A FINITE ELEMENT FORMULATION FOR FLUID-STRUCTURE INTERACTION
IN THREE-DIMENSIONAL SPACE

by

R. F. Kulak

MASTER

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Argonne National Laboratory
9860 South Cass Avenue
Argonne, Illinois 60439
Telephone (312) 998-6100
Telex 505050
Cable 505050
Post Office Box 1617
Argonne, Illinois 60439

ARGONNE NATIONAL LABORATORY, ARGONNE, ILLINOIS

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R. E. Kulak

Reactor Analysis and Safety Division
Argonne National Laboratory
Argonne, Illinois

ABSTRACT

In this paper a development is presented for a three-dimension hexahedral hydrodynamic finite-element. Using trilinear shape functions and assuming a constant pressure field in each element, simple relations were obtained for internal nodal forces. Because the formulation was based upon a rate approach it was applicable to problems involving large displacements. This element was incorporated into an existing plate-shell finite element code. Diagonal mass matrices were used and the resulting discrete equations of motion were solved using explicit temporal integrators.

Results for several problems were presented which compare numerical predictions to closed form analytical solutions. In addition, the fluid-structure interaction problem of a fluid-filled, cylindrical vessel containing internal cylinders was studied. The internal cylinders were cantilever supported from the top cover of the vessel and were periodically located circumferentially at a fixed radius. A pressurized cylindrical cavity located at the bottom of the vessel at its centerline provided the loading.

INTRODUCTION

When assessing the integrity of three-dimensional structures which either contain or are embedded in a fluid it is often times necessary to account for the fluid structure interactions which occur under transient dynamic loadings. For the class of problems in which both the structure and fluid undergo large displacements closed form analytical solutions are not generally available. The finite element method has been accepted as an analytical tool for nonlinear structural analysis and is currently gaining acceptance in the area of nonlinear fluid-structure interaction dynamics.

In this paper, a Lagrangian finite element formulation for a three-dimensional hexahedral element is presented. The general forms for the element are developed and then specialized for the case of a compressible, inviscid fluid. Because the formulation uses a velocity-strain tensor it is applicable to large deformation problems. Also, by using low order displacement fields in conjunction with one point quadrature in each element, simple relations are obtained for nodal forces that in fact parallel Wilkin's finite difference equations [1]. However, the finite element format enhances the versatility of the method because of the ease of

coupling various types of elements, particularly fluid and structural elements. The fluid element developed here is an eight-node hexahedron. Within each element trilinear isoparametric interpolating functions are used to describe the unknown fields. Mass transfer between elements is not permitted in the present formulation; thus the element is of the Lagrangian type. The element is coupled to a three-dimensional plate-shell element for fluid-structure interaction calculations.

The semi-discrete equations of motion are numerically integrated with an explicit temporal integrator. Because the integration of these equations for undamped systems produces spurious oscillations, artificial viscosities are used here to produce numerical damping when needed.

In the last section, a problem set is presented. These include some comparisons between current numerical predictions and closed form analytical results or other numerical results. In addition, the fluid-structure interaction dynamics of a fluid filled, cylindrical vessel containing internal cylinders is studied. The internal cylinders are cantilever supported from the top cover of the vessel and are periodically located circumferentially at a fixed radius. A pressurized cylindrical cavity located at the bottom of the vessel at its centerline provides the loading.

GOVERNING EQUATIONS

In this section, the finite-element equations for a single element are derived based upon the principle of conservation of mechanical energy. The resulting form can be used to represent a Lagrangian hydrodynamic element. The principle of conservation of mechanical energy for an element is given in integral form as

$$\frac{d}{dt} \int_V \frac{1}{2} \rho v_i v_i dV + \int_V \sigma_{ij} \dot{\epsilon}_{ij} dV - \int_S T_i v_i dS - \int_V \rho F_i v_i dV = 0 \quad (1)$$

where

ρ = density
 v_i, T_i, F_i = components of velocity, surface traction, and body force (per unit mass), respectively
 $\sigma_{ij}, \dot{\epsilon}_{ij}$ = components of Cauchy stress and velocity strain tensors, respectively

