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Theory and Calculations of Synchrotron Instabilities
and Feedback-Mechanism

Thomas E. M. Meijssen

(Visitor, National Synchrotron Light Source,
Brookhaven National Laboratory, Summer 1981)

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NATIONAL SYNCHROTRON LIGHT SOURCE

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THEORY AND CALCULATIONS OF SYNCHROTRON
INSTABILITIES AND FEEDBACK-MECHANISM

by Thomas E.M. Meijssen

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June 2nd - August 12th 1981

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INTRODUCTION

In connection with my study Technical Physics (Technische Natuurkunde) at the University of Science and Technology Twente (Technische Hogeschool Twente) in the Netherlands, I have to fulfil an external term of probation. The purpose can be formulated as follows:

Concerning the level of knowledge and skills of the student at this moment, to give him the opportunity to contribute to research, development or production in practice and to increase his knowledge and skills.

In my opinion, my stay at National Synchrotron Light Source has been a very useful experience.

In the following report I start with giving the properties of the phenomenon synchrotron radiation (chapter 1) and some general theory on the basic processes and betatron and synchrotron oscillations (chapter 2).

Chapter 3 deals with a more extended theoretical view at transverse instabilities and the influence of a damping feedback system.

Chapter 4 concerns the longitudinal case.

For the calculations on the longitudinal case with M equally spaced point-bunches, with N electrons each, in the storage ring (chapter 6), we had to measure the parasitic modes of the radio-frequency cavity. A description of this is given in chapter 5.

The values of damping rates of the longitudinal feedback system found, are as expected, but too low to damp the longitudinal instabilities I calculated. This might be caused by the input data. The calculated growth rates are very sensitive to changes in frequency and width of the parasitic modes, which were measured under conditions differing slightly from the operating conditions.

I would like to end this introduction by thanking J. Galayda, my attendant, who has been a great help to me, and A. van Steenbergen, Head, who offered me the opportunity to work at NSLS.

Upton, August 4th, 1981

Thomas E.M. Meijssen

CHAPTER 1 : SYNCHROTRON RADIATION

1.1 Introduction

Definition:

Synchrotron radiation is the electromagnetic radiation emitted by relativistic charged particles (viz. high-energetic electrons) circulating in a magnetic field. (Ref.7)

Synchrotron radiation is not limited specifically to synchrotrons but it also occurs in nature. The earliest observation was in the year 1054 when Chinese astronomers recorded a super-nova, the remains of which are called the Crab Nebula. Recent astronomical observations established the existence of synchrotron radiation emitted by this source.

John P. Blewett was the first one who provided indirect observation of synchrotron radiation by correlating the observed shrinking of the electron orbit in General Electric's betatron with the radiation loss, which was predicted by Iwanenko and Pomeranchuk (1944). A few years later, also at General Electric, the first observation of the visible part of the synchrotron radiation spectrum was made. (Ref.1)

1.2 Properties of synchrotron radiation

Synchrotron radiation is emitted in a narrow cone around the velocity vector of the radiating electrons. The opening angle of the cone is:

$\theta \approx 1/\gamma = E_0/E$, where E_0 is rest energy electrons. (See fig.(1.1))

Fig.(1.1)

Synchrotron radiation angular distribution for relativistic electrons

By integrating over all azimuthal angles ψ , where ψ is the angle between the direction of photon emission and the orbital plane, the following expression for the radiated power versus wavelength is obtained:

$$P_k(\lambda) = 5.95 \cdot 10^{-6} \gamma^4 k G_2(\lambda/\lambda_c)/\rho \quad (\text{Watt}/k\lambda, \text{ mA}, \text{ mrad } \theta, \text{ all } \psi) \quad (1.1)$$

with: $k = \Delta\lambda/\lambda$ (See fig.(1.2))

Fig.(1.2)

Of great interest is the total magnitude of radiated power (at all wavelengths greater than λ) versus wavelength. About 95% of the radiated power is contained in a wavelength domain of $0.2 \lambda_c - 10 \lambda_c$. (See fig.(1.3))

The critical wavelength λ_c is given by:

$$\lambda_c = 18.6/BE^2 \quad (\text{\AA}) \quad (B \text{ in Tesla, } E \text{ in GeV}) \quad (1.2)$$

For spectroscopy the photon flux is of more interest. This is given in fig.(1.4) by means of a universal spectral distribution function for

Fig. (1.3)

synchrotron radiation:

$$N_k(\lambda) = k_\gamma F \text{ (Photons/k}\lambda, \text{ sec, mA, mrad } \theta, \text{ all } \psi) \quad (1.3)$$

Fig. (1.4)

The boundary of the long-wave side of the spectrum, is the high frequency acceleration system of the ring.

The short-wave bounds are set by the electron energy E and the magnetic induction B . See also formula (1.2).

The critical wavelength λ_c can also be decreased by using "wigglers".

A wiggler is a structure of several short sections of alternating polarity magnetic field which, in its totality, does not result in a net deflection of the electrons, but is intended to provide sources of lower critical wavelength.

A structure consisting of many low field wigglers in sequence is called an undulator.

Synchrotron radiation is polarized. In the orbital plane of the electron ($\psi=0$), it is 100% polarized with the $\vec{E}$ -vector parallel to the electron acceleration vector. Out of the orbital plane ($\psi \neq 0$), its degree of polarization depends upon the wavelength and the magnitude of angle ψ . (See fig.(1.5))

Fig.(1.5)

Note that in a synchrotron the light source is pulsed because of the high frequency acceleration system, which bunches the electrons. (See chapter 2)

In comparison with more conventional photon sources, synchrotron radiation sources are much brighter.

Especially in the X-ray domain of the wavelength region, improvements of 10^4 to 10^7 are reached. But also, typically in the VUV (vacuum ultra violet) domain, strong photon sources are now available where no other sources existed. By using precision tunable monochromators, we can create strong "line" sources and the full spectrum of electromagnetic radiation (from 0.1 Å to 5000 Å) is potentially available for research. (Ref.1)

1.3 Purposes and facilities

While the study of synchrotron radiation itself is of interest, the applications of it in advanced technological and industrial research are manifold:

- Solid state physics: topography, photon-electron spectroscopy (PES), crystallography
- Atom- and molecule-physics
- Analytical chemistry: fluorescence and absorption technics
- Physical chemistry: extended X-ray absorption fine line structure (EXAFS), electron emission spectroscopy for chemical analysis (ESCA), study of enzyme reactions
- Molecular biology: EXAFS, structure research, X-ray microscopy
- Medical physics: cell structure research, X-ray microscopy
- Lithography: development and production of masks and LSI-circuits

A listing of synchrotron radiation sources is given in table (1.1):

Table (1.1): Synchrotron radiation sources

<u>In operation:</u>		<u>Gev</u>	<u>λ_c</u>
USSR:	VEPP4 (Novosibirsk)	7.0	0.27 (1.1) _w
	VEPP3 (")	2.0	2.9 (1.0) _w
	VEPP2M (")	0.7	23
	ARUS (Erevan)(1)	4.5	1.5
	SIRIUS (Tomsk)(1)	1.4	9.4
	PAKHRA (Moscow)(1)	1.3	10
	FLAN, C60 (Moscow)(1)	0.7	28
	N-100 (Karkhov)	0.1	3100

		<u>Gev</u>	<u>λ_c</u>
Germany:	PETRA (Hamburg)	18.0	0.18
	DESY (") (1)	7.5	0.4
	DORIS (")	5.0	0.5
	BONN I (Bonn) (1)	2.5	2.7
	BONN II (") (1)	0.5	77
USA:	SPEAR (Stanford)	4.0	1.1 (1.6) _w
	SURF II (Washington, DC) (2)	0.25	344
	TANTALUS I (Wisconsin) (2)	0.24	258
France:	DCI (Orsay)	1.8	3.4
	ACO (") (2)	0.54	39
Japan:	INS.ES (Tokyo) (1)	1.3	10.1
	SOR (") (3)	0.4	95
Italy:	ADONE (Frascati)	1.5	8.3 (4.6) _w
Sweden:	LUSY (Lund) (1)	1.2	11.8

In construction:

Japan:	PH.FACT (Tsukuba) (3)	2.5	3.0 (0.6) _w
UK:	SRS (Daresbury) (3)	2.0	3.9 (0.9) _w
Germany:	BESSY (Berlin) (3)	0.8	20
USA:	PEP (Stanford)	18.0	0.16
	CESR (Cornell)	8.0	0.35
	ALADDIN (Wisconsin) (3)	1.0	11.6
	NSLS (Brookhaven Nat. Lab.) (3)	2.5	3.0 (0.6) _w
	NSLS (") (3)	0.7	31

with: (1) = Synchrotrons

(2) = Dedicated to synchrotron radiation research

(3) = Designed for synchrotron radiation research exclusively

At the National Synchrotron Light Source (NSLS), Brookhaven National Laboratory (BNL), two dedicated high photon flux facilities are under construction:

a. 2.5 GeV, 0.5 A storage ring for the X-ray domain

b. 700 MeV, 1 A storage ring for the VUV domain

In addition to the light from the bending magnets of the X-ray ring ($\lambda_c = 3 \text{ \AA}$), superconducting undulators will be incorporated ($\lambda_c = 0.6 \text{ \AA}$) for which five

special insertions have been reserved in the eight fold structure, providing for very high brightness photon sources into the wavelength regions of about $0.12 \text{ \AA}$ ($E \sim 100 \text{ keV}$).

For the VUV ring ($\lambda_c = 31 \text{ \AA}$) the incorporation of undulators is planned. Commissioning of both storage rings is scheduled for 1981. (Ref.1)

National Synchrotron Light Source Building, June 1981

More detailed information about NSLS is given in the appendix.

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CHAPTER 2 : GENERAL THEORY

2.1 Basic processes

First of all, I shall give a brief qualitative description of the basic processes in producing a stored electron beam. (See fig.(2.1))

Fig.(2.1): Schematic diagram of an electron storage ring

- A short pulse of a beam of electrons is injected into a vacuum chamber embedded in a more-or-less circular magnetic guide field. The guide field leads the electrons around in more-or-less closed paths to make a stored beam.
- The guide field has focussing properties which drive all electrons toward an ideal design orbit and cause them to execute lateral (radial and vertical) betatron oscillations about the ideal closed path.
- During each revolution an electron loses a small fraction of its energy by synchrotron radiation. For stored electrons this energy loss is compensated for by a corresponding gain of energy from a radio frequency cavity (RF-cavity) or from several cavities acting in concert.
- The periodic accelerating field collects the electrons into circulating bunches, within which the individual electrons oscillate in longitudinal position and in energy relative to an ideal reference particle at the center

of the bunch. The associated motions in longitudinal position and energy are called the synchrotron oscillations.

- The energy loss by synchrotron radiation together with the compensating energy gain from the RF-cavity gives rise to a slow radiation damping of all oscillation amplitudes; the trajectory of each electron tends towards that of an ideal reference particle at the center of the bunch (which moves with constant speed along the design orbit).

- Radiation damping does not conserve phase density, so it is possible to inject, successively, many pulses into the neighbourhood of the same ideal orbit and obtain high circulating currents from weak sources - for example with positron beams.

- The damping of all oscillation amplitudes is effectively arrested because of a continuous excitation of the oscillations by "noise" in the electron energy, which comes about from the fact that the synchrotron radiation is emitted in photons of discrete energy - the so-called quantum fluctuations of the energy loss.

- In stationary conditions a balance is reached between quantum excitation and radiation damping, leading to a statistically stationary distribution of the oscillation amplitudes and phases of the electrons in a bunch. The bunch then takes on the aspect of a traveling strip of ribbon which has a stationary "size" and "shape", with a Gaussian distribution of amplitudes in each of the transverse and longitudinal coordinates. (See fig.(2.2))

Fig.(2.2): Circulating bunches in a stored beam

- For each coordinate of an electron there is some maximum oscillation amplitude above which the electron no longer remains captured in the bunch. We may refer to the range of stable amplitude in each coordinate at its aperture. An electron is lost from a bunch when some disturbance increases the amplitude in any coordinate beyond the corresponding aperture limit. The aperture limit for each coordinate may be set by a physical obstacle which intercepts the electrons, or by nonlinear effects in the focussing forces which lead to unbounded trajectories for large displacements from the ideal reference electron.
- Electrons may be lost by scattering or energy loss in collisions with molecules of the residual gas in the vacuum chamber, or by a large statistical fluctuation in the quantum excitation of an oscillation amplitude.

The basic processes considered above are the single-particle effects which are primarily responsible for the intrinsic properties of a stored electron beam; we have considered a bunch as a collection of noninteracting electrons each of which moves as though it were alone in the storage ring. When the number of electrons in a circulating bunch is large enough (typically, greater than 10^9 or so) interactions among the electrons of a bunch, or among bunches, become important - and have, in fact, been a serious problem in all electron storage rings. A brief listing of the most significant collective effects:

- The AdA- or Touschek-effect. Two electrons oscillate within a bunch may Coulomb scatter, transferring some of the oscillation energy of each electron from one coordinate to the another. The new amplitude in the second coordinate may lie outside the available aperture, or may contribute to an increase of the bunch dimensions. The Touschek-effect is generally significant only at low energies - below 1 GeV or so.
- Coherent oscillations. Each electron in a circulating bunch produces electromagnetic fields in the vacuum chamber which influence the motion of the other stored electrons. Such collective interactions among the electrons can lead to unstable coherent oscillations, in which all of the electrons of a bunch oscillate in a collective mode whose amplitude grows exponentially with time. Such coherent oscillations may involve either the transverse or longitudinal motion of the electrons and can lead to a growth of the bunch size or to the loss of electrons from the bunch.

