COMPARISON OF BALLOONING MODE STABILITY PROPERTIES OF TOKAMAKS WITH CIRCULAR AND ELLIPTIC CROSS SECTIONS WITH THE SAME POLOIDAL FLUX

A. Miller, J. Manickam, and J. L. Johnson

Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543

ABSTRACT

The MHD ballooning mode stability limits for tokamak configurations with different cross section shapes but the same total poloidal flux are surprisingly similar.

PPPL--2639

DE89 016955

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

1. INTRODUCTION

The desire to confine high plasma pressures in tokamak reactors, while using only moderately high plasma currents, has provided an incentive to operate in the second stable region for ballooning modes [1-8]. The effects of configuration shape on this problem are important. It has generally been assumed that a tokamak with an elliptic cross section could have a better chance of getting into the second stability region for ballooning modes than one with a circular cross section since it lends itself to operation with higher q -values. However, ellipticity by itself is destabilizing, so that a quantitative study should be made to assess the relative importance of these two effects. The difficulty in this type of comparison is that the equilibria associated with the different configurations depend sensitively on many plasma parameters so that the results are strongly determined by the assumptions of what should be kept fixed. In this work we compare systems in which the poloidal flux between the magnetic axis and the plasma surface is kept fixed. This would appear to be a reasonable comparison since this flux is a measure of the effective poloidal field in the plasma. We also hold q_0 , the safety factor at the magnetic axis, fixed. This arbitrary choice should make the results as insensitive to the behavior of the plasma parameters near the magnetic axis as possible. The fixed poloidal flux constraint then makes it impossible to hold q_{edge} fixed as we change the shape of the plasma. Since the shape of the current profile is strongly dependent on the q -profile, we decided that it would make sense to fix the shape of the toroidal field function $g(\psi)$ at zero (β) to establish our base equilibria for the different configurations, rather than to prescribe the shape of the safety factor $q(\psi)$.

We introduce our model in the next section. Our results for high- n ballooning modes are given in Sect. 3 and for low- n kink modes in Sect. 4. Since the first and second stability regions are not connected for the configurations that we studied, we have considered the possibility of modifying one of them to get directly into the second stable regime. This is discussed in Sect. 5. We conclude with a few comments in Sect. 6.

2. PLASMA MODEL

We carried through the comparison with three sets of cases, first treating a series of equilibria with a circular plasma cross section, then a series with an elliptical cross section, and then one with a standard D -shape. In these cases we used the same pressure distribution,

$$p(\psi) = p_0(1 - \psi)^\alpha \quad (1)$$

with $\alpha = 2.0$. We prescribe a toroidal field function

$$g(\psi) = 1 - g_p(1 - \psi)^\gamma \quad (2)$$

with γ and g_p adjusted to get the correct value of q_0 and $(\Psi_{edge} - \Psi_0)$. Here

$$\psi = \frac{(\Psi - \Psi_0)}{(\Psi_{edge} - \Psi_0)} \quad (3)$$

with Ψ the poloidal flux. We prescribe the shape of the plasma surface to be

$$R = R_0 + a \cos(\theta + \delta \sin \theta), \quad (4)$$

$$Z = \kappa a \sin \theta. \quad (5)$$

We set the aspect ratio R_0/a equal to 5. The ellipticity of the configuration is set by κ and the triangularity by δ .

We settled on a value of $(\Psi_{\text{edge}} - \Psi_0) \approx 0.0051$ by making a $\langle \beta \rangle = 0\%$ run for a case with a circular cross section, adjusting the values of γ and g_p in Eq. (2) to be $\gamma = 2.80$, $g_p = 0.983$, to get $q_0 = 2.15$ and $q_{\text{edge}} = 7.93$. We then adjusted $\gamma = 3.30$ and $g_p = 0.975$ for the $\kappa = 1.4$, $\delta = 0$ elliptical case to recover the same poloidal flux $(\Psi_{\text{edge}} - \Psi_0)$ and the same axis safety factor $q_0 = 2.16$ for $\langle \beta \rangle = 0$. This led to an edge safety factor $q_{\text{edge}} = 14.43$. We did the same for a D -shaped case with $\kappa = 1.4$, $\delta = 0.5$; $\gamma = 3.17$, $g_p = 0.987$, $q_0 = 2.14$, $q_{\text{edge}} = 17.37$. We then ran a series of q -conserving cases to get a set of equilibria with increasing $\langle \beta \rangle$ for each of the configurations. The value of $(\Psi_{\text{edge}} - \Psi_0)$ remained constant, within 3%, as we increased $\langle \beta \rangle$ for all of the cases and no discernable discontinuity in the toroidal current at the plasma boundary was introduced by this procedure. We examined the high- n ballooning mode stability properties for each of the equilibria. The shapes of the magnetic surfaces for these configurations at a $\langle \beta \rangle$ -value just above where one enters the second stable region and the corresponding q and pressure profiles are shown in Figs. 1 and 2.

