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**The Application of the
J Integral to Fracture
under Mixed-Mode Loading**

Robert Allen Riddle
(PH.D. Thesis)

June 1981

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The Application of the J Integral
to Fracture under Mixed Mode Loading

By

Robert Allen Riddle

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DISSERTATION

Submitted in partial satisfaction of the requirements for the degree of

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OF THE

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..... Chairman Date
..... *SA*
..... *Jack Washburn*

.....

THE APPLICATION OF THE J INTEGRAL TO FRACTURE
UNDER MIXED MODE LOADING

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Ph.D.

Mechanical Engineering

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ABSTRACT

The calculation of the J integral proved to be a successful method for characterizing the stress and displacement fields around a crack tip under mixed mode loading. Separating the stress and displacement fields into their symmetric and antisymmetric parts, allowed a symmetric J integral due to the Mode I loading to be calculated, as well as an antisymmetric J integral value due to the Mode II loading. The ratio of these two J integral values, called J_I and J_{II} , defines the relative amount of Mode I to Mode II loading.

A computer program was written to determine the symmetric and antisymmetric J integral quantities, given the stresses and displacements at points along a specified path around a crack tip. The stress and displacement values were calculated for actual specimen geometries and material properties using finite element analysis. The stress intensity factors derived as a result of these J integral calculations were in excellent agreement with those calculated using

other established theoretical and numerical methods, for specimen geometries where these other solutions exist.

The compact shear specimen was chosen to be employed in this investigation into fracture under mixed mode loading. This specimen contains three loading holes, the load applied at the center hole being in the opposite direction to the load applied at the two outer holes. The compact shear specimen is fully symmetric about the loadline of the center hole, with two crack planes, halfway between the center and outer holes. As determined using finite element analysis and the J integral post processor, the J_I and J_{II} values at the crack tips of the compact shear specimen were very sensitive to the boundary conditions employed in the analysis. Depending on the loading direction and boundary conditions, the deformation at the crack tip could be either mostly Mode II or Mode I.

For 7075-T6 aluminum it was found that K_{IIc} was 1.9 times larger than K_{Ic} . In the brittle photoelastic material K_{IIc} was less than K_{Ic} . The failure of the 4330V steel compact shear specimens came as a result of the average shear stress in the region ahead of the crack tip exceeding the material flow shear stress, so that information for this material on the ratio of the Mode II to Mode I fracture toughness values was not obtained.

The experimental results suggest that the angle of crack growth is best predicted by the maximum tangential stress theory for brittle materials, but by the maximum J integral theory for ductile materials.

For ductile materials, there is reason to predict that, in general, fracture toughness values for Mode II loading will be larger than those for Mode I loading. Such a general statement for brittle materials cannot be made.

DEDICATION

* To my wife *

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THE APPLICATION OF THE J INTEGRAL TO FRACTURE
UNDER MIXED MODE LOADING

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I. INTRODUCTION

Fracture describes a particular way in which a material component may fail. This kind of failure is characterized by the growth of a crack or flaw in a part under the application of load. The crack grows until the part is not strong enough or stiff enough to fulfill its intended function.

In the fracture process, there are three basic loading modes as seen in Figure 1-1. These loading modes are defined by reference to the resulting crack surface displacements. Mode I is the crack opening mode, Mode II is the in-plane shear, or crack-sliding mode; and Mode III is the anti-plane shear mode.

Although many analytical studies have dealt with Mode III loading because of its simplicity, the vast majority of the reported fracture tests have been for Mode I loading. Without denying the great importance of Mode I fracture testing, there are many situations in which mixed mode loading is of concern.

The aircraft industry has shown interest in mixed mode fracture (1), (2), where it has been reported that the shear webs in aircraft structures experience nearly pure Mode II loading conditions (3).

Another example of mixed mode fracture is in angle-ply composite laminates, where, because of the material structure, both opening and sliding modes occur even in pure tensile loading (4).

Mode II fracture initiations have also been observed to occur in frangible highway luminaire supports during impact loading (5).

Lack of full penetration in welds is often a cause of subsequent mixed mode fracture. Stepjoint welds in spherical and cylindrical containers often retain unfused surfaces which may be subject to mixed mode loading when the containers are pressurized.

Another cause of mixed mode fracture in pressure vessels comes from changes in loading directions during the vessel's history. During low level cyclic loading, cracks may form with a certain orientation. Later when the vessel is at full pressure the large primary stress may be at an angle to these prior fatigue cracks.

An interesting example of the creation of a mixed mode crack comes from the arrest of a running crack in a brittle material. When a propagating crack in such a material reaches terminal velocity, the crack typically branches, momentarily decelerates, and then accelerates to a terminal velocity where it will branch again (6). The crack branches will be the sites of mixed mode fracture under subsequent loading.

Mixed mode fracture is also important in the failure of rock and graphite (7).

These examples show that the study of fracture under mixed mode loading has a wide range of application. The purpose of this investigation is, therefore, not only to aid in more complete understanding of the theory of fracture processes, but also to provide information useful in the day-to-day solution of fracture problems.

The mixed mode fracture considered in this investigation is a result of Mode I and Mode II loading. This is general in-plane loading, where the loading is constant through the thickness.

In addition to considering loads which are constant through the thickness, the analytical methods employed in this study predict deformation which is constant through the thickness. In particular, the analysis of loads and the subsequent deformation of parts containing cracks is based on plane strain, where the strain in the x direction (referring to Figure 1-1) is zero. The fracture process near the crack tip invariably includes complex three-dimensional deformation in this region, but the simplification to two-dimensional deformation makes the analytical task much more manageable, and is close to the real situation, when the component or part under study is reasonably thick.

A restriction on the scope of this study will be that the investigation of fracture under mixed mode loading will be focused on the initial measurable crack growth under a single load cycle. Thus, fatigue failure, where the crack grows a measurable amount only under the application of tens of thousands of load cycles, is not considered. Also, what happens after the crack begins to grow will receive only minor consideration.

Historically, there have been several different approaches used in attempting to solve the basic problem of fracture mechanics; that is, to predict the load at which a crack will begin to grow, and by what amount it will extend.

An important step forward in understanding the process of crack advance came in the work of Griffith (8), who used an energy method to predict the load level for crack growth.

The energy method is based on a fundamental axiom of solid mechanics, that the equilibrium configuration of a deformable body is that for which the potential energy is a minimum. The energy approach considers the question, for a given component with a crack, of the load at which it becomes energetically favorable for the crack to extend.

The energy approach is based on the assumption that there is some minimum energy required, or work which must be expended, in the creation of the new surface area which accompanies crack growth. The energy method balances the change in the work done on the body by the external forces and the strain energy stored in the body, with the energy required for the crack to advance.

Stated mathematically,

$$G = \frac{-\partial Pe}{\partial a}$$

where the energy available for crack extension G , per unit increase in crack area, is the negative rate of change of potential energy with respect to increasing crack length. The potential energy is the increase in strain energy in the body minus the work done by the external forces due to the presence of the crack. Because G has units of force per inch of thickness it may also be referred to as the crack driving force.

When the relationship between the deflection of a component and the load, i.e., the compliance, is linear, it is possible to derive a simple equation for the energy available for crack growth in terms of the change in compliance with increasing crack length.

At the load where the energy available for crack growth reaches a critical value, the crack will begin to grow.

The stress intensity approach to fracture mechanics came from solving the equations of elasticity for a semi-infinite region containing a sharp crack. Muskhelishvili (9) first solved these equations in a general form, although the results of his efforts were not widely known until others had independently obtained crack tip solutions in a similar, but more restricted form. Westergaard (10) obtained a solution using complex variable techniques, while William's solution (11) to the crack problem takes the form of an eigenequation.

For both solutions, taking only the lowest order terms of the eigenequation, the expressions for the stresses and displacements around the crack tip for Mode I and Mode II loading in plane strain are: (The specification of the coordinates is different from that commonly used. The reason for naming the coordinates as done here is to be compatible with the computer program used later in this investigation.)

Mode I

$$\sigma_{yy} = \frac{K_I}{(2\pi r)^{1/2}} \cos \frac{\theta}{2} \left[1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right]$$

$$\sigma_{zz} = \frac{K_I}{(2\pi r)^{1/2}} \cos \frac{\theta}{2} \left[1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right]$$

$$\tau_{yz} = \frac{K_I}{(2\pi r)^{1/2}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}$$

$$\sigma_{xx} = \nu (\sigma_{yy} + \sigma_{zz}) \quad \tau_{xy} = \tau_{zx} = 0$$

$$v = \frac{2(1+\nu)K_I}{E} \left[\frac{r}{2\pi} \right]^{1/2} \cos \frac{\theta}{2} \left[1 - 2\nu + \sin^2 \frac{\theta}{2} \right]$$

$$w = \frac{2(1+\nu)K_I}{E} \left[\frac{r}{2\pi} \right]^{1/2} \sin \frac{\theta}{2} \left[2 - 2\nu - \cos^2 \frac{\theta}{2} \right]$$

$$u = 0$$

Mode II

$$\sigma_{yy} = \frac{-K_{II}}{(2\pi r)^{1/2}} \sin \frac{\theta}{2} \left[2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right]$$

$$\sigma_{zz} = \frac{K_{II}}{(2\pi r)^{1/2}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}$$

$$\tau_{yz} = \frac{K_{II}}{(2\pi r)^{1/2}} \cos \frac{\theta}{2} \left[1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right]$$

$$\sigma_{xx} = \nu (\sigma_{yy} + \sigma_{zz}) \quad \tau_{xy} = \tau_{zx} = 0$$

$$v = \frac{2(1+\nu)K_{II}}{E} \left[\frac{r}{2\pi} \right]^{1/2} \sin \frac{\theta}{2} \left[2\nu - 2 + \cos^2 \frac{\theta}{2} \right]$$

$$w = \frac{2(1+\nu)K_{II}}{E} \left[\frac{r}{2\pi} \right]^{1/2} \cos \frac{\theta}{2} \left[-1 + 2\nu + \sin^2 \frac{\theta}{2} \right]$$

$$u = 0$$

where the coordinate directions y , z , and x as well as r and θ are given in Figure 2. Also, ν is Poisson's ratio and σ_{yy} , σ_{zz} , and

σ_{xx} are the normal stresses in the direction of the subscript, while v , w , and u are the corresponding displacements. τ_{yz} , the only non-zero shear stress in plane strain, is the stress on the y face in the z direction (the y face being that face normal to the positive y direction vector). K_I and K_{II} are the stress intensity factors for Modes I and II.

Irwin (12), extended the energy method of Griffith, and later was able to show equivalence between the energy and stress intensity methods.

The equivalence between the energy method and the stress intensity approaches may be stated in the form

$$G_I = \frac{K_I^2 (1-\nu^2)}{E}$$

and

$$G_{II} = \frac{K_{II}^2 (1-\nu^2)}{E}$$

where ν is Poisson's ratio as before and E is Young's modulus of the material.

These equations relate the amount of energy available for crack growth in Modes I and II to the appropriate stress intensity factors. Implicit in these relationships is the idea of self similar crack growth. That is, this is the energy available for crack growth, assuming that the crack grows as a simple extension of its previous configuration. The possibility of the crack kinking, or changing directions as it grows, lies outside the range of applicability of these equations.

Both the energy method and the stress intensity approach contain the mathematical anomaly of infinite stresses at the crack tip. This stress singularity is a fiction of the continuum elasticity theory used to obtain the stress solutions, but has a useful interpretation.

The effect of the stress singularity is that for any load infinite stresses are predicted at the crack tip. However, for any finite distance away from the crack tip, the stresses are finite, and their magnitude depends on the geometry of the component with the crack and on the applied loading. How rapidly these stresses ascend to infinity as the distance to the crack tip decreases is a measure of the strength, or magnitude, of the singularity. In mixed mode loading K_I and K_{II} quantify the strength of the stress singularities.

Because the energy method and stress function solutions are based on elasticity theory, they are limited in application to situations where the load versus deflection record of a component is essentially linear up to the onset of crack growth. Exactly how much nonlinearity is permissible for the experimental determination of K_{IC} , the critical stress intensity for crack growth in Mode I, is the subject of a standard of the American Society for Testing and Materials (ASTM) (13). The study of elastic behavior to the onset of crack growth in components of various materials is the realm of Linear Elastic Fracture Mechanics (LEFM), and encompasses a large and widely used body of knowledge.

When the region of plastic deformation at the crack tip becomes large, the load versus deflection curve of a component becomes increasingly non-linear. Then the energy dissipated in plastic

deformation becomes a larger and larger percentage of the stored strain energy in the load and component system. The concepts of LEFM then no longer apply, and methods of analysis for elastic-plastic fracture mechanics need to be applied.

The J integral, as introduced by Rice (14), has found wide application as a method to characterize the singularity of a crack tip in elastic-plastic fracture mechanics. One outstanding feature of the J integral is that for elastic material behavior, it equals exactly the energy available for crack growth, G, and it is, therefore, also directly related to the stress intensity factor approach. Because the J integral extends the compliance testing method of LEFM to nonlinear materials, it lends itself to straightforward techniques of experimental measurement, at least for Mode I crack growth (15).

Chapter Two of this dissertation explores the value of the J integral as a ductile fracture parameter, particularly as it applies to Mode II and mixed mode loading.

One difference in mixed mode fracture, as contrasted to pure Mode I or Mode II fracture, is that two quantities are needed to characterize the stress singularity at the crack tip. In Chapter Two various candidates for these two parameters which describe mixed mode fractures are discussed. The parameters chosen are the J integral and a mixity parameter which expresses the ratio of Mode I to Mode II loading.

It became clear for mixed mode fracture that the J integral and mixity parameter as a function of load on a component were most easily calculated using finite element analysis. Chapter Three contains an

explanation of the determination of the J integral and the mixity parameter using the stresses and displacements around the crack tip as calculated by this numerical technique.

In Chapter Four various possibilities are presented for specimen designs for Mode II and mixed mode fracture, the thinking behind the choice of specimen for this study is explained, and values of the J integral and mixity parameter for various loading configurations of the specimen are given.

Chapter Five outlines the materials chosen for the mixed mode fracture investigation, gives the results of mechanical tests to determine their relevant mechanical material properties, and shows the results of the mixed mode fracture toughness tests.

Chapter Six contains an assessment of the failure surface produced by the mechanical tests, using the scanning electron microscope as the imaging technique.

In Chapter Seven failure criteria are discussed for mixed mode fracture.

Chapter Eight contains the author's opinion of the significance of the results obtained in this investigation, and considers what future efforts might be undertaken to expand our knowledge of fracture under mixed mode loading.

II. THE J INTEGRAL

The increasing acceptance of the J integral as a ductile fracture parameter has led to a careful scrutiny of its theoretical basis in solid mechanics. Since Rice's first introduction of the J integral, it has been realized that it is part of the fundamental conservation laws of solid mechanics.

Using the coordinates of Figure 2-1, the J integral, as originally defined by Rice (14), is

$$J = \int_S W dz - \tilde{T} \cdot \frac{\partial \tilde{u}}{\partial y} ds$$

where S is a curve surrounding the crack or notch tip, and the integral is evaluated along the curve in the counter-clockwise direction. W is the strain energy density

$$W = \int_0^{\epsilon} \sigma_{ij} d\epsilon_{ij}$$

Here ϵ_{ij} is the infinitesimal strain tensor and σ_{ij} is the Cauchy stress tensor. The body with the crack is assumed to be homogeneous and subject to a two-dimensional deformation field.

$\tilde{T}$ is the traction vector, whose components are $T_i = \sigma_{ij} n_j$ (using the convention of summation over repeated indices) and $\tilde{n}$ is the outward normal vector to curve S , whose increment is ds . $\tilde{u}$ is the displacement vector.

Rice showed for both linear and nonlinear elastic materials that the J integral is path independent. That is, for all paths in the material beginning at the lower crack surface and ending at the upper crack surface, the J integral evaluated along any such path has a constant value. Further, for any closed path in the body, the J integral evaluated around this path has a zero value. For the path shown in Figure 2-1, the portions of the path at the crack surface have zero value to the integration because dz and T_{ij} are both zero for these segments. This, then, implies that the value of the J integral from the inner path traversed in a clockwise direction is equal but opposite in sign to the J integral value of the outer path taken in a counter-clockwise direction. Therefore, changing the direction of integration changes the sign of the integral.

For a set of curves such as S , which are at all points a finite distance away from the crack tip singularity, the J integral has a constant value for each curve, taken in the counter-clockwise direction, and, therefore, may be considered as a measure of the far field effect of the singularity.

In the context of this investigation into fracture under mixed mode loading, where the fracture event occurs under a single application of load to the onset of crack growth, the elastic-plastic behavior of a material may be modelled using the deformation theory of plasticity, provided that the principal stresses in the material remain in fixed proportion during the course of the loading history (16,17). Since the deformation theory of plasticity is equivalent to

nonlinear elasticity, the results derived for linear and nonlinear elastic materials also apply to the elastic-plastic behavior of ductile metals, if the assumptions of deformation theory of plasticity are satisfied (18).

By proving that the value of the J integral equals the negative rate of change in potential energy of a body with respect to crack length

$$J = \frac{-\partial Pe}{\partial a} \quad (2-2)$$

for both linear and nonlinear elastic materials, Rice was able to extend the compliance testing methods of LEFM to nonlinear materials (19). Several methods for testing the fracture toughness of elastic-plastic materials have been developed based on these compliance methods, but they are all solely concerned with Mode I self-similar crack growth.