- Constructive interference of the radiation fields of electrons in a bunch may give rise to coherent synchrotron radiation, which can increase the energy loss if individual electrons. (Taken from M. Sands, Ref.6)

2.2 Oscillations

Electrons are held in a storage ring by the forces from the magnetic guide field. Magnets are disposed along an ideal orbit which is generally a smooth, roughly circular closed curve. When the magnet currents are adjusted to any particular set of consistent values, the designed fields are intended to be such that an electron of a nominal energy E_0 , once properly started, will move forever along the ideal orbit. All stored electrons are constrained by the guide field to move in quasi-periodic, stable trajectories in the neighbourhood of the ideal orbit. (Ref.6)

We consider two kinds of oscillations:

- a. Betatron oscillations, whose behaviour is governed by the properties of the guide field and which are independent of the accelerating field.
- b. Synchrotron oscillations, arising from the accelerating process.

We can consider these oscillations separately as long as the frequency of betatron oscillations is large compared to that of synchrotron oscillations, which is generally true in most synchrotrons. (Ref.4)

2.2.a Betatron oscillations

Betatron oscillations are transverse oscillations (vertical as well as radial) of the charged particles, due to a certain initial displacement and are independent of the accelerating field.

The betatron oscillation is stable if the particle, whose momentum is appropriate to the given equilibrium orbit, is started with a small initial displacement and a small initial angle from the equilibrium orbit, will remain near the equilibrium orbit for all time. (Ref.4)

It is convenient to describe the motion of the electron in terms of coordinates related to the ideal orbit. (See fig.(2.3))

In order to get stable trajectories near the equilibrium orbit, the magnetic field is described by:

Fig.(2.3): Coordinates for describing the trajectories

$$B_z(s, x, z) = B_0(s) + \left[\frac{\partial B}{\partial x} \right]_{os} \cdot x \quad (2.1)$$

$$B_x(s, x, z) = \left[\frac{\partial B}{\partial x} \right]_{os} \cdot z \quad (2.2)$$

For various reasons we choose to define the (linear) properties of the guide field by the two functions:

$$G(s) = \frac{e c B_0(s)}{E_0} \sim 1/\rho_s \quad (2.3)$$

$$K_1(s) = \frac{e c}{E_0} \left[\frac{\partial B}{\partial x} \right]_{os} \quad (2.4)$$

The differential equations of the electron trajectories (the motions in x and z are independent because of linearization) are given by:

$$x'' = K_x(s) x + G(s) \frac{e}{E_0} \quad (2.5)$$

$$z'' = K_z(s) z \quad (2.6)$$

$$\text{with: } K_x(s) = -(G^2(s) + K_1(s)) \quad (2.7)$$

$$K_z(s) = K_1(s) \quad (2.8)$$

Suppose we write for x :

$$x = x_c + x_\beta \quad (2.9)$$

with: x_e = displacement due to off-energy

x_β = free betatron oscillation

Equation (2.5) is satisfied if both of the following equations are true:

$$x_e'' = K_x(s) x_e + G(s) \frac{E}{E_0} \quad (2.10)$$

$$x_\beta'' = K_x(s) x_\beta \quad (2.11)$$

Equations (2.6) and (2.11) describe the free vertical and radial betatron oscillations. Consider now their representative form:

$$x'' = K(s) x \quad (2.12)$$

The solutions of this differential equation are:

$$K > 0 : x = a \cos(\sqrt{K} s + b)$$

$$K = 0 : x = a s + b \quad (2.13)$$

$$K < 0 : x = a \cosh(\sqrt{-K} s + b)$$

Since equation (2.12) is linear in x , any linear combination of C and S will also describe a possible trajectory:

$$x(s) = C(s, s_0) x(s_0) + S(s, s_0) x'(s_0) \quad (2.14)$$

and:

$$x'(s) = C'(s, s_0) x(s_0) + S'(s, s_0) x'(s_0) \quad (2.15)$$

It is often convenient to write the last two equations in a matrix notation:

$$\begin{bmatrix} x(s) \\ x'(s) \end{bmatrix} = M(s, s_0) \begin{bmatrix} x(s_0) \\ x'(s_0) \end{bmatrix} = \begin{bmatrix} C(s, s_0) & S(s, s_0) \\ C'(s, s_0) & S'(s, s_0) \end{bmatrix} \begin{bmatrix} x(s_0) \\ x'(s_0) \end{bmatrix} \quad (2.16)$$

$$\text{with: } K > 0 : M(s, s_0) = \begin{bmatrix} \cos\sqrt{K} s & 1/\sqrt{K} \sin\sqrt{K} s \\ -\sqrt{K} \sin\sqrt{K} s & \cos\sqrt{K} s \end{bmatrix}$$

$$K = 0 : M(s, s_0) = \begin{bmatrix} 1 & s - s_0 \\ 0 & 1 \end{bmatrix} ; \quad 1 = s - s_0 \quad (2.17)$$

$$K < 0 : M(s, s_0) = \begin{bmatrix} \cosh\sqrt{-K} s & 1/\sqrt{-K} \sinh\sqrt{-K} s \\ \sqrt{-K} \sinh\sqrt{-K} s & \cosh\sqrt{-K} s \end{bmatrix}$$

Note that this matrix-formalism resembles the matrix-formalism of thin lenses. Because of this, one often refers to the "optics" of the accelerator. (Ref.4,6)

2.2.b Synchrotron oscillations

Synchrotron oscillations (energy oscillations) are the oscillations of the electrons caused by the deviation in energy around the nominal energy E_0 and the accelerating process.

Let us consider two examples:

1. Linear accelerator:

Fig.(2.4): Driving voltage RF-cavity versus $x-ct$

When an electron in the bunch has too much energy (1), it will go faster and will move towards point (2). Here, the electron gains less energy and finally it moves "backwards".

So synchrotron oscillation is here longitudinal.

Note that electrons are gathered in bunches by the RF-system (see page 2-1).

2. Synchrotron:

Fig.(2.5): Driving voltage RF-cavity versus $x-ct$

When an electron has too much energy (ϵ), it will move outwards, because of its higher momentum. Now, the rotation time is longer than the rotation time of the reference point. (Note that the reference point of the synchrotron differs from that of the linac.)

So it moves "backwards" when compared to the reference point. Synchrotron oscillation here is both longitudinal and transverse.

An important equation in longitudinal oscillations is:

$$\frac{dl}{L} = \alpha \frac{d\epsilon}{E_0} \quad (2.18)$$

with: dl = deviation in orbit length (m)

L = length of ideal orbit (m)

α = momentum compaction

ϵ = deviation in energy (J)

E_0 = energy of the ideal orbit (J)

All the important effect of the transverse motion are included in the momentum compaction α , which is positive for most synchrotrons.

Some remarks:

1. Without damping, the trajectory of the electron in the phase space is the same as an harmonic oscillator.
2. Synchrotron radiation causes shrinking of the beam. This is because the energy is taken at random from the electrons, whereas the RF-cavity puts a well defined momentum into the beam.
The net effect is a shrinking beam.
3. Synchrotron radiation also causes synchrotron oscillations. Suppose we consider a particle which is moving at a proper orbit concerning its energy. When it radiates synchrotron light, it loses energy and its position is no longer an equilibrium one.
4. We have neglected the variation of velocity with energy for highly relativistic electrons. If we include this effect, we find that at low energy, the change in pathlength specified by α , is compensated by a change in velocity of the particles. Below "transition-energy", the stable phase in a synchrotron is on the downward slope of the RF-voltage, as in a linac.

CHAPTER 3 : TRANSVERSE INSTABILITIES AND FEEDBACK

3.1 Transverse instabilities

As we have seen in chapter 2, transverse instabilities mainly are betatron oscillations, which are driven by electromagnetic fields produced by the beam. The beam thus excites itself in a positive feedback loop which can result in loss of the beam.

There are two different ways of describing these instabilities, namely:

1. Matrix equation, in time domain (see viz. chapter 2)
2. Differential equation, in time and frequency domain.

The former formalism is easier to understand but the latter is used more frequently because of its easiness to handle.

Theory of transverse instabilities described by differential equations can be found in S. Krinsky's notes. (Ref.32,34,35)

3.2 Transverse feedback

Very useful in understanding something about (the theory of) accelerators are the formalisms which describe the feedback control of transverse instabilities.

We also deal with two kinds of formalisms, namely:

1. Matrix equation, in time domain
2. Differential equation, in time and frequency domain.

After looking at these two formalisms, I shall try to compare these for the coupled multi-bunch case.

3.2.a Matrix equation

The matrix-formalism of transverse feedback is investigated by S. Kheifets. You can find the theory on the single and multi-bunch case (the latter with two counter-rotating beams) with memory in PEP-notes 317 and 322, respectively. (Ref.15,16)

I shall now consider the matrix-formalism of the feedback system with memory for a single beam consisting of M bunches with N particles each.

The kicker signal is damped only slowly and affects many bunches: memory. The action of the kicker depends on the displacement of the first (intended) bunch, thus establishing coupling between the motion of different bunches. The motion of the bunches is described by a $2M$ -dimensional vector $\vec{X}$:

$$\vec{X} = \begin{bmatrix} x_1 \\ x_1' \\ \vdots \\ x_M \\ x_M' \end{bmatrix} \quad (3.1)$$

Fig.(3.1): Synchrotron schematically

The action of the kicker can be described by the matrix-operator $\hat{L}$:

$$\hat{L} \vec{X} = \hat{I} \vec{X} + \sum_{m=1}^{\infty} \hat{P}(\hat{f}_m) \hat{F} \hat{M}^{-m} \vec{X} \quad (3.2)$$

with:

$$\text{Projection operator } \hat{P}(\hat{f}_m) = \begin{bmatrix} 0 & 0 & \cdot & 0 & 0 \\ f_{m11} & 0 & \cdot & f_{m1M} & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & 0 & 0 \\ f_{mM1} & 0 & \cdot & f_{mMM} & 0 \end{bmatrix} \quad (3.3)$$

The meaning of the coefficients of the projection operator $\hat{P}(\hat{f}_m)$ can be seen if we write explicitly the action of the kicker on the j th bunch on the n th passage:

$$\begin{aligned} \Delta x_j(n) &= 0 \\ \Delta x_j'(n) &= \sum_{l=1}^M \sum_{m=1}^{\infty} f_{mj2} x_j^{(d)}(n-m) \end{aligned} \quad (3.4)$$

with: m = "memory"

j = action on j th bunch

l = bunch which is the source of the action

d = detector

The full matrix of one revolution $\hat{T}$ is:

$$\hat{T} = \hat{L} \hat{M} = \left(\hat{I} + \sum_{m=1}^{\infty} \hat{P}(\hat{f}_m) \hat{F} \hat{M}^{-m} \right) \hat{M} \quad (3.5)$$

with: Matrices $\hat{F}$ and $\hat{M}$ quasi-diagonal with 2x2 submatrices F and M respectively and zeros elsewhere.

and:

$$\hat{F} = \begin{bmatrix} F_{11} & F_{12} & & & 0 \\ F_{21} & F_{22} & & & \\ & & \ddots & & \\ & 0 & & \boxed{\begin{matrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{matrix}} & \\ & & & & \end{bmatrix} \quad \text{--- } F \quad (3.6)$$

$$\hat{M} = \begin{bmatrix} M_{11} & M_{12} & & & 0 \\ M_{21} & M_{22} & & & \\ & & \ddots & & \\ & 0 & & \boxed{\begin{matrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{matrix}} & \\ & & & & \end{bmatrix} \quad \text{--- } M \quad (3.7)$$

The submatrices F and M are the transfer-matrices of a single bunch, one revolution, which have the following general form:

$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix} \quad (3.8)$$

$$\begin{aligned} \text{with: } N_{11} &= \sqrt{\beta_2/\beta_1} (\cos\mu_{12} + \alpha_1 \sin\mu_{12}) \\ N_{12} &= \sqrt{\beta_1\beta_2} \sin\mu_{12} \\ N_{21} &= \frac{(\alpha_1 - \alpha_2) \cos\mu_{12} - (1 + \alpha_1\alpha_2) \sin\mu_{12}}{\sqrt{\beta_1\beta_2}} \\ N_{22} &= \sqrt{\beta_1/\beta_2} (\cos\mu_{12} - \alpha_2 \sin\mu_{12}) \end{aligned} \quad (3.9)$$

According to Kheifets we 'll get the following expression for $\hat{T}$:

$$\hat{T} = \begin{bmatrix} M_{11} & M_{12} & & & 0 \\ M_{21} & M_{22} & & & \\ & & \ddots & & \\ & 0 & & \boxed{\begin{matrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{matrix}} & \\ & & & & \end{bmatrix} + \begin{bmatrix} 0 & 0 & \cdot & 0 & 0 \\ R_{11} & S_{11} & & R_{1M} & S_{1M} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & 0 & 0 \\ R_{M1} & S_{M1} & \cdot & R_{MM} & S_{MM} \end{bmatrix} \quad (3.10)$$

$$\begin{aligned}
\text{with: } R_{j\ell} &= \sqrt{\beta_d/\beta_k} \left(\sum_{m=1}^{\infty} f_{mj\ell} \{ \cos((m-1)\mu_0 - \Delta\mu) - \alpha_k \sin((m-1)\mu_0 - \Delta\mu) \} \right) \\
S_{j\ell} &= -\sqrt{\beta_d\beta_k} \left(\sum_{m=1}^{\infty} f_{mj\ell} \sin((m-1)\mu_0 - \Delta\mu) \right) \\
\Delta\mu &= \mu_0 - \mu_k = \mu_0 + \mu_d - \mu_{k+1}
\end{aligned} \tag{3.11}$$

By calculating the eigenvalues $\lambda_i = \exp(-i\Omega_i)$ of $\hat{T}$, the growthrate of the damped instabilities can be found by taking the imaginary part of Ω_i .

