a^{2.53} R^{0.45} q_a^{-2.60} B_T^{1.01} \langle n \rangle^{0.30} Z^{-0.17} p_T^{-0.53} \kappa_*^{0.53} \quad (64)$$

$$\langle \chi_e^* \rangle = 13.490 a^{-0.84} R^{0.05} q_a^{-3.49} B_T^{-0.56} \langle n \rangle^{-0.89} Z^{0.21} p_T^{0.63} \kappa_*^{-0.63} \quad (65)$$

$$\langle \chi_{\eta i} \rangle = 201.432 a^{0.51} R^{-1.91} q_a^{-3.04} B_T^{-0.38} \langle n \rangle^{-1.30} Z^{-0.32} p_T^{0.81} \kappa_*^{-0.81} \quad (66)$$

$$\langle \chi_{neo} \rangle = 0.124 a^{-1.71} R^{1.79} q_a^{2.73} B_T^{-2.52} \langle n \rangle^{1.40} Z^{-2.10} p_T^{-0.26} \kappa_*^{0.26} \quad (67)$$

$$T_{e0} = 3.376 a^{0.66} R^{-0.51} q_a^{-1.98} B_T^{0.96} \langle n \rangle^{-0.59} Z^{-0.14} p_T^{0.42} \kappa_*^{-0.42} \quad (68)$$

$$T_{i0} = 3.994 a^{0.42} R^{-0.58} q_a^{-2.38} B_T^{1.05} \langle n \rangle^{-0.80} Z^{-0.20} p_T^{0.51} \kappa_*^{-0.51} \quad (69)$$

2

$$\tau_E = 7.146 a^{2.70} R^{0.22} q_a^{-3.53} B_T^{1.24} \langle n \rangle^{0.02} Z^{-0.31} P_T^{-0.55} \kappa_*^{0.55} \quad (70)$$

$$\langle \chi_e^* \rangle = 57.246 a^{-0.71} R^{-0.32} q_a^{-5.00} B_T^{-0.18} \langle n \rangle^{-1.39} Z^{-0.43} P_T^{0.64} \kappa_*^{-0.64} \quad (71)$$

$$\langle \chi_{\eta i} \rangle = 818.479 a^{0.96} R^{-2.25} q_a^{-4.26} B_T^{-0.08} \langle n \rangle^{-1.61} Z^{-0.53} P_T^{0.74} \kappa_*^{-0.74} \quad (72)$$

$$\langle \chi_{neo} \rangle = 0.076 a^{-1.83} R^{1.91} q_a^{3.17} B_T^{-2.63} \langle n \rangle^{1.52} Z^{2.17} P_T^{-0.24} \kappa_*^{0.24} \quad (73)$$

$$T_{e0} = 8.765 a^{0.74} R^{-0.75} q_a^{-2.99} B_T^{1.21} \langle n \rangle^{-0.93} Z^{-0.29} P_T^{0.43} \kappa_*^{-0.43} \quad (74)$$

$$T_{i0} = 10.121 a^{0.67} R^{-0.81} q_a^{-3.24} B_T^{1.26} \langle n \rangle^{-1.04} Z^{-0.34} P_T^{0.48} \kappa_*^{-0.48} \quad (75)$$

where units are $a(m)$, $R(m)$, $B_T(T)$, $\langle n \rangle (10^{20} m^{-3})$, $P_T(MW)$, $\tau_E (s)$, $\langle \chi_e^* \rangle (m^2/s)$, $\langle \chi_{\eta i} \rangle (m^2/s)$, $\langle \chi_{neo} \rangle (m^2/s)$, $T_{e0} (keV)$, and $T_{i0} (keV)$.

Note that for the parameters given by Eqs. (51) - (57) $\langle \chi_{\eta i} \rangle$ is greater than $\langle \chi_e^* \rangle$ by a factor of about 5.3. This implies that the ion channel is the dominant loss channel in this two fluid model. The η_i mode has a threshold (i.e., $\eta_i > \eta_{ic} = 1-2$); therefore shutting off the η_i mode by making $\eta_i < \eta_{ic}$ would have a big effect on τ_E . This leverage in τ_E could be useful to a reactor designer (see Part 2).

The two fluid model's τ_E scaling is compared to Kaye-Goldston scaling in Table 1.