3. BALLOONING MODE RESULTS

3.a. Circular Case

Our measure of the growth rate of the most unstable ballooning mode is given as a function of $\langle \beta \rangle$ in Fig. 3 and as a function of $\epsilon\beta_p$ in Fig. 4. Instability sets in at $\langle \beta \rangle \approx 0.22\%$, $\epsilon\beta_{\text{poloidal}} \approx 0.43$. The regions in the plasma that are unstable for different values of $\langle \beta \rangle$ are shown in Fig. 5a. The point in the middle denotes the surface with the fastest growing mode. The modes become weaker and more confined towards the outside as $\langle \beta \rangle$ is increased. This configuration reaches the second stable regime at $\langle \beta \rangle \approx 1.17\%$, $\epsilon\beta_{\text{poloidal}} \approx 0.96$.

3.b. Elliptical Case

The results for the elliptical cross section are also given in Figs. 3 and 4. The growth rates of the modes do not differ much from those of the circular case. In this case the first stable region ends near $\langle \beta \rangle \approx 0.22\%$, $\epsilon\beta_{\text{poloidal}} \approx 0.80$. The second stability regime is reached at $\langle \beta \rangle \approx 1.0\%$, $\epsilon\beta_{\text{poloidal}} \approx 1.4$. The unstable regions are shown in Fig. 5b. Again, the location of the unstable surfaces shifts outward as $\langle \beta \rangle$ is increased.

3.c. D -shape case

The growth rates for a D -shaped cross section with $\kappa = 1.4$ and $\delta = 0.5$ are also shown in Figs. 3 and 4, and the regions of instability in Fig. 5c. In this case the unstable region

is bounded between $\langle\beta\rangle \approx 0.21\%$, $\epsilon\beta_{\text{poloidal}} \approx 0.66$, and $\beta \approx 0.82\%$, $\epsilon\beta_{\text{poloidal}} \approx 1.23$. In this case there is a smaller shift of the region of instability towards the outside as $\langle\beta\rangle$ is increased than in the other cases.

4. LOW- n INSTABILITIES

For each of the cases, we picked the equilibrium that had the lowest $\langle\beta\rangle$ value in the second stable ballooning region and looked at its low- n stability properties. Both the circular and elliptical shaped cases are unstable with respect to the $n = 1$ free-boundary kink. The eigenfunctions for these two cases are given in Figs. 6 and 7. The modes have a lot of structure and are very nonlocal. Conducting walls with the same shape as the plasma surface stabilize these modes, stability being obtained with $b/a \sim 0.25$, where a is the distance from the magnetic axis to the plasma surface on a line directed outwards from the axis of the system and b is the distance outwards from the plasma surface to the wall. The D -shaped case is stable with respect to these low- n modes for $\langle\beta\rangle$ near the onset of ballooning second stability. However, at lower values of β where the D -shaped configurations were strongly ballooning unstable, a low- n global instability was found. The instability, shown in Fig. 8, is an 'infernal' mode, driven primarily by the interaction of pressure gradients with magnetic field line curvature, and has a much smaller growth rate than the kinks found in the other configurations. It can only be stabilized by placing a conducting wall at the plasma surface.

