OCTOBER 1979

PPPL 1597
UC-20g

THE MAGNETIC DRIVING ENERGY OF THE TEARING MODE

BY

E. A. ADLER, R. M. KULSRUD, AND R. B. WHITE

PLASMA PHYSICS LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

This work was supported by the U. S. Department of Energy Contract No. EY-76-C-02-3073.

The Magnetic Driving Energy of the Tearing Mode

E. A. Adler,* R. M. Kulsrud, and R. B. White
Plasma Physics Laboratory, Princeton University
Princeton, New Jersey 08544

ABSTRACT

The change in the magnetic energy density produced by a tearing mode is calculated exactly. The driving energy for the mode is found to come entirely from the region inside the tearing layer, although there is also a displacement of energy in the outer region which integrates to zero. The total change in magnetic energy is exactly equal to the change in a quadratic form related to a variational principle for the full resistive equations.

I. Introduction

The tearing mode plays an important role in the history of a plasma discharge in present-day tokamaks. The associated change in the magnetic energy density has long been understood to be the driving mechanism of the mode in its linear[1] and nonlinear[2] behavior, although this change has not been calculated previously except in the external, ideally conducting region. In this work we evaluate the magnetic energy density change exactly. The integrated change in energy is shown to be equal to the change in a quadratic form derived by Furth,[1] related to a variational principle for the finite resistivity analysis. In Section II we review the linear theory of the tearing mode, and in Section III the associated magnetic energy is calculated. Section IV consists of an explicit calculation of the equation of energy conservation for the mode.

II. Linear Tearing Mode Theory

The theory of resistive instabilities for an infinite plane current layer has been given in detail previously.[3] We briefly review here the analysis of the tearing mode.

The initial field is taken to be of the form

$$B_{yo} = \bar{B}F(x)/L$$

$$B_z >> \bar{B} \qquad (1)$$

$$B_{xo} = 0$$

where B_z is constant in space and $F(x) = \vec{k}\cdot\vec{B}_o/kB_o \simeq x$ for $x << L$, L being the shear length of the initial field (thickness of the current layer) and $F(x)$ is odd in x with $F(x) \to 1$ for $x >> L$. It is convenient to use $\vec{\nabla}\cdot\vec{B} = 0$ to introduce a flux function $\psi(x, y)$ with

$$\vec{B} = \vec{\nabla}\psi \times \hat{z} + B_z\hat{z} \tag{2}$$

satisfying $\vec{B}\cdot\vec{\nabla}\psi = 0$. From Maxwell's equation $\nabla \times \vec{B} = (4\pi/c)\vec{J}$ we find

$$\nabla^2\psi = -(4\pi/c)\ J_z\ . \tag{3}$$

Ohm's law is assumed to have a simple form;

$$\vec{E} + \frac{\vec{u} \times \vec{B}}{c} = \eta\vec{J}\ . \tag{4}$$

Taking the $\hat{z}$ component and using the Maxwell equation $\vec{\nabla} \times \vec{E} = -(\partial\vec{B}/\partial t)/c$ we find for ψ

$$\frac{\partial\psi}{\partial t} + (\vec{u}\cdot\vec{\nabla})\psi = \frac{\eta c^2\nabla^2}{4\pi}\ \psi\ . \tag{5}$$

For the velocity, use

$$\rho\ \frac{\partial\vec{u}}{\partial t} = \frac{\vec{J} \times \vec{B}}{c} - \nabla p\ . \tag{6}$$

and operate on this equation with $\hat{z}\cdot\nabla\times$, also using the incompressibility of the plasma in the x - y plane (large B_z) to introduce the velocity stream function ϕ with $\vec{u} = \vec{\nabla}\phi \times \hat{z}$. This gives

$$\rho \frac{\partial}{\partial t} \nabla^2\phi = -\frac{1}{4\pi} \hat{z}\cdot[\nabla\psi \times \nabla(\nabla^2\psi)] \ . \tag{7}$$