Table 1: Comparison of two fluid transport model's scaling of τ_E with the empirical Kaye Goldston τ_E scaling

	Kaye-Goldston	Two fluid theory $\ell = -1.5$
Coefficient exponents	0.532	1.832
a	1.99	2.45
R	0.41	0.59
q_a	-1.24	-2.07
B_T	1.15	0.91
$\langle n \rangle$	0.26	0.46
P_T	-0.58	-0.55
Z	0	-0.09
κ_*	2.14	0.55

If the τ_E scaling for central heating (i.e., $\ell = -1.5$) given by Eq. (58) is compared with the empirical Kaye-Goldston scaling [6] (i.e., $\tau_E = 0.532$

$$a^{1.99} R^{0.41} q_a^{-1.24} B_T^{1.15} \langle n \rangle^{0.26} P_T^{-0.58} \kappa_*^{0.28} (A_1/1.5)^{0.5} \left(\frac{1 + \kappa_*^2}{2} \right)^{1.24}$$

the following observations can be made:

- The scalings with respect to a, R, B_T , $\langle n \rangle$, and Z are in close agreement.
- The strongest discrepancy between the two scalings is in the q_a and κ_* scaling and the magnitude of the coefficient.
- For $q_a \sim 1$ and $\kappa_* \sim 1$ the ratio of the theoretical to empirical value of τ_E , assuming the τ_E scaling with respect to a, R, B_T , $\langle n \rangle$, P_T , and Z is the same for both τ_E scalings is:

$$\frac{\tau_E \text{ two fluid}}{\tau_E \text{ Kaye-Goldston}} = \frac{1.832 q_a^{-2.07} \kappa_*^{0.55}}{0.532 q_a^{-1.24} \kappa_*^{0.28} \left(\frac{1 + \kappa_*^2}{2} \right)^{1.24}} \approx 1.384 \quad (76)$$

If the empirical scaling $\tau_E(0) \propto \left(\frac{P_T}{n} \right)^{0.4} B_T^{0.8}$ [2] is compared with Eq. (62)

we can also conclude that the τ_{eo} scalings with respect to n_0 , P_T and B_T from the theoretical model agree with the empirical τ_{eo} scalings assuming $\tau_E(0)$

$\propto \tau_{eo}$.

The TFTR shot #14727 had the following reactor parameters [7]:

$$a = 0.82 \text{ m} \quad (77)$$

$$R = 2.56 \text{ m} \quad (78)$$

$$q_a = 2.80 \quad (79)$$

$$B_T = 4.80 \text{ T} \quad (80)$$

$$\langle n \rangle = 0.48 (10^{20} \text{ m}^{-3}) \quad (81)$$

$$Z = 2.50 \quad (82)$$

$$P_T = 5.30 \text{ MW} \quad (83)$$

$$\kappa_* = 1 \quad (84)$$

The empirical value of global τ_E from this shot was 200 ms [7]. For these reactor parameters the two fluid transport model predicts the value of τ_E to be 254 ms. The conclusion is that the theoretical prediction of global τ_E is in good agreement with the empirical value of τ_E . The Joint European Torus (JET) has the following reactor parameters:

$$a = 1.25 \text{ m} \quad (85)$$

$$R = 2.96 \text{ m} \quad (86)$$

$$q_a = 3.50 \quad (87)$$

$$B_T = 3.40 \text{ T} \quad (88)$$

$$\langle n \rangle = 0.275 \times 10^{20} \text{ m}^{-3} \quad (89)$$

$$Z = 1.50 \quad (90)$$

$$P_T = 17.90 \text{ MW} \quad (91)$$

$$\kappa_* = 1.68 \quad (92)$$

The empirical value of global τ_E from JET experiments was 390 ms. For these reactor parameters the two fluid transport model predicts the value of τ_E to be 198 ms. The conclusion is that the theoretical prediction of global τ_E is smaller than the empirical value by a factor of about 2.0. A major contribution to this discrepancy comes from the discrepancy in the theoretical and empirical scaling of τ_E with respect to κ_* .

The conclusion from comparing the theoretical transport model's scaling trends for τ_E with the empirical τ_E scaling trends is that the theoretical transport model can only predict the empirical τ_E values when $q_a \sim 3$ and $\kappa_* \sim 1$. For $q_a > 3$ and $\kappa_* > 1$ a correction factor has to be attached to the theoretical scaling to compensate for the discrepancy in the q_a and κ_* scaling of τ_E .

The subject of determining the magnitude of such a correction factor and its use in designing the smallest size tokamak fusion reactor will be taken up in the next part where the two fluid transport model's scaling trends for τ_E and T_0 will be used to determine the parameters of such a reactor.