3.2.b Differential equation

We now investigate the feedback formalism with memory of transverse instabilities of a single beam, consisting of M bunches, N particles each, by means of differential equations.

The differential equation of the 1 st bunch is given by:

$$\begin{aligned}
\xi_1''(t) + \omega_0^2 Q^2 \xi_1(t) &= \sum_n \left(\xi_1(t-t_0-nT) g(nT) + \xi_2(t-t_0-nT-\frac{T}{M}) g(nT+\frac{T}{M}) + \right. \\
&\quad \left. + \dots + \xi_M(t-t_0-nT-\frac{(M-1)T}{M}) g(nT+\frac{(M-1)T}{M}) \right)
\end{aligned} \tag{3.12}$$

with: ξ_ℓ = transverse displacement of ℓ th bunch (m)

ω_0 = revolution frequency (rad/sec)

Q = betatron tune

T = revolution time (sec)

t_0 = starting time (sec)

The terms behind the summation sign describe the influence of memory of the feedback system. In other words, the field of the feedback kick (supposed to be meant for a certain bunch), is felt by the other bunches somewhat later, because of the narrow-banded feedback system.

Considering all bunches at once, we obtain a cyclic matrix, which has the following form:

$$\text{Cyclic matrix } C_M = \begin{pmatrix} a_1 & a_2 & \dots & a_M \\ a_M & a_1 & \dots & a_{M-1} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ a_2 & a_3 & \dots & a_1 \end{pmatrix} \tag{3.13}$$

Part of the solution of the differential equation is given by the eigenvalues of this matrix, which are:

$$\lambda_n = \sum_{\ell=1}^M a_{\ell} \exp\left(\frac{2\pi i(\ell-1)\mu}{M}\right) \quad (3.14)$$

with: $\mu = \text{mode } (0,1,2,\dots,M-1)$, (compare oscillation modes of coupled, mechanic oscillators)

Suppose the solution for the ℓ th bunch (which also includes time dependence) is of the following form:

$$\xi_{\ell}(t) = \exp\left(\frac{2\pi i(\ell-1)\mu}{M}\right) \exp(-i\Omega t) \quad (3.15)$$

$$\text{and: } g(t) = A e^{-\Gamma t} \cos m_0 \omega_0 t \quad (3.16)$$

Then the imaginary part of Ω , which represents the growthrate of the damped transverse instability, is given by:

$$\text{Im}(\Omega) = \frac{-A}{4\omega_0 Q} \left[\sum_{\pm} \frac{\sin\Delta\psi - e^{-\alpha} \sin\left(\frac{Q+\mu\pm m_0}{M} + \Delta\psi\right)}{1 + e^{-2\alpha} - 2e^{-\alpha} \cos\left(\frac{Q+\mu\pm m_0}{M} + \Delta\psi\right)} \right] \quad (3.17)$$

with: $\alpha = \Gamma T/M$

$$\Omega \sim \omega_0 Q$$

The solution in frequency domain can be found using the relation :

$$\begin{aligned} g(t) &= \int d\omega e^{-i\omega t} G(\omega) \\ G(\omega) &= \int \frac{dt}{2\pi} e^{-i\omega t} g(t) \end{aligned} \quad (3.18)$$

3.2.c Comparison

I shall now try to compare the matrix formalism by S. Kheifets with the differential formalism by S. Krinsky for this special case.

Suppose:

1. ξ_1 "feels" the other particles and itself in the past (memory), starting with itself one turn ago: $n = 1, 2, 3, \dots$
2. Detector and kicker in the same place:

$$N = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \beta_d = \beta_k = \beta; \alpha_d = \alpha_k = \alpha; \Delta\mu = 0$$

3. $\alpha = 0$
4. Number of bunches $M = 3$
5. We consider bunch $j = 1$

The feedback coefficient is given by:

$$f_{ml2} = f_o \exp\left(-\zeta(m-1 + \frac{l-1}{M})\right) \cos\left((m-1 + \frac{l-1}{M})\Delta\phi + \delta\right) \quad (3.19)$$

with: ζ = decrement of tune amplifier in one turn

$\delta = 72$ nd harmonic of orbital frequency, $\delta = \delta(\Delta\phi, \zeta)$

$\Delta\phi = \Delta\phi(T_o, \Delta F)$

This results in:

$$R_{jl2} = \frac{f_o}{2} e^{-\zeta(\frac{l-1}{M})} \left\{ \sum_{\pm} \frac{\cos[\pm(\frac{l-1}{M})\Delta\phi \pm \delta] - e^{-\zeta} \cos[\mu_o \pm (\frac{l-1}{M})\Delta\phi \pm \delta]}{1 + e^{-2\zeta} - 2e^{-\zeta} \cos\{\mu_o \mp \Delta\phi\}} \right\} \quad (3.20)$$

$$S_{jl2} = \frac{f_o \beta}{2} e^{-\zeta(\frac{l-1}{M})} \left\{ \sum_{\pm} \frac{\sin[\pm(\frac{l-1}{M})\Delta\phi \pm \delta] - e^{-\zeta} \sin[\mu_o \pm (\frac{l-1}{M})\Delta\phi \pm \delta]}{1 + e^{-2\zeta} - 2e^{-\zeta} \cos\{\mu_o \mp \Delta\phi\}} \right\} \quad (3.21)$$

By calculating the eigenvalues $\lambda_i = e^{-1\Omega_i}$ of $\hat{T}$ (see (3.10)), we get the following expression for Ω_i :

$$\Omega_1 = i \ln(S_1 + S_2 + S_3 + 1); \quad \Omega_i = k 2\pi \quad (i=2,3,\dots,6) \quad (3.22)$$

with: $S_1 = S_{1j}$, since it makes no difference which j you take

The growthrate of the damped transverse oscillations is - as we have seen in part 3.2.a - given by the imaginary part of Ω_i . Only $\text{Im}(\Omega_1)$ differs from zero:

$$\begin{aligned} \text{Im}(\Omega_1) &= \ln(S_1 + S_2 + S_3 + 1) \sim S_1 + S_2 + S_3 \sim \\ &\sim \frac{3}{2} f_o \beta \left[\sum_{\pm} \frac{\sin[\pm\delta \pm \Delta\phi/3] - e^{-\zeta} \sin[\mu_o \mp 2/3 \Delta\phi \pm \delta]}{1 + e^{-2\zeta} - 2e^{-\zeta} \cos\{\mu_o \mp \Delta\phi\}} \right] \end{aligned} \quad (3.23)$$

For the M bunch case we get:

$$\text{Im}(\Omega_1) = \frac{M}{2} f_o \beta \left(\sum_{\pm} \frac{\sin\{\pm\delta \pm 1/M \Delta\phi\} - e^{-\zeta} \sin\{\mu_o \pm (1/M-1)\Delta\phi \pm \delta\}}{1 + e^{-2\zeta} - 2e^{-\zeta} \cos\{\mu_o \mp \Delta\phi\}} \right) \quad (3.24)$$

This equation corresponds to the equation found by S. Krinsky (3.17) if:

$$\begin{aligned} f_o &= \frac{-A}{2\omega_o MQ\beta} \\ \zeta &= \alpha \\ \mu_o &= \frac{Q + \mu}{M} \\ \Delta\phi &= \pm \frac{m_o}{M} \\ \delta &= \mp \frac{m_o}{M^2} \end{aligned} \quad (3.25)$$

Remarks:

1. The solution is found after linearization.
2. There are two solutions ($\pm$).
3. In linearized form, the following expression holds:

$$K \sin \phi \sim \sin K\phi$$

But taking the left part instead of the right one, results in a different final solution (3.25).

4. $\Delta\psi = 0$

Conclusion:

I am not totally satisfied about the solution of the comparison of matrix/differential formalism. The nearly identical solution of both formalisms, however, stresses justification.

CHAPTER 4 : LONGITUDINAL INSTABILITIES AND FEEDBACK

We now deal with the theory of longitudinal instabilities of electrons in accelerator rings and their feedback system.

Since I have been working rather a long time during my stay at NSLS with these properties, I shall give some more information than just a brief introduction.

There exist no accurate matrix-formalism to describe the theory of longitudinal instabilities and feedback, so we use the differential-formalism, which can be found in extend in S. Krinsky's notes. (Ref.33,34,35,41)

4.1 Longitudinal instabilities

As we have seen in chapter 2, longitudinal instabilities are growing synchrotron oscillations, which are oscillations of the electrons due to the deviation in energy around the nominal energy E_0 and the accelerating process.

Using the differential formalism, we get the following basic differential equations of the energy and phase of the electrons in the accelerator ring:

$$\frac{d\varepsilon}{dt} = \frac{e \omega_0}{2\pi} \left[V_{rf}(\phi) + V_1(\phi) e^{-i\Omega t} \right] \quad (4.1)$$

$$\frac{d\phi}{dt} = -\alpha \omega_0 \frac{\varepsilon}{E_0} \quad (4.2)$$

with: E_0 : nominal energy of electrons (J)

$\varepsilon = E - E_0$: energy deviation (J)

$\theta = s/R$: azimuthal position

$\omega_0 = c/R$: revolution frequency (rad/sec)

α : momentum compaction ϕ : reference phase

h : harmonic number ϕ : actual phase of oscillating bunch

V_{rf} : voltage RF-cavity (V)

V_1 : beam-induced voltage (V)

Ω : complex frequency (1/sec)

The time dependent part of the voltage oscillation is given by:

$$V_1(\phi) e^{-i\Omega t} = \int d\tau W(t-\tau) i(\theta - \omega_0 \tau) e^{-i\Omega \tau} \quad (4.3)$$

and:

$$W(t-\tau) = - \int \frac{d\omega}{2\pi} \bar{z}(\omega) e^{-i\omega(t-\tau)} \quad (4.4)$$

$$i(\phi) = \omega_0 \rho(\phi) = \omega_0 \sum_n \rho_n e^{in\phi} \quad (4.5)$$

with: $W(t)$: wake function (Ω/sec)
 $\bar{z}(\omega)$: longitudinal impedance (Ω)
 $i(\phi)$: current (A)
 $\rho(\phi)$: charge per radian (C)

Remarks:

1. Ω = Complex frequency of the unstable oscillation: We assume that V_1 , the current i and the synchrotron oscillation coordinates (ϵ, ϕ) are all modulated at this frequency. This is the consistency condition, which leads to self-driven, growing synchrotron oscillations, the longitudinal instability.
2. $V_1(\phi) \exp(-i\Omega t)$ = The accelerating potential created by the oscillations of the beam itself, e.g., the current of the bunched beam may excite a resonant mode in the RF-cavity at some frequency higher than the frequency of the accelerating system. If the frequency and characteristic electric field pattern of the mode is such that it is strongly excited by the beam current, the higher mode can both absorb energy from the beam and induce beam oscillations which further excite the higher mode. This is the essence of the instability!
3. $W(t-\tau)$ = The wake potential: A Greens function, which expresses the change in energy of a part of the beam at time t , due to electromagnetic fields in the beam pipe, which were created by the passage of another part of the beam at an earlier time τ . To find the total energy change, we must include the effects of current passing at all earlier times τ :

$$e V_1 = e \int d\tau W(t-\tau) i(\tau) \quad (4.6)$$

After linearizing of (4.1) and (4.2), and solving (4.3) with (4.4) and (4.5), we get the following set of differential equations:

$$\frac{d\phi}{dt} = -k_0 \varepsilon \quad ; \quad k_0 = \frac{\alpha \omega_0}{E_0} \quad (4.7)$$

$$\frac{d\varepsilon}{dt} = \frac{\omega_s}{k_0} \tilde{\phi} + \frac{e\omega_0}{2\gamma} v_1(\phi) e^{-i\Omega t} \quad (4.8)$$

$$v_1(\phi) = -\omega_0 \sum_n \rho_n e^{in\phi} \mathcal{Z}(n\omega_0 + \Omega) \quad (4.9)$$

If we take a certain electron density distribution (viz. waterbag distribution), then we have to take into account the requirement of conservation of phase space density and the problem can be solved by solving the remaining Vlasov-equation.