The two coupled second order partial differential equations, Eqs. (5) and (7), are the starting point of the analysis of the tearing mode. Linearizing these equations in terms of $\psi_1(x)$, $\phi_1(x)$, where

$$\begin{aligned} \psi(x, y) &= \psi_o(x) + \psi_1(x) \cos ky \\ \phi(x, y) &= (\gamma/k\bar{B})\ \phi_1(x) \sin ky \end{aligned} \tag{8}$$

and $\psi_1(x)$ and $\phi_1(x)$ are assumed to vary in time as $\exp(\gamma t)$, we find

$$\psi_1(x) - \phi_1(x)F(x) = (\gamma\tau_R)^{-1}[\psi_1''(x) - k^2\psi_1(x)] \tag{9}$$

$$-\gamma^2\tau_A^2[\phi_1''(x) - k^2\phi_1(x)] = F(x)[\psi_1''(x) - k^2\psi_1(x)] - F''(x)\psi_1(x) \tag{10}$$

where the primes denote d/dx and $\tau_A = (4\pi\rho)^{1/2}/(k\bar{B})$, $\tau_R = (4\pi L^2/\eta c^2)$ are the characteristic Alfvén and resistive times for the problem.

These equations are solved using a boundary layer analysis,[3] the resistivity being negligible except in the interior region $x < x_T$, where the tearing layer width is given by

$$x_T \sim (\gamma \rho \eta c^2)^{1/4}/(k\bar{B})^{1/2} \; . \tag{11}$$

In the exterior region the solution is given by $\phi_1(x) = \psi_1(x)/F(x)$, and $\psi_1(x)$ satisfies

$$\psi_1'' - k^2\psi_1 - (F''/F)\psi_1 = 0 \; . \tag{12}$$

The external solution ψ_1 has a discontinuous derivative at $x = 0$, and the quantity $\Delta' = [\psi_1'(0+) - \psi_1'(0-)/\psi_1(0)]$ characterizes the current profile as far as linear tearing mode behavior is concerned.

In the interior region $F(x) \simeq x$, and it is assumed that $\Delta' x_T << 1$ which means that $\psi_1(x)$ is approximately constant (the "constant ψ" approximation), giving

$$\gamma^2 \tau_A^2 \phi_1''(x) = -(x/L)\psi_1''(x) \quad , \tag{13}$$

$$\psi_1(0) - (x/L)\phi_1(x) = [\psi_1''(x)/\gamma\tau_R] \quad , \tag{14}$$

which upon substituting $(x/L) = (\gamma\tau_A^2/\tau_R)^{1/4} z$ and $\phi_1 = -(\tau_R/\gamma\tau_A^2)^{1/4}$ $\psi_1(0)\ X(z)$ leads to the equation $X'' - z^2 X = z$ which can be solved in closed form.[4] From Eq. (13) we obtain the condition necessary for matching to the exterior solution

$$\frac{\gamma^2\tau_A^2 L}{\psi_1(0)} \int_{-x_T}^{x_T} \frac{\phi_1''(x)}{x}\, dx = -\Delta' \quad . \tag{15}$$

This equation determines the growth rate; solution of Eqs. (13) and (14) in the interior region giving[3]

$$\gamma = \left[\frac{2}{\pi}\,\frac{\Gamma(5/4)\Delta' L}{\Gamma(3/4)}\right]^{4/5} \tau_R^{-3/5}\tau_A^{-2/5} \quad . \tag{16}$$

In the following we will make x_T precise by defining it arbitrarily through

$$x_T = L^2\Delta'(\gamma\tau_R)^{-1} \quad . \tag{17}$$

At this point, $\phi_1'(x)$ is approximately zero and thus u_x reaches its peak value, and u_y changes sign. As we will see in the next section, x_T is also a natural boundary point with regard to the magnetic energy density.

III. Magnetic field energy

In this section we calculate the magnetic energy released during island formation. In terms of the notation of Section II the energy $M = (1/8\pi) \int \delta(\vec{B}^2)\, dxdy$ is given correctly to second order by

$$M = \frac{1}{16\pi} \int dx \, \{[\psi_1'(x)]^2 + k^2\psi_1^2(x) - 4F\psi_{20}'(x)\} \quad , \qquad (18)$$

where $\psi(x, y) = \psi_o(x) + \psi_1(x)\cos ky + \psi_{20}(x) + \psi_{22}(x)\cos 2ky$ and terms linear in $\psi_1(x)$ and $\psi_{22}(x)$ have vanished due to the integration over y. We now determine the second order y independent part of $\psi(x, y)$, $\psi_{20}(x)$.