The velocity strain tensor (rate-of-deformation tensor) is defined by

$$\dot{\epsilon}_{ij} = \frac{1}{2} (v_{i,j} + v_{j,i}) \quad (2)$$

where the comma denotes a spatial derivative and the superposed dot a temporal derivative. It should be noted that $\dot{\epsilon}_{ij}$ is not a strain rate. The first two terms of equation (1) represent the time rate of change of the kinetic and internal energies of the element, respectively; the last two terms are the rates of work done on the element. Equation (1) is valid for any type of material, and there is no restriction on the size of the deformation gradients. Both the stress and velocity strain tensors can be divided into hydrostatic and deviatoric components

$$\sigma_{ij} = S_{ij} - p \delta_{ij} \quad (3)$$

$$\dot{\epsilon}_{ij} = \dot{e}_{ij} + 1/3 \dot{\epsilon}_{kk} \delta_{ij} \quad (4)$$

where

- $S_{ij}, \dot{\epsilon}_{ij}$ - deviatoric components of the stress and velocity strain tensors, respectively
 p, ϵ_{kk} - hydrostatic pressure and cubical dilatation, respectively

Using the above relations in equation (1) we have

$$\begin{aligned} \frac{d}{dt} \int_V \frac{1}{2} \rho v_i v_i dV - \int_V p \dot{\epsilon}_{kk} dV + \int_V S_{ij} \dot{\epsilon}_{ij} dV \\ - \int_S T_i v_i dS - \int_V \rho F_i v_i dV = 0 \end{aligned} \quad (5)$$

The second integral on the left-hand side of equation (5) represents a contribution due to the hydrostatic behavior of the material, and the third integral represents a contribution due to the deviatoric behavior of the material. Thus, for an inviscid fluid, the third integral is not calculated, but for a viscous fluid both integrals must be evaluated.

In the finite-element method, the velocity field for each element is approximated by

$$v_i = \phi_I \dot{d}_{iI} \quad (6)$$

where v_i is the velocity in the i th direction, ϕ_I is the interpolation function for the I th node, and $\dot{d}_{iI}$ is the velocity of the I th node in the i th direction. Using the above approximation for the velocity in the conservation of mechanical energy expression equation (5) we have

$$\begin{aligned} \frac{d}{dt} \int_V \frac{1}{2} \rho \dot{d}_{kI} \phi_I \phi_J \dot{d}_{kJ} dV + \int_V p \phi_{I,k} \dot{d}_{kI} dV \\ + \int_V S_{ij} G_{IIJk} \dot{d}_{kI} dV - \int_S T_k \phi_I \dot{d}_{kI} dS \\ - \int_V \rho F_k \phi_I \dot{d}_{kI} dV = 0 \end{aligned} \quad (7)$$

where

$$G_{IIJk} = \frac{1}{2} (\phi_{I,j} \delta_{ik} + \phi_{I,i} \delta_{jk}) - \frac{1}{3} \phi_{I,k} \delta_{ij} \quad (8)$$

and δ_{ij} the Kronecker delta. Now taking the indicated time derivatives we obtain for a single finite element

$$\dot{d}_{kI} (\omega_{IJ} \dot{d}_{kJ} + f_{kI}^{\text{int},h} + f_{kI}^{\text{int},d} - f_{kI}^T - f_{kI}^b) = 0 \quad (9)$$

where the element's consistent mass is given by

$$m_{IJ} = \int_V \rho \phi_I \phi_J dV \quad (10)$$

the hydrostatic internal nodal force matrix by

$$f_{kI}^{int,h} = \int_V p \phi_{I,k} dV \quad (11)$$

the deviatoric internal nodal force matrix by

$$f_{kI}^{int,d} = \int_V S_{IJ} C_{IIJJk} dV \quad (12)$$

and the external nodal load matrices by

$$f_{kI}^T = \int_S T_{k,I} dS \quad (13)$$

and

$$f_{kI}^b = \int_V \rho F_k \phi_I dV \quad (14)$$

Since equation (9) must be valid for arbitrary motions, it must be true for arbitrary d_{kI} ; thus, the term in parentheses must vanish. The general equation of motion of a finite-element is given by

$$m_{IJ} \ddot{d}_{kJ} + f_{kI}^{int,h} + f_{kI}^{int,d} - f_{kI}^T - f_{kI}^b = 0 \quad (15)$$