5. DIRECT APPROACH TO SECOND STABILITY

The fact that second stability can be achieved at reasonable plasma parameters even for simple configurations with circular cross sections makes it useful to ask how one could achieve direct stable access to this regime. Since the purpose of our investigation was to investigate the effects of configuration shaping rather than to propose specific experiments, we have done little work in this direction. We looked at several circular cross section configurations with γ and g_p adjusted to raise q_0 , keeping α_{edge} fixed. Extrapolation of these indicated that the critical β -values for departure from the first stability regime and entry into the second should coalesce at $\langle\beta\rangle \approx 0.2\%$ when $q_0 \approx 5.5$ if the shape of the pressure distribution is not optimized. Since this occurs at fairly low pressure, it should be possible to use a current drive technique to obtain this profile shaping in order to enter the second stable regime and then peak the distributions. The task should actually be much easier if the profiles are more carefully adjusted.

6. DISCUSSION

The results for the calculation of ballooning modes are summarized in Figs. 3 and 4. The stability properties of these differently shaped plasmas are surprisingly similar. We see little difference between the locations of the unstable modes or of the magnitude of the eigenvalue for the most unstable mode for cases with circular or elliptical cross sections. Even the values of the parameter used to represent the pressure at the edges of the unstable region are similar. The curves for a D -shape configuration are qualitatively similar. We believe that the reduced shift of the unstable region towards the outside for this case, as

observed in a comparison of Fig. 5c with Fig. 5a or 5b, is associated with a difference in the shape of the q -profile and could be modified easily by adjusting the shape of $p(\psi)$.

We found the behavior of the low- n modes to be more surprising. The instabilities observed in systems with circular or elliptic cross sections at the onset of second stability look like ordinary current-driven free boundary modes. Since the value of q_{edge} is significantly higher in the elliptic case than in the circular one, it would be reasonable to expect to find a lower growth rate in this case. The fact that this is not observed provides one of many illustrations of the sensitivity of the MHD stability properties of tokamaks to the exact plasma profiles. We see no sign of a low- n instability near the onset of second stability in our D -shaped model. Since the growth rates for these kink modes decrease rapidly with decreasing $\langle\beta\rangle$, it is reasonable to conjecture that an instability would set in even in this case as this parameter is increased. As is illustrated in Fig. 8, we can find much weaker pressure-driven 'infernal' modes at lower values of $\langle\beta\rangle$ where the Shafranov shift has not produced a deep enough well to provide sufficient stabilization. We found that conducting surfaces near the plasma edge can control all of these low- n modes.

The possibility of using current drive or bootstrap techniques to operate with $q_0 > 1$ should make operation of tokamaks in the second stable region possible even with little or no configuration shaping. The ease of generating interesting equilibria is tied to the plasma shape. If the amount of auxiliary heating that is available provides the limiting effect so that the value of $\langle\beta\rangle$ at which the second stable region is entered is most important, the D -shaped plasma looks most encouraging. On the other hand, if the problem is primarily one of maintaining an equilibrium so that the value of $\epsilon\beta_p$ or the size of the Shafranov shift is the limiting factor, the circular cross section model appears to be very reasonable. It is worth noting that the second stability region can be approached and entered at relatively small values of $\langle\beta\rangle$ so that existing devices should be capable of investigating the behavior of plasmas in this regime.

ACKNOWLEDGMENTS

We are indebted to Dr. H. P. Furth for suggesting this problem and for providing valuable insight and comments. Drs. M. S. Chance, D. A. Monticello, and N. Pomphrey also contributed to the effort and to our understanding. Dr. Pomphrey's assistance in the early stages of this work was extremely useful. The work was supported by the US Department of Energy under Contract No. DE-AC02-76CHO3073 with Princeton University.

REFERENCES

- [1] Mikhailovskii, A. B., and Shafranov, V. D., *Sov. Phys.-JETP*, **39** (1974) 88.
- [2] Sykes, A., and Turner, M. F., in *Controlled Fusion and Plasma Physics* (Proc. 9th European Conf., Oxford, 1979) UKAEA, Culham Lab., Abingdon (1979) p. 161.
- [3] Coppi, B., *Comments in Plasma Physics* **6** (1981) 109.
- [4] Grimm, R. C. *et al.*, *Nuclear Fusion* **25** (1985) 805.
- [5] Chance, M. S., in *Theory of Fusion Plasmas* (Proc. Varenna Workshop, Varenna, Italy, September, 1987).
- [6] Bonoli, P. T., Porkolab, M., Takase, Y., and Knowlton, S. K., *Nuclear Fusion* **28** (1988) 991.
- [7] Naitou, H., and Yamazaki, K., *Nuclear Fusion* **28** (1988) 1751.
- [8] Sabbagh, S. A., Hughes, M. H., Phillips, M. W., Todd, A. M. M., and Navratil, G. A., *Nuclear Fusion* **29** (1989) 423.