Averaging the second order part of Eq. (5) over y we find

$$2\psi_{20}(x) + \frac{1}{2}\frac{d}{dx}\left(\frac{\psi_1(x)\phi_1(x)}{\bar{B}}\right) = (\gamma\tau_R)^{-1}\frac{d^2\psi_{20}}{dx^2} \quad , \qquad (19)$$

where we have taken η = constant for simplicity. The sclution to Eq. (19) is

$$\psi_{20}(x) = -\frac{1}{2\alpha}\int_{-\infty}^{\infty} dx' \, e^{-\alpha|x-x'|}\,\frac{\partial}{\partial x'}K(x') \quad , \qquad (20)$$

where $K(x) = (\gamma\tau_R/2\bar{B})\,\psi_1(x)\,\phi_1(x)$, $\alpha = (2\gamma\tau_R)^{1/2}/L$. The asymptotic form of Eq. (20) is readily evaluated. The contribution to the integral is small provided $|x - x'| > \delta$, where $\delta \gg (x_T/\Delta')^{1/2}$ and thus for $x \gg \delta$ we can write

$$\psi_{20}(x) \approx -(1/2\alpha)\int_{x-\delta}^{x+\delta} dx' \, \exp(-\alpha|x - x'|)(\partial/\partial x')\,K(x') \ .$$

Using the fact that $K(x')$ is slowly varying in the integration range, ($\delta << 1$) we find $\psi_{20}(x) = -(1/\alpha^2)K'(x)(1 - e^{-\alpha\delta})$, which upon substituting the external solution for $\phi_1(x)$ and using $\alpha\delta >> 1$ gives

$$\psi_{20}(x) \approx -\frac{1}{4}\frac{d}{dx}\left(\frac{\psi_1^2(x)}{\bar{B}F(x)}\right) , \tag{21}$$

which is also directly obtained form Eq. (19) by neglecting the resistivity. Note that, unlike the exterior solutions for $\psi_1(x)$, $\phi_1(x)$, this expression is only valid for $x >> (x_T/\Delta')^{1/2}$ and cannot be used in the vicinity of $x = x_T$. The validity of Eq. (21) only when x is large compared to the effective skin depth of the current sheet, $(x_T/\Delta')^{1/2}$, reflects the fact that for smaller scale lengths than this resistive diffusion will be important.

To evaluate the change in the magnetic energy M, first consider the last term in Eq. (18),

$$M_3 = \frac{1}{4\pi} \int_{-\infty}^{\infty} \psi_0'(x)\psi_{20}'(x)\ dx , \tag{22}$$

and split it into two parts

$$M_3 = -\frac{1}{2\pi} \int_0^{\varepsilon} \frac{\bar{B}x}{L}\psi_{20}'(x)\ dx + \frac{1}{8\pi} \int_{\varepsilon}^{\infty} dx\ F(x)\ \frac{d^2}{dx^2}\left(\frac{\psi_1^2(x)}{F(x)}\right)$$

where $(x_T/\Delta')^{1/2} << \varepsilon << L$, and we have used $F(x) \simeq x$ in the interior region and used the asymptotic expression for $\psi_{20}(x)$ in the exterior region. To evaluate the first term of Eq. (23), integrate once by parts;

$$-\frac{1}{2\pi}\int_0^{\varepsilon} x\psi_{20}'(x)\ dx = -\frac{\varepsilon}{2\pi}\psi_{20}(\varepsilon) + \frac{1}{2\pi}\int_0^{\varepsilon}\psi_{20}(x)\ dx\ . \qquad (24)$$

Integrating Eq. (19) we find

$$\int_0^{\varepsilon}\psi_{20}(x)\ dx = \frac{1}{2\gamma\tau_R}\psi_{20}'(\varepsilon) - \frac{\psi_1(\varepsilon)\phi_1(\varepsilon)}{4\bar{B}}\ . \qquad (25)$$