The formulation of this equation differs from the usual approach because the internal nodal forces are divided into hydrostatic and deviatoric parts. Equation 15 is the equation of motion of a single finite element; however, the global form of the equations of motion for the entire structure can be obtained by adding, in a consistent manner, the equations of motion for all the elements. Since this procedure is well known, it is not presented here.

We will now develop the specific form of the internal nodal force matrix for a hydrodynamic element. The element is an 8-node hexahedron (see Fig. 1). This

Fig.1 Eight-Node Hexahedron. (a) Arbitrary Hexahedron in Global (x,y,z) Coordinate System. (b) Hexahedron in Natural (ξ,η,ζ) Coordinate System.

element has a node located at each of its eight corners. The element interpolation functions ϕ_I used here are the trilinear functions described by Ergatoudis et al. [2]:

$$\phi_I = \frac{1}{8} (1 + \xi \xi_I) (1 + \eta \eta_I) (1 + \zeta \zeta_I) \quad (16)$$

where ξ , η , and ζ are the natural coordinates of the element (Fig. 1). For example, the shape function for node 1 is

$$\phi_I = \frac{1}{8} (1 + \xi) (1 + \eta) (1 - \zeta) \quad (17)$$

The method for computing the hydrostatic internal nodal forces proposed by Kennedy and Belytschko [3] will be followed. From equations (9) and (11) the following relation for the rate-of-internal work for an element is given by

$$- \int_V p \dot{\epsilon}_{kk} dV = \dot{d}_{kI} f_{kI}^{int,h} \quad (18)$$

Now we treat the pressure field within each element as a constant which reduces the integral in equation (18) to the following form

$$\int_V p \dot{\epsilon}_{kk} dV = p \int_V \dot{\epsilon}_{kk} dV = p \dot{V} \quad (19)$$

where $\dot{V}$ is the rate-of-volume change. The rate-of-volume change can be expressed in linear form by

$$\dot{V} = H_{kI} \dot{d}_{kI} \quad (20)$$

where H_{kI} depends only upon the nodal coordinates. From equations (18) to (20) we obtain the following expression for the hydrostatic internal nodal forces

$$f_{kI}^{int,h} = p H_{kI} \quad (21)$$

We will obtain the form for these forces by determining H_{kI} from equation (20) as follows: first, the expression for the volume of a generic element is determined and second, the temporal derivative of the volume gives the form for H_{kI} . Because of the difficulty in computing the volume of an element arbitrarily oriented in the global coordinate system the volume is computed in the natural coordinate (ξ, η, ζ) system with a one point Gaussian quadrature rule

$$V = \int_V dx dy dz = \int_{\Omega} J d\xi d\eta d\zeta = WJ(0) \quad (22)$$

in which J is the Jacobian determinant of the isoparametric transformation and W is a weighting factor. The above procedure gives the following form for the hydrostatic internal nodal forces

$$f_{iI}^{int,h} = \frac{p}{16} \left[X_{jPJ} (X_{kKM} - X_{kNL}) - X_{jKM} (X_{kPJ} - X_{kNL}) + X_{jNL} (X_{kPJ} - X_{kKM}) \right] \quad (23)$$

where, for example, X_{jPJ} is defined to be

$$X_{jPJ} \equiv X_{jP} - X_{jJ} \quad (24)$$

Here X_{jp} is the j th component of the coordinate of node P. The appropriate node numbers are given in the permutation table (Table 1).