FIGURE CAPTIONS

- Fig. 1. Shapes of magnetic flux surfaces for equilibria in the second stability region: (a) Circular cross section, $\langle\beta\rangle = 1.11$; (b) Elliptical cross section, $\langle\beta\rangle = 1.03$; (c) *D*-shaped cross section, $\langle\beta\rangle = 0.80$.
- Fig. 2. Pressure and q profiles for the configurations of Fig. 1: (a) $q(\Psi)$ for circular cross section; (b) $q(\Psi)$ for elliptic cross section; (c) $q(\Psi)$ for *D*-shaped cross section; (d) $p(\Psi)$ for all cross sections.
- Fig. 3. Ballooning mode eigenvalue as a function of $\langle\beta\rangle$.
- Fig. 4. Ballooning mode eigenvalue as a function of $\epsilon\beta_p$.
- Fig. 5. Ballooning mode unstable surfaces for different values of $\langle\beta\rangle$ for tokamaks with (a) a circular cross section, (b) an elliptic cross section, and (c) a *D*-shaped cross section. The central point denotes the surface with the largest eigenfunction.
- Fig. 6. (a) Fourier components of ξ_ψ , the component of the instability eigenfunction perpendicular to the magnetic surfaces, for a free-boundary $n = 1$ mode in a circular tokamak with $\langle\beta\rangle = 1.11\%$; (b) eigenfunction ξ . The growth rate of this mode, normalized to the time it would take a shear Alfvén wave to propagate across the minor radius is 1.7.
- Fig. 7. Fourier components of ξ_ψ , the component of the instability eigenfunction perpendicular to the magnetic surfaces, for a free-boundary $n = 1$ mode in an elliptic tokamak with $\langle\beta\rangle = 1.03\%$; (b) eigenfunction ξ . The growth rate of this mode, normalized to the time it would take a shear Alfvén wave to propagate across the minor radius is 2.0.
- Fig. 8. Fourier components of ξ_ψ , the component of the instability eigenfunction perpendicular to the magnetic surfaces, for a free-boundary $n = 1$ mode in a *D*-shaped tokamak with $\langle\beta\rangle = 0.39\%$; (b) eigenfunction ξ . The growth rate of this mode, normalized to the time it would take a shear Alfvén wave to propagate across the minor radius is 0.14.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