Now integrate the second term in M_3 by parts twice, giving as a contribution to M_3 from the region $x > \varepsilon$

$$M_{3>} = \frac{1}{8\pi}\left[F\left(\frac{\psi_1^2}{F}\right)'\Bigg|_{\varepsilon}^{\infty} - \frac{F'}{F}\psi_1^2\Bigg|_{\varepsilon}^{\infty} + \int_{\varepsilon}^{\infty}\frac{\psi_1^2}{F}F''\ dx\right]\ , \qquad (26)$$

which includes a singular term $\psi_1^2(F'/F) \sim (1/\varepsilon)$. This singular surface term is, however, canceled in the full expression for M by a similar contribution from the region $x < \varepsilon$. The lower limit of integration in the last term of Eq. (26) can be replaced by zero since F''/F is well behaved. Using $(x_T/\Delta')^{1/2} \ll \varepsilon \ll L$ we substitute $F(x) \simeq x$ and use the exterior solution for $\psi_{20}(x)$ to find

$$M = \frac{1}{16\pi}\int_{-\infty}^{\infty} dx\ \{[\psi_1'(x)]^2 + k^2\psi_1^2(x) + [F''\psi_1^2(x)/F]\} + R\ , \qquad (27)$$

and

$$R = \frac{\Delta' \psi_1^2(0)}{4\pi} \left[\frac{x_T}{2\Delta' \varepsilon^2} - \frac{x_T}{2(\Delta')^2 \varepsilon^3} - \frac{x_T}{8\varepsilon} + \frac{x_T}{\varepsilon(\Delta')^2} \right] \tag{28}$$

where we have again used the constant ψ approximation. The integral term in Eq. (27) is equal to $-[\Delta' \psi_1^2(0)/16\pi]$, and $|R| \ll \Delta' \psi_1^2(0)$ provided $\varepsilon \gg x_T^{1/3} (\Delta')^{-2/3}$. Thus

$$M = - \frac{\Delta' \psi_1^2(0)}{16\pi} \quad . \tag{29}$$

The magnetic energy density can be directly evaluated in the interior region. Differentiate Eq. (20), obtaining

$$\frac{d\psi_{20}}{dx} = - \frac{1}{2\alpha} \int_{-\infty}^{\infty} dx' \, e^{-\alpha|x-x'|} \frac{\partial^2}{\partial x'^2} K(x') \tag{30}$$

which, upon integration by parts twice gives

$$\frac{d\psi_{20}}{dx} \approx K(x) \tag{31}$$

plus terms of order αx, which are negligible provided $x < x_T$. We then find for the magnetic energy density

$$\frac{\delta(\vec{B}^2)}{8\pi} = - \frac{x\Delta'}{8\pi x_T} \psi_1(0) \phi_1(x) \quad , \tag{32}$$

which, numerically evaluating $\phi_1(x)$ in the interior region,[4] has value of $(-\Delta'/8\pi x_T)\ \psi_1^2(0)$ at $x = x_T$. The total magnetic ene change for $|x| < x_T$ is given by

$$M(x_T) = \frac{-\Delta'}{8\pi x_T}\ \psi_1(0) \int_0^{x_T} x\phi_1(x)\ dx \quad , \qquad ($$

which upon using Eqs. (13), (14), and (15), gi $M(x_T) = -\psi^2(0)\Delta'/16\pi$; i.e., the entire change in magnetic ene In Fig. 1 is shown schematically the change in magnetic ene density as a function of x, as found numerically using Eq. (18 The surface terms in the integrations for $|x| < \varepsilon$ and $|x| >$ result from the energy transfer which takes place for $|x| > x$ The total contribution from this region is, however, zero. Als shown is the divergent energy density obtained by using t exterior solution for ψ_{20} for $|x| < \varepsilon$.

IV Energy Conservation

It is instructive to evaluate the total rate of change energy inside a volume $a < x < b$ due to the change in kinetic ar magnetic energy. We show that the sum of these terms is equal the work done by the pressure at the two surfaces plus the ene radiated through the two surfaces.

The kinetic energy is given by $(1/2)\rho\vec{u}^2$, and expressing this in terms of the velocity potential we find

$$\frac{d}{dt} KE = \frac{\gamma}{8\pi} \gamma^2 \tau_A^2 \int_a^b [(\phi_1')^2 + k^2\phi_1^2]\, dx \tag{34}$$

which upon integrating the first term by parts and substituting Eq. (10) gives

$$\frac{d}{dt} KE = \frac{\gamma}{8\pi} \int_a^b F\phi[\psi'' - k^2\psi - (F''/F)\psi]\, dx \quad , \tag{35}$$

where we have neglected the surface term, which is of order $\gamma^2\tau_A^2$ smaller than our final result.