If Eqs. (58), (64), and (70) are compared with each other, the conclusion is that for the same reactor parameters a , R , q_a , B_T , n_{e0} , Z , and P_T the value of τ_E predicted for the $\ell = -1.5$ (i.e., centrally peaked heating deposition profile) case is the greatest. The same is true for T_{i0} when Eqs. (53), (69), and (75) are compared. From these two observations we can conclude that, according to our model, in order to maximize the achievable τ_E and T_0 , and therefore to optimize the size of a tokamak, the profile for auxiliary heating required to heat the plasma to ignition temperatures should be peaked towards the center of the plasma. As a result the τ_E and T_0 scalings for the $\ell = -1.5$ case will be selected to design the smallest size tokamak fusion reactor in the next part. There is reason to believe that this reactor would maintain ignition since the alpha heating which is proportional to the square of the density ($p_{\alpha} = \frac{n^2}{4} \langle \sigma v \rangle E_f$) will also be peaked towards the center [see Eq. 10].

PART 2

In this section we will attempt to design the smallest size tokamak fusion reactor that can achieve ignition and compare it to the TIBER-II design. The design will be based on the τ_E and T_0 scalings predicted by the two fluid theoretical transport model developed in Part 1, and the TIBER-II values for aspect ratio, edge safety factor, magnetic field strength, effective atomic number, elongation, triangularity, and average ion temperature.

The design of this reactor will have two basic sources of constraints: (1) plasma physics and (2) technology. With a primary emphasis on plasma confinement and heat transport we shall simplify the other plasma physics constraints as follows:

$$(1) \quad P_{\alpha} \geq P_{\text{cond}} + P_{\text{brem}} \quad (93)$$

or

$$\int \frac{n^2}{4} \sigma v \langle T_i \rangle E_f dV \geq \frac{\int \frac{3}{2} n (T_i + T_e) dV}{\tau_E} + \int 5.35 \times 10^{-37} n^2 Z T_e^{1/2} dV \quad (94)$$

where $E_f = 3.52$ MeV and $\sigma v \langle T_i \rangle$ is used to avoid integrating $\langle \sigma v \rangle$ over the volume.

$$(11) \quad \frac{0.04 I_T}{a B_T} \geq \langle \beta \rangle \quad (95)$$

where

$$\langle \beta \rangle = \frac{\int [n (T_i + T_e) \times 1.6 \times 10^{-16} / (B^2 / 2 \mu_0)] dV}{\int dV} \quad (96)$$

is the volume averaged beta.

$$(iii) \quad q_a \geq 2.5 \quad (97)$$

$$(iv) \quad \langle T_i \rangle = \frac{\int n T_i dV}{\int n dV} \geq 8 \text{ keV} \quad (98)$$

In Eqs. (93) - (98) the ion density is assumed to be equal to the electron density ($n = n_i = n_e$) for the sake of simplicity.

The first constraint, Eq. (94), on which we focus primarily is known as the Lawson Criterion and it has to be satisfied for ignition to occur.

The second constraint, Eq. (95), is known as the Troyon limit [8] and it gives an upper threshold for the average beta that a tokamak plasma can hold for a given minor radius, current, and toroidal field. In Eq. (95) units are I_T (MA), a (m), and B_T (T). The current is related to B_T and q_a by:

$$I_T = \frac{5a^2 B_T}{q_a R} \left(\frac{1 + \kappa_z^2}{2} \right) \quad (99)$$

It is worthy of noting that some experiments on tokamaks have shown that there is an upper Murakami Limit ($\langle n \rangle = 1.5 B_T/q_a R$) [8] where units are R (m), B_T (T), and $\langle n \rangle$ (10^{20} m^{-3}). This would also set an upper threshold for $\langle \beta \rangle$ through Eq. (96). However, we shall use the Troyon limit in our calculations because for typical values of B_T , q_a , and R in a fusion reactor design the Troyon Limit is more restricting.

Equation (97) is a rule of thumb in tokamaks to avoid $q < 1$ through most of the plasma, which would bring on the highly disruptive $m = 1$ kink instability.

Equation (98) is another rule of thumb to ensure that the plasma ions are hot enough to cause fusion (i.e., $\sigma v(T)$ is high enough to produce ignition at a useful fusion power density). It is important to point out here that if an economically optimal fusion reactor were being designed $\langle T_i \rangle$ would also be a free parameter that would have to be optimized.

The two fluid theoretical transport model puts the following two constraints on the energy confinement time and central electron temperature for $\ell = -1.5$:

$$(v) \quad \tau_E = 1.832 a^{2.45} R^{0.59} q_a^{-2.07} B_T^{0.91} \langle n \rangle^{0.45} p_T^{-0.55} Z^{-0.09} \kappa_*^{0.55} \quad (58)$$

$$(vi) \quad T_{e0} = 2.363 a^{0.67} R^{-0.38} q_a^{-1.38} B_T^{0.85} \langle n \rangle^{-0.32} p_T^{0.37} Z^{-0.07} \kappa_*^{-0.37} \quad (62)$$

Technology puts two basic constraints on a tokamak. These are:

$$(vii) \quad B_T \leq 6 \text{ T} \quad (100)$$

$$(viii) \quad R/a \geq 3 \quad (101)$$

Equation (100) illustrates the technological limit on the toroidal field that can be induced in a tokamak plasma, based on present day superconductors and Eq. (101) accounts for the fact that a tokamak fusion reactor must provide room for a neutron multiplication blanket and poloidal superconducting coils around the plasma to induce the toroidal magnetic field.