Since the approximation is accurate enough, we solve the problem by taking M equally spaced point-bunches, for which the electron density is given by:

$$\begin{aligned} \rho(\theta, t) &= \sum_{l=0}^{M-1} \delta p(\theta - \omega_0 t - \frac{2\pi l}{M} - \phi_l) \sim \\ &\sim \sum_{l=0}^{M-1} \delta p(\theta - \omega_0 t - \frac{2\pi l}{M}) - \sum_{l=0}^{M-1} \tilde{\phi}_l \delta p'(\theta - \omega_0 t - \frac{2\pi l}{M}) \end{aligned} \quad (4.10)$$

$$\text{with: } \delta p(\theta) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} e^{in\theta} \quad ; \quad \delta p'(\theta) = \frac{i}{2\pi} \sum_{j=-\infty}^{\infty} n e^{in\theta} \quad (4.11)$$

Note: Often B is used instead of M .

So the electron density for all bunches $l=0$ to $M-1$ is approximated by:

$$\rho_1(\theta, t) = \frac{-i}{2\pi} \sum_{l=0}^{M-1} \tilde{\phi}_l \sum_n i n e^{in(\theta - \omega_0 t - \frac{2\pi l}{M})} \quad (4.12)$$

By using:

$$\tilde{\phi}_l = \tilde{\phi} e^{2\pi i \mu l / M} \quad (4.13)$$

(See also eq.(3.13) and (3.14))

$$n = \mu + Mj \quad (4.14)$$

We get:

$$\rho_1(\theta, t) = -\frac{\tilde{\phi} M}{2\pi} \sum_j i(\mu + Mj) e^{i(\mu + Mj)\phi} \quad (4.15)$$

with: $\tilde{\phi}$ = phase of the 0 th bunch in the oscillation mode μ

By using (4.6), (4.8), (4.9) and (4.15) we get:

$$V_1(\phi) = I_{av} \oint \sum_j i(\mu + Mj) e^{i(\mu + Mj)\phi} z[(\mu + Mj)\omega_0 + \Omega] \quad (4.16)$$

with: $I_{av} = \frac{Ne\omega_0 M}{2\pi}$

and:

$$\ddot{\phi}'' = -\omega_s^2 \phi - \frac{e\omega_0}{E_0} \frac{e\omega_0}{2\pi} I_{av} \oint \sum_j i(\mu + Mj) e^{i(\mu + Mj)\phi} z[(\mu + Mj)\omega_0 + \Omega] \quad (4.17)$$

Suppose the solution has the following form:

$$\phi = \exp(-i\Omega t) \quad ; \quad \Omega \sim \omega_s \quad (4.18)$$

Then the growthrate of the longitudinal instability is given by the following expression:

$$\text{Im}(\Omega) = \frac{e\omega_0^2 I_{av}}{4\pi\omega_s E_0} \sum_j (\mu + Mj) \text{Re}\{z[(\mu + Mj)\omega_0 + \omega_s]\} \quad (4.19)$$

4.2 Longitudinal feedback

Here we also deal with M equally spaced, point-bunches.

Suppose we determine radial displacement of the beam at a pickup electrode and relate this to the energy deviation via:

$$\Delta x = \eta \frac{\epsilon}{E_0} \quad (4.20)$$

with: η = dispersion function at electrode

The voltage applied to the feedback cavity is given by:

$$\Delta V_{\text{appl.}} = \left(\frac{\Delta V}{\Delta x} \right)_{\text{appl.}} \cdot \Delta x \quad (4.21)$$

Note that $\left(\frac{\Delta V}{\Delta x} \right)_{\text{appl.}}$ is related to $\frac{\Delta V}{\Delta x}$ in S. Krinsky's notes by:

$$\left(\frac{\Delta V}{\Delta x} \right)_{\text{appl.}} = N \frac{\Delta V}{\Delta x} \quad (4.22)$$

with: N = number of particles per bunch

The differential equations for energy and phase of the 1 st bunch are:

$$\dot{\Phi}_1' = k_0 \varepsilon_1 \quad (4.23)$$

$$\begin{aligned} \varepsilon_1'(t) = & \frac{\omega_s^2}{k_0} \phi_1(t) + \frac{e\omega}{2\pi} \left(\frac{\Delta V}{\Delta x} \right)_{\text{appl.}} \frac{n}{E_0} \sum_n \left(\varepsilon_1(t-nT) h(nT) + \right. \\ & \left. + \varepsilon_2(t-nT - \frac{T}{M}) h(nT + \frac{T}{M}) + \dots + \varepsilon_M(t-nT - \frac{(M-1)T}{M}) h(nT + \frac{(M-1)T}{M}) \right) \end{aligned} \quad (4.24)$$

The equations for bunches 2,...M are obtained by making a cyclic replacement. Using (4.23) and (4.24) we get the next differential equation:

$$\begin{aligned} \Phi_1''(t) + \omega_s^2 \Phi_1(t) = & \frac{e\omega}{2\pi} \left(\frac{\Delta V}{\Delta x} \right)_{\text{appl.}} \frac{n}{E_0} \sum_n \left(\Phi_1'(t-nT) h(nT) + \right. \\ & \left. + \Phi_2'(t-nT - \frac{T}{M}) h(nT + \frac{T}{M}) + \dots + \Phi_M'(t-nT - \frac{(M-1)T}{M}) h(nT + \frac{(M-1)T}{M}) \right) \end{aligned} \quad (4.25)$$

Suppose:

$$\Phi_l(t) = \exp\left(\frac{2\pi i(l-1)\mu}{M}\right) \exp[-i(\omega_s + i\delta)t] \quad (\delta \ll \omega_s) \quad (4.26)$$

Then:

$$\delta = \frac{e\omega}{4\pi} \left(\frac{\Delta V}{\Delta x} \right)_{\text{appl.}} \frac{n}{E_0} \operatorname{Re} \left\{ \sum_j \exp\left(i \frac{2\pi j(\nu_s + \mu)}{M}\right) h\left(j \frac{T}{M}\right) \right\} \quad (4.27)$$

with: $\nu_s = \omega_s/\omega_0$

ω_s = synchrotron frequency (rad/sec)

ω_0 = revolution frequency (rad/sec)

Using the transition:

$$\begin{aligned} h(t) &= \int d\omega e^{-i\omega t} H(\omega) \\ H(\omega) &= \int \frac{dt}{2\pi} e^{i\omega t} h(t) \end{aligned} \quad (4.28)$$

$$\text{and: } I_{av} = M I_0 \quad ; \quad I_0 = \frac{e N \omega_0}{2\pi} \text{ (current of one bunch)} \quad (4.29)$$

We get:

$$\delta = \frac{1}{2} \frac{I_{av}}{N} \left(\frac{\Delta V}{\Delta x} \right)_{\text{appl.}} \frac{n}{E_0} \omega_0 \sum_{j=-\infty}^{\infty} \text{Re} \{ H[(Mj + \mu + v_s)\omega_0] \} \quad (4.30)$$

Suppose the field of the feedback kick is given by:

$$h(t) = e^{-\Gamma \omega_0 v_s t} \cos(m_0 \omega_0 t) \quad (4.31)$$

Then the damping rate of the longitudinal oscillations due to the feedback system is:

$$\delta = \frac{1}{2} \frac{I_{av}}{N} \left(\frac{\Delta V}{\Delta x} \right)_{\text{appl.}} \frac{n}{E_0} \sum_{\pm} \sum_{j=-\infty}^{\infty} \left(\frac{1}{4\pi} \frac{\Gamma v_s}{(Mj + \mu + v_s + m_0)^2 + (\Gamma v_s)^2} \right) \quad (4.32)$$

Note that we have neglected the contribution of the feedback cavity to $Z(\omega)$ because Q is very low (~ 25).

CHAPTER 5 : MEASUREMENTS OF PARASITIC MODES

As we have seen in part 4.1, the essence of the instability is that the current of the bunched beam may excite a resonant mode in the RF-cavity at some frequency higher than the frequency of the accelerating system. And if the frequency and the characteristic electric field pattern of the mode is such that it is strongly excited by the beam current, the higher mode can both absorb energy from the beam and induce beam oscillations which further excite the higher mode.

Therefore - especially because the growthrates of the longitudinal instabilities depend strongly upon them - it is important to have accurate data upon the parasitic modes of the RF-cavity: the resonant frequency, the loaded Q-value and the longitudinal impedance (R/Q).

These have been measured before by K. Batchelor, J. Galayda and R. Hawrylak for the VUV RF-cavity, but because this was done in atmospheric conditions (not in vacuum), these data might have changed. (Ref.40)

By using a Hewlett-Packard Tracking Generator 8444A-H59 (.5-1500 MHz), a Hewlett-Packard Spectrum Analyzer 8555A and 8555B, a Hewlett-Packard Electronic Counter 5245L, two antennes placed in the RF-cavity and the older data, we were able to recognize and measure the most important, former parasitic frequencies, who have changed slightly and also measure the loaded Q-values, who are larger than the result obtained during (R/Q)-measurements, as one would expect.

For the longitudinal impedance (R/Q) we had to assume that its value wasn't affected by working under vacuum.

Fig.(5.1):
VUV RF-cavity

The results are listed in table (5.1). (The former data are given between parentheses.)

Table (5.1) : Measurements of parasitic modes in the VUV cavity

<u>Resonant frequency (MHz)</u>	<u>Loaded Q-value</u>	<u>Long. impedance (R/Q) (Ω)</u>
52.88 (52.88)	5300 (5300)	130
272.677 (273.41)	18178 (828)	32
506.872 (506.94)	24137 (20278)	10
578.469 (578.67)	3196 (2184)	7
859.545 (859.41)	6367 (5371)	36
1100.836 (1101.23)	3058 (2831)	11
1153.253 (1153.19)	19304 (8606)	20
1175.287 (1175.95)	13058 (7304)	9
1282.926 (1282.08)	14255 (3561)	15
1299.465 (1300.05)	14767 (14723)	16
1365.506 (1365.31)	1706 (3969)	9
1447.021 (1447.40)	14470 (9295)	43
1478.982 (1478.11)	924 (8351)	14
1538.015 (1538.35)	19718 (9615)	43

Note that the temperature when measuring ($\sim 80^{\circ}\text{C}$) is different from the operating temperature ($\sim 40^{\circ}\text{C}$) because the RF-cavity was being baked (to outgas) at that moment.

CHAPTER 6 : CALCULATIONS ON LONGITUDINAL CASE

6.1 Calculations longitudinal instabilities

As we have found in part 4.1, the growthrate of the longitudinal instability is given by expression (4.19):

$$\text{Im}(\Omega) = \frac{e\alpha\omega_0 I_{av}}{4\pi\omega_s E_0} \sum_{j=-\infty}^{\infty} (\mu + Mj) \text{Re}\{ \tilde{z}((\mu + Mj)\omega_0 + \omega_s) \} \quad (4.19)$$

with: e = electron charge (C)

α = momentum compaction

ω_0 = revolution frequency (rad/sec)

ω_s = synchrotron frequency (rad/sec)

I_{av} = total current (A)

μ = mode of oscillation

M = number of bunches

N = number of electrons per bunch

$\tilde{z}$ = longitudinal impedance (Ω)

The relations to calculate ω_0 , ω_s and I_{av} are:

$$\omega_0 = \frac{2\pi\nu_0}{h} \quad (6.1)$$

with: ν_0 = accelerator frequency of RF-cavity (1/sec)

h = maximum number of bunches in storage ring

$$\omega_s = \nu_s \omega_0 \quad (6.2)$$

$$\nu_s = \sqrt{\frac{\alpha h V}{2\pi E_0}} \quad (6.3)$$

with: V_{rf} = voltage of RF-cavity (V)

E_0 = energy of storage ring (J)

$$I_{av} = \frac{N e \omega_0 M}{2\pi} \quad (6.4)$$

The RF-cavity resonates at many discrete frequencies, the lowest of which is used for accelerating the beam. The higher modes of the cavity are the most important contributors to the coupled-bunch instability, because they

have high Q and because the cavity is designed to apply strong forces to the beam.

Measurements show that each of the higher modes makes a contribution to the wake potential and its fourier transform $Z(\omega)$, which is well approximated by the responce of a parallel LRC-circuit. Each mode is characterized by the three parameters ω_r , Q , R or equivalently L , C , R . (See fig.(6.1))

Fig.(6.1): Model for long. impedance $Z(\omega)$

$$\text{Then: } \frac{1}{Z(\omega)} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \quad (6.5)$$

$$\text{And: } \text{Re}\{Z(\omega)\} = \frac{(\omega L)^2 R}{(R - \omega^2 RCL)^2 + (\omega L)^2} \quad (6.6)$$

$$\text{With: } Q = R \sqrt{\frac{C}{L}} : \text{quality} \quad (6.7)$$

$$\omega_r = 1/\sqrt{LC} : \text{parasitic frequency (rad/sec)}$$

Equation (6.6) becomes:

$$\text{Re}\{Z(\omega)\} = \frac{R}{1 + Q^2 \left\{ \left(\frac{\omega_r}{\omega} \right)^2 + \left(\frac{\omega}{\omega_r} \right)^2 - 2 \right\}} \quad (6.8)$$

Because we are only interested in investigating the growthrates of the longitudinal instabilities, I have designed several programs in BASIC that calculate - for a given value of the voltage of the RF-cavity and Q - these growthrates and the sums as a function of the parasitic frequency and mode. (See fig.(6.2))

The change in the unloaded Q is given by a Q -factor by which the Q is multiplied. Also included in the program is the option to change the parasitic frequencies by a certain amount (See column 2, fig.(6.2)).