Table 1. Permutation Table

I	J	K	L	M	N	P
1	2	3	4	5	6	8
2	3	4	1	6	7	5
3	4	1	2	7	8	6
4	1	2	3	8	5	7
5	8	7	6	1	4	2
6	5	8	7	2	1	3
7	6	5	8	3	2	4
8	7	6	5	4	3	1

The state of stress in a deforming fluid is given by

$$\sigma_{ij} = -p\delta_{ij} + f_{ij}(\dot{\epsilon}_{ij}) \quad (25)$$

where p is the thermodynamic pressure and f_{ij} is a tensor valued function. The thermodynamic pressure is determined by the kinetic equation of state which is of the form

$$p = p(\rho, T) \quad (26)$$

where ρ is the density and T is the absolute temperature. For the class of problems we are interested in we can consider the fluid to be isotropic. Furthermore, we are interested in problems in which the wave propagation aspect dominates so we can neglect the viscous behavior of the fluid. Thus, the stress state becomes isotropic and it is given by

$$\sigma_{ij} = -p\delta_{ij} \quad (27)$$

Now the equation of state equation (26) for our frictionless fluid reduces to the following form

$$p = p(\rho) \quad (28)$$

The explicit form used in the code is

$$p = K \frac{\Delta\rho}{\rho_0} \quad (29)$$

where K is the bulk modulus of the fluid and the naught subscript refers to the original state. From the equation of mass conservation for a Lagrangian element we have

$$\rho V = \rho_0 V_0 \quad (30)$$

In terms of the element volume, the pressure is given by

$$p = K \left(\frac{V_0}{V} - 1 \right) \quad (31)$$

The above form of the equation of state for a fluid element is coded in the program. Other forms, when needed, can be readily incorporated.

ARTIFICIAL VISCOUS STRESSES

The numerical solution of certain transient problems result in high frequency oscillation which are not present in the analytical solution. These spurious oscillations are due to frequency cut off of a discrete mesh and can be eliminated by viscosities. In this report we are using an explicit central difference method which has no artificial viscosities so that spurious oscillations with a frequency equal to the highest mesh frequency are produced. Also, our discretized equations of motion do not contain any type of damping. Therefore, in order to reduce or eliminate spurious oscillations when they occur we will follow a two step approach and introduce artificial viscous stresses, a form of artificial viscosity.

Belytschko et al. [4] presented the following form for a viscous stress to damp out the highest mode in a 2D element

$$\dot{\sigma}_{ij}^v = d_L \rho \sqrt{A} \left[(C_1 - C_2) \dot{\epsilon}_{kk} \delta_{ij} + 2C_2 \dot{\epsilon}_{ij} \right] \quad (32)$$

where C_1 and C_2 are the dilatational and shear wave speeds, respectively, A the area of the element, ϵ_{ij} the strain and d_L a fraction of critical damping for the highest mode. Here we will use a similar form for the 3D hexahedral element. Since our stresses are separated into hydrostatic and deviatoric components, we will also separate the viscous stresses into these components. Specifically, the forms used for the hydrostatic and viscous stresses are given respectively by

$$p^v = -\frac{1}{3} \sigma_{ii}^v = -d_L \rho h \left(C_1 - \frac{1}{3} C_2 \right) \dot{\epsilon}_{ii} \quad (33)$$

and

$$S_{ij}^v = 2d_L \rho h C_2 \dot{\epsilon}_{ij} \quad (34)$$

where h is a characteristic length and the remaining symbols retain their previous definitions. The characteristic length here was taken to be the length of the smallest side of the hexahedron. The deviatoric viscous stress equation (34) is linearly related to the velocity strain which is an objective tensor. Therefore, the deviatoric viscous stress is also objective (i.e. frame-indifferent) and it can be used for large deformation problems. Similarly, since the viscous pressure is a scalar quantity, it is also frame-indifferent and thus applicable to large deformation problems.