There would also be a minimum dimension R associated with the minimum thickness of shielding determined by neutron mean free paths, but this is not a primary constraint for $R \geq 3 \text{ m}$.

Once the constraints have been laid out, the exercise of optimizing on the size becomes one of determining the smallest value of plasma minor radius that will satisfy all the constraints. This is done as follows:

a) First the toroidal field strength, the aspect ratio, the edge safety factor, and the average ion temperature are fixed as:

$$B_T = 6 \text{ T} \quad (102)$$

$$R/a = 3.6 \quad (103)$$

$$q_a = 2.57 \quad (104)$$

$$\langle T_i \rangle = 8.2 \text{ keV} \quad (105)$$

in accordance with TIBER-II values for these parameters.

b) Next the integrals in Eqs. (94), (95), and (98) are solved using the parabolic density profile given by Eq. (10). In these integrals the ion and electron temperature profiles are given by Eq. (7) and (8) in the $q > 1$ region and by $T = T_0 \exp^{-\frac{2}{3} (0.5 + q_a)/q_a^2}$ within the $q = 1$ surface (i.e., consistent temperature profiles with flat topping in the sawteeth region). The results of the integration are given below:

$$\int \frac{n^2}{4} \sigma v \langle T_i \rangle E_f dV = \frac{0.333}{4} n_0^2 \sigma v \langle T_i \rangle E_f \times \text{volume} \quad (106)$$

$$\frac{\int \frac{3}{2} n(T_i + T_e) dV}{\tau_E} = \frac{\frac{3}{2} \times 0.388 q_a^{-0.40} n_0 (T_{i0} + T_{e0}) \times \text{volume}}{\tau_E} \quad (107)$$

$$\int 5.35 \times 10^{-37} n^2 Z T_e^{1/2} dV = 5.35 \times 10^{-37} \times 0.303 q_a^{-0.18} n_{e0}^{2-1/2} Z \times \text{volume} \quad (108)$$

$$\frac{\int n (T_i + T_e) dV}{\frac{B^2/2 \mu_0}{\int dV}} = \frac{2 \mu_0 n_0 \times 0.388 q_a^{-0.40} (T_{i0} + T_{e0}) \times 1.6 \times 10^{-16}}{B_T^2} \quad (109)$$

$$\frac{\int n T_i dV}{\int n dV} = 0.776 q_a^{-0.4} T_{i0} \quad (110)$$

In Equation (109) B is assumed to be equal to B_T for the sake of simplicity.

c) Equations (106) - (108) are substituted into Eq. (94) and the resulting equation is solved for $n \tau$:

$$n_0 \tau_E \geq \frac{1.5 \times 0.3888 q_a^{-0.40} (T_{i0} + T_{e0})}{\frac{0.333}{4} \sigma v (0.739 q_a^{-0.62} T_{i0}) E_f - 5.35 \times 10^{-37} \times 0.303 q_a^{-0.18} Z T_{e0}^{1/2}} \quad (111)$$

(d) Equations (99) and (109) are substituted into Eq. (95) and the resulting equation is solved for n_0 :

$$n_0 \leq \frac{5 a^2 B_T}{q_a R} \left(\frac{1 + \kappa_*^2}{2} \right) \times \frac{B_T^2 \times 10^{-20}}{2 \mu_0 \times 0.388 q_a^{-0.40} (T_{i0} + T_{e0}) \times 1.6 \times 10^{-16}} \quad (112)$$

where units are n_0 (10^{20} m^{-3}).

(e) Equation (110) is substituted into Eq. (98) and the resulting equation is solved for T_{i0} :

$$T_{i0} = \frac{8.2}{0.776 q_a^{-0.4}} \quad (113)$$

(f) The total input power, P_T , in Eqs. (58) and (62) is rewritten in terms of n , T_e , T_i , and τ_E as:

$$P_T = \frac{\int \frac{3}{2} n (T_i + T_e) dV}{\tau_E} \quad (114)$$

$$= \frac{1.5 \times 0.388 q_a^{-0.40} n_0 (T_{i0} + T_{e0}) \times \text{volume}}{\tau_E} \quad (115)$$

where we take as the volume $2\pi^2 R a^2 \kappa_*$ in order to take into account noncircular cross-section in an approximate way as before.