Schematically:

Input data: Voltage RF-cavity (V)

Q-factor

Growthrates longitudinal instabilities (1/sec):

		modes	
parasitic freq.	alteration freq.	growthrates	sum
*****	*****	sum	*****

Fig.(6.2)

I further wrote "automatic"-programs who calculate these growthrates for different sets of V_{rf} and Q.

The "A"-programs put all the information gained in the output file, the "B"-programs only the sum over the frequencies at a certain mode.

This is done for the VUV and X-ray storage ring, partly and totally filled with bunches.

Schematically:

Table (6.1): Listing of different programs

	VUV		X-ray	
# bunches:	3	9	10	30
Manual program	VUV3	VUV9	X10	X30
"A"-program	VUV3A	VUV9A	X10A	X30A
"B"-program	VUV3B	VUV9B	X10B	X30B

The names of the respectively output files are: "(name).OUTPUT"

6.2 Calculations longitudinal feedback

As we have seen in part 4.2, the damping rate of the longitudinal oscillations due to the feedback system is given by (4.32):

$$\delta = \frac{1}{2} \frac{I_{av}}{N} \left(\frac{\Delta V}{\Delta x} \right)_{\text{appl.}} \frac{\eta}{E_0} \sum_{\pm} \sum_{j=-\infty}^{\infty} \frac{1}{4\pi} \frac{\Gamma v_s}{(Mj + \mu + \nu + m_0)^2 + (\Gamma v_s)^2} \quad (4.32)$$

with: η = dispersion function, a given value for each storage ring

The maximum voltage which can be applied by the feedback system is related to the maximum power output of the system P, by:

$$P = \frac{(\Delta V)^2}{2R} \quad (6.9)$$

$$\text{So: } \Delta V = \sqrt{2 P \left(\frac{R}{Q} \right) Q} \quad (6.10)$$

$$\text{And: } Q = R \sqrt{\frac{R}{L}} = \frac{m_0}{2 v_s \Gamma} \quad (6.11)$$

Reasonable values for the feedback system parameters are:

$$P = 1000 \text{ (W)} ; \left(\frac{R}{Q} \right) = 200 \text{ } (\Omega) ; \Delta x = 2 \text{ (mm)}$$

Δx is the beam displacement at operating current I_{av} which calls for full power. Beyond that the power supply is saturated.

Note that $\left(\frac{\Delta V}{\Delta x} \right)_{\text{appl.}}$ also depends upon the voltage of the RF-cavity because v_s does (see equation (6.3)).

I designed a manual program "FEEDBACK" which calculates - given the system (VUV3, VUV9, X10 or X30), voltage of the RF-cavity, Γ and m_0 - the damping rates of the longitudinal oscillation modes and the sum over them.

Schematically:

Input data: System (VUV3, VUV9, X10, X30)

Voltage RF-cavity

Γ

m_0 (rather fixed; for VUV ring, $m_0 = 83$)

Damping rates long. instab. due to feedback (1/sec):

mode	damping rate
*****	sum

Fig. (6.3)

Listing and output of VUV3, VUV3A, VUV3B and FEEDBACK can be found in the appendix.

CHAPTER 7 : CONCLUSIONS

In the previous chapters I have tried to give an overview of synchrotron radiation, its properties and instabilities.

After having examined the theory of transverse instabilities and feedback in different formalisms, I have tried to compare the matrix and the differential formalism of transverse feedback with memory of a single beam with M equally spaced bunches, N particles each. I am not totally satisfied about this comparison, but one might say that because of the resemblance, both formalisms will give comparable results.

In chapter 4, I gave the theory of longitudinal instabilities and feedback for M equally spaced point-bunches, N particles each, which results in the expressions for the growthrates of longitudinal instabilities (4.19) and the damping rates due to the feedback system (4.32).

Using these expressions and the data upon parasitic modes of the VUV RF-cavity (parasitic resonance frequency, loaded Q , longitudinal impedance Z), I was able to calculate the growthrates, as a function of the voltage of the RF-cavity, Q -factor (by which the unloaded Q of the RF-cavity is multiplied), parasitic resonance frequency and mode for the VUV and the X-ray storage ring, totally and partly (1/3) filled with bunches of electrons.

In order to prevent the longitudinal oscillations from growing, we must be sure that the feedback system damps them for all coupled-bunch modes enough.

Since the parasitic modes of the RF-cavity of the VUV and X-ray ring are not exactly the same, and because I only measured the RF-cavity of the VUV ring (under vacuum, at $\sim 80^\circ\text{C}$ instead of the operation temperature which is $\sim 40^\circ\text{C}$), I shall now only discuss the results of the VUV ring.

After careful examination of the results, the following can be said:

1. Instabilities:

- a. The results of the growthrates of the longitudinal instabilities are highly sensitive to the value of the parasitic resonance frequency. By changing a parasitic resonance frequency 1MHz, the growth-rate can easily become 20-50 times bigger or smaller.
- b. We choose:
 $V_{\text{rf}} = 100 \text{ (kV)}$; because this is the design value for the VUV ring and its influence upon the growthrates isn't significant.

Q-factor = 1 ; because in order to do better we would need:

- for VUV3: $Q < 0.05$
- for VUV9: $Q < 0.007$

values who are hard to reach.

- c. Taking $V_{rf} = 100$ (kV) and Q-factor = 1, the major contribution to the growth rates was given by 2 modes (who are because of symmetry about the same in magnitude):

- for VUV3: mode 1 and 2
- for VUV9: mode 2 and 7

In both cases the major contribution to these modes was at the frequency $\nu^* = 1175.287$ (MHz). (See table (7.1))

VOLTAGE RF-CAVITY = 100 kV , Q-FACTOR = 1

NU (MHZ)	ALT (MHZ)	MODE 0	MODE 1	MODE 2	SUM
52.83+	0.00+	15.37+	0.05+	0.05-	15.37+
272.63+	0.00+	0.00+	5.91+	5.74-	0.17+
506.87+	0.00+	0.07-	30.34-	39.70+	1.38+
578.47+	0.00+	1.26-	95.17-	97.36+	0.93+
859.54+	0.00+	4.82-	2135.36-	2205.54+	65.36+
1100.84+	0.00+	0.39+	1409.47+	1368.61-	41.24+
1158.25+	0.00+	1.13-	4944.70-	5292.07+	347.04+
1175.28+	0.00+	0.54-	56721.85-	74701.49+	17979.00+
1282.93+	0.00+	3.07-	897.89-	923.19-	21.41+
1299.46+	0.00+	1.09-	4599.73-	4057.56+	255.94+
1365.51+	0.00+	0.71+	2343.86+	3291.00-	63.49+
1447.02+	0.00+	202.16+	584.98+	595.08-	192.03+
1470.98+	0.00+	696.54-	3031.06-	3083.39+	645.01-
1530.01+	0.00+	0.26+	7450.96+	7756.94-	299.73-
*****	*****	403.24-	60661.76-	79193.58+	*****

NU (MHZ)	ALT (MHZ)	MODE 0	MODE 1	MODE 2	MODE 3
52.80+	0.00+	15.37+	0.09+	0.04+	0.02+
272.68+	0.00+	0.01+	11.42+	5.57+	0.67+
506.87+	0.00+	0.00+	0.20-	0.73-	5.13-
578.47+	0.00+	1.27-	101.70-	6.91-	1.60-
859.54+	0.00+	0.14+	97.35+	2342.93+	379.23+
1100.84+	0.00+	0.53-	612.47-	1993.03-	120.30-
1158.25+	0.00+	1.20-	4999.53-	56.93-	11.84-
1175.28+	0.00+	0.06+	50.84+	74760.55+	60.87+
1282.93+	0.00+	0.00+	54.69+	1004.39+	279.21+
1299.46+	0.00+	0.01-	7.04-	25.64-	162.73-
1365.51+	0.00+	1.61-	1979.00-	4287.09-	311.87-
1447.02+	0.00+	0.10+	56.95+	250.45+	6453.02+
1470.98+	0.00+	697.04-	3391.40-	360.13-	92.76-
1530.01+	0.00+	0.55+	7673.75+	229.00+	52.33+
*****	*****	677.34-	3131.40-	71062.90+	6519.04+

MODE 4	MODE 5	MODE 6	MODE 7	MODE 8	SUM
0.01+	0.01-	0.02-	0.04-	0.09-	15.37+
0.14+	0.14-	0.60-	5.65-	11.17-	0.17+
30.97-	40.24+	5.06+	0.73+	0.20+	1.38+
0.35-	0.36+	1.61+	6.96+	103.91+	0.93+
40.45+	40.00-	384.10-	2273.15-	96.51-	65.36+
21.46-	21.69+	121.31+	2043.41+	603.54+	41.28+
2.60-	2.62+	11.91+	57.43+	5347.17+	347.05+
3.64+	0.73-	61.47-	56791.47-	50.33-	17979.01+
27.19+	27.40-	203.15-	373.76-	54.23-	21.42+
4610.10-	4076.20+	160.85+	25.49+	7.00+	255.95+
56.07-	57.46+	314.22+	4379.83+	1948.56+	63.37+
704.47+	796.96-	6250.94-	256.42-	56.56-	192.10+
21.12-	21.42+	93.37+	370.02+	3430.14+	644.77-
11.90+	11.99-	52.61-	231.67-	7974.74-	289.69-
3006.67-	4133.02+	6324.71-	53643.44-	3196.00+	*****

Table (7.1): Growth rates for VUV3 and VUV9

2. Feedback:

- a. We choose: $m_0 = 83$; design value
 $V_{rf} = 100$ (kV) ; design value
 $P = 1000$ (W)
 $\left(\frac{R}{Q}\right) = 200$ (Ω) See page 6-4
 $\Delta x = 2$ (mm)

So the only variables left are the system (VUV3, VUV9) and Γ .

- b. The maximum Q-value $Q_{max} = 10,000$, the corresponding $\Gamma = 1.895$
 (See equation (6.11))

- c. Starting from Q_{max} , modes 1 and 2 for the 3 bunch case and modes 2 and 7 for the 9 bunch case are highly damped, but the others are hardly damped at all. (See table (7.2))

By decreasing the loaded Q of the feedback system, the peak damping of these modes smoothes out over the others.

At about $Q = 15$ ($\Gamma \sim 1500$), the damping rates are all the same and decreasing Q even more, causes an overall decrease of the damping rates.

Table (7.2): Damping rates feedback for VUV3 and VUV9

FEEDBACK		Q			
		10	100	1,000	10,000
VUV3					
MODE	0	5.723E+02+	9.555E+02+	3.396E+02+	1.072E+02+
MODE	1	5.729E+02+	2.746E+03+	6.768E+04+	1.671E+06+
MODE	2	5.729E+02+	2.744E+03+	6.768E+04+	1.671E+06+
VUV9					
MODE	0	5.913E+02+	6.274E+02+	2.056E+02+	6.504E+01+
MODE	1	5.920E+02+	1.167E+03+	4.211E+02+	1.334E+02+
MODE	2	5.891E+02+	6.592E+03+	2.022E+05+	5.014E+06+
MODE	3	5.799E+02+	1.121E+03+	4.053E+02+	1.283E+02+
MODE	4	5.708E+02+	4.897E+02+	1.592E+02+	5.037E+01+
MODE	5	5.705E+02+	4.905E+02+	1.595E+02+	5.045E+01+
MODE	6	5.799E+02+	1.128E+03+	4.083E+02+	1.293E+02+
MODE	7	5.891E+02+	6.592E+03+	2.022E+05+	5.014E+06+
MODE	8	5.920E+02+	1.161E+03+	4.192E+02+	1.324E+02+

VOLTAGE RF-CAVITY = 100 (kV)

 $m_0 = 83$

Results:

By using $V_{rf} = 100$ (kV), $Q\text{-factor} = 1$, $m_0 = 83$, I was able to find the Q -domain of the feedback system in which the two major coupled-bunch modes were damped sufficiently:

- for VUV3: $17.0 < Q < Q_{max} = 10,000$
- for VUV9: $37.8 < Q < Q_{max} = 10,000$

But now the other modes were not damped enough (a factor of 10 too little damping).

In order to get rid of the major contributors of the instability, I assumed that we are able to damp v^* by a factor of 10 by a physical alteration in the cavity.

Now the Q -domain for the 3 bunch case is:

$$308 < Q < 492$$

The lower boundary is set because of the need of high Q -values in order to damp the major modes 1 and 2, the upper by mode 0 and the "peak"-damping process.

For the 9 bunch case, there exist no Q -domain in which all modes are damped properly.

Since the results are highly sensitive to changes in the values of the parasitic resonance frequencies, and because these were measured under conditions differing slightly from the operation conditions, the parasitic modes should be remeasured with the cavity at the right temperature, under vacuum, if possible with operating conditions duplicated as closely as possible and repeat the calculations in order to decide whether one or more parasitic resonance frequencies must be damped to get a reasonable Q -domain for the feedback system, or to decide whether a more powerful (and also more expensive) feedback device is needed.