The change in the magnetic energy is, from Eq. (18),

$$\frac{d}{dt} M = \frac{\gamma}{8\pi} \int_a^b [(\psi_1')^2 + k^2\psi^2 - 4F\psi_{20}']\, dx \quad . \tag{36}$$

Integrate the first term by parts and add the kinetic and magnetic energies, giving

$$\begin{aligned} \frac{d}{dt} (KE + M) &= \frac{\gamma}{8\pi} \int_a^b (\phi_1 F - \psi_1)(\psi_1'' - k^2\psi_1)\, dx \\ &- \frac{\gamma}{8\pi} \int_a^b (4F\psi_{20}' + \phi_1 F''\psi_1)\, dx + \frac{\gamma}{8\pi} \psi_1\psi_1' \Big|_a^b \quad . \end{aligned} \tag{37}$$

The first term on the right-hand side of this equation is negligible. To see this, first note that for $|x| > x_T$ $\phi_1 F = \psi_1$ so the contribution is from the interior region only. Without loss of generality, we can take $0 < a < x_T < b$ since there is symmetry about zero. The two terms proportional to k^2 are readily seen to be of order x_T and thus negligible. Thus, we are left with $I = \int_a^{x_T} (\phi_1 x\psi'' - \psi_1\psi'')\, dx$. Integrate the first term by parts twice and then use the constant ψ approximation, giving $I = (x\phi_1 - \psi_1)\psi_1'\Big|_a^{x_T}$, which is negligible by the vanishing of the first factor in the exterior region and by the vanishing of ψ_1' at $x = 0$. We thus find

$$\frac{d}{dt} (KE + M) = - \frac{\gamma}{8\pi} \int_a^b (4F\psi_{20}' + \phi F''\psi)\, dx + \frac{\gamma}{8\pi} \psi\psi'\Big|_a^b \quad . \tag{38}$$

To evaluate the first term on the right-hand side, split the range of integration into two parts according to whether $x > \varepsilon$ with $\varepsilon >> x_T^{1/2}/\Delta'$, and use the exterior solution for ψ_{20} for $x > \varepsilon$. Integrating by parts then gives

$$\int_\varepsilon^b (4F\psi_{20}' + \phi F''\psi)\, dx = \left[-F \frac{d}{dx}\left(\frac{\psi_1^2}{F}\right) + F' \frac{\psi^2}{F}\right]_\varepsilon^b \tag{39}$$

In the interior region use $F \simeq x$ and integrate both terms by parts to find

$$\int_a^\varepsilon (4F\psi_{20}' + \phi F''\psi)\, dx = \left[4x\psi_{20}\right]_a^\varepsilon - \int_a^\varepsilon 4\psi_{20}\, dx \quad . \tag{40}$$

Now use Eq. (25) and discard the term $(\gamma\tau_R)^{-1}\psi_{20}'$, which is negligible, giving

$$\frac{d}{dt}\,(KE + M) = \frac{\gamma}{8\pi}\left[F\,\frac{d}{dx}\left(\frac{\psi^2}{F}\right) - \frac{F'\psi^2}{F}\right]_\varepsilon^b$$

$$+\ (\gamma/8\pi)\left[-4x\psi_{20} - \psi_1\phi_1\right]_a^\varepsilon + (\gamma/8\pi)\,\psi\psi'\Big|_a^b\ . \qquad (41)$$

Using $x^{1/2}/\Delta' \ll \varepsilon \ll 1$ we note that the terms evaluated at ε vanish as they must, giving

$$\frac{d}{dt}\,(KE + M) = (\gamma/8\pi)\left[-4F\psi_{20} - F'\psi_1\phi_1 + \psi\psi'\right]_a^b\ . \qquad (42)$$