The above viscous stresses are added to the total stresses as determined from the constitutive relations. Thus, for problems in which viscous stresses are used the internal forces equations (11) and (12) are given by

$$f_{kI}^{int,h} = \int_V (p + p^v) \phi_{I,k} dV \quad (35)$$

and

$$f_{kI}^{int,d} = \int_V (S_{ij} + S_{ij}^v) G_{Iijk} dV \quad (36)$$

With the above approach it is seen that the inclusion of viscous stress, in effect, adds another set of internal forces to the equations of motion.

In addition to the above type of viscosity there is need for another type called antihourglass viscosity. The need for this viscosity arises because a one point quadrature formula was used in the nodal force calculation which produces zero nodal forces for deformation modes in which there is no volume change. This results in a rapid deterioration of the finite element mesh.

Several schemes have been developed to control this unwanted response. One of the simpler schemes was developed by Chan [5] for the two-dimensional case and extended by Pracht and Brackbill [6] to the three-dimensional case. This is the form we have incorporated in our computations. The antihourglass nodal force for the i th node in the i th direction is given by

$$f_{ii}^a = \frac{\rho V C}{24 \Delta t^2} \left[2 (\dot{d}_{ij} + \dot{d}_{im} + \dot{d}_{il}) - (\dot{d}_{in} + \dot{d}_{ip} + \dot{d}_{ik}) - 3 \dot{d}_{ii} \right] \quad (37)$$

in which ρ and V are the current density and volume, respectively, of the element, C is a constant with a recommended value of 20 and Δt is the computational time step. Thus, it is seen that the above approach simply leads to another set of internal nodal forces which are added to those previously developed.

PLATE ELEMENT

A previously developed triangular plate shell element [7] is used for structural representation. A brief description of this element is presented here for clarity. The plate element is formulated using a rotational coordinate system which enables the use of single strain-nodal displacement and nodal stress-force relations. This plate element can be subjected to linear in-plane and cubic transverse displacements. The orientation of lumped masses is described by unit vectors so that arbitrarily large rotations can be treated. The material response is taken to be elastoplastic. The integration of the elastoplastic constitutive equation employs a subincrementation procedure and a yield-surface return scheme so that the end-of-step stress lies on the yield surface.

SOLUTION OF EQUATIONS

Equation 10 defines the consistent mass matrix for finite elements; it is a non-diagonal matrix. When the consistent mass matrix is used in the equations of motion they become a set of coupled equations whose solution requires matrix inversion. In order to circumvent matrix inversion we will use a diagonal matrix which is formed by lumping the masses at the nodes. Now the equations become uncoupled and their solution is quite efficient. It should be noted that the plate element which is being used is also formulated with a diagonal mass matrix. In support of the use of a diagonal mass matrix the numerical studies of Freig and Key [8] showed that the combination of a diagonal mass matrix and an explicit temporal integrator, which we are using, leads to accurate and economical solutions.

For convenience the semi-discretized equations (10) of motion are recast into the following form

$$m_{IJ} \ddot{d}_{jJ}(t) = f_{jI}^c(t) - f_{jI}^i(t) \quad (38)$$

in which f_{jI}^c is the sum of all the external forces acting on node I in the j th direction and f_{jI}^i is the sum of all the internal forces. It should be noted that equation (38) is a set of uncoupled equations.

The Newmark- β temporal integrator [9] is used to integrate equation (38). By using an explicit form ($\beta = 0$) of the integrator the nodal displacements and velocities are updated according to the formulas

$$d_{iI}(t) = d_{iI}(t - \Delta t) + \dot{d}_{iI}(t - \Delta t) \Delta t + 0.5 \ddot{d}_{iI}(t - \Delta t) \Delta t^2 \quad (39)$$

and

$$\dot{d}_{iI}(t) = \dot{d}_{iI}(t - \Delta t) + 0.5 [\ddot{d}_{iI}(t) + \ddot{d}_{iI}(t - \Delta t)] \Delta t \quad (40)$$

For each time step the computations proceed in the following manner. First, the external and internal nodal forces are computed at time t . Since the internal

forces depend on the displacement field at time t , the nodal displacements are computed by equation (34). Next, the nodal accelerations are computed by equation (35). Finally, the nodal velocities are determined from equation (36). The above computation cycle continues until the final problem time is reached.