Theoretical studies have been undertaken to investigate the fundamental mathematical characteristics of the J integral in the context of continuum mechanics. Knowles and Sternberg (20), as well as Chen and Shield (21), have shown that the J integral is related to fundamental conservation laws of finite strain, three-dimensional elastostatics. Specifically, it was shown that the J integral, as introduced by Rice, is one component of a more general three-dimensional vector quantity.

$$J_k = \int_S \left(w n_k - T_i \frac{\partial u_i}{\partial x_k} \right) ds \quad (2-3)$$

where k is the vector index taking the values 1, 2, and 3 -- corresponding to the vector quantities in the three coordinate directions, n_k represents the components of the outward normal of the path of integration, and W , T_i , u_i , x_i and ds are as defined before. This three-dimensional vector J_k was shown to be path independent for both finite and infinitesimal deformations. In addition to J_k , two other path independent integrals were obtained.

As clarified by Budiansky and Rice (22), these three path independent integrals may be related to changes in potential energy of a body containing a defect. Noting that this defect may be either a crack or a cavity in the material, the three-dimensional J integral is the energy release rate when the crack tip or void is translated in position relative to the body.

This concept of the J integral as a three-dimensional vector giving the energy release rate as the crack tip advances in an arbitrary direction is very significant to the problem of fracture under two-dimensional mixed mode loading. Under this condition it may be energetically favorable for the crack to extend at an angle to its previous plane. This process known as crack kinking has been observed experimentally many times under mixed mode loading for a variety of materials (1,2,23).

It should be noted, however, that when the crack kinks, the formulation of the J integral around the crack tip acquires additional difficulties. A simple explanation of these difficulties may be found in re-examining the original assumptions in proving the path independence of the J integral. Since the J integral is zero around

any closed path, the two paths in Figure 2-1 have equal but opposite values only because the portion of the paths along the crack surfaces make a zero contribution. This is because dz and T_i are zero at the crack faces.

Consider now the kinked crack in Figure 2-2. If the reference coordinates are rotated with the crack kink so that dz and T_i are zero along the crack kink faces, then for paths along the axis of the original crack, although T_i is still zero, dz has a nonzero value, and the J integral paths ending in this region of the crack surface are no longer path independent. Only paths ending on the kink surfaces are path independent.

Because of this, the J integral may be thought of as the negative rate of change of potential energy with increasing crack length only if the crack extension involves no kinking -- that is, self similar crack growth. For cracks that kink, the new boundary conditions violate the assumptions made in proving that

$$J = \frac{-\partial Pe}{\partial a} .$$

The restriction in the interpretation of the J as an energy release to self-similar crack growth has been noted by several authors studying mixed mode fracture (24,25).

The relationship between J integral values and the energy release rate of kinking cracks is clear only if the corresponding paths of integration end at the kink surface. However this does not preclude the possibility of using the J integral vector to predict the onset of crack growth, whether kinked or not, under mixed mode loading.

For the purpose of this study, where the mixed mode loading is in plane, the J integral vector has two components, J_1 and J_2 , where J_1 is the J integral originally defined by Rice. Calculating J_1 and J_2 along paths surrounding the planar crack allows one to predict the direction of the crack driving force. However, if the crack extends with a kink, the energy release as predicted by the previous calculation of the J integral for the straight crack will differ from the actual energy release due to the formation of the kink.

The idea of a crack extending at an angle to its original axis compels one to think of a crack in a different perspective. In this new perspective, the crack tip singularity is an entity separate from the rest of the crack, and able to move or translate independently of the rest of the crack. This crack tip singularity might represent not only the physical crack tip but also a group of dislocations, a cavity, or other defect in this highly stressed region.

This way of looking at the crack tip as a separate entity from the rest of the crack, is reinforced by the physical interpretation of the two other path independent integrals, denoted L and M by Budiansky and Rice.

The L integral is a three-dimensional vector which predicts the energy release rate as a cavity or crack tip defect rotates about the three coordinate axes. The M integral is a scalar which predicts the energy release rate as the cavity or crack tip defect expands uniformly.

The concept of a vector quantity, such as the J integral, representing the force on a defect, causing the defect to translate through a continuum lattice, existed prior to the introduction of the J integral to fracture mechanics.

The integrand of the three-dimensional J integral is mathematically identical to the energy-momentum tensor of Eshelby (26,27). The more general concept of Eshelby predated its more specific application in the form of the J integral by many years, though Rice was the first to apply it in solving fracture mechanics problems (28).

Eshelby describes the energy-momentum tensor as the force on a defect in a crystalline lattice, the defect being a dislocation, vacant lattice site, or an impurity or interstitial atom; the force being defined as the negative rate of increase of the total energy of the lattice system as the location of the defect is varied.

The J integral has found application in analyzing the diffusion of atoms in a stressed crystalline lattice (29), and in analyzing the force on a field of continuously distributed dislocations (30).

An example of a further generalization of the concept of J integral is contained in the work of Strifors (31). He derived a force, based on energy methods which tends to cause crack extension in solids under generalized loading. The crack singularity in the deformable solid is described by a surface of discontinuity or cohesive zone at the crack tip. The motion of particles in the cohesive zone may be discontinuous -- that is, a single point in the cohesive region of the underformed body may be mapped into two different points a finite distance apart in the deformed configuration. This discontinuous motion provides the possibility of crack growth in the solid, and underlines the concept of the crack tip as a unique region, allowing for the possibility of extension, expansion, and rotation of the crack tip region, without the entire crack doing so.

The crack extension force of Strifors is derived for very general circumstances, allowing for dynamic, heat flow, and surface energy effects, as well as mechanical loading. For the circumstances of interest in this investigation, plane strain and infinitesimal deformation, quasistatic loads, and no heat flux effects -- the generalized crack extension force reduces to

$$f_k = \int_S W(\cos \theta \, dz - \sin \theta \, dy) - \left(\cos \theta \frac{\partial u_i}{\partial y} + \sin \theta \frac{\partial u_i}{\partial z} \right) T_i \, ds \quad (2-4)$$

This may be shown to be equivalent to the J integral vector in two dimensions.

Referring to the coordinates in Fig. 2-3, where ξ is the direction of the crack extension force f_k , and n_k is the component of the outward normal of the path of integration in direction ξ , it follows that $n_k = n_1 \cos \theta + n_2 \sin \theta$, where n_1 and n_2 are the components of the path normal vector in the y and z coordinates.

Furthermore, $dz = n_1 ds$ and $-dy = n_2 ds$

$$\text{and} \quad \frac{\partial u_i}{\partial x_k} = \frac{\partial u_i}{\partial \xi}$$

Using the chain rule

$$\frac{\partial u_i}{\partial \xi} = \frac{\partial u_i}{\partial y} \frac{\partial y}{\partial \xi} + \frac{\partial u_i}{\partial z} \frac{\partial z}{\partial \xi}$$

$$\text{also } \frac{\partial y}{\partial \xi} = \cos \theta \quad \text{and} \quad \frac{\partial z}{\partial \xi} = \sin \theta$$

$$\text{then } \frac{\partial u_i}{\partial \xi} = \cos \theta \frac{\partial u_i}{\partial y} + \sin \theta \frac{\partial u_i}{\partial z}$$

making these substitutions into the general crack extension force one obtains

$$f_k = \int_S W(\cos \theta n_1 ds + \sin \theta n_2 ds) - \frac{\partial u_i}{\partial \xi} T_i ds \quad (2-5)$$

and collecting terms and further simplifying

$$f_k = \int_S \left(W n_k - T_i \frac{\partial u_i}{\partial x_k} \right) ds$$

and therefore $f_k = J_k$.

Further interesting results may be obtained by incorporating the elastic singularity terms for stresses and displacements in the vicinity of the crack tip, as given in Chapter 1, into a J integral vector calculation for a circular path.

For the component of the J integral in the y direction, assuming the crack lies in the same direction and self-similar crack growth, one obtains the equations given by Rice (14) for mixed mode loading,

That is,

$$J_1 = \int_S W dz - T_i \frac{\partial u_i}{\partial y} ds \quad (2-6)$$

$$= \frac{K_I^2 + K_{II}^2}{E} \frac{(1 - \nu^2)}{E} \quad (2-7)$$

where S is a circular path around the crack tip.

For the crack driving force in the direction perpendicular to self-similar crack growth one obtains

$$J_2 = \int -W dy - T_i \frac{\partial u_i}{\partial z} ds \quad (2-8)$$

$$= -2K_I K_{II} \frac{(1 - \nu^2)}{E} \quad (2-9)$$

For mixed mode loading, the crack extension driving force at an arbitrary angle θ equals $J_\theta = J_1 \cos \theta + J_2 \sin \theta$ (2-10)

and

$$J_\theta = \frac{(1 - \nu^2)}{E} \left[(K_I^2 + K_{II}^2) \cos \theta - 2K_I K_{II} \sin \theta \right] \quad (2-11)$$

This is exactly equivalent to equation 9 of Bergkvist and Guex, who evidently derived the equation using the generalized crack extension force of Strifors (32).

Note that by substitution of $K_I=0$ or $K_{II}=0$ in equation 2-11 the maximum crack driving force for either pure Mode I or Mode II loading is in the direction of self-similar crack growth. This detail will be discussed at some length in the Chapter on failure criteria for fracture under mixed mode loading.

A significant difference between self-similar crack growth in Mode I versus Mode II loading, for elastic material behavior, is found upon closer investigation into the derivation of equation (2-7). For Mode I loading and the circular path of integration going counter clockwise around the crack tip

$$J_I = r \int_{-\pi}^{\pi} (W \cos \theta - T_i \frac{\partial u}{\partial y}) d\theta \quad (2-12)$$

and calculating the separate quantities in the integral

$$r \int_{-\pi}^{\pi} W \cos \theta d\theta = K_I^2 \frac{(1 + \nu)}{E} \left[-\frac{\nu}{2} + \frac{1}{4} \right] \quad (2-13)$$

and

$$r \int_{-\pi}^{\pi} -T_i \frac{\partial u}{\partial y} d\theta = K_I^2 \frac{(1 + \nu)}{E} \left[-\frac{\nu}{2} + \frac{3}{4} \right] \quad (2-14)$$

While for Mode II loading the corresponding quantities are:

$$r \int_{-\pi}^{\pi} W \cos \theta d\theta = K_{II}^2 \frac{(1 + \nu)}{E} \left[\frac{\nu}{2} - \frac{1}{4} \right] \quad (2-15)$$

and

$$r \int_{-\pi}^{\pi} -T_i \frac{\partial u}{\partial y} d\theta = K_{II}^2 \frac{(1+\nu)}{E} \left(\frac{-3}{2} + \frac{5}{4} \right) \quad (2-16)$$

Addition of these four terms to form J_1 yields equation (2-7).

Now consider yet another slightly different interpretation of the J integral. While Griffith performed an energy balance over the entire body to derive the energy release rate with crack extension, it is possible to perform the same energy balance over a control volume near the crack tip and thereby derive the J integral, as shown by Finnie (33).

Giving this slightly different interpretation to the equality

$$\frac{-\partial Pe}{\partial a} = \int_S (W dz - T_i \frac{\partial u_i}{\partial y} ds) \quad (2-17)$$

The integral

$$\int_S W dz$$

gives the decrease in strain energy as the crack advances, and

$\int_S T_i \frac{\partial u_i}{\partial y} ds$ gives the work done on the material inside the control volume as the crack advances.

Since Poisson's ratio, ν , is always less than 1/2 for elastic material behavior, the strain energy of the region surrounding the crack tip always decreases as the crack advances in Mode I self-similar crack growth. In Mode II, however, the advance of the crack tip always increases the strain energy stored in the material region surrounding the crack tip.

This increase in stored energy as the crack advances in Mode II tends to increase the necessary work done on the material to continue the process of crack advance, and hence reduces the energy available for crack growth. Mathematically this may be stated as

$$G \Delta a = \left[J \Delta a - \int_0^{\Delta a} \left(\int_S W dz \right) da \right]_{II} \quad (2-18)$$

for a material of unit thickness in Mode II self-similar crack growth.

Only the singularity terms need to be included in the calculation of the change of potential energy with increasing crack length no matter what the size the control volume, because all the effects of the higher order terms in the stress and displacement eigen-functions integrate to zero. This is a necessary conclusion from the path independence of the J integral, as only those terms in which the product of stress and strain is proportional to the inverse of the distance from the crack tip - - the singular terms - - will contribute a path independent value to the J integral, as is evident from inspection of equation (2-12).

For plastic material behavior near the crack tip, the stresses and displacements are no longer satisfactorily described in terms of the singularity solution for elasticity, as given in Chapter 1.

Hutchinson, and Rice and Rosengren, concurrently but independently, solved for the stresses and displacements around the crack tip when there is plastic material behavior in this region (17, 18). Their solutions are equivalent, and the region near the crack tip where these analyses are valid is termed the HRR field.

In the HRR field, as in the corresponding elastic solution, the product of stress and strain varies as the inverse of the distance from the crack tip, so that the plastic singularity terms contribute in precisely the same manner as the elastic singularity terms in the calculation of the J integral for a path an arbitrary distance away from the crack tip.

For a material which deforms in simple tension according to the constitutive relation

$$\epsilon^P = \alpha \left(\frac{\sigma}{\sigma_0} \right)^{n-1} \frac{\sigma}{E}$$

where α is a material constant, σ_0 is the tensile yield stress of the material, E is again Young's Modulus, n is the strain-hardening exponent, and ϵ^P and σ are the uniaxial plastic strain and stress, then the stresses and strains in the HRR field have the form, in polar coordinates:

$$\sigma_{ij} \sim r^{-1/(n+1)} \tilde{\sigma}_{ij}(\theta)$$

and

$$\epsilon_{ij} \sim r^{-n/(n+1)} \tilde{\epsilon}_{ij}^P(\theta)$$

The functions for the angular variation of stress and strain for Mode I, Mode II, and mixed mode loading, in plane strain, were obtained numerically by Hutchinson and Shih (17,34,35).

Because the functions for the angular variation of stress and strain are independent of the magnitude of the singularity at the crack tip, the HRR field is proportional. This infers that in the plastically deformed region near the crack tip the normal stresses remain in fixed proportion during the load history, and the modelling of the elastic-plastic material behavior in the region using the deformation theory of plasticity is credible.

The singularity solutions for both elastic and plastic material behavior specify the stresses and displacements in a small crack tip region. A complete closed form analytical solution for a crack in a solid which includes both singular and non-singular terms, and which is necessary to determine the extent of the region in which the singularity terms predominate, does not exist.

In the crack tip region under mixed mode loading two parameters are needed to uniquely specify the stress and displacement fields. For elastic material behavior, these two parameters might appropriately be either the J integral and a ratio of the amount of Mode II to Mode I loading, or simply K_I and K_{II} . Equation (2-11) shows that these two methods of specifying the stress and displacement fields are equivalent.

For elastic-plastic material behavior, several options are available in choosing the set of two parameters needed to characterize the mixed mode loading.

One set of parameters suggested by Shih (34,35), is the J integral and a mixity parameter

$$M^P = \frac{2}{\pi} \tan^{-1} \lim_{r \rightarrow 0} \left[\frac{\sigma_{\theta\theta}(r, \theta=0)}{\tau_{r\theta}(r, \theta=0)} \right]$$

Shih's mixity parameter cannot be experimentally determined, but must be calculated using some numerical technique based on the combined mode load on the component of interest.

In making preliminary attempts to calculate this mixity parameter using finite element techniques, it became quickly clear that a detailed modelling of the crack tip region was necessary to obtain accurate values of M^P . In particular it was found that the M^P calculated was dependent on the zone size used in the crack tip region.

Shih overcame this difficulty in his finite element analysis of the crack tip region by using singularity elements and by assuming the HRR field solutions in a small region around the crack tip. An objection to this type of solution is that one must assume in advance the extent of the region where the HRR field stresses and displacements accurately depict the total solution.

Another set of parameters which characterize the crack tip singularity under mixed mode loading for elastic-plastic material behavior may be derived from results obtained by Ishikawa, Kitagawa, and Okamura (36). Ishikawa and colleagues present a straightforward analytical method to calculate a mixity parameter based on the symmetric and antisymmetric portions of the stress and displacement fields surrounding a crack tip.

The symmetric and antisymmetric stress and displacement fields correspond to the Mode I and Mode II loading contributions. The Mode I and Mode II stress and displacement fields are calculated using the following relations:

$$\sigma_{ij} = \sigma_{ij}^I + \sigma_{ij}^{II}$$

$$\begin{Bmatrix} \sigma_{11}^I \\ \sigma_{22}^I \\ \sigma_{12}^I \end{Bmatrix} = 1/2 \begin{Bmatrix} \sigma_{11} + \sigma_{11}' \\ \sigma_{22} + \sigma_{22}' \\ \sigma_{12} - \sigma_{12}' \end{Bmatrix}$$

and

$$\begin{Bmatrix} \sigma_{11}^{II} \\ \sigma_{22}^{II} \\ \sigma_{12}^{II} \end{Bmatrix} = 1/2 \begin{Bmatrix} \sigma_{11} - \sigma_{11}' \\ \sigma_{22} - \sigma_{22}' \\ \sigma_{12} + \sigma_{12}' \end{Bmatrix}$$

and for the displacements

$$u_i = u_i^I + u_i^{II}$$

$$\begin{Bmatrix} u_1^I \\ u_2^I \end{Bmatrix} = 1/2 \begin{Bmatrix} u_1 + u_1' \\ u_2 - u_2' \end{Bmatrix}$$

$$\begin{Bmatrix} u_1^{II} \\ u_2^{II} \end{Bmatrix} = 1/2 \begin{Bmatrix} u_1 - u_1' \\ u_2 + u_2' \end{Bmatrix}$$

Here the primed quantities are those values of stress and displacement at point P' which is the symmetrical about the $y=x_1$ axis to point P where the unprimed quantities are taken.

Using these Mode I and Mode II stress and displacement fields the J integral quantities, J_I and J_{II} , are calculated, where these are both J_I integral quantities for Mode I and Mode II deformation, and not to be confused with J_1 and J_2 , the J integral vector components in the axial directions.