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L-4

APPENDIX

*All dimensions are in millimeters
(Scale 1=500)
Center @ BOOSTER*

SPATIAL RELATIONSHIP BETWEEN BOOSTER AND STORAGE RINGS

SYNCHROTRON RADIATION SPECTRA FOR THE NSLS DESIGN PARAMETERS

e⁻ LINEAR ACCELERATOR INJECTOR

Accelerator type	S band, $2\pi/3$ mode
Frequency (at 39°C)	2855.78 MHz
Pulse repetition rate	1 pps
Number of accelerating sections	3
Rf power source	2 klystrons. $P = 21$ MW/unit
Nominal output beam parameters:	
Energy	100 MeV
Energy spread, $\Delta p/p$	$\pm 0.25\%$
Emittance	2×10^{-5} rad. m.
Design beam current	20 mA
Beam modulation frequency	52.88 MHz
Chopped beam modulation frequency	10.576 MHz

BOOSTER SYNCHROTRON PARAMETERS

Energy	70-700 MeV
Design Current	20 mA
Cyclr Rate	1 Hz
τ Touschek ($V_{RF} = 25$ kV)	70 MeV undamped $> 10^6$ sec 700 MeV damped 5000 sec
B (ρ)	1.22 T (1.91 m)
Circumference	28.347 m
τ Orbit	94.6 nsec
Damping Times	$\tau_x = .006$ sec $\tau_y = .012$ sec $\tau_z = .013$ sec
Damped Emittances @ 700 MeV	ϵ_x 4.8×10^{-8} m. rad
(K = 0.1)	ϵ_y 4.8×10^{-10} m. rad
	$\sigma (\frac{\Delta p}{p})$ 6.4×10^{-4}
Lattice Structure	modified FODO combined function horizontal defocusing dipoles, separate focusing quadrupoles
Number of Superperiods	4
Magnet Compliment	8 Dipoles (1.5 m) 8 Quadrupoles (.3 m) 4 Sextupoles (.2 m)
Momentum Compaction	$\alpha = .097$
Damping Partition D	-1.06
Structure Parameters	
Tunes ν_x, ν_y	2.42, 2.36
Maximum β_x, β_y	12.15 m, 4.45 m
Minimum β_x, β_y	.908 m .636 m
Maximum, Minimum η_x	1.33 m, 0.36 m
Injection (e^- per pulse)	Multiturn, ~ 20 mA ($\sim 10^{10}$)
Rf system, frequency	52.88 Hz
Harmonic number	5
Peak radiation loss per turn	11 keV
Rf peak voltage, minimum	20 kV
Radiated power loss (700 MeV, $10^{10} e^-$)	160 W
Peak power, rf drive system	0.4 kW
Booster cycle: injection dwell time	0.1 sec
rise time	0.4 sec
high energy dwell time	0.2 sec
fall time	0.3 sec

VUV STORAGE RING PARAMETERS

Energy	0.7 GeV
Design Current	1 A ($1.06 \cdot 10^{12} e^-$)
Charging Time	100 sec (100 Booster-VUV Charge Transfers)
λ_c (e_c)	31.6 Å (390 eV)
τ_{Touschek}^*	≥ 1.5 hr ($\phi_{\text{rf}} \approx 100$ kV)
B (ρ)	1.22 T (1.91 m)
Circumference	51.02 m
τ_{orb} (h)	170.2 nsec (9)
Damping times	$\tau_x \approx \tau_y \approx 21$ nsec; $\tau_z \approx 11$ nsec
Lattice structure	Separated function, doublets
Number of superperiods	4
Long straights	3.26 m
Magnet complement	8 B (1.5 m) 24 Q (0.3 m) 12 S (0.20 m)
Structure Parameters	"High β "
Betatron Tunes: ν_x, ν_y	3.3; 1.3
β 's in long straights	11.6; 5.5 m
β_x, β_y in dipole (center)	1.5; 11.2 m
Maximum β_x, β_y values	11.9; 14.0 m
Maximum X_p value	; 1.5 m
Damped Emittance	$8.7 \cdot 10^{-8}, 8.8 \cdot 10^{-10}$ m-r.

* Without bunch lengthening. Assuming a factor of 2 bunch dilution a value of $\tau_{\text{Touschek}} \approx 3$ hr is calculated.

RF PARAMETERS, VUV STORAGE RING

Energy loss per turn	$11.1 + N_w(0.5) \text{ keV}^*$
Synchrotron radiation power (1A)	12.1 kW^{**}
Frequency	52.88 MHz
Harmonic number	9
Number of cavities	1
Shunt impedance/unit	$\approx 1 \text{ M}\Omega$
Cavity Q	$\approx 10,000$
Power source (tetrode)	50 kW
Quantum life time	$\gg 100 \text{ hrs}$
Over voltage (eV/U)	9
Effective peak cavity voltage	100 kV
Synchronous phase angle	83.6 Degr.
Synchrotron oscillation freq.	12.8 KHz
ν_s	$2.2 \cdot 10^{-3}$
Natural bunch length (2σ)	0.26 nsec
Number of tetrodes	1
Power available at cavity	50 kW
Cavity power dissipation	$\approx 10 \text{ kW}$
Beam current	1 A
Beam power	$\leq 20 \text{ kW}$

* Noncoherent wiggler: 0.1 keV/period of the wiggler; example, 0.5 keV total per wiggler. Coherent wiggler, typ. $n = 81$ periods, $B = 4 \text{ kG}$, $\lambda = 2.5 \text{ cm}$, total 0.1 keV per wiggler.

** $N_w = 2$

X-RAY STORAGE RING PARAMETERS

Energy range	2.5 GeV - 0.7 GeV	
Design current	1 A (2 GeV) ($3.9 \cdot 10^{12} e^-$)	
	0.5 A (2.5 GeV) ($1.9 \cdot 10^{12} e^-$)	
$\lambda_c (e_c)$ at 1.22 T (B)	2.48 Å (5 keV)	
4.0 T (W)	0.74 Å (16.8 keV)	
τ_{Touschek} (0.7 GeV, 1A)	≈ 0.6 hrs ($\hat{V}_{\text{rf}} = 800$ kV)	
(2.5 GeV, 0.5A)	≈ 8 hrs (same)	
$\hat{B} (\rho)$	1.22 T (6.834 m)	
Circumference	170.20 m	
τ_{orb} (h)	567.7 nsec (30)	
Damping times (0.7 GeV)	$\tau_x = \tau_y = 256$ msec; $\tau_z \approx 129$ msec	
Lattice structure	Separated function, triplets	
Number of superperiods	8	
Long straights	4.50 m	
Magnet complement	16 B ⁻ (2.7 m)	
	40 Q (0.45 m)	
	16 Q (0.80 m)	
	32 S (0.20 m)	
Structure parameters	"Optics"	"Day One"
Betatron tunes: ν_x, ν_y	9.7; 5.7	5.7; 3.7
Minimum β 's in long straights	1.4; 0.3	15.0; 13.0 m
β_x, β_y in dipoles (center)	0.8; 14.0 m	2.0; 11.8 m
Maximum β_x, β_y values	26.2; 27.6 m	16.6; 28.5 m
Maximum K_p value	1.38 m	1.0 m

RF PARAMETERS, X-RAY RING

Energy loss per turn, 2.5 GeV	$505 + N_w^* (17.6) \text{ keV}^{**}$
Synchrotron radiation power (0.5A)	252 kW ($N_w = 0$) 297 kW ($N_w = 5$)
Frequency	52.88 MHz
Harmonic number	30
Number of cavities	1
Shunt impedance/unit	4.3 M Ω
Cavity Q	≈ 15000
Power source (tetrode)	500 kW
Quantum lifetime	$\gg 100 \text{ hrs}$
Overvoltage (eV/U)	1.6 ($N_w = 0$)
Effective peak cavity voltage	800 kV
Synchronous phase angle	51 degr.
Synchrotron oscillation freq.	4.9 kHz
v_s	$2.8 \cdot 10^{-3}$
Natural bunch length	0.35 nsec
Number of tetrodes	1
Power available at cavities	500
Cavity power dissipation	160 kW
Beam power, design value, maximum	300 kW

* W ($\lambda = 0.14 \text{ m}$, 8.8 keV/period, typ. 17.6 keV/wiggler, at 2.5 GeV)

** $207 + N_w (7.3) \text{ keV}$ at 2.0 GeV

TABLE I
NSLS X-ray Beam Lines

X 1.	- X8 -	
X 9.	Johnson Res. Found. (Chance)	A) EXAFS (Biology) B) Scattering
X10.	EXXON (Cramer)	A) EXAFS B) X-Ray Spectroscopy
X11.	N C. State/Conn./BNL/G.E.(Sayers, Budnick, Pick, Wong)	EXAFS
X12.	NSLS/BNL/Bio. (Schoenborn)	A) Small Angle Scattering B) Protein Crystallography
X13.	NSLS/BNL/Chem. (Koetzle, Thomlinson) NSLS/BNL/Physica/Penn/SUNY /Allied Chem. (Cox, Egami, Prewitt, Hasegawa)	Crystallography/Diffuse Scattering Energy Dispersive Diffraction
X14.	ORNL (Sparks)	A) Diffuse Scattering B) Microprobe C) Topography
X15.	Bell Labs	A) Scattering
X16.	Bell Labs (Eisenberger, Golovchenko, Kincaid, Citrin, Moncton)	B) Interferometry C) EXAFS/Scattering D) Spectroscopy
X17.	SUNY (Coppens)	Scattering
X18.	Purdue/Midwest (Liedl)	Diffraction
X19.	NSLS (Hautinga) NSLS/SUNY (Bilello)	EXAFS/SEXAFS/XPS Topography
X20.	IBM/MIT (Horn, Birgeneau)	A) Scattering (Low Q Resolution) B) Scattering (High Q Resolution)
X22.	BNL/Physica (Shirane)	A) Scattering (High Q Resolution) B) Scattering (High E Resolution)
X23.	NRL/NBS	A) Topography
X24.	NRL/NBS (Kabler, Nagel, Madden, Kuriyama)	B) SAXS C) EXAFS/SEXAFS D) Crystallography E) XPS/UPS
X25.	Wiggler Test	
X26.	Diagnostics	
Unassigned:		
BNL (Gordon) Microprobe		
BNL (Jones) Atomic Physics		

TABLE II			
NSLS UV Beam Lines			
		$\lambda(\text{\AA})$	Monochromator
U 1. NSLS Diagnostics			
U 2. -			
U 3. -			
U 4. Bell Labs (Rowe, Smith)	A) SEXAFS	12-1200	PGM
	A) ARUPS	80-500	TGM
	C) Refl/Mod. Spect.	400-6000	NIM
	D) Lithography	White	
U 5. NSLS Free Electron Laser			
U 6. IBM (Grobman)	Lithography	White	
U 7. BNL/SUNY/Penn/Xerox (Strongin, Perlman) (Jona, Plummer)	A) ARUPS/XPS/SEXAFS	15-1200	PGM
	B) ARUPS	80-2500	TGM
	C) Infrared	10^4 - 10^6	NIM
U 8. IBM (Eastman, Spiller)	A) ARUPS	18-2000	TGM
	B) ARUPS	80-2500	TGM
	C) EXAFS/SEXAFS/Micr'y.	8-100	Fresnel Zone Plate
U 9. NSLS/BNL-Bio. (Sutherland)	Dichroism/Fluorescence	1200-300,000	NIM
NSLS/BNL-Chem. (Weston)	Fluorescence Lifetime	1050-12,0000	NIM
U10. -			
U11. NSLS/BNL-Chem. (Grover)	Gas Phase Spectroscopy	300-2000	NIM
U12. -			
U13. -			
U14. NSLS (Howells, Ginter)	A) ARUPS/XPS/SEXAFS	15-1200	PGM
	B) ARUPS	80-500	TGM
	C) 7M. Spectrograph	300-3000	
U15. NSLS (Howells)	TGM Monochromator	10-80	TGM
U16. -			