RESULTS

In this section the solution to several problems are presented. Some of the results have been obtained during the verification phase of the program and others have been run to demonstrate some of the analysis capabilities of the present program.

The first problem is one of wave propagation. The physical problem considered was that of a pressure wave emanating from a pressurized spherical cavity. The cavity radius, r_1 , is 10 cm and the fluid medium, r_2 , at the outer boundary is 20 cm (see Fig. 2a). The finite-element model, shown in Fig. 2b, consists of three-dimensional fluid elements. The elements were defined by the nodes 1 and 48; both of these nodes were fixed to 0. It should be noted that since this problem is reduced to a one-dimensional problem in the radial direction, it is a three-dimensional problem in Cartesian coordinates. All of the nodes in the finite element mesh underwent motion in the x , y , and z directions. The pressure attenuation ability of the element was used in this problem as a measure of the accuracy for that element. For the spherical-wave problem, the pressure attenuates inversely as its radial distance, that is,

$$\frac{P_2}{P_1} = \frac{r_1}{r_2} \quad (41)$$

Fig.2 Pressure Wave Emanating From a Pressurized Spherical Cavity. (a) A Pressurized Spherical Cavity in a Fluid Medium. (b) Finite Element Model.

Fig.3 Comparison Between Analytical and Computed Results for Pressure Attenuations in a Spherical Configuration. Calculations Were Performed Using Artificial Viscosity.

A comparison between analytically predicted and computed pressure attenuations at various radial locations (see Fig. 3) gave good agreement.

The second problem studied was a compression wave traveling through a fluid contained within an infinite cylinder (see Fig. 4) with an outer radius of 40 cm. A cylindrical cavity of 20 cm radius was located in the center of the fluid cylinder. At time zero, a step pressure was applied and sustained at the inner fluid interface. Analytically, this problem reduces to a one dimensional wave

Fig.4 Compression Wave Propagation Within an Infinite Fluid Cylinder. Cylindrically Contained Fluid With an Internal Pressurized Cavity. (b) Finite Element Model.

propagation problem in cylindrical coordinates. However, in three-dimensional Cartesian space the problem was two-dimensional, that is, motion occurred in the x and y directions. The finite-element model for this problem is shown in Fig. 4b. The model consisted of 136 fluid elements with a height of 0.01 to 1 cm. The cavity radius was 1.7 cm and the outer fluid boundary radius was 4 cm. The fluid model was taken to be a 400- μ s pulse = 0.01 of the potential function. The measure used to evaluate the accuracy of the fluid element response to the prescribed loading was the geometric pressure attenuation. From the analytical solution it was known that the fluid pressure attenuated inversely as the square root of the radial distance, that is,

$$\frac{P_2}{P_1} = \sqrt{\frac{r_1}{r_2}} \quad (42)$$

where P_1 is the fluid pressure at radius r_1 . The computer results for the fluid pressure are compared at several locations with the analytical solution in Fig. 5. The computer-predicted pressures at various radial locations compared well with the analytical predictions.

Fig. 5 Comparison Between Analytical and Computed Results for Pressure Attenuations in a Cylindrical Configuration. Calculations Were Performed Using Artificial Viscosity.

The combined fluid-structural problem presented here is the dynamic response of two, finite-length, concentric cylinders (see Fig. 6a) separated by a sodium layer, and loaded by a pressure pulse acting over part of the inner cylinder. Both cylinders were 30.48 cm long and 1.016 cm thick; the nominal radius of the inner cylinder was 5.08 cm. A layer of sodium 2.54 mm thick separated the cylinders. A 100- μ s triangular pulse (see Fig. 6b) with a peak pressure of 20.7 MPa (at 50 μ s) was applied to a central 15.24-cm-long region around the inner circumference of the inner cylinder. Because of symmetry in both loading and geometry, the finite-element model (see Fig. 7) for this problem consisted of a 10° sector 15.24 cm high. Twenty-four (24) triangular plate elements were used to model the cylinders (12 for the inner and 12 for the outer) and six (6) fluid-hexahedral elements for the sodium layer. The material properties are given in Table 2.