Ishikawa and colleagues also prove that the J integral quantities J_I and J_{II} are path independent, and that

$$J = J_I + J_{II} \quad (2-19)$$

Therefore, knowing any two of the quantities in equation 2-19 allows one to calculate the third, and the stress and displacements fields around the crack tip may be thereby uniquely characterized.

Just as the J integral value is a far field effect of the singularity at the crack tip, J_I and J_{II} are calculated for paths a finite distance away from the singularity, and do not require for finite element numerical calculation a detailed modelling of the crack tip.

The calculation of J_I and J_{II} was chosen as the best way to characterize the crack tip singularity under mixed mode loading, given the computational tools and desired results of this investigation.

For the sake of completeness, two other methods to characterize crack tip loading conditions under mixed mode fracture will be mentioned, with only brief mention as to advantages and limitations.

Stern, Becker, and Dunham present another method of contour integral computation for the determination of mixed mode stress intensity factors (37), but the use of their integral is limited to elastic material behavior.

More empirical methods for characterization of loads on components with cracks, based on resulting deformation changes in the crack tip profile, exist and include the crack opening displacement (COD) and the crack opening angle (COA) methods. Although these methods are concerned solely with Mode I loading, the extension to mixed mode loading which would include measuring both crack opening and crack sliding displacements is definitely possible, and results of an experimental investigation into measuring crack surface relative motion under mixed mode loading exist (38). However, the experimental problems associated with such measurements appear to be very challenging.

III. J INTEGRAL CALCULATIONS.

Mathematical closed form solutions for stress singularities at crack tips as a function of load are available for only the most highly idealized situations, such as infinite plates. To determine stress singularities in components of a practical nature invariably requires numerical methods (39, 40).

A wide variety of numerical methods, such as finite difference, finite element, and boundary collocation, are available for solution of crack problems in solid mechanics. For ease of use and cost effectiveness the finite element method (FEM) was chosen for this investigation.

The FEM computer program used in all the numerical analyses of this investigation was NIKE2D (41). Other authors have already given fairly thorough accounts of various aspects of the computational techniques used in NIKE2D (42,43) so that only some general characteristics of the program will be mentioned here.

Elastic and elastic-plastic material behavior and finite deformations may be analyzed using NIKE2D. Slide lines, which model frictional sliding interfaces between material surfaces, represent an important capability of this FEM computer program (44). Four node isoparametric, constant strain, and compatible elements were used throughout. Although NIKE2D has dynamic response and time dependent material properties capability, only static analysis was performed in this work.

NIKE2D analyzes nonlinear material behavior by approximating it with a number of linearized steps to the final load. Because of this, when significant plasticity developed around the crack tip, a number of load steps preliminary to the final load were necessary in order to get the FEM solution to converge to within a prescribed error tolerance.

Within the extensive literature on finite element analysis applied to crack growth problems at least four methods of characterizing the crack tip singularity in terms of the applied loads are identifiable.

One method in FEM uses special elements near the crack tip to insure a $1/r^{1/2}$ singularity in the crack tip region (45, 46, 47, 37). A special singular element for mixed mode loading has also been developed (48). The disadvantage of these elements is that they lack constant strain and rigid body modes, and are not compatible, and hence do not insure convergence (49). The necessity for using these singular elements to obtain the $1/r^{1/2}$ singularity is disputed (50), and the equivalent effect is obtained by using 8 noded isoparametric elements with the midside node at the quarter point (51, 50, 49).

A second method for determining stress intensity factors for crack tips, particularly where plastic material behavior is expected, is to constrain displacements within a small crack tip region to conform to the HRR solution, and to use these displacements as a boundary condition for the outer elements (52, 34). This is the method used by Shih, as described in the previous chapter, to insure a singularity in the plastic region of the anticipated form (34).

A third method is used primarily for elastic material behavior. The compliance of the component is calculated as a function of crack length and load, the crack driving force G is calculated

$$G = \frac{p^2}{2B} \frac{dc}{da} ,$$

and the stress intensity factor is thereby indirectly found using equation (2-7) (53, 54, 55). The extension of this method to elastic-plastic material behavior, where J is calculated using the load verses deflection curve of the component, has also been documented (56).

Finally, the J integral, as directly calculated on contours surrounding the crack tip, has been used to characterize the stress singularity at the crack tip for both elastic and elastic-plastic material behavior (57, 58, 59, 60). Advantages of the J integral method to determine the elastic or elastic-plastic stress intensity are that only a single component configuration is considered, and that no extrapolations of the stress or displacement fields around the crack tip are necessary. McMeeking (58) calculated the J integral on essentially circular contours around the crack tip, and found the value of the integral to be path independent as long as the contour was not taken through elements immediately adjacent to the crack tip.

McMeeking also discusses at some length (58, 59) the fact that the J integral values are path dependent in the region immediately adjacent to the crack tip. However, path dependence of J near the crack tip should not be considered a serious obstacle in using FEM to calculate the J integral. One reason for this is that there are inherent inaccuracies in using the FEM to model the material behavior

in the region of large stress and displacement gradients near the crack tip with a limited number of mesh nodal points. For example in the Mode I stress singularity, where the singularity solution predicts that the stress perpendicular to the crack axis approaches infinity just ahead of the crack tip and drops to zero just behind the crack tip, the FEM solution predicts a finite stress ahead of the crack tip, and the stress behind the crack tip experiences overshoot-- that is, it drops to a negative value and then oscillates back to zero. Figures 3-1 and 3-2 are plots of the z direction normal and shear stress as functions of the number of elements from the crack tip, which show this oscillatory behavior near the crack tip, from an analysis of a compact tension specimen which will be discussed in further detail in later sections of this chapter. However, this non-physical material behavior predicted by FEM is limited to regions near the crack tip and other regions of large stress gradients, such as free surfaces, and for intermediate regions FEM predictions for stress and displacement are very accurate. Therefore, path dependence for J integral values for paths in elements adjacent to the crack tip is not surprising, and should be discounted in favor of J integral values and path independence for paths in regions away from the crack tip. McMeeking calculated that for contours at distances greater than $5J/\sigma_0$ away from the crack tip, the J integral evaluated along such contours will be invariant, although this distance will depend on mesh size.

It should be realized that no modelling of material behavior in the crack tip region based on continuum theory will duplicate the actual situation there. The microprocesses of fracture in this region include nonhomogeneous and anisotropic effects, such as dislocation

motion on preferred slip planes, dislocation-inclusion interactions and shear band formations. These effects, which will not be important in the far field material regions, will determine fracture behavior near the crack tip. The attempt of the continuum theory, as pursued in this investigation, is to characterize the complex fracture processes in the crack tip region by the applied stresses and displacements of the far field. For fracture under mixed mode loading, the symmetric and antisymmetric J integral quantities J_I and J_{II} describe the stresses and displacements of the far field, and are assumed to control the material behavior at the crack tip, even though a precise specification of exactly what happens there is not attempted.

With this view of the fracture process, a post-processor for NIKE2D was written which reads the stresses and displacements along paths surrounding the crack tip and calculates the quantities J , J_I and J_{II} . The NIKE2D output files, used as input to the post-processor, contain the stresses at the gaussian integration points for all elements and the displacements at the corresponding nodal points. Other information such as the coordinates of the nodal points and the element-nodal connectivity array is also contained in the output files, and is used in calculation of the J integral quantities. The information in the NIKE2D output files is organized so that for multiple time steps, all the information for a particular time step is contained in one file. This feature makes it easy to figure the file and location in the file where the needed stresses and displacements for a particular load level are to be found. The post

processor, named MMJINT, also requires an input file which specifies the angle of the crack with respect to the global cartesian coordinates, the elastic material properties, the load step, and a description of the path along which the J integral is to be calculated.

The fact that the J integral may be calculated for a crack at an arbitrary angle to the global coordinates also implies that the vector quantity J_k may be calculated at any angle to the crack. In connection with this it is interesting to note that the M integral of Chen and Shield (21) as applied in (61) is simply the mixed mode term of the Strifor's generalized J_k vector. It may be determined by this post processor by calculating J_k in a direction perpendicular to the crack axis, where the independent equilibrium states are the symmetric and anti-symmetric portions of the stress-strain field.

The contour along which the J integral is calculated consists of line segments connecting the selected gaussian integration points within the elements. The description of the paths includes the number of the path points, and the element number and gauss point number through which the path goes.

NIKE2D employs the displacement method where the displacements are calculated at the nodal points, but the stresses are calculated at the gauss points. It was decided to take the path through the gauss points where the stresses are known, and use the natural coordinates and shape functions of the elements to calculate the displacements at the gauss points from the nodal displacements. McMeeking's J integral method calculated the path through the elements's centroids-- but while this would smooth the variability of stresses with the element, it would also require the interpolation of not only the displacements

but also the stresses to values at the centroid (58). The approach of calculating the J integral values at the gauss point, while requiring less interpolation, requires more input as both the element and a gauss point must be specified. The two methods should be comparably accurate, and as indicated, the J integral quantities were calculated at the gauss points. Both of these methods should be superior to taking the J integral path through the nodal points, where the stresses from four elements would have to be interpolated and then averaged to obtain representative stresses.

In this investigation the J integral paths were always taken through regions of the component which were elastically deformed. This was always possible since in all the configurations analyzed there was no gross yielding of the cross-section. Since the J integral values are independent of the region of evaluation, no further essential information should be obtained by taking paths through the plastically deformed regions.

When the symmetric and antisymmetric J integral quantities are to be calculated, the path around the crack tip must be symmetric with respect to the crack axis. As explained in Chapter 2, not only quantities at path point P must be known, but also the corresponding stress and displacement values at point P', which is symmetric to P through the axis of the crack, in order for the symmetric and antisymmetric stress and displacement fields to be calculated. Clearly the most expeditious way to obtain the stress and displacement values at both P and P' is to have them both be points on the J integral path. Taking this required symmetry of the J integral path one step further infers that the finite element mesh in the region

where the J integral values are to be calculated must also be symmetric with respect to the crack axis. The achievement of symmetry about the crack for both the mesh and J integral path presents no serious difficulty.

The appendix is a listing of the MMJINT program statements in FORTRAN. Two examples were used to verify the accuracy and path independence of J integral values calculated using MMJINT. The first example involved a FEM analysis of a compact tension specimen. The mesh for this specimen is seen in Figures 3-3 and 3-4. The nodes lying between points AB and B are constrained to have zero deflection in the z direction. The load is applied between points P and O, with point OP constrained to have zero deflection in the y direction. The load is distributed along the nodes between P and O using a half-sine wave variation, point OP being the point of maximum load, points O and P having one-hundredth the maximum load. Point AB is the crack tip, the distance between A and AB modelling a 0.1 inch fatigue pre-crack. The distance between the load line IJ and the back of the specimen BC is 2.0 inches, and is the W dimension. The crack length a is the horizontal distance from IJ to AB and is 1.2 inches and therefore $a/w = 0.6$. There are 1455 elements and 1558 nodal points in the mesh, the smallest element near the crack tip having dimensions .006 by .0156 inch.

For a 1,060 lb. load on a steel specimen, the J integral was calculated along six paths shown in figure 3-5. The J values for each path is also shown in the figure, the variation in the J values for the 3 longest paths being less than 1% from the average, which is very

good. The variation for J integral values near the crack tip is somewhat larger, partly due to the numerical variation in using a fewer number of path points.

The average value of the J integral for these last three paths is 12.7 in-lb/in². This corresponds to a K_I value of 20.46 ksi-in^{1/2}. From reference 13 the predicted K_I for the same load and a/w ratio is 20.57 ksi - in^{1/2}. The less than 1% difference between the two values demonstrates the ability of the post processor to accurately predict K_I values, in agreement with those of this widely accepted standard.

A second analysis of the same compact tension specimen was performed, this time with a finer mesh near the crack tip-- the smallest element was .003 by .004 inches. For paths in the same region as the last three of the previous analysis, the variability in the J values was comparable, and the average predicted K_I value was 20.54 ksi - in^{1/2}. Hence, for paths some distance from the crack tip, the element zone size at the crack tip has a minimal effect on the calculated J values.

As a further check on the FEM analysis, the compliance of the compact tension specimen, as determined using the applied load and deflection at point J from NIKE2D output, was compared against an experimentally measured compliance. The experimental compact tension specimen was made of 4330V steel, and had the same dimensions as the specimen in the FEM analysis. The material properties used as input to NIKE2D were those of the 4330V steel. The finite element solution predicted a compliance for the compact tension specimen which was 4%

higher than that actually measured. This was felt to be excellent agreement. Also, the compliance as experimentally measured was within 1% of that calculated for a compact tension specimen of the same dimensions in the proposed J integral fracture toughness method of evaluation (15).

With the confidence that both NIKE2D and MMJINT were working well in predicting Mode I stress intensity values, a second example problem was solved to verify their use in mixed mode crack problems. A finite element mesh was constructed which modelled a 5 inch square plate with a 1 inch angled center crack. The crack was at an angle of 60 degrees to the vertical axis, measured clockwise in the first quadrant. Using NIKE2D, a biaxial load, with the horizontal load 30% of the vertical load, was applied and the stresses and displacements around the crack tips were calculated. Taking a path around the upper-right crack tip, MMJINT was used to calculate J , J_I and J_{II} . Using equation 2-7, the corresponding values $(K_I^2 + K_{II}^2)^{1/2}$, K_I , and K_{II} were calculated.

The values calculated using MMJINT was 1.7% higher than the theoretical solution, and the ratio K_I/K_{II} was 1% higher in the numerical than in the theoretical solution.

For essentially elastic material behavior it has been demonstrated that the use of NIKE2D and MMJINT allow one to accurately characterize the crack tip singularities for both Mode I and mixed mode loading.

IV. SPECIMEN DESIGN AND ANALYSIS

In contrast to fracture toughness testing in Mode I, where standard specimens and testing procedures are established or have been proposed (13,15), for fracture under mixed mode or Mode II loading, no such guidelines exist. As a result, a wide variety of mixed mode and Mode II specimen designs are reported in the literature, each specimen being adapted to the particular needs of the author.

These mixed mode and Mode II specimen designs include variations of the center cracked plate (1, 2, 3, 23, 24, 63, 64, 65, 66, 67), the single edge cracked plate (68, 69), the center cracked plate with concentrated shear loads acting at the crack surface (23), a cylindrical tube with circumferential through crack (23, 2), and the compact symmetric shear specimen (5, 70, 71, 72).

While the center cracked plate has some advantages in studying mixed mode fracture, as evidenced by the amount of use it has received, it has certain other features which make it inappropriate for the current investigation.

In this investigation the desire was to study both pure Mode II and mixed mode fracture, but pure Mode II loading in the center cracked plate requires the use of biaxial loading rams, and such equipment was not available.

Also, in order to fatigue pre-crack the notch machined in the center of the plate, a second set of loading holes are needed in this specimen, as in (2). For both the fatigue pre-crack loading holes, which would load the crack in pure Mode I to insure self-similar crack growth, and the primary load holes, which would allow the crack to be

loaded at the proper ratio of modes I and II, a significant distance between the holes and the crack tip is required. If the distance between the loading holes and the crack tip is too small, local loading effects, which are difficult both to control and analyze, may significantly affect the fracture processes at the crack tip.

In order to minimize the stress and displacement concentration effects of the loading holes on the crack tip fracture process, the distance between the crack tip and the loading holes should be several times the plate thickness, and probably several times the characteristic fracture distance

$$2.5 \left(\frac{K_{Ic}}{\sigma_0} \right)^2 ,$$

as discussed for similar situations in (56). Because of this the ideal center cracked plate fracture toughness specimen is large and thin. For a given material volume, the center cracked panel is more suited for testing in plane stress rather than plane strain conditions.

Plane strain fracture toughness values for fairly strong, ductile metals were of interest. In order to use modest amounts of material for specimens, and testing systems with moderate loading capability, the use of the center-cracked plate was avoided.

The possibility of using a variation of the center-cracked plate, a beam with a center crack, similar to that used in (63), was explored. For pure Mode II loading of this specimen, four point bending would be used, which would produce a pure bending moment in the cracked region of the beam.

Because the crack is at, and aligned with, the neutral axis of the beam, it is in a low stress region, and large moments would be required to produce the Mode II stress intensity factor necessary for crack growth. A FEM analysis of this specimen showed that for elastic material behavior, the effective stress in the crack tip region was less than one-half the stress in the outer fiber.

The compact shear specimen, reported in the literature to produce pure Mode II deformation, appeared to meet the requirements of providing plane strain fracture toughness, with a large ratio of K_{II} to applied load, and in a relatively small material volume. Straightforward variations of the specimen should be also capable of producing a continuous variation from pure Mode II to mixed mode to pure Mode I conditions.

The compact shear specimen, shown in Figure 4-1, is symmetric with respect to the centerline, and therefore only one-half of the specimen needs to be modelled in an analytic study. The appropriate symmetry boundary conditions are applied at the centerline.

The length of the compact shear specimen W is measured from the centerline of the loading holes to the end of the specimen ahead of the crack tip. The crack length " a " is measured from the centerline of the loading holes to the crack tip. The material regions outside of the two cracks are called the outer arms or tangs, the dimension " H " being referred to as the tang width. The center region may also be referred to as the center tang. For the loading shown in Figure 4-1 the outer tangs are in tension and the center tang is in compression.

Jones and Chisholm, who first introduced the compact shear specimen in the literature (70), used the boundary collocation method, employing a complete William's type stress function, to calculate Mode II stress intensity factors as functions of a/W and H/W for the specimen.

They analyzed a constant thickness specimen with distributed pressure and shear loads along rectangular boundaries of the specimen, omitting any modelling of the loading holes. The shear load, acting in the positive z direction, on the y face of the outer arm, was introduced in addition to the normal load P , to simulate the restraining effect of the specimen loading device. The magnitude of the shear load was adjusted to insure that the analytical results provided K_I two orders of magnitude less than K_{II} .