(g) Equation (115) is substituted for P_T in Eqs. (58) and (62) to give:

$$\tau_E = 1.832 a^{2.45} R^{0.59} q_a^{-2.07} B_T^{0.91} (0.5 n_0)^{0.46} Z^{-0.09} \\ \times \left(\frac{1.5 \times 0.388 q_a^{-0.40} n_0 (T_{i0} + T_{e0}) \times 2\pi^2 a^2 R \kappa_*}{\tau_E} \right)^{-0.55} \kappa_*^{0.55} \quad (116)$$

$$T_{e0} = 2.363 a^{0.67} R^{-0.38} q_a^{-1.38} B_T^{0.85} (0.5 n_0)^{-0.38} Z^{-0.07} \\ \times \left(\frac{1.5 \times 0.388 q_a^{-0.40} \times n_0 (T_{i0} + T_{e0}) \times 2\pi^2 a^2 R \kappa_*^{0.37}}{\tau_E} \right) \kappa_*^{-0.37} \quad (117)$$

where $\langle n \rangle$ has been rewritten in terms of n_0 (i.e., $\langle n \rangle = 0.5 n_0$).

(h) Finally the values of R/a , q_a , B_T , Z , κ_* , and E_F along with the expression for $\sigma v(T)$ (i.e., $\sigma v = 5.1 \times 10^{-22} [2n T - 2.1]$) from Ref. [8] are substituted into Eqs. (111), (112), (113), (116), and (117) to yield five simultaneous equations for a , n_0 , T_{e0} , T_{i0} , and τ_E . Note that, in identifying τ_E in Eq. (116) with τ_E in the ignition condition, Eq. (111), we implicitly assume that alpha heating in the reactor has the same effect on plasma confinement as the auxiliary heating in the experiments against which Eq. (116) was calibrated in Part 1. This assumption is strengthened by the good agreement between the experiments and our theoretical model that shows that the apparent dependence on heating power in the experiments is actually a dependence on the ion and electron temperatures, however they are maintained.

Before we solve Eqs. (111), (112), (113), (116), and (117) simultaneously we have to resolve the issue of the discrepancy in the q_a and κ_* scaling of τ_E in Eq. (58) compared to the empirical τ_E scaling given by Eq. (99). To do this we compare the coefficient and exponents of both scalings and find that the two τ_E scalings can be written as a ratio:

$$\frac{\tau_E \text{ two fluid}}{\tau_E \text{ Kaye-Goldston}} \sim \frac{1.832 q_a^{-2.07} \kappa_*^{0.55}}{0.532 q_a^{-1.24} \kappa_*^{0.28} \left(\frac{1 + \kappa_*^2}{2} \right)^{1.24}} \quad (118)$$

where it has been assumed that the τ_E scalings with respect to a , R , B_T , $\langle n \rangle$, P_T , and Z are approximately equal for both the theoretical and empirical scalings of τ_E .

For $\kappa = 1$, $\delta = 0$, $\kappa_* = 1$, and $q_a = 2.5$, Eq. (118) becomes:

$$\frac{\tau_E \text{ two fluid}}{\tau_E \text{ Kaye-Goldston}} = 1.610 \quad (119)$$

The TIBER-II design values of R/a , q_a , B_T , and Z , and k_* are used to solve Eqs. (111), (112), (113), (116), and (117) simultaneously for a , $\langle n \rangle$, τ_E , P_T and T_{eo} .

This optimization exercise was carried out for the TIBER II design for steady state $\beta = 5$, and the pulsed ignition case. The two fluid model's design predictions are compared with the actual TIBER II design values in Table 2:

<u>Parameter</u> <u>State</u>	<u>Two Fluid</u>	<u>Tiber Ignition</u>	<u>Two Fluid</u>	<u>Time Steady</u>
a	0.88m	0.834m	0.998m	0.834
R	3.196m	3.0m	3.590m	3.0
q_a	2.57	2.57	2.57	2.57
B_T	6T	6T	6T	6T
$\langle n \rangle \text{ m}^{-3}$	3.55×10^{20}	4.30×10^{20}	1.35×10^{20}	1.18×10^{20}
Z	1.56	1.56	1.56	1.56
P_T	223.854 MW	108.8 MW	593.565 MW	185.8 MW
k_*	2.871	2.871	2.871	2.871
$\langle T_i \rangle \text{ keV}$	8.2	8.2 keV	24.6	24.6
$\langle T_e \rangle \text{ keV}$	6.42	8.2 keV	13.7	24.6
$\tau_E \text{ s}$	0.80	0.7 sec	0.43	

Table 2

Table 2 indicates that for ignition case the two fluid theoretical transport model predicts the smallest size tokamak fusion reactor to have about the same dimensions as the TIBER II design. The steady state case is of designing on Kaye Goldston Scaling is supported by a theoretical τ_E scaling which has been developed in the several keV temperature range. Therefore, the motivation to develop the theoretical transport model as described in the first quarterly report has been satisfied i.e., Kaye-Goldston scaling has theoretical support at high temperatures which gives support to the TIBER-II design.