There are two contributions to the energy balance at the surface, one due to the Poynting flux and one due to the work done by the pressure. To evaluate the Poynting flux use

$$P_x = (c/4\pi)(E_1 \times B_1)_x + (c/4\pi)(E_2 \times B_0)_x\ . \qquad (43)$$

Using Eq. (4) to evaluate E_z, and Eq. (3) to evaluate j_z we find that $E_{1z}B_{1y} = \gamma\psi_1'\psi\ \cos^2 ky$ and $E_{2z}B_{oy} = -2\gamma\psi_{20}F$, giving

$$P_x = (\gamma/8\pi)(-\psi_1\psi_1' + 4F\psi_{20})\ . \qquad (44)$$

The work done by the pressure is given at each surface by pu_x since the fluid is incompressible. To evaluate the pressure, use $\nabla p = j \times B$ to find

$$\frac{dp}{dx} = \frac{c}{4\pi} \frac{d}{dx} (F' \psi_1) \quad , \tag{45}$$

where we have used Eq. (10) to simplify $j \times B$ and have discarded a term of order $\gamma^2 \tau_A^2$ smaller. The work done is then $pu_x = (\gamma/8\pi)\phi_1 F' \psi_1$ and we thus find

$$\frac{d}{dt} \int_V (KE + M)\, d\tau = - \int_S \vec{P} \cdot ds - \int_S p\vec{u} \cdot ds \quad , \tag{46}$$

as was to be shown.

Note from Eq. (35) that the kinetic energy density is zero for $x > x_T$. We also note from Eq. (44) that the Poynting flux is outward provided $4F\psi_{20} > \psi_1 \psi_1'$.

Furth[1] derived a quadratic form which leads to a variational principle for the resistive Eqs. (9) and (10). In the exterior region it is given by

$$V_o = (1/8\pi) \int_o^\infty dx \left[(\psi_1')^2 + k^2 \psi_1^2 + (F''/F)\psi_1^2 \right]$$

which is equal to the total change in magnetic energy from Eq. (27). The resulting variational "energy" density D_v is shown in Fig. 1 and it is seen to agree with the magnetic energy density when x is comparable to the shear length. The infinite conductivity energy principle[5] .

$$W_\infty = (1/8\pi) \int_\varepsilon^\infty [(\psi_1')^2 + k^2\psi_1^2 - F' (d/dx)(\psi_1^2/F)] \qquad (48)$$

is simply related to V_o by an integration by parts. Both the resulting term and W are singular in the limit $\varepsilon \to 0$.

In conclusion, we have calculated in closed form the magnetic energy density, kinetic energy, Poynting vector, and pressure for the tearing mode in slab geometry, and explicitly verified that these terms account for all the energy of the mode. The total change in magnetic energy can be ascribed to the interior region, and all of the kinetic energy is also located in this region.

ACKNOWLEDGMENTS

This work was supported by United States Department of Energy Contract No. EY-76-C-02-3073 and by United States Air Force Office of Scientific Research Contract No. F 44620-75-C-0037.

REFERENCES

*Present address: Center for Plasma Physics and Fusion Engineering, University of California, Los Angeles, California.

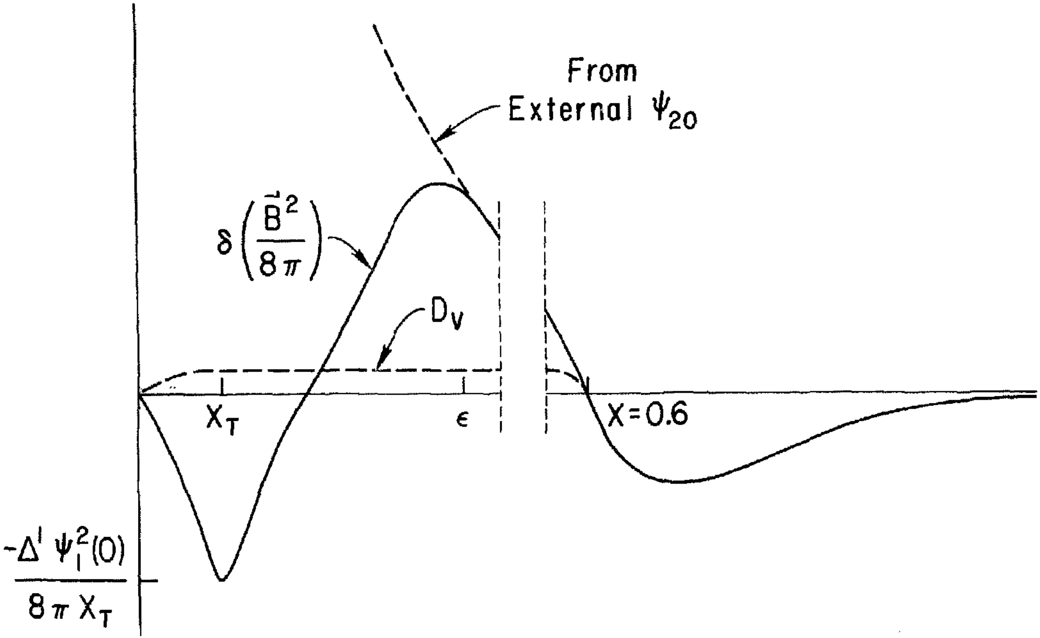
1. H. P. Furth in Propagation and Instabilities in Plasmas, edited by W. I. Futterman (Stanford University Press, Stanford, California, 1963) p. 87.