Hoyniak and Conway produced a finite element analysis of similar versions of the compact shear specimen. Exactly what boundary conditions were used in the analysis is unclear, as those listed in the paper are incorrect, but the values of K_{II} they predicted correlate fairly well with those of Jones and Chisholm, as seen in Figure 4-2. However, this agreement appears to be fortuitous. The correlation between the two sets of results is seen to be much less satisfactory when the tang width H is non-dimensionalized by W .

Also seen in Figure 4-2 are the nondimensionalized K_{II} versus a/W curves resulting from a simple compliance calculation of the specimen. Assuming the outer and inner tang act in simple extension with fixed ends and no rotation, the compliance is simply

$$C = \frac{a}{BHE}$$

The energy available for crack growth has been shown to be:

$$G = \frac{1}{2} \frac{P^2}{B} \frac{dc}{da}$$

$$= \frac{1}{2} \frac{P^2}{B^2} \frac{1}{HE}$$

and is independent of crack length.

The Mode II stress intensity factor is then

$$K_{II} = \frac{P}{B} \left(\frac{1}{\sqrt{2H}} \right)$$

and the nondimensionalized value plotted is

$$\frac{K_{II}}{\sigma \sqrt{a}} = \sqrt{\frac{H}{2a}}$$

where $\sigma = \frac{P}{BH}$, is the average longitudinal stress in the outer tang.

The fact that the compliance computed Mode II stress intensity values consistently lie below those computed using more sophisticated techniques indicate that, except for the smallest a/W ratio, the rotation and shear deformation in the region of the specimen ahead of the crack tip make a significant contribution to the overall compliance.

In designing a compact shear specimen, one wishes to insure that if any plastic deformation occurs in the specimen, that it happens in the regions near the crack tip, and not at the loading holes. To

prevent loading hole plasticity, Jones and Chisholm originally reduced the material thickness of the specimen in regions along and ahead of the crack tip, as seen in Figure 7 of (70). This has the undesirable effect of constraining the direction of crack growth.

In subsequent studies both Chisholm and Jones (73), and Pook and Greenan (72), made the compact shear specimen thicker in the region of the loading holes, with constant thickness ahead of the crack tip. This greater thickness around the loading holes prevents plasticity around the loading hole, and also increases the transverse stiffness of the outer tang. In the present work it was seen that this increases the relative amount of Mode II deformation for a given shear loading.

The required thickness in the crack tip region to insure plane strain deformation under Mode II loading is much less than that required for Mode I loading. For Mode I loading, the effective fracture toughness varies greatly with specimen thickness (74). While no experimental studies have been found which relate specimen thickness to effective fracture toughness under Mode II loading, the effect of thickness on Mode II fracture toughness should be negligible. The reason for the foregoing statement is based on the elastic solution for stresses and displacements in the crack tip region as given in Chapter I.

In plane strain, the strain in the out-of-plane direction is zero, and the out-of-plane stress σ_{xx} , is

$$\sigma_{xx} = \nu(\sigma_{yy} + \sigma_{zz})$$

In plane stress, the out-of-plane stress is taken to be zero.

In Mode I loading, for plane strain deformation

$$\sigma_{xx} = \nu \left(\frac{K_I}{(2r)^{1/2}} (2 \cos \frac{\theta}{2}) \right)$$

Thus σ_{xx} has a maximum at $\theta = 0$, directly ahead of the crack tip.

In Mode II loading, for plane strain deformation

$$\sigma_{xx} = \nu \left(\frac{K_{II}}{(2r)^{1/2}} (2 \sin \frac{\theta}{2}) \right)$$

Here σ_{xx} is a minimum at $\theta = 0$, and is equal to zero. In the fracture process zone ahead of the crack tip under Mode II loading, the difference between plane strain and plane stress is negligible. Therefore, the required thickness for plane strain deformation under Mode II loading is much less than Mode I, and the thickness used in this experimental study is conservatively based on the required Mode I loading plane strain thickness.

Figure 4-3 shows the drawing used in the manufacture of the compact shear specimens for this study. For all specimens, the thickness of the crack tip region was 0.5 inch and was one-half the thickness of the loading hole region.

Finite element analyses of the compact shear specimen were performed using NIKE2D, and the increased thickness of the loading hole region was accounted for in the specification of the material properties. The mesh for the first analysis is seen in Figures 4-4 and 4-5. Referring to the points defined in Figure 4-5, the nodes

along the symmetry boundary from A to R were constrained to have zero deflection in the z direction. The crack tip is at point QQ, and the length from PP to QQ simulates a fatigue pre-crack of 0.1 inch, extending from the machined notch.

In the first analysis a pressure load was applied between points K1 and K2, and the point midway between K1 and K2 was fixed in the z direction. The points between C and C1 were constrained in both the y and z directions.

In all analyses the tang width H was 1.0 inch, and W, the length between the center line of the loading hole and the back edge of the specimen, was 2.45 inch. For the first study the crack length "a" was 1.55 inch. In additional studies crack lengths of 1.75, 1.95, and 2.15 were analyzed.

At first, attention was confined to load ranges where material plasticity effects were small, and values of J_I and J_{II} were calculated using MMJINT. The resulting values of K_{II} for the specimen were plotted in comparison to the results obtained by Jones and Chisholm and Hoyniak and Conway using nondimensionalized quantities in Figure 4-6.

To achieve an analytical situation closer to those of Jones and Chisholm and Hoyniak and Conway, a constant thickness specimen was also analyzed for the same crack lengths, and these results are also shown in Figure 4-6. For the longer crack lengths the difference between the values of the Mode II stress intensity factor for the constant thickness and the variable thickness compact shear specimen vanish, as would be expected.

The agreement between the results of this investigation and those of Jones and Chisholm is fairly good, even though the method of applying the load and the modelling of the loading device transverse motion restraint are different in the finite element and boundary collocation analyses.

Perhaps the largest difference between the results of this investigation and those of Jones and Chisholm is in the amount of Mode I deformation predicted at the crack tip for this shear loading of the compact shear specimen.

As explained, Jones and Chisholm adjusted the amount of shear load on the outer tang to control the amount of Mode I loading to be small, that is $K_I/K_{II} < 0.01$. In the current investigation, the transverse, or z direction, displacement at the center of the applied pressure was set equal to zero. It was felt that this more realistically modelled the restraint of the loading devices, but the amount of Mode I loading near the crack tip was somewhat larger. The ratio of K_I/K_{II} in this analysis varies with crack length, from 0.30 for the shortest crack to 0.083 for the longest crack, as seen in figure 4-7.

In this loading arrangement both K_{II} and K_I are negative. A negative K_I implies that the Mode I deformation is acting to close the crack. The superposition of the Mode I crack closing forces and the Mode II crack-sliding forces bring about a sliding interface with friction between the two surfaces of the fatigue pre-crack.

The slide-line friction between the two crack surfaces was modelled using NIKE2D, and the friction coefficient was estimated to be 0.78, based on values in the literature for dry metallic rubbing surfaces.

Jones and Chisholm, in a continuation of their Mode II fracture study, examined the fracture surfaces of the compact shear specimen, and found in all cases evidence of abrasion due to the sliding frictional interfaces between the crack surfaces (5). However, the results were inconclusive in determining whether the crack tip abrasion occurred during the crack initiation process, or during the final unstable fracture.

The assertion of Jones and Chisholm that they were able to obtain pure Mode II deformation in the compact shear specimen for the loading shown in Figure 4-1 and the boundary conditions, as described, was based on photoelastic observations of the part.

Examination of the photographs of their photoelastic specimens under load, as seen in Figure 5 of (70) and Figure 8 of (73), shows indeed isochromatic patterns with a significant Mode II character. However, their efforts to quantify results from the isochromatic fringe patterns and calculate the ratio K_{II}/K_I are in error, because no account was taken for the effect of the far field stresses on the fringe pattern near the crack tip. The far-field stresses affect the isochromatic patterns near the crack tip, and neglecting their effect leads to errors in calculating K_I and K_{II} , shown by Dally and Sanford (75). Compressive stresses behind the crack tip further complicate the resulting isochromatic fringe patterns in the compact shear specimen. Precise quantification of mixed mode stress intensity factors from the isochromatic patterns appears to be beyond the current state of the art in photoelasticity.

In this investigation, a comparison was made between the shear stresses at the crack tip, as predicted by the finite element analysis, and photoelastic results for compact shear specimens of the same configuration. Figure 4-8 shows contours of constant shear stress, while Figure 4-9 shows contours of constant shear stress in the form of a photoelastic isochromatic-fringe pattern. While no quantitative comparison is attempted, qualitatively the contour patterns show good agreement. The computer-generated contours of Figure 4-8 also agree qualitatively with the photoelastic patterns of Jones and Chisholm, although analysis of the computer results predicts $K_I/K_{II} = 0.30$. The effect of the negative K_I does not seem to be distinguishable in the stress contour pattern.

As the finite element analysis of the compact shear specimen proceeded, it became clear that the boundary conditions and the loading direction had a profound effect on the relative amounts of Mode I and Mode II deformation in the crack tip region. Four different cases of boundary conditions and load direction for the compact shear specimen were identified, and subsequently analyzed.

The loading and boundary conditions on the compact shear specimen already discussed, with the outer tangs in tension, the center tang in compression, and the outer tangs fixed at the load center node to have zero deflection in the transverse z direction, is classified as case 1.

In case 2, the loads are again applied in the same directions as those in Figure 4-1, so that the outer tangs are in tension and the center tang is in compression, but the constraint of zero deflection in the z direction is removed, allowing the tangs to rotate.

In case 3, the load directions are reversed, so that the outer tangs are in compression and the center tang is in tension. In this case, the nodes at the load center in the outer tangs are constrained to have zero z deflection.

In case 4, the load directions are the same as case 3, but the restraint of the load center is removed, and the outer tangs may rotate.

Figures 4-10 through 4-16 show the loading devices, or clevises, used to obtain these loading and boundary conditions on the compact shear specimen.

Figures 4-17 and 4-18 show the ratios $|K_{II}|/\sqrt{K_I^2 + K_{II}^2}$ and $|K_{II}|/\sigma\sqrt{a}$ versus a/W for each of the four loading cases and crack lengths analyzed, and essentially elastic material behavior. When plasticity effects are included, the relative amounts of Mode II and Mode I deformation change with load level for a given specimen configuration, as does also the ratio $|K_{II}|/\sigma\sqrt{a}$.

Certain trends of elastic specimen behavior are evident from these graphs-- particularly noticeable is that when the z direction motion constraint on the outer tangs is released, as in cases 2 and 4, the relative amount of Mode II deformation is greatly reduced. Very close to pure Mode II deformation occurs in case 3, while case 1 approaches pure Mode II only at longer crack lengths. Figure 4-18 shows that case 3 loading produced the greatest amount of Mode II deformation per given load, though case 1 is not far behind.

The absolute value of K_{II} is taken in these graphs for comparison, because there is no physical difference between positive and negative for the Mode II stress intensity factor, the sign of the

factor being simply a matter of definition. This is in contrast to Mode I loading where positive K_I causes crack growth due to crack opening forces, whereas negative K_{II} causes compression between the crack faces. In cases 1 and 2, both K_I and K_{II} are negative. In cases 3 and 4, both K_I and K_{II} are positive.

Figure 4-19 shows the singularity strength as a function of crack length and load case. For this specimen, Mode I loading at the crack tip due to the unconstrained motion of the outer tangs, as in cases 2 and 4, is clearly much more effective in producing a stress singularity at the crack tip. The positive K_I of case 4 is also more effective than the negative K_I of case 2.

A comparison of the stress contour patterns, and isochromatic fringe patterns for cases 2, 3, and 4, as produced using finite element analysis and photoelasticity, show good qualitative agreement, as shown in Figures 4-20 through 4-25.

A further important consideration in the design of a Mode II or mixed mode fracture toughness specimen is the necessity of obtaining a fatigue pre-crack ahead of the machined notch.

A fatigue pre-crack is necessary in order to discount the notch radius effects in the material. Many materials are notch sensitive--for components with notch root tip larger than a certain radius, the material in this notch root region will plastically deform under load application, and crack growth will not occur until a much higher load than would be necessary if the notch root tip radius were sufficiently small. In order to get crack growth at the smallest load, a notch of infinitely small radius, i.e., a sharp crack, as produced by fatigue cycle loading, is necessary.

As discussed in Chapter 2, the fatigue pre-crack should be aligned with the notch plane, in order to preserve path-independence of the J integral around the crack tip. It is desired to be able to view the fatigue pre-crack as a simple extension of the machined notch with a vanishingly small root tip radius.

Jones and Chisholm, in their early study (70), had the notch tips in their specimens filed to a 0.004 inch radius. While it is not known whether this introduced notch tip radius effects in their results, the uncertainty would be eliminated if one were able to cyclically load the specimen to obtain a fatigue pre-crack.

Pook and Greenan, using a specimen with $H/W = 0.25$ and $a/W = .078$, loaded horizontally between the outer and center holes to fatigue pre-crack their specimens.

Using finite element analysis and the J integral post-processor, the possibility of Mode I loading, both between the center and outer holes, and between the outer holes, to obtain a fatigue pre-crack, was investigated. The objective was to find the loading configuration which would produce as nearly as possible a pure Mode I stress singularity at the crack tip, to avoid crack tip kinking. For the specimen configuration of this investigation, where $H/W = .408$ and $a/W = 1.45$, considering the machined notch, horizontal loading between the center and outer holes produced a stress singularity with mixity $K_I/K_{II} = 7.2$.

Loading between the outer holes produced a slightly more favorable Mode I situation, with $K_I/K_{II} = 7.5$, where K_I was positive and K_{II} was negative.

However, using equation 2-11 to figure the angle of fatigue crack extension, assuming that fatigue crack growth occurs in the direction of maximum crack driving force, then

$$\frac{\partial J}{\partial \theta} = 0 \text{ implies that}$$

$$\tan \theta = \frac{-2K_I K_{II}}{K_I^2 + K_{II}^2}$$

yielding an angle of approximately 16 degrees. It was felt that a fatigue pre-crack at an angle of 16 degrees to the notch was unacceptable, and that the fatigue pre-crack should be at a maximum of 5 degrees from the notch axis.

The possibility of moving the center hole in or out along the y axis, to increase the resulting K_I/K_{II} ratio for Mode I loading between the center and outer holes, was investigated using finite element analysis. However, changing the location of the center hole had no significant effect for the hole positions considered, and this approach was abandoned.

A successful method of obtaining near Mode I loading at the crack tip consisted of a combination of unconstrained shear loading, as in case 4, and horizontal loading. Since horizontal loading between the outer holes produced K_I positive and K_{II} negative, and case 4 shear loading was also predominantly Mode I, with K_I and K_{II} positive, it became clear that some combination of case 4 shear loading and horizontal loading between the holes would produce a stress singularity with $K_{II} = 0$. The proper combination resulted in loading the outer tangs at 45 degrees, for the 1.45 inch machined notch, 50 degrees for the 1.65 inch machined notch, and 55 degrees for

the 2.05 inch machined notch, where the angle is measured from an axis parallel to the notch through the center of the outer hole. As shown in Figure 4-26, zero degrees is the angle of loading for case 4 and 90 degrees is the horizontal loading angle, as clarified in Figure 4-26. Figure 4-27 shows the loading plate used in connection with the other loading devices of case 4, to apply the load to the specimen at the required angle.

In summary, using the specimen configuration shown in Figure 4-3, it was possible to obtain stress and displacement conditions at the crack tips which varied from nearly completely Mode II to Mode I, by changing the loading and boundary conditions. This specimen became the basis for the Mode II and mixed mode fracture studied in this investigation.

Further variations of this specimen design could include notches machined at various angles to the loading directions in cases 1 and 3, and would produce crack tip mixity parameters unattainable in the current design. However, these possibilities were left for some future investigation.

V. MATERIAL PROPERTIES, TEST PROCEDURES, AND RESULTS:

Three different materials, 7076-T6 aluminum, AISI 4330V steel, and a photoelastic material PSM-5, were tested in this investigation of fracture under mixed mode loading. The three materials have very different fracture characteristics. The aluminum alloy has been heat-treated to near maximum strength for an alloy of this type, but as a result of the precipitation-hardening mechanism, a microstructure with a fairly low fracture toughness is created. The steel, on the other hand, has a moderate yield strength for a martensitic, low alloy steel, and has a high fracture toughness. The photoelastic material has a low yield strength, and is brittle.

The aluminum alloy received the most attention in this investigation, both in terms of number of specimens tested, and in the analysis of the test results. This alloy is used in the aircraft industry for highly stressed structural components because of its high strength to weight ratio in the T-6 heat-treated condition. The chemical composition, typical mechanical properties, and references for the heat-treatment procedures for this material may be found in (76).

The 7075-T6 aluminum has directional properties because during its manufacturing process, as it is rolled into plates, elongated stringers of impurities are formed in the aluminum matrix in the rolling (longitudinal) direction. As a result, the fracture toughness for crack propagation in the longitudinal direction (T-L) is less than that for propagation in the transverse direction (L-T). However, Young's modulus was found to be isotropic to within a few percent, so

that Poisson's ratio was assumed also to be independent of direction. The stress-strain curves beyond the elastic range were very similar and average material properties were used as input to the FEM analysis.

For Mode I fracture toughness testing there is a standard method for identifying the crack plane orientation with respect to the rolling direction, as in Figure 9 of (13). For Mode II fracture toughness testing in this investigation, the same designation for the relationship between the crack plane orientation and the rolling direction will be used.

AISI 4330V steel is a low alloy, heat treatable, martensitic steel. Although classified as a ultra-high-strength steel (77), the tempering temperature used in the heat treatment, 1200⁰F, reduced the tensile yield strength in favor of increasing the fracture toughness. The chemical composition and typical physical properties of this material are given in (77). Double-redundant forging was used to fabricate the material to insure isotropic material behavior.