Fig.6 (a) Concentric Cylinders Separated by a Sodium Layer. (b) Applied Pressure Pulse.

Table 2. Material Properties

	Cylinders	Fluid (sodium)
Density (g/cm^3)	7.8	0.826
Young's Modulus (GPa)	152	--
Bulk Modulus (GPa)	--	4.05
Yield Stress (MPa)	517	--
Tangent Modulus (MPa)	414	--

The numerically predicted response was compared with the results obtained with the WHAM [10] code. The radial displacement history of the inner cylinder at the line of symmetry is shown in Fig. 8. The results from both computational methods compare reasonably well in displacement amplitude, response frequency, and amplitude variation.

As a demonstrative problem, consider the fluid-structure interaction of a fluid-filled cylindrical vessel containing internal cylinders (Fig. 9). The internal cylinders are cantilever supported from the top cover of the vessel and are periodically located 90° apart circumferentially at a fixed radius. A pressurized cylindrical cavity located at the bottom of the vessel at its centerline provides the loading. To study this complex problem, we developed a simple model which has many of the salient features of this fluid-structural component system. To begin, we modeled the vessel and inner cylinders as deformable elastoplastic structures made of 304 SS, the fluid as inviscid and compressible. Both the top and bottom of the vessel are considered to be rigid. Our model is developed by assuming a 45° repeated symmetry of the system. Therefore a model (Fig. 10) which includes a 45° sector of the fluid pool, one-half of an in-tank component and a 45° sector of the tank is sufficient. Because of these symmetries we can define the symmetry planes shown in Fig. 10. Symmetry plane S1 is the plane OACED which originates along the axial centerline of the tank and passes through the axial centerline of the in-tank component. The symmetry plane S2 OBDEO also originates along the tank's axial centerline but it passes halfway between adjacent in-tank components.

The finite-element mesh for our model is shown in Fig. 11. The entire mesh (Fig. 11) consists of 122 triangular plate/shell elements and 337 hydro-dynamic elements. The in-tank component model consists of 38 triangular plate elements which simulates the entire length but only one-half of the circumference of an in-tank component. The boundary conditions at the top are zero displacements and rotations. The nodes which lie in the symmetry plane S1 are confined to motion

Fig.7 Finite Element Model for Two Concentric Cylinders Separated by a Sodium Layer.

Fig.8 Comparison Between Computed Results and WHAM [10] Results for the Radial Displacement of the Inner Cylinder at the Line of Symmetry.

Fig.9 Fluid-Filled Cylindrical Vessel Containing Internal Cylinders. (a) Elevation View. (b) Plan View.

only in that plane. The remaining nodes of the component model are free to move arbitrarily in three-dimensional space. The model for the primary tank consists of 84 plane elements. The boundary conditions at the top and bottom of the tank are zero displacement and rotations. The nodes which are connected entirely by fluid elements are free to move arbitrarily in three-dimensional space with the exception of those which are restricted to motion in a symmetry plane.

For our preliminary study a pressure-volume relationship was applied to the pressurized cavity shown in Fig. 9. The resulting displacement history for the in-tank component was obtained. The radial displacement time history of the bottom of the in-tank component is given in Fig. 12 and shows a maximum displacement of 23.4 cm at 41 ms after the initiation of cavity pressurization. At this time the clearance between the in-tank component and the tank is 94 cm. Figure 13 shows the deformed mesh at the peak displacement time (41 ms).

Fig.10 Model of In-Tank Component, Sodium Pool and Primary Tank.

Fig.11 Elevation View of Finite Element Mesh. For the Purpose of Clarity Only Part of the Mesh is Shown.

Fig.12 Radial Displacement Histories of the Bottom of the In-Tank Component and Primary Tank.

Fig.13 Deformed Mesh at the Peak Displacement Time (41 ms).

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