References

1. M. Greenwald et al., Physical Review Letters **53** (1984) p. 352.
2. W. M. Tang, Microinstability Based Model for Anomalous Thermal Confinement in Tokamaks, PPPL-2311 (1986).
3. S. I. Braginskii, Transport Process in a plasma, Review of Plasma Physics, Vol. 1, New York (1965) p. 205.
4. G. S. Lee and P. H. Diamond, Theory of Ion Temperature Gradient Driven Turbulence in Tokamaks, IFSR-209 (1986).
5. Kenro Miyamoto, Plasma Physics for Nuclear Fusion Research, MIT Press, Cambridge (1976).
6. S. M. Kaye, Physics of Fluids **28** (1985) p. 2327.
7. W. M. Tang et al., Microinstability-Based Models for Confinement Properties and Ignition Criteria in Tokamaks Plasma Physics and Controlled Nuclear Fusion Research, IAEA-CN-47/A-V1-1-2, Kyoto, Japan (1986).
8. T. J. Dolan, Fusion Research **1**, Pergamon Press (1982).

APPENDIX A. TABLE OF SYMBOLS

<u>Symbol</u>	<u>Quantity</u>	<u>Units</u>
a	Minor Radius	m
A	Amperes	Amperes
B	Magnetic field	Tesla
B_θ	Poloidal field	Tesla
B_T	Toroidal field	Tesla
C_μ	Constant	
D_{amb}	Ambipolar diffusion coefficient	m^2/s
E	Electric field	Volts/m
$J_\parallel$	Parallel current density	A/m^2
k	Thermal conductivity	$m^{-1}s^{-1}$
L	Coulomb logarithm	
m	Mass	kg
n	Density	m^{-3}
P_T	Total input power	MW
Q	Heat flux	$J/m^2 s$
q_a	Edge safety factor	
q_e	Charge of particle	C
R	Major radius	m
T	Temperature	keV
W	Free energy	keV
Z	Effective atomic number	
a_n	Density peakedness	
β	Ratio of thermal to magnetic pressure	
γ	Growth rate of instability	s^{-1}
δ	Triangularity of plasma	
ϵ	Inverse aspect ratio	
ϵ_0	Permittivity of free space	F/m
κ	Elongation of plasma	
κ_*	Effective noncircularity factor	
χ	Thermal diffusivity	m^2/s
η_1	Ratio of density to temperature gradient length	

Appendix B

The program titled "CHAP5" given below is in the APL computer language [10]. It has been run on the CMS system at UC Berkeley.

"CHAP5" solves the simultaneous set of equations (45) and (46) for T_{i0} and T_{e0} . There are two input variables ' χ ' and ' W '. ' χ ' is a vector of 99 numbers giving the radial position in the plasma at $0.01a$ where a is the plasma minor radius. " W " is a vector of 20 numbers in the following order: $W[1]$ is the plasma minor radius a , $W[2]$ is the plasma major radius R , $W[3]$ is the edge safety factor q_a , $W[4]$ is the toroidal magnetic field strength B_T , $W[5]$ is the profile consistent parallel current density on axis $J_{||0}$, $W[6]$ is the plasma density on axis n_0 , $W[7]$ is the effective atomic number Z , $W[8]$ is the value of α_n (set equal to one), $W[9]$ is the upper limit on the guess range of α_n (set equal to one), $W[9]$ is the upper limit on the guess range of T_{e0} , $W[11]$ is $0.01 a$ (i.e. dr in the integral equations), $W[13]$ is the total power P_T , $W[14]$ is the lower limit on the guess range for T_{e0} , $W[15]$ is the value of ℓ (i.e. peakedness of the heating profile), $W[16]$ is the value of the increment in testing for the correct T_{e0} , $W[17]$ is the value of $\bar{C}$ (set at 0.15), $W[18]$ is the value of H_0 in Eq. (4.35), $W[19]$ is the value of η_{ic} (set at 1.5), and $W[20]$ is the value of the mesh point where $\eta_i = \eta_{ic}$.