PSM-5 is a proprietary commercial photoelastic material. It is an epoxy, with quite a high modulus of elasticity and good stress-optic sensitivity. Some pertinent information regarding its optical and mechanical properties may be found in (78,79).

Tensile tests were performed on aluminum and steel specimens to determine Young's modulus, tensile strength, ultimate strength, and reduction of area at fracture. The reduction of area at fracture is a measure of the ductility of the material. Table 5-1 summarizes the tensile test results. The material orientation for the aluminum tensile specimens is L-T, when the axis of the specimen was taken from

the longitudinal direction, and T-L for the transverse direction. The experimentally determined values of Young's modulus were within the range specified by handbook values, and Poisson's ratio values were taken from the handbooks to be 0.33 for aluminum and 0.3 for steel.

The failure surface of the aluminum tensile specimens is seen in Figure 5-1. The 45° slant of the failure surface suggests that formation of localized shear bands was the cause of final separation. The failure surfaces of the steel tensile specimens as seen in Figure 5-2 show a cup and cone separation. Figure 5-3 shows a close-up of the steel tensile specimen failure surface. The facets in the surface, as well as the dimpled regions indicate the presence of a non-homogeneous phase in the material.

Mechanical properties for the photoelastic material were taken from the previously cited references. For this material, Young's modulus is 450,000 psi, the tensile strength is 9,000 psi, and Poisson's ratio is 0.36.

Mode I fracture toughness tests were performed on aluminum, steel, and photoelastic specimens, in order to be able to compare the Mode I fracture toughness values to those for mixed mode and Mode II fracture.

For the aluminum Mode I fracture toughness tests, the specimens and test procedures were in accordance with ASTM standard test procedures (13). The results are summarized in Table 5-2. The fracture toughness values in the L-T direction are approximately 10% more than those in the T-L direction. Figures 5-4 and 5-5 show the fracture surfaces for L-T and T-L specimens. The T-L fracture surface is completely flat, where the L-T specimen fracture surface shows a slight amount of shear lip at the outer edge. In both cases the deformation is two-dimensional to a great extent.

The 4330V steel was too tough to be tested in accordance with (13) for specimens taken from the one inch thick plate, and, therefore, the J_{IC} proposed test method and specimen configuration (15) were employed to obtain Mode I fracture toughness values for this material. Figure 5-6 shows the resulting J versus increment of crack growth relationship, with average J_{IC} value of 694 in-lb/in². This corresponds to a K_{IC} value of 150 ksi-in^{1/2}.

Specimen 5 was 25% side-grooved, and, as seen in Figure 5-7, out of plane deformation in the fracture surface of this specimen was extremely limited, which would explain the lower apparent fracture toughness derived from this specimen. Figure 5-8 shows the fracture surface of a specimen without side-grooves, and although the out-of-plane deformation was considerably more than that of specimen 5, the deformation was again largely two-dimensional.

The photoelastic material PSM-5 was too brittle to fatigue pre-crack in the given loading system, and hence failed to meet the criteria for testing in accordance with (13). Although it is known that there are significant notch root radius effects in this material (80), the notch root radius was .010 inches in both the Mode I and Mode II tests, so that direct comparison should be possible. The Mode I fracture toughness specimen for the photoelastic material was similar to that of (13), with the as-machined notch tip as the stress concentrator. Results for the Mode I photoelastic material fracture toughness tests are shown in Table 5-3.

For Mode II fracture toughness testing, the specimens were loaded using the loading devices corresponding to cases 1, 2, 3, and 4 as explained in Chapter 4. In every case, three displacement gages were

used to monitor the deflection of the specimen as a function of load. The response of the displacement gages and load transducer were electronically amplified and autographically recorded. For all load cases displacement gages were attached, as shown in Figure 5-9. Two displacement gages were fixed between the outer and center loading pins to measure deflection in the z or horizontal direction. For measuring the deflection in the y direction, a bar was attached to the loading pins on the side opposite where the z deflection displacement gages were attached. The displacement between this bar and the center loading pin was measured by attaching the displacement gage to both the bar and the center pin.

The displacement measuring system was calibrated, and repeatable results were obtained. It was found, however, that the specimen compliance, as predicted by the displacement measuring system, was much larger than the compliance predicted by the finite element analysis. The reason for this discrepancy is that the deflections, as determined by the displacements at the clip gages, included the effects of hertzian compression between the pins and clevises and pins and specimen and the pin bending, which were not accounted for in the finite element analysis. In the cases of near Mode II deformation, the specimen is stiff, and these normally small deflections associated with the loading system become significant. The single point of agreement between the experimentally measured deflections and the computer calculated predictions was in the z deflection of the outer tangs in case 4, where the results differed by about 10%.

In establishing Mode II fracture toughness test procedures, it is desirable that these be related to the corresponding

procedures in the Mode I fracture toughness testing standards. Then K_{Ic} and K_{IIc} , for primarily elastic material behavior, and J_{Ic} and J_{IIc} , for elastic-plastic material behavior, will be directly comparable.

In the test procedure for determining K_{Ic} , the load-displacement record plays an integral part. As the Mode I fracture toughness specimen is loaded, the load-displacement curve is initially linear, corresponding to reversible elastic material behavior. The reversibility of the material behavior implies that if the load were removed, the specimen would return to its original configuration. As the load is increased beyond the linear range of the load-displacement record, two irreversible, non-linear processes may occur simultaneously -- crack growth, and plastic deformation.

Since the object of the test is to find the load at which crack growth begins, and because the effect of increasing crack length on the change in compliance is known, allowing for a measurable change in compliance specifies a certain amount of permissible crack growth. For the Mode I fracture toughness specimens, the relationship between the change in compliance and the amount of crack growth is

$$\frac{\Delta c}{c} \cong 2.5 \left(\frac{\Delta a}{a} \right) \quad (5-1)$$

where Δc and c are the change in compliance and compliance respectively, and Δa and a are the change in crack length and crack length (81). Therefore, the five percent change in slope of the load-displacement record allowed by the ASTM Mode I fracture standard corresponds to a limit of 2% on the amount of slow crack growth prior

to final fracture. The maximum load level on the load-displacement curve which occurs prior to the intersection of this curve with a line at a slope at 5% offset to the initial linear portion of the load-displacement curve, is taken as the load for fracture toughness computation.

Plastic deformation in the specimen will also cause a change in compliance. In order to insure that a least part of the 5% change in compliance in the K_{Ic} test occurs as a result of crack growth and is not all due to plastic deformation, the ratio of the maximum load on the load-displacement record to the load at which crack growth is assumed to initiate must be less than 1.1. Since for plane-strain deformation in these specimens crack growth begins no later than at maximum load, the amount of error in determining the load at which crack growth initiates is set at an acceptable level. Plastic deformation must be limited to a reasonable level because in the K_{Ic} test a linear relationship between the applied load in the specimen and the resulting K_I is assumed.

For determining J_{Ic} with elastic-plastic material behavior, very similar specimens to those in the K_{Ic} test standard are employed. The J_{Ic} test procedure specifies interrupted loading. The Mode I elastic-plastic fracture toughness specimen is loaded to a level where crack growth is presumed to have occurred, and then it is either partially unloaded to determine the change in compliance due to crack growth, or it is completely unloaded, heat-treated and broken, and the amount of crack growth is directly measured. By using either several unloading compliance measurements, or several specimens, at least four measurements of change in crack length at the corresponding load

levels are tabulated. Knowing the area under the load-deflection curve to the load where the loading was interrupted, values of the J integral are derived. The values of J and Δa are plotted, as in Figure 5-6, and, assuming a linear relationship between J and Δa , the value of J_{IC} is extrapolated back to where physical Δa is zero. This, then, is the critical J integral value for initiation of crack growth.

Elastic material behavior prior to the onset of crack growth was anticipated in both the aluminum and photoelastic specimens. In considering Mode II fracture test procedures, it was determined that the relationship between the change in compliance and the increase in crack length was different in the compact shear specimen, as compared to that for the Mode I fracture toughness specimens.

For the compact shear specimens, in the range $0.6 < a/w < 0.8$, and load cases 1 and 3, the elastic finite element solutions and J integral post-processor calculations show

$$\frac{\Delta c}{c} \approx 0.6 \left(\frac{\Delta a}{a} \right) \quad (5-2)$$

This implies that for a two percent increase in crack length, a change of compliance of only about one percent results. If the load at a change of compliance of five percent were used to calculate K_{IIC} , it would be overestimated because a change in crack length of more than eight percent or substantial plastic deformation would be allowed.

Because of this relationship between the change of compliance and change in crack length for the elastic behavior of the compact shear specimen, either the load at departure of linearity from the

load-displacement record or the maximum load are logical choices for determining K_{IIc} . The inaccuracy of the displacement measuring system did not influence the determination of both the load at departure of linearity and maximum load from the load-displacement records of the specimens.

Ten aluminum compact shear specimens were tested in this investigation. The results of the fatigue-precracking and the material orientation of the crack plane are given in Table 5-4. In cases where one crack length was measurably larger than the other in a specimen, the longer crack failed first.

As load was applied to the specimens, it was noted from the load-displacement record that there was some small rotation of the center tang, with respect to the outer tangs, in the direction of the tang with the longer crack. The amount of the rotation was so small, about .07 degree, that its effect on the results was neglected. At final fracture there was also some rotation of the center tang, as the tang with the longer crack failed first. The actual crack lengths for cases 2 and 3, where the crack symmetry is most critical, were within 1% of each other for a given specimen, and were also within 1% of the crack length values used in the corresponding computer analyses. In post-fracture examination of the failure surfaces, it was seen that the angle of the fatigue crack was overestimated by the surface measurement shown in Table 5-4. A very small shear lip had formed at the surface during fatigue pre-cracking, distorting the angle of the fatigue crack at the surface. The fatigue crack angle in the interior of the material was closer to zero degrees than indicated.

Figures 5-10 through 5-17 show pictures of the fractured compact shear specimen for each material orientation and each load case. In load cases 1 and 2, both tangs were broken off in all 4 specimens tested, but the final fracture event did not occur simultaneously in both tangs. From observation of the tests, and from examination of the broken pieces of the specimens, it was evident that first the side with the longer crack failed in shear, and then second tang rotated, and was broken in a more nearly Mode I type of fracture. The force for the second, or sympathetic, failure come from the reflex action of the loading ram, caused by the sudden decrease in load-carrying capacity of the specimen, due to the first failure. For case 1 loading conditions, as seen in Figures 5-10 and 5-11, the initial fracture is on the left-hand side, and the fracture surfaces here are featureless, indicating a sliding mode of fracture. The Mode I fracture surface of the sympathetic failure has a rough, granular appearance.

Figure 5-12 and 5-13 show the specimens failed in case 2. In this case, K_{II} is negative, and there is also a large negative K_I which acts to close the gap between the crack surfaces. The effect of this large negative K_I is evident from examining the failure surfaces on specimens 1D and 1T. For both specimens, the original shear failure, on the right in the figures, began at the root of the machined notch and not at the tip of the fatigue crack. In the failures of case 2, the presence of the fatigue crack had negligible influence on the final fracture.

Four specimens were loaded to failure in load case 3. Pictures of two of the broken specimens are seen in Figures 5-14 and 5-15. In each case here, only one side originally failed. The side which

originally failed is on the right side in both figures. On the side that did not fail in these tests, it was noticed that there appeared to be a substantial amount of crack growth ahead of the fatigue pre-crack. It was assumed that this was shear crack growth which occurred prior to the final fracture on the other side. Visual observations of the specimens while they were being loaded during the tests indicated crack growth prior to final fracture.

In order to determine the amount of shear crack growth on the side that did not fail, the remaining part of the specimen was loaded until failure between the two remaining holes. The regions of the fatigue crack growth, shear crack growth, and the final fracture were distinguishable to the eye. However, under an optical microscope, it was difficult to distinguish between the fatigue and the shear crack growth regions. The amount of this sympathetic shear crack growth was approximately measured in the four specimens tested in case 3. An interesting feature of this crack growth was that it was a minimum at the interior of the specimen, and a maximum at the outside edges, in contrast to the fatigue pre-crack.

In case 4, failure occurred at one side only, as seen in Figures 5-16 and 5-17. The crack trajectory had a definite, though different, angle for the crack plane in the different material orientations.

The results of the tests on the aluminum compact shear specimens support the idea that here there are two competing failure theories to explain the fracture process. The first is that theory already presented, that the applied loading causes a certain stress amplification at the crack tip described by Mode I and Mode II stress intensity factors, and that at a critical combination of K_I and

K_{II} , crack growth will occur. The second is a much simpler theory, and is that the average applied shear stress will cause failure when the average shear stress in the remaining ligament equals the material flow shear stress. For the aluminum, with a relatively small amount of work hardening, the material flow shear stress is taken to be one-half the average of the yield and ultimate strengths.

Examining the simpler theory first, it is seen in Table 5-5 that for load cases 1 and 2 the failure of the specimens may be explained in terms of the average shear stress in the region ahead of the crack tip.

In case 2 there is a large negative K_I compressing the fatigue pre-crack surfaces against each other, which promotes crack tip plasticity and inhibits crack growth. The compressive K_I also tends to shorten the effective crack length in the specimen, so that the average failure shear stresses in Table 5-5 for specimens 1T and 1D represent a slight over-estimate, because they are based on the remaining ligament ahead of the fatigue pre-crack. The load-displacement records of the case 2 specimen tests show considerable non-linearity prior to maximum load, due to plastic deformation in the region ahead of the end of the machined notch. The final fracture surface in the specimens did not initiate at the fatigue pre-crack, and there was no evidence for crack growth prior to final fracture.

In loading case 1, however, where the applied loading is largely Mode II, with only a small negative K_I , there was some evidence in specimens 1V and 1E for a small amount of crack growth prior to final fracture. This evidence consisted of observations of the failure

surfaces on the side of the sympathetic failure, which showed about .02 inch of shear crack growth at the center of the thickness in specimen 1V, and about three times as much in specimen 1E. These increments of crack growth represent about 2.5 percent of the original crack length, and indicate the possibility the onset of crack growth occurs at less than maximum load.

However, equation 5-2 based on elastic solutions suggests that for this amount of crack growth the change in compliance would be between 1 and 2 percent. Much larger changes in compliance were found in the elastic-plastic computer analysis of this load case. These results support the conclusion that the crack growth occurs only in conjunction with significant plastic deformation at the crack tip. Because the applied shear stress at final fracture is such a large fraction of the material flow stress, the stress amplification due to the crack tip singularity must be damped out by this plasticity. Therefore, it seems that the simpler theory of a maximum applied shear stress best explains the failure of the aluminum compact shear specimens in case 1, as well as in case 2.

More specimens with shorter crack lengths would be necessary to generalize these results and infer that the material flow shear stress is always the failure criteria, for these types of loading conditions. This is because in the compact shear specimen the ratio $K_{II} / \tau \sqrt{a}$, where τ is the applied shear stress in the remaining ligament, increases with decreasing crack length. Therefore, for a specimen with a shorter crack length, larger values of K_{II} are achieved before the material flow shear stress is reached. This would tend to promote crack growth. However, in cases

1 and 2 the negative K_I introduces the complication of a compressive normal stress and a frictional shear stress at the crack tip. For case 1, these effects also increase with decreasing crack length. Because of this, Mode II or mixed mode crack growth is more easily studied where K_I is positive, as in load cases 3 and 4.

In load case 3, four specimens were failed, with two specimens each at two different crack lengths. From Table 5-5 it is seen that failure in this case could be described in terms of the ratio of the average applied shear stress to the material flow shear stress, as this ratio is very nearly constant for the two crack lengths. The fact that the specimens fail at a lower shear stress than in cases 1 and 2, could be explained by the positive rather than negative K_I . In this case, crack growth was observed to occur prior to final fracture, so the possibility of K_{IIc} controlled crack growth was investigated.

For the compact shear specimen, even for essentially elastic material behavior to the onset of crack growth, the final fracture in the specimen, which occurs at maximum load, is controlled by the maximum shear stress in the remaining ligament. If the shear stress at which failure occurs is a material constant, this implies

$$\frac{P_{1f}}{A_{1f}} = \frac{P_{2f}}{A_{2f}}$$

where P_{1f} and A_{1f} are respectively the maximum load and area of the remaining ligament at maximum load for the specimens with the 1.55

inch initial crack length, l_W and l_G . P_{2f} and A_{2f} are the corresponding quantities for specimens 1X and 1A with 1.75 inch crack lengths. Then one obtains

$$\frac{P_{1f}}{P_{2f}} = \frac{A_{1f}}{A_{2f}} \quad (5-3)$$

Suppose there is some unspecified amount of crack growth prior to maximum load, then

$$A_{1f} = B (W - a_1 - \Delta a_1)$$

where B , W , a_1 , Δa_1 are as previously defined.

Also,
$$A_{2f} = B (W - a_2 - \Delta a_2).$$

Substituting these relations for A_{1f} and A_{2f} into equation (5-3), and using the appropriate values for W , a_1 , a_2 , P_{1f} , and P_{2f} , one obtains

$$\Delta a_1 = 1.3 \Delta a_2$$

Also, assume that crack growth begins at a constant value of K_{II} , and at the load at deviation from linearity of the load-displacement record. Then, knowing the relationship between the

Mode II stress intensity factor and the relative crack length from the numerical computations

$$\frac{K_{II}}{\sigma\sqrt{a}} = 0.4 + 1.56 \frac{a}{w} \quad (5-4)$$

for the specimens with the 1.55 and 1.75 inch crack lengths, the ratio of the loads at departure from linearity must be

$$\frac{P_{1d}}{P_{2d}} = 1.16$$

Finally, with the linearized relationship of equation 5-2, values for Δa_1 and Δa_2 are obtained, assuming that the change in compliance is due completely to increase in crack growth.