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1000 "0003"
1115
0010 PRINT
0020 PRINT
0030 PRINT "0003  GROWTH-RATES LONG INSTABILITIES IN VUV-STORAGE RING WITH 3 POINT-PUNCHES "
0040 PRINT "      FOR PARASITIC FREQUENCIES AND MODES 0-2, B, C, D, E, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z"
0050 PRINT
0060 PRINT "INPUT DATA "
0070 PRINT
0080 PRINT "NUMBER OF BUNCHES B = 3"
0090 PRINT "ACCELERATION COEFFICIENT ALPHA = 0.02345"
0100 PRINT "STORED ENERGY E0 = 700 (MEV)"
0110 PRINT "FREQUENCY RF-CAVITY = 52.58 (MHz)"
0120 PRINT "REVOLUTION FREQUENCY NO = 36917204.34 (RAD/SEC)"
0130 PRINT "CURRENT = 1 (A)"
0140 PRINT
0150 PAGE = 142
0160 DIM P(14, 3), T(14, 6)
0170 MAT READ P
0180 DATA 52.58, 5300, 130, 272, 677, 18178, 32
0190 DATA 506, 872, 24137, 10, 578, 469, 3196, 7
0200 DATA 859, 545, 6767, 36, 1100, 836, 3058, 11
0210 DATA 1158, 253, 19604, 20, 1175, 287, 13058, 9
0220 DATA 1282, 826, 14255, 15, 1299, 465, 14767, 16
0230 DATA 1345, 506, 1706, 9, 1447, 821, 14470, 43
0240 DATA 1478, 992, 9, 4, 14, 1538, 815, 19719, 43
0250 REM
0260 REM FILLING 1ST AND 2ND COLUMN OF T-MATRIX
0270 FOR R=1 TO 14
0280   LET T(R, 1) = P(R, 1)
0290   LET T(R, 2) = 0
0300 NEXT R
0310 REM
0320 REM PRINTING INPUT
0330 PRINT "PARASITIC FREQUENCIES "
0340 PRINT
0350 PRINT "NU (MHz)      0      R 0"
0360 PRINT
0370 MAT PRINT P
0380 PRINT
0390 PRINT "OUTPUT "
0400 PRINT
0410 LET B=3
0420 LET JO=200
0430 INPUT "Q-FACTOR = ? ".Q
0440 INPUT "VOLTAGE RF-CAVITY (MV) = ? ".U2
0450 GOTO 0540
0460 PRINT "Q-FACTOR = ".Q
0470 INPUT "VOLTAGE RF-CAVITY (MV) = ? ".U2
0480 GOTO 0540
0490 INPUT "Q-FACTOR = ? ".Q
0500 PRINT "VOLTAGE RF-CAVITY (MV) = ".U2
0510 GOTO 0540
0520 PRINT "Q-FACTOR = ".Q
0530 PRINT "VOLTAGE RF-CAVITY (MV) = ".U2
0540 PRINT
0550 INPUT "ALTER FREQUENCY (Y/N) ? ".S4
0560 IF S4="N" THEN GOTO 0590
0570 INPUT "LINE NUMBER, ALTERATION (MHz) = ? ".X, T(X, 2)
0580 GOTO 0550
0590 PRINT

```

```

0600 LET H1=200*U1
0610 LET H2=327.264*U1
0620 LET H1=255703.642*U1
0630 LET H2= 614207.242*U1
0640 REM
0650 REM CALCULATION OF GROWTHRATE FOR (PARASITIC) FREQUENCIES AND MODES 0-2
0660 FOR R=1 TO 14
0670 LET H2=U1*(R-1)+(R-2)*1000000002*SYS(15)
0680 LET Q3=0*H1*(R-2)
0690 LET R2=0*H1*(R-3)
0700 FOR N=0 TO 2
0710 LET T0=R2+1+Q3*(H2*(H1*H1)+2*(H1*H2*H1)+H2*(2-2))
0720 LET Q=0*H1*(2)
0730 FOR J=1 TO 10
0740 LET M3=M+H1
0750 LET M4=M+H1
0760 LET Z1=R2+1+Q3*(H2*(H3*H2*H1)+2*(H3*H2*H1)+H2*(2-2))
0770 LET Z2=R2+1+Q3*(H2*(H4*H2*H1)+2*(H4*H2*H1)+H2*(2-2))
0780 LET Q=0*H1*(H3*H1+H4*H2)
0790 NEXT J
0800 LET T1=R*(N+3)=0
0810 NEXT N
0820 NEXT R
0830 REM
0840 REM SUB OVER COLUMNS AT CERTAIN FREQUENCY
0850 FOR R=1 TO 14
0860 LET T1=R*(N+3)=0
0870 FOR A=3 TO 5
0880 LET T1=R*(N+3)=0
0890 NEXT A
0900 NEXT R
0910 REM
0920 REM SUB OVER ROWS AT CERTAIN MODE
0930 FOR A=3 TO 5
0940 LET T1=15*(N+3)=0
0950 FOR R=1 TO 14
0960 LET T1=15*(N+3)=0
0970 NEXT R
0980 NEXT A
0990 REM
1000 REM FILLING RESTING ELEMENTS WITH #####
1010 LET T1=15*(N+3)=0
1020 LET T1=15*(N+3)=0
1030 LET T1=15*(N+3)=0
1040 REM
1050 REM PRINTING OUTPUT
1060 PRINT "GROWTHRATES (1 SEC) "
1070 PRINT
1080 PRINT " NO (H1) HLT (H2) MODE 0 MODE 1 MODE 2 SUB"
1090 PRINT
1100 FOR N=1 TO 15
1110 FOR J=1 TO 6
1120 PRINT USING "##### " "T,X,Y".
1130 NEXT J
1140 PRINT
1150 NEXT N
1160 PRINT
1170 INPUT "STOP = 0 ALTER MODE = 1. ALTER Q-FACTOR = 2 ALTER VOLTAGE = 3 ALTER BOTH = 4 " G
1180 PRINT
1190 IF G=1 THEN GOTO 0520
1200 IF G=2 THEN GOTO 0430
1210 IF G=3 THEN GOTO 0460
1220 IF G=4 THEN GOTO 0430
1230 END
1

```

0003 COUNTERS LONG INELIBILITIES IN NON-STORAGE RING WITH 3 POINT-BUNCHES
FOR PARASITIC FREQUENCIES AND MODES 0-2. BY THOMAS E. M. NELLSSEN

INPUT DATA

NUMBER OF SUMMES $E = 3$
MOMENTUM COMPACTED $(\Delta P/P) = 0.02345$
STORED ENERGY $E_0 = 1.00 \text{ (GeV)}$
FREQUENCY RF-CWITY $= 92.88 \text{ (MHz)}$
REVOLUTION FREQUENCY $H_0 = 36917204.34 \text{ (RAD/SEC)}$
CURRENT $I \text{ (A)}$

(PARASITIC) FREQUENCIES

NU (MHZ)	Q	R Q
52.88	5300	150
272.677	18173	32
506.872	34137	10
528.469	2156	7
859.545	6367	36
1100.536	3058	11
1158.253	19304	20
1175.287	13050	9
1282.926	14255	15
1293.465	14767	16
1365.506	1706	9
1447.021	14470	43
1478.982	933	14
1538.015	19718	43

OUTPUT

Q-FACTOR = 2.1

VOLTAGE RF-CWITY (MU) = 2.1

ALTER FREQUENCY (V) = 0.0

COUNTERS (T SEC)

NU (MHZ)	ALT (MHZ)	MODE 0	MODE 1	MODE 2	SUM
52.88	0.00	15.37	0.05	0.05	15.37
272.677	0.00	0.00	5.91	5.91	0.17
506.872	0.00	0.07	38.37	39.20	1.50
528.469	0.00	1.26	95.17	97.36	0.93
859.545	0.00	4.52	2135.36	2205.54	65.36
1100.536	0.00	0.39	1409.47	1368.61	41.24
1158.253	0.00	1.13	4944.70	5392.87	347.04
1175.287	0.00	0.54	56721.95	74701.49	17929.00
1282.926	0.00	3.87	837.83	933.13	31.41
1293.465	0.00	1.59	4539.73	4857.86	255.94
1365.506	0.00	0.71	2313.86	2281.08	63.43
1447.021	0.00	202.16	584.98	595.08	192.05
1478.982	0.00	686.54	3031.86	3083.39	645.01
1538.015	0.00	8.26	7458.90	7756.34	289.73
00000000	00000000	493.24	60661.76	79193.53	00000000

STOP = 0 MODES MODE 1 ALTER Q-FACTOR = 2.0 PER VOL TAGE = 3.0 ALTER 6074 = 4.1

```

/
/
/ LONG"0003a"
/ LIST
0010 OPEN FILE (1,1) "0003a OUTPUT"
0020 PRINT FILE (1,1)
0030 PRINT FILE (1,1)
0040 PRINT FILE (1,1) "0003a GROWTHRATES LONG INSTABILITIES IN QWO-STORAGE RING WITH 3 POINT-BUNCHES."
0050 PRINT FILE (1,1) "AUTOMATIC CALCULATION FOR SEVERAL Q-FACTORS VOLTAGES RF-CAVITY."
0060 PRINT FILE (1,1) " (PARASITIC) FREQUENCIES AND MODES 0-2. BY THOMAS E. M. HELGREN"
0070 PRINT FILE (1,1)
0080 PRINT FILE (1,1)
0090 PRINT FILE (1,1) "INPUT DATA "
0100 PRINT FILE (1,1)
0110 PRINT FILE (1,1) "NUMBER OF BUNCHES B = 3"
0120 PRINT FILE (1,1) "COMPRESSION COEFFICIENT ALFA = 0.0345"
0130 PRINT FILE (1,1) "STORED ENERGY E0 = 700 (MEV)"
0140 PRINT FILE (1,1) "FREQUENCY RF-CAVITY = 52.88 (MHZ)"
0150 PRINT FILE (1,1) "REVOLUTION FREQUENCY W0 = 36917204.34 (RAD SEC)"
0160 PRINT FILE (1,1) "CURRENT = 1 (uA)"
0170 PRINT FILE (1,1)
0180 PAGE = 132
0190 DIM P(14,3) : 1,15,6+
0200 MAT READ P
0210 DATA 52.88,5300.130,272.677,18178.32
0220 DATA 506.872,24137.10,578.469,3196.7
0230 DATA 959.545,6367.36,1100.836,3058.11
0240 DATA 1158.253,19304.20,1175.287,13058.9
0250 DATA 1282.826,14355.15,1299.465,14767.16
0260 DATA 1365.506,1706.4,1447.021,14470.43
0270 DATA 1478.932,214.14,1538.015,19718.43
0280 REM
0290 REM FILLING 1ST AND END COLUMN OF T-MATRIX
0300 FOR R=1 TO 14
0310 LET T(R,1)=P(R,1)
0320 LET T(R,2)=0
0330 NEXT R
0340 REM
0350 REM PRINTING INPUT
0360 PRINT FILE (1,1) "PARASITIC FREQUENCIES "
0370 PRINT FILE (1,1)
0380 PRINT FILE (1,1) " W0 (MHZ) Q R=0"
0390 PRINT FILE (1,1)
0400 MAT PRINT FILE (1,1) P
0410 PRINT FILE (1,1)
0420 PRINT FILE (1,1) "OUTPUT "
0430 PRINT FILE (1,1)
0440 LET B=3
0450 LET W0=200
0460 LET T=0
0470 FOR W2= 05 TO 15 STEP .05
0480 READ W0,W1
0490 DATA 001.01.01 1. 1.1
0500 DATA 001.01.01 1. 1.1
0510 DATA 001.01.01 1. 1.1
0520 FOR W=00 TO 01 STEP .00
0530 LET T=T+1
0540 PRINT "0003a" T
0550 PRINT FILE (1,1) "Q-FACTOR =" ,W
0560 PRINT FILE (1,1) "VOLTAGE RF-CAVITY =" ,W2
0570 LET W3=205.02
0580 LET W0=36917204.34
0590 LET W1=255703.6429003
0600 LET W0= 01420.2422 03