2. R. B. White, D. A. Monticello, M. N. Rosenbluth, and B. V. Waddell, Phys. Fluids 20, 800 (1977).

3. H. P. Furth, J. Killeen, and M. N. Rosenbluth, Phys. Fluids 6, 459 (1963).

4. R. D. Hazeltine, D. Dobrott, and T. S. Wang, Phys. Fluids 18, 1778 (1975).

5. I. B. Bernstein, E. A. Frieman, M. D. Kruskal, and R. M. Kulsrud, Proc. Roy. Soc. (London) A244, 17 (1958).



792422

Fig. 1. The change in magnetic energy density produced by a magnetic island. The integral of this density from x_T to ∞ is zero. Also shown is the divergent density obtained by using the asymptotic form of ψ_{20} for small x. The dotted line is the "energy" density obtained from a variational principle for the full resistive equations.

EXTERNAL DISTRIBUTION IN ADDITION TO TIC UC-20

ALL CATEGORIES

R. Askew, Auburn University, Alabama
S. T. Wu, Univ. of Alabama
Geophysical Institute, Univ. of Alaska
G.L. Johnston, Sonoma State Univ, California
H. H. Kuehl,Univ. of S. California
Institute for Energy Studies, Stanford University
H. D. Campbell, University of Florida
N. L. Oleson, University of South Florida
W. M. Stacey, Georgia Institute of Technology
Benjamin Ma, Iowa State University
Magne Kristiansen, Texas Tech. University
W. L. Wiese, Nat'l Bureau of Standards, Wash., D.C.
Australian National University, Canberra
C.N. Watson-Munro, Univ. of Sydney, Australia
F. Cap, Inst. for Theo. Physics, Austria
Ecole Royale Militaire, Bruxelles, Belgium
D. Palumbo, C. European Comm. B-1049-Brussels
P.H. Sakanaka, Instituto de Fisica, Campinas, Brazil
M.P. Bachynski, MPB Tech., Ste. Anne de Bellevue, Quebec, Canada
C. R. James, University of Alberta, Canada
T.W. Johnston, INRS-Energie, Vareenes, Quebec
H. M. Skarsgard, Univ. of Saskatchewan, Canada
Inst. of Physics, Academia Sinica, Peking, People's Republic of China
Inst. of Plasma Physics, Hefei, Anhwei Province, People's Republic of China
Library, Tsing Hua Univ., Peking, People's Republic of China
Soules Inst. of Physics, Leshan, Sechuan Province, People's Republic of China
Librarian, Culham Laboratory, Abingdon, England (2)
A.M. Dupas Library, C.E.N.-G, Grenoble, France
Central Res. Inst. for Physics, Hungary
S. R. Sharma, Univ. of Rajasthan, JAIPUR-4, India
R. Shingal, Meerut College, India
A.K. Sundaram, Phys. Res. Lab., India
M. Naraghi, Atomic Energy Org. of Iran
Biblioteca, Frascati, Italy
Biblioteca, Milano, Italy
G. Rostagni, Univ. Di Padova, Padova, Italy
Preprint Library, Inst. de Fisica, Pisa, Italy
Library, Plasma Physics Lab., Gokasho, Uji, Japan
S. Mori, Japan Atomic Energy Res. Inst.,Tokai-Mura
Research Information Center, Nagoya Univ., Japan
S. Shioda, Tokyo Inst. of Tech.,Japan
Inst. of Space & Aero. Sci., Univ. of Tokyo
T. Uchida, Univ. of Tokyo, Japan
H. Yamato, Toshiba R. & D. Center, Japan
M. Yoshikawa, JAERI, Tokai Res. Est., Japan
N. Yajima, Kyushu Univ., Japan
R. England, Univ. Nacional Auto-noma de Mexico
B. S. Liley, Univ. of Waikato, New Zealand
S. A. Moss, Saab Univas Norge, Norway
J.A.C. Cabral, Univ. de Lisboa, Portugal
O. Petrus, AL.I. CUZA Univ., Romania
J. de Villiers, Atomic Energy Bd., South Africa
A. Maurech, Comisaria De La Energy y Recoursos Minerales, Spain
Library, Royal Institute of Technology, Sweden
Cen. de Res. En Phys.Des Plasmas,Switzerland
Librarian, Fom-Instituut Voor Plasma-Fysica, The Netherlands
V. E. Golant, A.F. Ioffe Physical-Tech. Inst.,USSR
B.B. Kadomtsev, Kurchatov Inst. of Atomic Energy, USSR
The Kharkov Physical-Tech. Inst., USSR
M. S. Rabinovich, Academy of Sci, USSR
Bibliothek, Stuttgart, West Germany
R.D. Buhler, Univ. of Stuttgart, West Germany
Max-Planck-Inst. fur Plasmaphysik, W. Germany
Nucl. Res. Estab., Julich, West Germany
K. Schindler, Inst. Fur Theo. Physik, W. Germany