Based on these assumptions,

$$\Delta a_1 = 0.08 \text{ inch}$$

and

$$\Delta a_2 = 0.06 \text{ inch}$$

By measuring the amount of sympathetic shear crack growth which occurred on the side which did not fail in shear, and taking the ratio of the loads for departure from linearity from the load-displacement records of these specimens, an indication may be obtained as to how accurate these assumptions are.

The measured amounts of sympathetic shear crack growth and loads at departure from linearity are seen in Table 5-6. The ratios for the loads at departure from linearity for the different material orientations are also seen in this table.

The experimental values are fairly close to those predicted by the analysis, assuming the change in compliance is due entirely to elastic material behavior and crack growth. However, the difference is enough to indicate that there is an effect of plastic deformation as well.

The K_{IIc} value calculated from the load at departure from linearity represents a lower bound on the actual K_{IIc} value, since in the linear range of the load-displacement record no crack growth occurs. On the other hand, since there is experimental evidence to suggest there was crack growth prior to maximum load, the K_{IIc} value based on maximum load is an upper bound on the real K_{IIc} . From the load case 3 tests, then, the value of K_{IIc} is known within certain limits.

The results of case 4 were the most straightforward to interpret. The load-displacement record was linear to maximum load, where crack growth occurred, and the part failed.

The values of K_I and K_{II} calculated from the J integral values at maximum load, and also at load of departure from linearity, are seen in Table 5-7. These values from cases 1 and 2 are for comparison only, since significant plastic behavior has occurred in these tests. Figure 5-18 shows the test results for the aluminum alloy using the values of Table 5-7. The K_I and K_{II} values at fracture are

normalized by K_{IC} . For the different material orientations the corresponding values of K_{IC} are used. The results of case 2 are not plotted, but lie far to the left. From the figure it can be seen that the ratio K_{IIc}/K_{IC} is about 1.9.

Finally, the angle at which crack growth initiates is of interest in terms of deciding on an appropriate failure criterion, as discussed in Chapter 7. In cases 1 and 3 the shear crack growth was straight ahead of the original crack, that is $\theta = 0$, and the crack growth is self-similar. In case 4 self similar crack growth did not occur. The angle of crack growth in specimen 1Y, where the crack plane was in the L-T orientation, was -50 degrees, using the sign conventions of Figure 4-1. In specimen 1B, where the crack plane was in the T-L orientation, the angle of crack growth was -35 degrees.

Figures 5-19, 5-20, and 5-21 show the failed 4330V steel specimens. The specimens, with a 2.15 inch crack length, were tested in load cases 1, 3, and 4. In load cases 1 and 3, where Mode II loading predominated, failure of the specimens was due to the applied shear stress reaching the material flow shear stress. The applied J_{II} values were not large enough to initiate crack growth, indicating that specimens with shorter crack lengths are necessary to increase the ratio of the Mode II stress singularity to the applied shear stress. Even with the shorter crack lengths though, much larger loads will have to be applied to achieve a critical J_{II} value. Table 5-8 shows the calculation of the ratio of the applied shear stress to an estimate of the material flow shear stress. In this case, the material flow shear stress was taken to be one-half the ultimate tensile strength. However, because of significant

strain-hardening in the material, the material flow shear stress is underestimated, as indicated by the fact that ratio of the applied shear stress to the flow shear stress is larger than unity for the failures in cases 1 and 3.

In case 4 the specimen was loaded to 10,000 lbs. and then heat-tinted and broken to reveal the amount of crack growth. The crack growth was measured to be .070 inch at a J_I value of approximately 2,600 in-lb/in². Because of the larger amount of plastic deformation ahead of the crack tip, convergence of the finite element analysis was not obtained at the maximum load, and the J_I value had to be extrapolated from lower load levels.

When this single point of J versus Δa information is compared to the J versus Δa curves for Mode I shown in Figure 5-6, it is seen that the value of $J=2600$ in lb/in² at Δa of 0.070 inch lies above the other curves by about 30%. Since the compact shear specimen was only one-half the thickness of the Mode I specimens, this J value may be due to the thinner specimen or to the presence of a Mode II component of loading. The angle of crack growth in specimen 3, loaded in case 4, was zero degrees.

Clearly, additional tests would have to be performed to adequately assess the mixed mode fracture behavior of the 4330V steel.

Four photoelastic specimens, with 1.55 inch crack length, were loaded to failure in load cases 1, 2, 3, and 4. The optical properties of the material aided in the interpretation of the results. The amount of mixity in the specimen as it was loaded could at least be compared qualitatively to that expected for the particular load case by viewing the isochromatic fringe patterns. Also, the attainment of symmetry in the specimen could be verified as the load

was applied. Finally, plastic deformation in the material causes residual stresses, which remain in the part after it is loaded, so that the existence of plastic deformation may be detected after failure in the polariscope.

The results of the test are summarized in Table 5-9. In load case 2, values of K_I and K_{II} were not obtained, because the test was interrupted when plastic deformation began to occur at the crack tip. In the other load cases the material behavior was brittle, with no visible plastic deformation occurring in conjunction with the crack growth. In these specimens, since they were difficult to fatigue pre-crack, a .010 inch thick jeweler's saw was used to create an 0.100 inch extension of the machined notch, to approximate a fatigue pre-crack of the same length. Because of the width of the saw cut, the crack faces did not touch in case 1, even though a negative K_I was applied.

Figure 5-22 is a plot of the results of Table 5-9, where the critical stress intensity factors have been non-dimensionalized by dividing through by the average K_{IC} for the material, 2.8 ksi-in^{1/2}. Admittedly, four points are far from sufficient to establish definitive results. However, it does appear that in this brittle material K_{IIC} is less than K_{IC} , and that the influence of the Mode II loading is much more pronounced in this material, than in the more ductile aluminum.

Another difference between failure in the brittle photoelastic epoxy, as contrasted to that of the aluminum is the angle at which crack growth occurred. Figures 5-23, 5-24, and 5-25 are pictures of the failed photoelastic specimens, and show that self-similar crack growth did not occur in near Mode II loading.

Figure 5-23 shows one specimen failed in load case 1, which is near Mode II loading. The angle of crack propagation at the initial failure is seen to be approximately 85 degrees, with the angle of fracture being slightly less near the crack tip. For load case 3, which also produces nearly pure Mode II loading, Figure 5-24 shows the initial angle of fracture to be approximately -70 degrees. For load case 4, which is mixed mode loading with predominant Mode I character, Figure 5-25 shows the angle of initial crack to be about -45 degrees.

VI. FRACTOGRAPHY:

The fracture surfaces of both the Mode I and compact shear specimens, for the aluminum and steel alloys, were examined using the scanning electron microscope (SEM). The purpose of the examination of these surfaces was to gain qualitative information about the microstructural mechanisms for failure, contrasting the fracture surfaces formed under Mode I or near Mode I loading to those formed as a result of near Mode II loading.

Figure 6-1 is a picture of the fracture surface of the aluminum compact tension specimen 2A. A very similar fracture surface is seen in Figure 6-2, which is the failed surface of the aluminum compact shear specimen 1B at the same magnification. Specimen 1B was tested in load case 4, which is near Mode I loading conditions. The surfaces in both figures are rough and irregular, showing that the failure occurred on planes with many different orientations. Two different types of fracture surfaces are noticeable in Figure 6-3, which shows the fracture surface of specimen 1B at a higher magnification. There are regions where completely ductile fracture has occurred, which produced a very finely dimpled surface. There are also regions where failure appears to have occurred along the grain boundaries, producing grain boundary facets bearing small, very shallow dimples. The fracture surface of Figure 6-3 is very similar to that seen in (88), which also depicts the Mode I fracture surface of 7075-T6 aluminum.

In contrast, the fracture surfaces of the specimens loaded in near Mode II loading are practically featureless. Figures 6-4 and 6-5 are pictures of fracture surfaces formed under near Mode II loading at the

same magnification as Figures 6-1 and 6-2. Figure 6-4 is a surface image of specimen 1V, while Figure 6-5 depicts part of the fracture surface of specimen 1A; both are loaded in case 3 conditions. The failure surfaces here are very near planar, the shear crack growth occurring within a very narrow band in the material.

At higher magnification, as in Figure 6-6 from specimen 1V, the nearly featureless flat fracture predominates. Only shallow ridges and gullies vary the topography. Occasionally, though, small regions of shear ductile-dimple are found, as seen in Figure 6-7, which is from specimen 1A. A higher magnification of these dimples shown in Figure 6-8 shows that they have a parabolic shape, being elongated in the direction of shear crack growth. These shear ductile-dimple regions indicate a local Mode I character in the microstructural failure mechanisms, as evidenced by the increased amount of out-of-plane deformation.

The 4330V steel has a finer microstructure than the aluminum, but precisely the same differences are seen between the Mode I and Mode II fracture surfaces. Figure 6-9 shows the Mode I fracture surface of the side-grooved compact tension specimen. Here the surface is rough, with large valleys and peaks. Under higher magnification of this surface, as seen in Figure 6-10, regions of ductile-dimple and cleavage fracture are readily distinguishable.

Again, though, as in the aluminum, the Mode II fracture surface in the steel has the appearance of being scraped clean. Figure 6-11 shows the failure surface of compact shear specimen 1

loaded in case 1. The fracture surface is nearly flat, though slightly pockmarked. At higher magnification, as shown in Figure 6-12, the pockmarks are more readily seen, but otherwise low ridges are the only surface features.

VII. MIXED MODE FRACTURE FAILURE CRITERIA

Two important questions regarding fracture failure criteria under mixed mode loading will be considered in this Chapter. First, given an initially straight crack under mixed mode loading, at what angle will the crack begin to grow? Second, is there a relationship between the critical Mode I and Mode II fracture toughness values, so that knowledge of one will infer the other?

Many theories have been proposed which predict the angle of initial crack growth under mixed mode loading. Both Bergkvist and Guex (32), and Streit (82) give a thorough discussion of many of these theories, and both come to the same conclusion. They conclude that the differences in the predictions of the theories are very small for near Mode I loading, and that the use of the maximum tangential stress theory is totally acceptable.

The maximum tangential stress theory, as proposed by Erdogan and Sih (23), is based on the assumptions that crack extension starts at the tip in a radial direction, and that crack extension starts in the plane perpendicular to the direction of greatest tension. Many experiments using center-cracked plate specimens and linear elastic, brittle materials have been performed to substantiate the maximum tangential stress theory, and the agreement between experiment and theory has generally been quite acceptable (23, 64, 65, 68).

The use of the direction of the maximum J integral value to predict the angle of crack growth under mixed mode loading, has been proposed more recently in connection with near Mode I loading

(54, 32), but has received little attention. The maximum J integral theory is based on equations 2-11 and 4-1 and predicts crack growth in the direction of maximum J, where $\partial J / \partial \theta$ equals zero.

Figure 7-1 shows a comparison between the predicted angle of crack growth as calculated by both the maximum J integral and maximum tangential stress theories. The agreement between the two theories for near Mode I loading, i.e., near a loading mixity of 90 degrees as defined in Figure 7-1, is remarkable. Not until the loading mixity is at 65 degrees, corresponding to a K_I/K_{II} ratio of about 2.0, do the predictions for crack angle differ by more than one degree. Therefore, for calculating the angle of predicted crack growth for near Mode I fatigue pre-crack loading, as done in Chapter 4, the two theories are equivalent. However, as the amount of Mode II loading increases, the predictions of the two theories begin to differ greatly. At Mode II loading, when K_I/K_{II} equals zero, the maximum J integral theory predicts an angle of extension of zero degrees -- i.e., self-similar crack growth. The maximum tangential stress theory predicts an angle of crack growth of -70 degrees.

Although at near Mode II loading there is a clear-cut difference in the prediction of the angle of crack growth between the two theories, the results of this investigation suggest that both theories may be correct.

For the observed crack growth at zero degrees in the aluminum specimens loaded in case 3, where the loading mixity was about 5 degrees, this data point is seen in Figure 7-1 to lie somewhat below the prediction of the maximum J integral theory. For crack growth in the photoelastic specimen loaded in case 3, the level of mixity was

also about 5 degrees, but the angle of crack growth was -70 degrees. This data point is seen to lie slightly above the prediction of the maximum tangential stress theory. The loading in case 4 for both materials resulted in loading mixity of about 60 degrees, where the two theories are in substantial agreement. The angle of crack growth in the photoelastic material of 45 degrees, and the average angle of crack growth in the aluminum specimens of 43 degrees, are close to the predicted angles from both theories. Whether the maximum J integral theory or maximum tangential stress theory is appropriate to predict the angle of crack growth in a material, seems to depend on the materials relative ductility.

Based on the results of this investigation, it also appears that K_{IIc} may be less than K_{Ic} for brittle materials, but that K_{IIc} is greater than K_{Ic} in materials where some plastic deformation is possible. Figure 4 of reference (83) supports this type of relationship between K_{IIc} and K_{Ic} for brittle and more ductile materials.

The crystalline structures of brittle solids vary greatly, and it would seem to be almost impossible to predict a priori if K_{IIc} is always less than K_{Ic} for these materials. However, for ductile materials there is a rational basis for predicting that K_{IIc} or J_{IIc} will be greater than the corresponding K_{Ic} or J_{Ic} .

Rice, in a thorough review of the current state of elastic-plastic fracture (84), presents the idea that there are two competing effects in the process of ductile crack growth. These are void growth and coalescence, and shear band localization of plastic strain near the crack tip. Mode I loading causes large hydrostatic stresses at the

crack tip, and these stresses promote void growth and coalescence. As previously noted for Mode II loading, the state of stress at the crack tip is pure shear (there is no hydrostatic component) so that crack growth should be caused by local shear band formation. This suggests that in the material microstructure, the onset of crack growth should be best predicted by a critical strain criterion.

Mackenzie, Hancock, and Brown (85), in discussing failure criteria based on critical effective plastic strains, show the influence of the state of stress in the region of plastic deformation in affecting the critical strain at which crack growth initiates. Figure 19 of their paper shows the effective plastic strain to failure as a function of the ratio of the hydrostatic stress to the effective stress for 7075 aluminum.

For Mode II loading the stress state parameter is zero, and the effective plastic strain to failure reaches a maximum of about 0.5, according to this figure. From the stress intensity solution for stresses and displacements around the crack tip, as given in Chapter 1, it may be shown that in the region ahead of the crack tip, the ratio of the hydrostatic stress to the effective stress is 2.6. This value lies well above the results of Mackenzie, Hancock, and Brown, but an extrapolation of their information would suggest that the effective plastic strain to failure is much less in Mode I loading than in Mode II.

Assuming that the work to fracture is related to a product of the effective stress and strain in the crack tip region, then the much larger effective strain to failure in Mode II loading can explain why K_{IIc} is larger than K_{Ic} for relatively ductile materials. This

failure theory, based on the product of the effective stress and strain, would hypothesize that since the effective stress is limited in magnitude to be roughly equal to the flow stress of the material, in both Mode I and Mode II loading, that a greater effective plastic strain to failure would infer a larger energy required for crack growth. This type of fracture failure theory is similar to a cumulative-strain-damage model of ductile fracture propounded by others (86, 87). Therefore, there does appear to be a rational basis to expect that Mode II fracture toughness values should be greater than Mode I, for materials such as the aluminum and steel tested in this investigation.

VIII. DISCUSSION OF RESULTS AND CONCLUSION

The calculation of the J integral proved to be a successful method for characterizing the stress and displacement fields around a crack tip under mixed mode loading. Separating the stress and displacement fields into their symmetric and antisymmetric parts allowed a symmetric J integral value, due to the Mode I loading, to be calculated, as well as an antisymmetric J integral value, due to the Mode II loading. The ratio of these two J integral values, called J_I and J_{II} , defines the relative amount of Mode I to Mode II loading. For essentially elastic material behavior, knowing J_I and J_{II} allows one to calculate the corresponding stress intensity factors K_I and K_{II} . Because the J integral is a vector quantity, the direction of the maximum crack driving force may be determined by knowing values of the J integral vector in orthogonal directions.

A computer program was written to determine the symmetric and antisymmetric J integral quantities, given the stresses and displacements at points along a specified path around a crack tip. The stress and displacement values were calculated for the actual specimen configurations and material properties using finite element analysis. Because loading and boundary conditions can be accurately modelled in finite element analysis, their sometimes unexpected effects on the stress singularity at the crack tip may be determined. For both Mode I loading on a compact tension specimen and mixed mode loading on a center-cracked plate, the finite element analysis, coupled with the J integral post-processor, provided stress intensity factors which agreed very well with those calculated using established

numerical and theoretical methods on the same geometries. The J integral quantities calculated were shown to be independent of the finite element mesh size and path of integration within certain constraints.

The compact shear specimen was chosen from among several possibilities to be used in this investigation into fracture under mixed mode loading. Using the finite element analysis, and the J integral post-processor, it was found that the J_I and J_{II} values at the crack tip were very sensitive to the boundary conditions employed in the analysis. Depending on the loading direction and the degree of constraint on the motion of the outer tangs, the resulting deformation at the crack tip could be either mostly Mode I or Mode II. For further study, it is suggested that the loading conditions which produce crack opening displacements at the crack tip, that is positive K_I , be used. The effects of a negative K_I , or crack closing displacements, include frictional shear and compressive surface stresses in the region behind the fatigue crack tip, which produce unnecessary complications in calculating mixed mode fracture toughness values. To investigate fracture at ratios of Mode I to Mode II loading unattainable in the compact shear specimen, straightforward variations of the specimen may be designed. These variations of specimen design would include putting the crack planes at an angle to the loading direction. For the applied loading and boundary conditions on these specimens, the J_I and J_{II} values may be calculated using finite element analysis and the J integral post-processor.