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0610  REM (Calculation of GROWTHRATE FOR (FRACTIONAL) FREQUENCIES AND MODES 0-2
0620  FOR R=1 TO 14
0630  LET Q2=FOR 1 TO R 2 * (100000000/2457567390)
0640  LET Q2=Q2 * A **
0650  LET Q2=Q2 * R **
0660  FOR M=0 TO 2
0670  LET Q2=Q2 * (100.24 * Q2 * (M+M+M1) ** 2 * ((M+M+M1) * M2) ** 2 - 2)
0680  LET Q2=Q2 * Q2
0690  FOR J=1 TO 30
0700  LET M3=M+Q2
0710  LET M4=M+Q2
0720  LET M5=M+Q2
0730  LET M6=M+Q2
0740  LET M7=M+Q2
0750  LET M8=M+Q2
0760  LET M9=M+Q2
0770  LET M10=M+Q2
0780  LET T-R M3=0
0790  NEXT H
0800  NEXT R
0810  REM
0820  REM SUM OVER COLUMNS of CERTAIN FREQUENCY
0830  FOR R=1 TO 14
0840  LET T-R C=0
0850  FOR A=3 TO 5
0860  LET T-R C=T-R C + T-R A
0870  NEXT A
0880  NEXT R
0890  REM
0900  REM SUM OVER ROWS of CERTAIN MODE
0910  FOR I=3 TO 5
0920  LET T-I F=0
0930  FOR R=1 TO 14
0940  LET T-I A=T-I F + T-R A
0950  NEXT R
0960  REM
0970  REM FILLING RESTING ELEMENTS WITH 00000000
0980  LET T-I 1=1234567390
0990  LET T-I 2=1234567390
1000  LET T-I 6=1234567390
1010  REM
1020  REM PRINTING OUTPUT
1030  PRINT FILE (1) "GROWTHRATES (1-SEC) "
1040  PRINT FILE (1)
1050  PRINT FILE (1)
1060  PRINT FILE (1) " NO (M2) "
1070  PRINT FILE (1)
1080  FOR Y=1 TO 15
1090  PRINT FILE (1) USING "***** 000 " T-X,Y
1100  NEXT Y
1110  PRINT FILE (1)
1120  PRINT FILE (1)
1130  NEXT X
1140  PRINT FILE (1)
1150  NEXT U
1160  IF U=1 THEN GOTO 0480
1170  NEXT U2
1180  CLOSE
1190  PRINT "***** END OF 00000 000000"
1200  END

```


Q-FAC TOR = 005
VOLTAGE RF-CAPUTTY =

GROU TH RATES (1 SEC)

NU (MHz)	MODE 0	MODE 1	MODE 2	SUM
52.83*	0.05-	26.23*	25.32-	0.13-
272.68*	0.00*	525.95*	523.87-	2.44*
506.87*	0.00*	1298.72-	1304.64*	1.12*
578.47*	0.00*	0.72*	1.51*	3.16*
859.54*	0.00*	3.76*	9.62*	12.95*
1100.84*	0.00*	5.24*	10.65*	15.72*
1158.25*	0.00*	411.36-	417.43*	4.15*
1175.24*	0.00*	10.91-	14.27*	3.14*
1282.93*	0.00*	24.63-	31.08*	6.04*
1289.46*	0.00*	32.46-	39.36*	6.41*
1365.51*	0.00*	17.44*	27.07*	40.33*
1447.02*	0.00*	28.89*	34.27*	34.27*
1478.93*	0.00*	45.96*	92.16*	137.39*
1539.01*	0.00*	331.44*	310.91-	20.60*
#####	9.62-	796.44-	1053.10*	#####

Q-FAC TOR = 003
VOLTAGE RF-CAPUTTY =

GROU TH RATES (1 SEC)

NU (MHz)	MODE 0	MODE 1	MODE 2	SUM
52.83*	0.03-	21.33*	21.47-	0.13-
272.68*	0.33*	952.46*	845.05-	7.71*
506.87*	0.00*	2370.39-	2334.63*	6.69*
578.47*	0.07-	0.29*	1.15*	1.37*
859.54*	0.00*	26.45-	35.12*	8.03*
1100.84*	0.00*	3.43*	7.05*	10.28*
1158.25*	0.00*	2489.09-	2439.81*	2.40*
1175.24*	0.84-	176.05-	173.73*	1.91*
1282.93*	1.79-	352.06-	387.53*	3.69*
1289.46*	2.13-	474.35-	480.38*	3.91*
1365.51*	0.00*	9.99*	19.15*	26.94*
1447.02*	0.54*	429.75*	415.12-	15.03*
1478.93*	0.27-	31.77*	61.82*	95.32*
1539.01*	11.19*	3297.07*	3295.70-	12.57*
#####	11.05-	1272.24-	1479.05*	#####

Q-FAC TOR = 004
VOLTAGE RF-CAPUTTY =

GROU TH RATES (1 SEC) OAC

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4938 GRADUATES AND DOCTORS: IN CONNECTION WITH 3 POINT-SCALE:
AUTOMATIC CALCULATION FOR GENERAL D-FACTORS, VOLTAGES RE-CAVITY,
(PARASTIC) FREQUENCIES AND MODES D-2. BY THOMAS E M. HEIJSEN

INPUT DATA

NUMBER OF LOUNCHES B = 3
MOMENTUM COMPACTON $\alpha F_0 = 0.0345$
STORED ENERGY $E_0 = 700$ (Joule)
FREQUENCY RF-CAVITY = 52.50 (MHz)
REVOLUTION FREQUENCY $\omega_0 = 56917204.34$ RAD/SEC.
CURRENT = 1 (A)

• PARASITIC • FREQUENCIES

$80 \pm 82^\circ$	0	50
52 88	5300	130
272 677	13178	70
506 872	24137	32
573 469	3196	35
589 545	6767	38
1100 836	3058	11
1158 293	19309	20
1175 237	13083	9
1282 826	14255	15
1299 465	14767	18
1365 306	1706	7
1447 921	14470	43
147 8 2	374	14
172 015	1471	14

Output

GROWTH RATE : 1 %

0047-1022/87/0005-0000\$01.50/0

[illegible]

VOLTAGE RF-CAVITY (KV) =

1

Q-FACTOR	MODE 0	MODE 1	MODE 2	WORST
0.001+	5.265-	60.163+	2.95.533+	2.000+
0.002+	9.690-	563.442-	765.542+	2.000+
0.003+	11.047-	598.079-	1047.362+	2.000+
0.004+	2.554-	8.952-	139.504+	1.000+
0.005+	16.213+	1972.303+	1375.504+	1.000+
0.006+	43.147+	4434.146+	4365.500-	1.000+
0.007+	75.097+	6832.325+	6731.803-	1.000+
0.008+	113.703+	8805.334+	8807.750-	2.000+
0.009+	155.343+	10200.884+	10243.336-	2.000+
0.010+	201.104+	10962.153+	11046.288-	2.000+
0.013+	301.103+	10962.153+	11046.288-	2.000+
0.020+	863.027+	5441.385-	5170.431+	1.000+
0.030+	1749.593+	34657.837-	34909.773+	2.000+
0.040+	1940.195+	59000.451-	60193.823+	2.000+
0.050+	2073.743+	76453.829-	79631.816+	2.000+
0.060+	2061.375+	88852.651-	91357.645+	2.000+
0.070+	1879.433+	96080.346-	101922.396+	2.000+
0.080+	1369.738+	105369.149-	109980.788+	2.000+
0.090+	1752.534+	111416.436-	116815.945+	2.000+
0.100+	1637.528+	116593.893-	123827.261+	2.000+
0.100+	1637.528+	116593.893-	123827.261+	2.000+
0.200+	833.761+	140163.033-	156765.904+	2.000+
0.300+	334.738+	134257.485-	159460.413+	2.000+
0.400+	65.903+	113809.457-	146636.347+	2.000+
0.500+	156.973-	105352.267-	131827.978+	2.000+
0.600+	504.145-	92691.472-	117875.360+	2.000+
0.700+	395.267-	82280.560-	105703.345+	2.000+
0.800+	447.338-	73704.814-	95366.793+	2.000+
0.900+	473.627-	66681.575-	86613.550+	2.000+
1.000+	493.244-	60631.763-	79193.578+	2.000+

VOLTAGE RF-CAVITY (KV) =

15

Q-FACTOR	MODE 0	MODE 1	MODE 2	WORST
0.001+	5.265-	53.150+	240.469+	2.000+
0.002+	9.690-	468.219-	634.928+	2.000+
0.003+	11.047-	732.316-	856.115+	2.000+
0.004+	2.554-	5.779-	108.074+	1.000+
0.005+	16.213+	1611.534+	1510.182+	1.000+
0.006+	43.147+	3630.234+	3564.550-	1.000+
0.007+	75.097+	5572.230+	5547.735-	1.000+
0.008+	113.703+	7184.386+	7196.531-	2.000+
0.009+	155.343+	8320.779+	8371.857-	2.000+
0.010+	201.108+	8938.934+	9030.595-	2.000+
0.010+	201.108+	8938.934+	9030.595-	2.000+
0.020+	863.024+	4469.562-	4193.703+	1.000+
0.030+	1549.610+	28274.368-	28523.592+	2.000+
0.040+	1940.242+	48064.108-	49255.439+	2.000+
0.050+	2073.894+	62226.547-	64352.853+	2.000+
0.060+	2061.482+	72270.363-	75313.980+	2.000+
0.070+	1879.686+	79731.739-	83572.299+	2.000+
0.080+	1370.003+	85611.603-	90221.855+	2.000+
0.090+	1752.916+	90476.137-	95873.974+	2.000+
0.100+	1637.934+	94626.566-	108857.800+	2.000+
0.100+	1637.934+	94626.566-	108857.800+	2.000+
0.200+	839.847+	113987.382-	124537.157+	2.000+
0.300+	334.985+	102622.453-	131826.714+	2.000+
0.400+	65.946+	96898.982-	122556.375+	2.000+
0.500+	158.587-	33886.072-	110535.150+	2.000+
0.600+	507.168-	73771.405-	99031.644+	2.000+
0.700+	398.558-	65424.519-	88934.335+	2.000+
0.800+	451.334-	58567.879-	80313.265+	2.000+
0.900+	477.773-	52899.031-	72939.297+	2.000+
1.000+	497.412-	49164.964-	65776.550+	2.000+


```

0610 LET B=M*1.E-4
0620 LET E0=M*1.E-11*1000000
0630 LET I=M*5.E-4
0640 LET H=M*4.E-4
0650 LET H0=M*5.E-4
0660 LET P=M*6.E-4
0670 PRINT
0680 IF C=1 THEN GOTO 0730
0690 IF C=2 THEN GOTO 0730
0700 IF C=5 THEN GOTO 0730
0710 PRINT "VOLTAGE RF-CAPUTTY: H0 = " H0
0720 GOTO 0740
0730 INPUT "VOLTAGE RF-CAPUTTY: H0 = ? " H0
0740 IF C=1 THEN GOTO 0730
0750 IF C=3 THEN GOTO 0790
0760 IF C=5 THEN GOTO 0790
0770 PRINT "GAMMA = " G
0780 GOTO 0800
0790 INPUT "GAMMA = ? " G
0800 IF C=1 THEN GOTO 0850
0810 IF C=4 THEN GOTO 0850
0820 IF C=5 THEN GOTO 0850
0830 PRINT "H0 = " H0
0840 GOTO 0860
0850 INPUT "H0 = ? " H0
0860 LET H=B
0870 IF K=1 THEN LET H=316
0880 IF K=3 THEN LET H=316
0890 LET U2=0.211000000
0900 LET U1=SRH*H4H*U2*(215)S(15+0E0)
0910 LET P0=1000
0920 LET R2=200
0930 LET U=SOURCE*H21H0*(116)
0940 PRINT "VOLTAGE FEEDBACK: VOLT CM) =".U
0950 LET U=6444444444*(112)S(15)
0960 FOR M=0 TO 8-1
0970 LET J0=50
0980 LET F1=0
0990 LET F2=0
1000 FOR J=(J0) TO 10
1010 LET F1=F1+G1U1*(111)H*H*U1*H0*(2)G1U1*(2)
1020 LET F2=F2+G1U1*(111)H*H*U1*H0*(2)G1U1*(2)
1030 NEXT J
1040 LET H*H*(1)=1110P0*F1+F2*(315)S(15+0E0)
1050 NEXT M
1060 LET H*H*(1)=0
1070 FOR T=1 TO 8
1080 LET H*H*(1)=H*H*(1)+H*H*(1)
1090 NEXT T
1100 PRINT
1110 FOR T=1 TO 8
1120 PRINT "MODE":T-1
1130 PRINT USING "B ###.###" H*H*(1)
1140 NEXT T
1150 PRINT
1160 PRINT "SUM" " 1"
1170 PRINT USING "B ###.###" H*H*(1)
1180 PRINT
1190 INPUT "STOP=0 SYSTEM=1 VOLT RF CAP =2 GAMMA=3 H0=4 ALL=5 " C
1200 IF C=1 THEN GOTO 0540
1210 IF C=2 THEN GOTO 0610
1220 IF C=3 THEN GOTO 0610
1230 IF C=4 THEN GOTO 0610
1240 IF C=5 THEN GOTO 0610
1250 END

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FEEDBACK FEEDBACK SYSTEM FOR LONG INTELIGILITIES IN COO-STORAGE RING (3 AND 7 POINT-BUNCHES)
AND (N-KM) RING (10 AND 30 POINT-BUNCHES) BY THOMAS E. N. MEIJSEN

INPUT DATA

	0003	0009	X10	X30
NUMBER OF BUNCHES B	3	9	10	30
STORED ENERGY EU (MEV)	700	700	700	700
AVERAGE CURRENT I (A)	1	1	5	5
MOMENTUM COMPACTION ALFA	02345	02345	006516	006516
REV FREQ (RAD/SEC)	36917204	36917204	11075161	11075161
PCT	1 4	1 4	1 3	1 3

OUTPUT

SYSTEM (0003, 0009, X10, X30) ? 0009

VOLTAGE RF-CAVITY (CNU) = ? 1

GAMMA = ? 500

MO = ? 83

VOLTAGE FEEDBACK (VOLT CH) = 3893 0697

MODE	0	8 473E+02*
MODE	1	1 156E+03*
MODE	2	1 780E+03*
MODE	3	1 100E+03*
MODE	4	6 887E+02*
MODE	5	6 894E+02*
MODE	6	1 104E+03*
MODE	7	1 780E+03*
MODE	8	1 153E+03*

SUM 1 1 030E+04*

STOP=0. SYSTEM=1. VOLT RF CNU =2. GAMMA=3. MO=4. ALL=5 1

SYSTEM (0003, 0009, X10, X30) ? 0003

VOLTAGE RF-CAVITY (CNU) = ? 1

GAMMA = ? 500

MO = ? 83

VOLTAGE FEEDBACK (VOLT CH) = 3893 0697

MODE	0	1 011E+03*
MODE	1	1 202E+03*
MODE	2	1 202E+03*

SUM 1 3 416E+03*

STOP=0. SYSTEM=1. VOLT RF CNU =2. GAMMA=3. MO=4. ALL=5 3

VOLTAGE RF-CAVITY (CNU) = ? 1

GAMMA = ? 1500

MO = ? 83

VOLTAGE FEEDBACK (VOLT CH) = 2247 6648

MODE	0	6 466E+02*
MODE	1	6 476E+02*
MODE	2	6 476E+02*

SUM 1 1 942E+03*