EXPERIMENTAL THEORETICAL

M. H. Brennan, Flinders Univ. Australia
H. Barnard, Univ. of British Columbia, Canada
S. Screenivasan, Univ. of Calgary, Canada
J. Radet, C.E.N.-B.P., Fontenay-aux-Roses, France
Prof. Schatzman, Observatoire de Nice, France
S. C. Sharma, Univ. of Cape Coast, Ghana
R. N. Aiyer, Laser Section, India
B. Buti, Physical Res. Lab., India
L. K. Chavda, S. Gujarat Univ., India
I.M. Las Das, Banaras Hindu Univ., India
S. Cuperman, Tel Aviv Univ., Israel
E. Greenspan, Nuc. Res. Center, Israel
P. Rosenau, Israel Inst. of Tech., Israel
Int'l. Center for Theo. Physics, Trieste, Italy
I. Kawakami, Nihon University, Japan
T. Nakayama, Ritsumeikan Univ., Japan
S. Nagao, Tohoku Univ., Japan
J.I. Sakai, Toyama Univ., Japan
S. Tjotta, Univ. I Bergen, Norway
M.A. Hellberg, Univ. of Natal, South Africa
H. Wilhelmson, Chalmers Univ. of Tech., Sweden
Astro. Inst., Sonnenborgh Obs.,The Netherlands
N.G. Tsintsadze, Academy of Sci GSSR, USSR
T. J. Boyd, Univ. College of North Wales
K. Hubner, Univ. Heidelberg, W.Germany
H. J. Kaeppeler, Univ. of Stuttgart, West Germany
K. H. Spatschek, Univ. Essen, West Germany

EXPERIMENTAL ENGINEERING

B. Grek, Univ. du Quebec, Canada
P. Lukac, Komenskeho Univ., Czechoslovakia
G. Horikoshi, Nat'l Lab for High Energy Physics, Tsukuba-Gun, Japan
V. A. Glukhikh, D.V. Efremov Sci. Res. Instit.of Elect. App., USSR

EXPERIMENTAL

F. J. Paoloni, Univ. of Wollongong, Australia
J. Kistemaker, Fom Inst. for Atomic & Molec. Physics, The Netherlands

THEORETICAL

F. Verheest, Inst. Vor Theo. Mech., Belgium
J. Teichmann, Univ. of Montreal, Canada
T. Kahan, Univ. Paris VII, France
R. K. Chhajlani, India
S. K. Trehan, Panjab Univ., India
T. Namikawa, Osaka City Univ., Japan
H. Narumi, Univ. of Hiroshima, Japan
Korea Atomic Energy Res. Inst., Korea
E. T. Karlson, Uppsala Univ., Sweden
L. Stenflo, Univ. of UMEA, Sweden
J. R. Saraf, New Univ., United Kingdom