Equation 45 is solved for T_{i0} in terms of T_{e0} . The resulting equation is substituted into Eq. (46) to yield a polynomial equation in T_{e0} . The computer program determines the root of this equation in the range fixed by $W[14]$ and $W[9]$ by an iterative procedure with a precision set by $W[16]$. The program then substitutes the computed value of T_{e0} into the equation where T_{i0} is in terms of T_{e0} to determine T_{i0} . Once T_{e0} and T_{i0} are determined the program generates the T_i and T_e profiles using Eqs. (2.36) and (2.27) in the $q < 1$ and $q > 1$ regions respectively. The program uses these temperature profiles to compute τ_E , $\langle \chi_e \rangle$, $\langle \chi_{n1} \rangle$, $\langle \chi_{neo} \rangle$ according to Eqs. (4.45), (3.18), (5.8) and (5.9). The output Z gives τ_E , $\langle T_e \rangle$, $\langle T_i \rangle$, $T_e(0)$, $T_i(0)$, $T_i(0)$, T_{e0} , T_{i0} , P_T , I_T , H_0 , ℓ , $\langle \chi_e \rangle$, $\langle \chi_{n1} \rangle$, $\langle \chi_{neo} \rangle$ according to Eqs. (4.45), (3.18),

(5.8) and (5.9). The output Z gives τ_E , $\langle T_e \rangle$, $\langle T_i \rangle$, $T_e(0)$, $T_i(0)$, T_{eo} , T_{io} , P_T , I_T , H_0 , ℓ , $\langle x_e \rangle$, $\langle x_{\eta i} \rangle$, and $\langle x_{neo} \rangle$.

The program is run with the following fixed values:

$$W[8] = 1 \quad W[9] = 30 \text{ keV} \quad W[14] = 100 \quad W[16] = 0.01 \quad W[17] = 0.15 \quad W[19] = 1.5.$$

The τ_E , T_{eo} , T_{io} , $\langle x_e \rangle$, $\langle x_{\eta i} \rangle$, $\langle x_{neo} \rangle$ scalings vs a , R , q_a , B_T , n_0 , Z , and P_T are determined by running the program for different values of a , R , q_a , B_T , n_0 , Z and P_T changed one at a time. This creates a data base for τ_E , T_{eo} , T_{io} , $\langle x_e \rangle$, $\langle x_{\eta i} \rangle$, and $\langle x_{neo} \rangle$ vs a , R , q_a , B_T , n_0 , Z , and P_T . The data base is developed around a fixed set of a , R , q_a , B_T , n_0 , Z , and P_T given below:

$$\begin{aligned} W[1] \quad a &= 0.75\text{m} \\ W[2] \quad R &= 3.0\text{m} \\ W[3] \quad q_a &= 3 \\ W[4] \quad B_T &= 6\text{T} \\ W[6] \quad n_0 &= 5 \times 10^{20} \text{m}^{-3} \\ W[7] \quad Z &= 1.5 \\ W[11] \quad 0.01a &= 0.0075\text{m} \\ W[13] \quad P_T &= 1 \times 10^{24} \text{keV/sec} \end{aligned}$$

Statistical regression analysis [11] on the data base gives the scalings of τ_E , T_{eo} , T_{io} , $\langle x_e \rangle$, $\langle x_{\eta i} \rangle$, and $\langle x_{neo} \rangle$ vs a , R , q_a , B_T , n_0 , Z , and P_T .

VZ* χ CHAP5 W;A;B;C;D;E;F;G;H;I;J;K;L;M;N;P;Q;R;S;U;V;Y;A1;B1;C1;
D1;E1;F1;AT1;AT2;CT;CU;TE;TO;TI;T1;T2;TP;K1;K2;K3;K4

$$W[18] + W[13] \times (W[15] + 2) \div 39.478 \times W[2] \times W[1] \div 2$$

$$A \leftarrow 0.5 + W[3]$$

$$B \leftarrow (W[1]^2) - \chi^2$$

$$W[20] \leftarrow W[19] \leq 2xAxB + 3x(W[1]^2)xW[8]$$

$$C \leftarrow 2.718 * A x (\chi^2) \div W[1]^2$$

$$D \leftarrow LW[1] \div W[11] x W[3]$$

$$E \leftarrow +/2 \geq ((\chi + W[1])^2) x W[3] x (1 - 2.718^* - 1xA) \div (1 - C^* - 1)$$

$$W[5] + 2xAxW[4] + 1.257E = 6xW[2]xW[3]1 - 2.718^* - 1xA$$

$$F \leftarrow 0.375x(W[1]^4 + 2xW[8]) \times 3.567EBx(W[5]^2)xW[7] \div (A^2)xW[6]$$

$$G \leftarrow \chi x (C * 0.667) \div (B * W[8])$$

$$H \leftarrow ((0.5x2.718^*A \div W[3]^2) + (0.5x2.718^* - 1xA \div W[3]^2)) - 2.8718^* - 1xAx(\chi^2) \div W[1]^2$$