Because of the number of tests performed with near Mode II loading in the aluminum alloy, a high degree of confidence was obtained in the result that K_{IIc} is about 1.9 times K_{Ic} in this material. To better define the failure envelope in mixed mode loading, critical stress intensity factors intermediate to those of cases 3 and 4 could be obtained using the modified specimens.

The results of this investigation indicate that the procedures employed by Jones and Chisholm (5,70,73), in using the compact shear specimen to study Mode II fracture, lead to an overestimation of Mode II fracture toughness values. Their method of loading the compact shear specimen, termed load case 1 in this investigation, produces a negative K_I at the crack tip. The effect of this negative K_I is to increase plastic deformation near the crack tip and retard crack growth initiation, and hence K_{IIc} is overestimated. Also, Jones and Chisholm use the 5 percent secant offset method to determine the load at which the critical K_{IIc} value will be computed. This allows for too much change in compliance, due to either crack growth or plasticity, for their K_{IIc} results to be directly comparable to the K_{Ic} values of the ASTM standard test method.

It would also be interesting to perform J integral interrupted load cycle tests. However, due to the small change in compliance with change in crack length for the compact shear specimen, unloading compliance crack measurement techniques would probably not be useful. However, by loading several specimens to different load levels where different amounts of crack growth has occurred, one could obtain a

plot of J integral versus change in crack length. The value of the J integral, and hence, K_{IIc} and K_{Ic} , at which crack growth initiated, could be extrapolated.

For the 4330V steel material more tests are required to determine the Mode II fracture toughness values. Compact shear specimens with shorter crack lengths, to increase the ratio of the Mode II stress intensity factor to the applied shear stress, are required. For both the steel and the aluminum, the effect of thickness on the predicted mixed mode fracture toughness values should be investigated, although the effect of thickness in Mode II fracture toughness should be small.

Particularly because of the statistical nature of fracture in brittle materials, more tests are needed to determine the critical mixed mode stress intensity factors for the photoelastic material. It was very interesting, though, to contrast the very different mixed mode fracture behavior of the brittle photoelastic epoxy and the more ductile aluminum.

It would be interesting to investigate the fracture behavior of more materials, which range in ductility between the photoelastic material and the aluminum, to find which materials have crack growth in accordance with the maximum tangential stress theory, and which follow the maximum J integral theory for near Mode II loading.

For materials which exhibit fatigue, it would be particularly interesting to determine the direction of fatigue crack growth under cyclic, near Mode II loading. Since fatigue crack growth is thought to occur because of shear deformation, it might be expected that the fatigue crack growth would be in accordance with the maximum J integral theory.

Using the methods presented in this investigation, Mode II and mixed mode fracture toughness values may be determined for various materials, both ductile and brittle, as needed to predict the failure of components containing cracks.

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TABLE 5-1: SUMMARY OF TENSILE TEST DATA
YIELD AND ULTIMATE STRENGTHS AND YOUNG'S MODULUS IN PSI UNITS

MATERIAL	SPECIMEN IDENTIFICATION	MATERIAL ORIENTATION	YIELD STRENGTH	ULTIMATE STRENGTH	REDUCTION IN AREA %	YOUNG'S MODULUS
Aluminum	3A	L-T	78,100	85,700	13.0	9.633x10 ⁶
	3B	L-T	78,300	86,000	14.9	10.04x10 ⁶
	3X	T-L	79,700	88,100	15.35	10.33x10 ⁶
	3Y	T-L	79,100	88,000	15.35	10.34x10 ⁶
Average Aluminum Properties			78,800	87,000	14.7	10.25x10 ⁶ *
4330V Steel	T1		148,900	149,900	62.3	30.6x10 ⁶
	T2		144,400	149,600	62.3	29.9x10 ⁶
	T3		150,900	153,600	54.3	30.5x10 ⁶
Average Steel Properties			148,000	151,000	60	30.0x10 ⁶

*Average of Last Three Values Only

TABLE 5-2: ALUMINUM MODE I FRACTURE TOUGHNESS VALUES
UNITS ARE KSI-IN^{1/2}

SPECIMEN IDENTIFICATION	MATERIAL ORIENTATION	K _{Ic}
2A	L-T	28.79
2B	L-T	26.73
2C	L-T	29.51

AVERAGE K_{Ic}

FOR L-T ORIENTATION = 28.3

2X	T-L	26.53
2Y	T-L	26.31
2Z	T-L	24.89

AVERAGE K_{Ic}

FOR T-L ORIENTATION = 25.9

TABLE 5-3: PHOTOELASTIC PSM-5 MODE I FRACTURE
TOUGHNESS VALUES UNITS ARE KSI-IN^{1/2}

SPECIMEN IDENTIFICATION	K _{Ic}
4P	2.78
5P	2.91
6P	2.78

TABLE 5-4 THE CONFIGURATION OF THE ALUMINUM COMPACT
SHEAR SPECIMENS PRIOR TO FINAL TESTING

<u>SPECIMEN IDENTIFICATION</u>	<u>MATERIAL ORIENTATION</u>	<u>CRACK LENGTH (1) - (2)</u>	<u>FATIGUE ANGLE (1) (2)</u>	<u>LOAD CASE</u>
1A	T-L	2.300 2.304	-3.9 -2.3	3
1B	T-L	2.319 2.248	-5.0 1.7	4
1X	L-T	2.282 2.279	1.9 -4.25	3
1Y	L-T	2.279 2.281	-3.4 -0.3	4
1D	T-L	2.074 2.081	1.65 0.65	2
1E	T-L	2.102 2.096	-1.9 -3.19	1
1G	T-L	2.10 2.10	-3.0 3.1	3
1T	L-T	2.095 2.10	2.85 1.65	2
1V	L-T	2.105 2.115	0.4 1.95	1
1W	L-T	2.09 2.09	4.05 -1.35	3

TABLE 5-5 ALUMINUM MAXIMUM LOAD VALUES AND CALCULATION
OF FLOW SHEAR STRESS

	<u>CASE</u>	<u>SPECIMEN</u>	<u>MAX. LOAD LBS.</u>	<u>τ_{FAILURE} PSI</u>	<u>$\frac{\tau_{\text{FAILURE}}}{\tau_{\text{FLOW}}}$</u>	<u>MATERIAL ORIENTATION</u>
a = 1.55	1	1V	36,000	40,000	.97	L-T
	1	1E	37,000	41,100	.99	T-L
	2	1T	38,000	42,200	1.02	L-T
	2	1D	39,000	43,300	1.05	T-L
	3	1W	33,400	37,100	.89	L-T
	3	1G	32,500	36,100	.87	T-L
a = 1.75	3	1X	26,100	37,300	.90	L-T
	3	1A	25,000	35,700	.86	T-L
	4	1Y	7,500	10,700	.26	L-T
	4	1B	6,925	9,900	.24	T-L

TABLE 5-6 MEASURED AMOUNTS OF SYMPATHETIC SHEAR CRACK GROWTH AND LOADS AT DEPARTURE FROM LINEARITY

	SPECIMEN	Δa , INCHES	P_d LBS.
$a_1 = 1.55$	1W	0.09	31,000
	1G	0.05	27,000
$a_2 = 1.75$	1X	0.03	22,500
	1A	0.05	23,500

$$\frac{P_{d,1W}}{P_{d,1X}} = 1.38$$

$$\frac{P_{d,1G}}{P_{d,1A}} = 1.15$$

TABLE 5-7 VALUES OF K_I AND K_{II} AT ONSET OF CRACK GROWTH
 BASED ON MAXIMUM LOAD, AT LOAD AT DEPARTURE FROM LINEARITY,
 UNITS ARE KSI-IN^{1/2}

	SPECIMEN IDENT	CASE NUMBER	MATERIAL ORIENTATION	MAXIMUM LOAD		DEPARTURE FROM LINEARITY	
				K_I	K_{II}	K_I	K_{II}
a = 1.55	1V	1	L-T	-18.6 *	-66.84 *		
	1E	1	T-L	-19.48 *	-69.82 *		
	1T	2	L-T	-67.29 *	-165.4 *		
	1D	2	T-L	-68.47 *	-185.7 *		
	1W	3	L-T	7.63	63.92	7.1	57.9
	1G	3	T-L	7.50	62.76	6.2	48.8
a = 1.75	1X	3	L-T	4.92	56.93	4.35	50.3
	1A	3	T-L	4.63	53.69	4.16	47.08
	1Y	4	L-T	27.62	13.17		
	1B	4	T-L	25.51	12.15		

* These values are for comparison only, and are not critical elastic stress intensity factors because of significant plastic deformation in the specimen prior to failure.

TABLE 5-8 RESULTS OF THE 4330V TESTS INCLUDING MAXIMUM
LOAD VALUES AND THE RATIO OF THE APPLIED SHEAR STRESS
TO THE MATERIAL FLOW SHEAR STRESS

<u>SPECIMEN IDENTIFICATION</u>	<u>LOAD CASE</u>	<u>MAX. LOAD LBS.</u>	<u>FAILURE PSI</u>	<u>$\frac{\tau_{\text{FAILURE}}}{\tau_{\text{FLOW}}}$</u>
1	1	25,700	85,670	1.12
2	3	29,000	96,670	1.27
3	4	10,000.*		

* No catastrophic failure occurred in this case.

This was simply the maximum load applied.

TABLE 5-9 RESULTS OF THE TESTS ON THE PHOTOELASTIC
COMPACT SHEAR SPECIMENS

SPECIMEN IDENTIFICATION	LOAD CASE	MAX. LOAD LBS.	K_I ksi-in ^{1/2}	K_{II} ksi-in ^{1/2}
4	1	1120	-.57	1.9
1	2	1580	*	*
3	3	675	.15	1.2
2	4	300	.76	.44

* In case 2, test was halted at the onset
of plastic deformation.

FIG. 1-1. The basic loading modes.

FIG. 1-2. Cartesian and polar coordinates, and stress components, with respect to the leading edge of the crack tip.

FIG. 2-1. Cartesian and polar coordinates and curves for definition of the J integral.

FIG. 2-2. Rotation of coordinates associated with a kinked crack and the J integral path along the unkinked portion of the crack surfaces.

FIG. 2-3. Coordinates for the definition of the J integral vector in two dimensions.

Fig. 3-1 The stress in the z direction as a function of element number along the boundary of symmetry in the J integral compact tension specimen. The crack tip is between elements 9 and 10. Units are in psi.

Fig. 3-2 The shear stress as a function of element number along the boundary of symmetry in the J integral compact tension specimen. The crack tip is between elements 9 and 10. Units are in psi.

Fig. 3-3 The outline and location of reference points for the finite element model of the J integral compact tension specimen.

Fig. 3-4 The finite element mesh for the J integral compact tension specimen.

PATH	J (IN-LB/IN ²)
1	12.270
2	13.462
3	12.998
4	12.782
5	12.702
6	12.610

Fig. 3-5 The six J integral paths and their corresponding values in the crack tip region of the J integral compact tension specimen.

FIG. 4-1. The compact shear specimen with associated coordinate system and dimensional definitions.

FIG. 4-2. Non-dimensionalized Mode II stress intensity factor vs a/W ratio for a tang width $H = 0.5$, 1.0 and 1.5 in.

FIG. 4-3. Drawing of the compact shear specimen with thicker section at loading holes. Dimensions in inches.

Fig. 4-4. The finite element mesh for the compact shear specimen with 1.55 inch crack length. Because of specimen symmetry, only half the specimen is modelled.

FIG. 4-6. Mode II stress intensity factor as a function of crack length and tang width.

FIG. 4-7. Variation of the relative amount of Mode I to Mode II deformation as a function of crack length in the compact shear specimen, Case 1.

Fig. 4-8. Computer-generated shear stress contours for case 1 loading on the compact shear specimen. Crack length is 1.55 inches.

Fig. 4-9. The isochromatic fringe pattern for the photoelastic specimen loaded in case 1. Dark field. Crack length is 1.55 inches.

Fig. 4-10 Case 1 loading conditions
front view

Fig. 4-11 Case 1 loading conditions side view.

Fig. 4-12 Case 2 loading conditions,
front view

Fig. 4-13 Case 3 loading conditions,
front view

Fig. 4-14 Case 3 loading conditions,
side view

Fig. 4-15 Case 4 loading conditions,
front view

Fig. 4-16 Case 4 loading conditions,
side view

FIG. 4-17. Relative amount of Mode II deformation as a function of crack length and loading case
 $H/W = 0.408$

FIG. 4-18. Mode II stress intensity factor as a function of crack length and load case. $H/W = 0.408$.

FIG. 4-19. Singularity strength as a function of crack length and load case $H/W = 0.408$.

Fig. 4-20. Computer-generated shear stress contours for case 2 loading on the compact shear specimen. Crack length is 1.55 inches

Fig. 4-21. The isochromatic fringe pattern for the photoelastic specimen loaded in case 2. Dark field. Crack length is 1.55 inches.

Fig. 4-22. Computer-generated shear stress contours for case 3 loading on the compact shear specimen. Crack length is 1.55 inches.

Fig. 4-23. The isochromatic fringe pattern for the photoelastic specimen loaded in case 3. Dark field. Crack length is 1.55 inches.

Fig. 4-24. Computer-generated shear stress contours for case 4 loading on the compact shear specimen. Crack length is 1.55 inches.

Fig. 4-25. The isochromatic fringe pattern for the photoelastic specimen loaded in case 4. Dark field. Crack length is 1.55 inches.

Figure 4-26. Explanation of loading directions for compact shear specimen fatigue pre-cracking.

Fig. 4-27 Lower tie plate used to fatigue pre-crack the compact shear specimens

Fig. 5-1 Aluminum tensile test specimens
showing slanted failure surface

Fig 5-2 Cup and cone failure surfaces
for 4330V steel tensile test
specimen

Fig. 5-3 Close-up of steel tensile specimen failure surfaces

Fig. 5-4 Fracture surface of aluminum compact tension specimen; crack plane in T-L orientation

Fig. 5-5 Fracture surface of aluminum compact tension specimen; crack plane in L-T orientation

Figure 5-6. Values of J integral versus plastic crack growth - 4330 V compact tension

Fig. 5-7 Fracture surface of 25% side grooved steel compact tension specimen

Fig. 5-3 Fracture surface of steel compact tension specimen without side grooves

Fig. 5-9 Attachment of displacement gages to compact shear specimen loading devices, Case 1

Fig. 5-10 Specimen 1E, Case 1

Fig. 5-11 Specimen 1V, Case 1

Fig. 5-12 Specimen 1D, Case 2

Fig. 5-13 Specimen 1T, Case 2

Fig. 5-14 Specimen 1A, Case 3

Fig. 5-15 Specimen 1W, Case 3

Fig. 5-16 Specimen 1B, Case 4

Fig. 5-17 Specimen 1Y, Case 4

Figure 5-18. Estimates of critical stress intensity factors for mixed mode loading in aluminum compact shear specimens.

Fig. 5-19 Specimen 1, load case 1

Fig. 5-20 Specimen 2, load case 3

Fig. 5-21 Specimen 3, load case 4.
The specimen has been heat-
tinted

Figure 5-22. Critical stress intensity factors for compact shear specimen tests in photoelastic material.

Fig. 5-23 Photoelastic specimen 4, load case 1.

Fig. 5-24 Photoelastic specimen 3, load case 3.

Fig. 5-25 Photoelastic specimen 2, load case 4.

Fig. 6-1 Fracture surface of aluminum compact tension specimen 2A.

Fig. 6-2 Fracture surface of aluminum compact shear specimen 1B, case 4 loading conditions.

Fig. 6-3 Fracture surface of aluminum compact shear specimen 1B at higher magnification, case 4 loading conditions.

Fig. 6-4 Fracture surface of aluminum compact shear specimen IV, case 1 loading conditions.

Fig. 6-5 Fracture surface of aluminum compact shear specimen 1A, case 3 loading conditions.

Fig. 6-6 Fracture surface of aluminum compact shear specimen 1V at higher magnification, case 1 loading conditions.

Fig. 6-7 Fracture surface of aluminum compact shear specimen 1A at higher magnification, case 3 loading conditions.

Fig. 6-8 Region of ductile-dimple failure under near Mode II loading conditions. Fracture surface of aluminum compact shear specimen 1A at yet higher magnification, case 3 loading conditions.

Fig. 6-9 Fracture surface of 4330V steel compact tension specimen with 25% side grooved.

Fig. 6-10 Fracture surface of 4330V steel compact tension specimen with side-grooves at higher magnification.

Fig. 6-11 Fracture surface of 4330V steel compact shear specimen, loaded in case 1.

Fig. 6-12 Fracture surface of 4330V steel compact shear specimen at higher magnification, loaded in case 1.

Figure 7-1. Comparison of predicted fracture angle between maximum J integral and maximum tangential stress failure criteria.