EXTERNAL DISTRIBUTION IN ADDITION TO UC-420

Dr. Frank J. Paoloni, Univ of Wollongong, AUSTRALIA
 Prof. M.H. Brennan, Univ Sydney, AUSTRALIA
 Plasma Research Lab., Australian Nat. Univ., AUSTRALIA
 Prof. I.R. Jones, Flinders Univ., AUSTRALIA
 Prof. F. Cap, Inst Theo Phys, AUSTRIA
 Prof. M. Heindler, Institut für Theoretische Physik, AUSTRIA
 M. Goossens, Astronomisch Instituut, BELGIUM
 Ecole Royale Militaire, Lab de Phys Plasmas, BELGIUM
 Commission-European, Dg-XII Fusion Prog, BELGIUM
 Prof. R. Boucqué, Rijksuniversiteit Gent, BELGIUM
 Dr. P.H. Sakanaka, Instituto Fisica, BRAZIL
 Instituto De Pesquisas Espaciais-INPE, BRAZIL
 Documents Office, Atomic Energy of Canada Limited, CANADA
 Dr. M.P. Bachynski, MPB Technologies, Inc., CANADA
 Dr. H.M. Skarsgard, University of Saskatchewan, CANADA
 Dr. H. Bernard, University of British Columbia, CANADA
 Prof. J. Teichmann, Univ. of Montreal, CANADA
 Prof. S.R. Sreenivasan, University of Calgary, CANADA
 Prof. Tudor W. Johnston, INRS-Energie, CANADA
 Dr. Bolton, Centre canadien de fusion magnetique, CANADA
 Dr. C.R. James, Univ. of Alberta, CANADA
 Dr. Peter Lukac, Komenského Univ, CZECHOSLOVAKIA
 The Librarian, Culham Laboratory, ENGLAND
 The Librarian, Rutherford Appleton Laboratory, ENGLAND
 Mrs. S.A. Hutchinson, JET Library, ENGLAND
 C. Mouttet, Lab. de Physique des Milieux Ionisés, FRANCE
 J. Radet, CEN/CADARACHE - Bat 506, FRANCE
 Ms. C. Rinni, Librarian, Univ. of Ioannina, GREECE
 Dr. Tom Muat, Academy Bibliographic Ser., HONG KONG
 Preprint Library, Hungarian Academy of Sciences, HUNGARY
 Dr. B. Das Gupta, Saha Inst of Nucl. Phys., INDIA
 Dr. P. Kaw, Institute for Plasma Research, INDIA
 Dr. Philip Rosenau, Israel Inst. of Tech, ISRAEL
 Librarian, Int'l Ctr Theo Phys, ITALY
 Prof. G. Rostagni, Istituto Gas Ionizzati Del CNR, ITALY
 Miss Clelia De Palo, Assoc EURATOM-ENEA, ITALY
 Dr. G. Grosso, Istituto di Fisica del Plasma, ITALY
 Dr. H. Yamato, Toshiba Res & Dev, JAPAN
 Prof. J. Kawakami, Atomic Energy Res. Institute, JAPAN
 Prof. Kyoji Nishikawa, Univ of Hiroshima, JAPAN
 Director, Dept. Large Tokamak Res. JAERI, JAPAN
 Prof. Satoshi Itoh, Kyushu University, JAPAN
 Research Info Center, Nagoya University, JAPAN
 Prof. S. Tanaka, Kyoto University, JAPAN
 Library, Kyoto University, JAPAN
 Prof. Nobuyuki Inoue, University of Tokyo, JAPAN
 S. Mori, JAERI, JAPAN
 H. Jeong, Librarian, Korea Advanced Energy Res Inst, KOREA
 Prof. D.I. Chol, The Korea Adv. Inst of Sci & Tech, KOREA
 Prof. B.S. Lilley, University of Waikato, NEW ZEALAND
 Institute of Plasma Physics, PEOPLE'S REPUBLIC OF CHINA
 Librarian, Institute of Phys., PEOPLE'S REPUBLIC OF CHINA
 Library, Tsing Hua University, PEOPLE'S REPUBLIC OF CHINA
 Z. Li, Southwest Inst. Physics, PEOPLE'S REPUBLIC OF CHINA
 Prof. J.A.C. Cebra, Inst Superior Tecnico, PORTUGAL
 Dr. Octavian Petrus, AL I CUZA University, ROMANIA
 Dr. Jan de Villiers, Fusion Studies, AEC, SO AFRICA
 Prof. M.A. Heilberg, University of Natal, SO AFRICA
 C.I.E.M.A.T., Fusion Div. Library, SPAIN
 Dr. Lennart Stenflo, University of UMEA, SWEDEN
 Library, Royal Institute of Tech, SWEDEN
 Prof. Hans Wilhelmson, Chalmers Univ of Tech, SWEDEN
 Centre Phys des Plasmas, Ecole Polytech Fed, SWITZERLAND
 Bibliotheek, Fom-Inst Voor Plasma-Fysica, THE NETHERLANDS
 Metin Durgut, Middle East Technical University, TURKEY
 Dr. D.D. Ryutov, Siberian Acad Sci, USSR
 Dr. G.A. Eliseev, Kurchatov Institute, USSR
 Dr. V.A. Glukhikh, Inst Electrophysical Apparatus, USSR
 Prof. O.S. Padichenko, Inst. of Phys. & Tech. USSR
 Dr. L.M. Kovrizhnykh, Institute of Gen. Physics, USSR
 Nuclear Res. Establishment, Julich Ltd., W. GERMANY
 Bibliothek, Inst. Für Plasmaforschung, W. GERMANY
 Dr. K. Schindler, Ruhr-Universität Bochum, W. GERMANY
 ASDEX Reading Rm, c/o Wagner, IPP/Max-Planck, W. GERMANY
 Librarian, Max-Planck Institut, W. GERMANY
 Prof. R.K. Janev, Inst of Phys, YUGOSLAVIA