$$A1 \leftarrow +/D1E \uparrow W[11] x F x G x H$$

$$K1 \leftarrow ((W[1]^2) \div 2 + 4xW[8]) x (2.718^*0.333xA \div W[3]^*W) x 1 - (1 - W[3]^* - 2)^*1 + 2xW[8]$$

$$K2 \leftarrow (2.718^*0.333xA + W[3]^2) x ((1 - W[3]^* - 2)^*1 + A2xW[8]) x W[1]^2 + 4xW[8]$$

$$K3 \leftarrow (2.718^*0.333xAx(\chi^2) \div W[1]^2) x B^*1 + 2xW[8]$$

$$K4 \leftarrow (1.5xW[1]^2 - 4xW[8]) \div (3x(W[1]^2)x1 + 2xW[8]) - Ax B$$

$$K \leftarrow K1 + (K2 - K3) * K4$$

$$B1 \leftarrow +/D1E \uparrow W[11] x J x G x K$$

$$L \leftarrow 0.0125xW[3]x(1 - 2.718^* - 1xA) \div W[7] x (W[4]^2)xW[17]x(W[1]^4)$$

$$M \leftarrow ((\chi^6) \div C - 1) x ((6x(W[1]^2)xW[8]) + (+4xAxB) + 8xW[2]x\chi xAxW[8] \div B$$

$$C1 \leftarrow +/D1E \uparrow W[11] x L x M$$

$$N \leftarrow 0.75x(W[1]^2 + 2xW[8])xW[18] \div AxW[6]x(W[15] + 2)x(W[1]^*W[15])$$

$$D1 \leftarrow +/D1E \uparrow W[11] x N x 6x(X^*W[15] + 2)$$

$$P \leftarrow 13.030x(W[1]^2) + (W[4]^2)xW[2]xW[3]x(1 - 2.718^* - 1xA)$$

$$Q \leftarrow (\chi^*1) x (C^* - 1) x (1 - C^* - 1)$$

$$R \leftarrow ((3x(W[1]^2)xW[8]) + 2xAxB) \div 3\chi(W[1]^2)xW[8]$$

$$E1 \leftarrow (W[20] \geq D) x +/D1(1/E, W[20]) \uparrow W[11] PxQx(R^2) x (2.718^*R)^2$$

$$S \leftarrow 1.422E - 21xW[6](W[7]^2) x (W[3]^2) x (W[2]^*1.5) x ((1 - 2.718^*1xA)^2)$$

$$+ (W[4]^2) x (W[1]^4 + 2xW[8])$$

```

U*(X*5.5)*(C*0.333)*(B*W[8])*((1-C*-1)*2)
F1+/(D+E*W[11])*S*U
TE+W[14]*W[16]*X/W[9]+W[16]
V+|((C1*TE*4)+(B1*TE)-A1)+B|
Y+|D|-((B1*TE*-1.5)*V-TE)+((E1*TE*-0.5)*V*3)+F1*X*V*0.5
T0+1+TE[Y]
T1+1+V[Y]
T1+T0*X1,C*-0.667
T2+T1*X1,C*-0.667
PT+W[13]+0*X39.478*XW[2]*(3.567E8*(W[5]*2)*(W[1]*2)*XW[7]+X*TO*1.5)*X-1+H
T1+(D*P[T1[D]]),D+T1
T2+(D*P[T2[D]]),D+T2
X*0,X
CTU(+/1.5*XW[6]*((W[1],B)*W[8])*(T1+T2)*X*XW[11]+2*X*W1*2*XW[8])+PT
+39.478*XW[2]
AT1+(+/2*X*X*T1*XW[11])/W[1]*2
AT2+(+/2*X*X*T2*XW[11])/W[1]*2
CU*(5*(W[1]*2)*XW[4]*1.77*XW[12]*1.5)/W[2]*(XW[3]
PT+1.6E-22*PT
XE*(+/D+E*W[11]))*2*(TO*1.5)*(X*2)*X*L/M)/((E*XW[1]+100)*2)
-(W[1]+W[3])*2
XI*(+/D+E*W[11])*2*(TI*2)*(TO*-0.5)*(X*-2)*X*P*X*Q*(R*2)*(2.718*R)*2
XI*(W[20]>D)*XI*((((L/E,W[20])*XW[1]+100)*2)-(W[1]+W[3])*2
XN*(+/D+E*W[11])*2*(X*-2)*(TI*-0.5)*S*X*U)+((E*XW[1]+100)*2)-(W[1]+W[3]+W[3]*2
Z+CT,AT1,AT2,(1+T1),(1+T2),TO,T1,PT,CU,W[18],W[15],XE,XI,XN

```