APPENDIX:

FORTRAN listing of the J integral post-processor MMJINT


```

c j integral version 3/11/81 only format change
c=cft l=mmjint, b=bj, l=lbj
c=ldr bin=(bj), lib=(fortlib), x=xmmj
dimension jel(200), jgauss(200), jelcon(200,4), yandz(200,4,2)
dimension stress(200,5), title(12), disp(200,4,2), je(4)
dimension y(200), z(200), pn1(200), pn2(200), dl(200)
dimension djinc(200), coeff(4), yx(200), zx(200), ds1(200), ds2(200)
dimension ya(2), dis(2), st(5)
dimension dudx1(200), dvdx1(200)
dimension dxidr(200), dxids(200), detadr(200), detads(200)
dimension dnidru(200), dnidsv(200), dnidsu(200), dnidrv(200)
dimension sdisp(100,4,2), edisp(100,4,2), sstres(100,4)
dimension astres(100,4), dmjinc(100)
call dropfile(0)
call link("unit59=tyy//")
call maglink(59,1)
write (59,5016)
read (59,5014) nw
write (59,5013)
read (59,5014) mm
write (59,5010)
read (59,5011) djin, drjout
write (59,5012)
read (59,5011) ppnike, ppnike01
call open(2,djin,2,len1,err1)
call create(50,drjout,2,-1)
call open(20,ppnike,4,len,err)
if ( len .eq. -1 ) goto 5000
if ( err .ne. 0.0 ) goto 5000
write (59,5007) len
read (2,1000) title
write (50,1010) title
read (2,1001) dtheta,njpp,ntime

c
c dimension jel(njpp),jgauss(njpp)
c
read (2,998) pnu, ymod
read (2,1002) (jel(jpp),jgauss(jpp),jpp=1,njpp)
write (50,999) ntime
write (50,997) pnu,ymod

c
c read preliminary information from ppnike file. to read from binary
c file place pointer one word before the first desired word.
c
call rdabs(20,numnp,1,12,errlab)
if (errlab .ne. 0.) goto 5000
call rdabs(20,nn,1,14,errlab)
if (errlab .ne. 0.) goto 5000
numelv=-nn
numel4=0
call rdabs(20,nummat,1,15,errlab)
if (errlab .ne. 0.) goto 5000
write (50,1003) dtheta,njpp
write (50,1004) numnp,numelv,nummat

c
c read connectivity array for j integral
c path point elements
c dimension jelcon(njpp,4)
c
iconb= 36+nummat+2*numnp

```

```

      write (50,1005)
c
      do 20 jpp=1,njpp
        icount=iconb+10*(jel(jpp)-1)
        call rdabs(20,je,4,icount,errlab)
        do 21 i=1,4
          jelcon(jpp,i)=je(i)
21      continue
        if (errlab .ne. 0.) goto 5000
20      continue
c
c read coordinate array for nodes connected with jpath elements
c dimension yandz(njpp,4,2)
c
      do 40 jpp=1,njpp
        do 30 it=1,4
          icoord= 36+nummat+2*(jelcon(jpp,it)-1)
          call rdabs(20,ya,2,icoord,errlab)
          do 31 i=1,2
            yandz(jpp,it,i)=ya(i)
31      continue
          if (errlab .ne. 0 ) goto 5000
30      continue
40      continue
c
c
c find the right binary plotfile to open to obtain
c displacements and stresses
c
      npw=37+nummat+2*numnp+10*numelv
      nwss=6+nummat+20*numelv+6*numnp
      ifw=npw+nwss+ntime*nwss
      if ( ifw .lt. 262144 ) goto 50
      call close(20)
      call open(20,ppnike01,4,len,err)
      if ( len .eq. -1 ) goto 5000
      if (err .ne. 0.0) goto 5000
      write (59,5009) len
      try= (262144-npw)/nwss
      itry=int(try)
      ithf=ntime-itry
      ibegin=(6+nummat+20*numelv)
      ibegin=ibegin+(ithf)*nwss
      ibegin=ibegin-(ithf+1)
      goto 54
c read displacements at nodes connected with the j path
c
50      continue
      ibegin= ntime*(6+nummat+5*numel4+20*numelv+6*numnp)
      ibegin=ibegin+36+nummat+2*numnp+10*(numelv)
      ibegin=ibegin+(6+nummat+20*numelv)
      ibegin=ibegin-(ntime+1)
54      continue
c
c dimension disp(njpp,4,2)
c
      do 60 jpp=1,njpp
        do 55 it=1,4
          idisp=ibegin+2*(jelcon(jpp,it)-1)
          call rdabs(20,dis,2,idisp,errlab)

```



```

        do 61 i=1,2
        disp(jpp,it,i)=dis(i)
61      continue
        if (errlab .ne. 0.) goto 5000
55      continue
60      continue
c
c read stresses at jpath points
c dimension stress(njpp,5)
c
        istart=ibegin-(20*numelv)
        do 80 jpp=1,njpp
        istres=istart+20*(j1(jpp)-1)+5*(jgauss(jpp)-1)
        call rdeba(20,at,5,istres,errlab)
        do 81 i=1,5
        stress(jpp,i)=st(i)
81      continue
        if (errlab .ne. 0.) goto 5000
        eps=stress(jpp,5)
        if (eps .gt. (0.001)) goto 5004
80      continue
c
c calculate symmetric and antisymmetric
c displacements and stresses
c
        if (mm .eq. 0) goto 118
        xy=njpp/2.0
        lxy=int(xy)
        if (xy .ne. lxy) goto 4998
        do 100 m=1,lxy
        k=njpp-m+1
        do 110 it=1,4
        kit=5-it
        sdisp(m,kit,1)=0.5*(disp(m,kit,1)+disp(k,it,1))
        sdisp(m,kit,2)=0.5*(disp(m,kit,2)-disp(k,it,2))
c
        adisp(m,kit,1)=0.5*(disp(m,kit,1)-disp(k,it,1))
        adisp(m,kit,2)=0.5*(disp(m,kit,2)+disp(k,it,2))
110      continue
c
        sstres(m,1)=0.5*(stress(m,1)+stress(k,1))
        sstres(m,2)=0.5*(stress(m,2)+stress(k,2))
        sstres(m,4)=0.5*(stress(m,4)+stress(k,4))
c
        astres(m,1)=0.5*(stress(m,1)-stress(k,1))
        astres(m,2)=0.5*(stress(m,2)-stress(k,2))
        astres(m,4)=0.5*(stress(m,4)-stress(k,4))
100      continue
c
c calculate y and z coordinates of j integral path points
c
118      continue
        do 140 jpp=1,njpp
        jg=jgauss(jpp)
        if (jg .eq. 1) goto 125
        if (jg .eq. 2) goto 126
        if (jg .eq. 3) goto 127
        if (jg .eq. 4) goto 128
125      r=-0.57735

```

```

      a=-0.57735
      goto 130
126    r=0.57735
      a=-0.57735
      goto 130
127    r=0.57735
      s=0.57735
      goto 130
128    r=-0.57735
      s=0.57735
130    continue
c
c dimension coeff(4) these are coefficients of
c transformation matrix between the natural and real coordinates.
      coeff(1)=0.25*(1.0-r)*(1.0-s)
      coeff(2)=0.25*(1.0+r)*(1.0-s)
      coeff(3)=0.25*(1.0+r)*(1.0+s)
      coeff(4)=0.25*(1.0-r)*(1.0+s)
      y(jpp)=0.0
      z(jpp)=0.0
      do 135 it=1,4
      yx(jpp)=coeff(it)*yandz(jpp,it,1)
      zx(jpp)=coeff(it)*yandz(jpp,it,2)
      y(jpp)=y(jpp)+yx(jpp)
      z(jpp)=z(jpp)+zx(jpp)
135    continue
140    continue
c
c
c
c calculate normals to j integral path segments
c
      jps=njpp-1
      do 160 jpp=1,jps
      jnp=jpp+1
      ds1(jpp)=y(jnp)-y(jpp)
      ds2(jpp)=z(jnp)-z(jpp)
      if (ds1(jpp).ne. 0.0) goto 145
      pn2(jpp)=0.0
      if (ds2(jpp).gt. 0.0) pn1(jpp)=1.0
      if (ds2(jpp).lt. 0.0) pn1(jpp)=-1.0
      goto 150
145    slope=ds2(jpp)/ds1(jpp)
      pn2(jpp)=sqrt(1.0/(1.0+slope**2))
      if (ds1(jpp).gt. 0.0) pn2(jpp)=-pn2(jpp)
      pn1(jpp)=-1.0*pn2(jpp)*slope
      continue
150    dl(jpp)=sqrt(ds1(jpp)**2+ds2(jpp)**2)
160    continue
c
c
c calculate displacement gradient in direction of self-similar
c crack growth
c
c
      xpl=3.141592654
      theta=(dtheta/360.0)*2.0*xpl
      do 180 jpp=1,njpp
      jg=jgauss(jpp)
      if (jg.eq. 1) goto 165

```

```

        if (jg .eq. 2 ) goto 166
        if (jg .eq. 3) goto 167
        if (jg .eq. 4) goto 168
165    r=-0.57735
        s=-0.57735
        goto 170
166    r=0.57735
        s=-0.57735
        goto 170
167    r=0.57735
        s=0.57735
        goto 170
168    r=-0.57735
        s=0.57705
170    continue
        cs1=1.0+s
        cs2=1.0-s
        cr1=1.0+r
        cr2=1.0-r
        y1=yandz(jpp,1,1)
        y2=yandz(jpp,2,1)
        y3=yandz(jpp,3,1)
        y4=yandz(jpp,4,1)
c
        z1=yandz(jpp,1,2)
        z2=yandz(jpp,2,2)
        z3=yandz(jpp,3,2)
        z4=yandz(jpp,4,2)
c
        cst=cos(theta)
        sst=sin(theta)
c
        dxldr(jpp)=0.25*(cst*(-y1*cs2+y2*cs2+y3*cs1-y4*cs1)
1+sst*(-z1*cs2+z2*cs2+z3*cs1-z4*cs1))
c
        detadr(jpp)=0.25*(-sst*(-y1*cs2+y2*cs2+y3*cs1-y4*cs1)
1+cst*(-z1*cs2+z2*cs2+z3*cs1-z4*cs1))
c
        dxlds(jpp)=0.25*(cst*(-y1*cr2-y2*cr1+y3*cr1+y4*cr2)
1+sst*(-z1*cr2-z2*cr1+z3*cr1+z4*cr2))
c
        detads(jpp)=0.25*(-sst*(-y1*cr2-y2*cr1+y3*cr1+y4*cr2)
1+cst*(-z1*cr2-z2*cr1+z3*cr1+z4*cr2))
c
        u1=disp(jpp,1,1)
        u2=disp(jpp,2,1)
        u3=disp(jpp,3,1)
        u4=disp(jpp,4,1)
c
        v1=disp(jpp,1,2)
        v2=disp(jpp,2,2)
        v3=disp(jpp,3,2)
        v4=disp(jpp,4,2)
c
        dnldr(jpp)=0.25*(cst*(-cs2*u1+cs2*u2+cs1*u3-cs1*u4)
1+sst*(-cs2*v1+cs2*v2+cs1*v3-cs1*v4))
c
        dnlds(jpp)=0.25*(-sst*(-cr2*u1-cr1*u2+cr1*u3+cr2*u4)
1+cst*(-cr2*v1-cr1*v2+cr1*v3+cr2*v4))

```

```

C      dnldsu(jpp)=0.25*(cst*(-cr2*u1-cr1*u2+cr1*u3+cr2*u4)
1      1+sst*(-cr2*v1-cr1*v2+cr1*v3+cr2*v4))
C      dnldrv(jpp)=0.25*(-sst*(-cs2*u1+cs2*u2+cs1*u3-cs1*u4)
1      1+cst*(-cs2*v1+cs2*v2+cs1*v3-cs1*v4))
C      j=jpp
      denom= dxidr(j)*detads(j)-dxids(j)*detadr(j)
      if (denom .eq. 0.0) goto 4999
      dudxi(jpp)=(detads(j)*dnldru(j)-detadr(j)*dnldsu(j))/denom
      dvdxj(jpp)=(detads(j)*dnldrv(j)-detadr(j)*dnldsv(j))/denom
180     continue
C      valu=0.0
      do 200 jpp=1,jps
      tudx=0.0
      wncr=0.0
      jnp=jpp+1
      xn1=pn1(jpp)
      xn2=pn2(jpp)
      do 201 i=1,2
      if (i .eq. 1) j=jpp
      if (i .eq. 2) j=jnp
      sig11=stress(j,1)
      sig22=stress(j,2)
      sig12=stress(j,4)
      sig21=sig12
      dud=dudxi(j)
      dvd=dvdxi(j)
C      stxi=cst*(sig11*xn1+sig12*xn2)+sst*(sig21*xn1+sig22*xn2)
      steta=-sst*(sig11*xn1+sig12*xn2)+cst*(sig21*xn1+sig22*xn2)
C      tudx=tudx+stxi*dud+steta*dvd
C      wsed=(-pnu/(2.0*ymod))*((1+pnu)**2)*((sig11+sig22)**2)
      ynumm=((1+pnu)/(2.0*ymod))
      snu=pnu**2
      wsed=wsed+ynumm*(sig11**2+sig22**2+snu*((sig11+sig22)**2))
      wsed=wsed+(((1+pnu)/ymod)*(sig12**2))
      wncr=wncr+wsed*(cst*xn1+sst*xn2)
C
C djinc is now the sum of the wncr and tudx quantities for the
C given path segment
C
201     continue
      djinc(jpp)=wncr-tudx
C
C calculation of the value of the J integral
C
      valu=valu+0.5*(djinc(jpp))*dl(jpp)
200     continue
C
      symj=0.0
      asymj=0.0
      if (mm .eq. 0) goto 400
C
C calculate displacement gradient in direction of self-similar

```

c crack growth

```

c
  do 300 l=1,2
  do 280 jpp=1,ixy
    jg=jgauss(jpp)
    if (jg .eq. 1) goto 205
    if (jg .eq. 2) goto 206
    if (jg .eq. 3) goto 207
    if (jg .eq. 4) goto 208
205  r=-0.57735
    s=-0.57735
    goto 210
206  r=0.57735
    s=-0.57735
    goto 210
207  r=0.57735
    s=0.57735
    goto 210
208  r=-0.57735
    s=0.57735
210  continue
    cs1=1.0+s
    cs2=1.0-s
    cr1=1.0+r
    cr2=1.0-r
    if (l .eq. 2) goto 220
c
    u1=sdisp(jpp,1,1)
    u2=sdisp(jpp,2,1)
    u3=sdisp(jpp,3,1)
    u4=sdisp(jpp,4,1)
c
    v1=sdisp(jpp,1,2)
    v2=sdisp(jpp,2,2)
    v3=sdisp(jpp,3,2)
    v4=sdisp(jpp,4,2)
c
    goto 225
c
220  continue
    u1=adisp(jpp,1,1)
    u2=adisp(jpp,2,1)
    u3=adisp(jpp,3,1)
    u4=adisp(jpp,4,1)
c
    v1=adisp(jpp,1,2)
    v2=adisp(jpp,2,2)
    v3=adisp(jpp,3,2)
    v4=adisp(jpp,4,2)
c
225  continue
c
    dnidru(jpp)=0.25*(cst*(-cs2*u1+cs2*u2+cs1*u3-cs1*u4)
1+sst*(-cs2*v1+cs2*v2+cs1*v3-cs1*v4))
c
    dnidsv(jpp)=0.25*(-sst*(-cr2*u1-cr1*u2+cr1*u3+cr2*u4)
1+cst*(-cr2*v1-cr1*v2+cr1*v3+cr2*v4))
c
    dnidsu(jpp)=0.25*(cst*(-cr2*u1-cr1*u2+cr1*u3+cr2*u4)
1+sst*(-cr2*v1-cr1*v2+cr1*v3+cr2*v4))

```

```

c      dnldrv(jpp)=0.25*(-sst*(-cs2*u1+cs2*u2+cs1*u3-cs1*u4)
1+cst*(-cs2*v1+cs2*v2+cs1*v3-cs1*v4))
c
      j=jpp
      denom= dxldr(j)*detads(j)-dxlds(j)*detadr(j)
      if (denom .eq. 0.0) goto 4999
      dudxl(jpp)=(detads(j)*dnldru(j)-detadr(j)*dnldsru(j))/denom
      dvdxl(jpp)=(detads(j)*dnldrv(j)-detadr(j)*dnldsv(j))/denom
280  continue
c
      valu=0.0
      jps=ixy-1
      do 399 jpp=1,jps
      tudx=0.0
      wncr=0.0
      jnp=jpp+1
      xn1=pn1(jpp)
      xn2=pn2(jpp)
      do 381 i=1,2
      if (i .eq. 1) j=jpp
      if (i .eq. 2) j=jnp
      if (i .eq. 2) goto 330
      sig11=sstres(j,1)
      sig22=sstres(j,2)
      sig12=sstres(j,4)
      sig21=sig12
      goto 335
c
330  continue
      sig11=astres(j,1)
      sig22=astres(j,2)
      sig12=astres(j,4)
      sig21=sig12
c
335  continue
c
      dud=dudxl(j)
      dvd=dvdxl(j)
c
      stx1=cst*(sig11*xn1+sig12*xn2)+sst*(sig21*xn1+sig22*xn2)
      steta=-sst*(sig11*xn1+sig12*xn2)+cst*(sig21*xn1+sig22*xn2)
c
      tudx=tudx+stx1*dud+steta*dvd
c
      wsed=(-pnu/(2.0*ymod))*((1+pnu)**2)*((sig11+sig22)**2)
      ynumm=((1+pnu)/(2.0*ymod))
      snu=pnu**2
      wsed=wsed+ynumm*(sig11**2+sig22**2+snu*((sig11+sig22)**2))
      wsed=wsed+(((1+pnu)/ymod)*(sig12**2))
      wncr=wncr+wsed*(cst*xn1+sst*xn2)
c
c d,jinc is now the sum of the wncr and tudx quantities for the
c given path segment
c
381  continue
      dmjinc(jpp)=wncr-tudx
c
c calculation of the value of the j integral
c

```



```

        valu= valu+0.5*(dmjinc(jpp))*dl(jpp)
        if (l .eq. 1) symj=valu
        if (l .eq. 2) asymj=valu
c
399  continue
300  continue
c
c now a few write statements
c
400  continue
      do 420 jpp=1,njpp
        write (50,1006) jcl(jpp),jgauss(jpp),(jelcon(jpp,it),it=1,4),
1y(jpp),z(jpp)
420  continue
c
      if (nw .eq. 0) goto 480
      write (50,1013)
      do 440 jpp=1,njpp
        write (50,1014) ((yandz(jpp,it,i),i=1,2),it=1,4)
440  continue
      write (50,1015)
      do 450 jpp=1,njpp
        write (50,1014) ((disp(jpp,it,i),i=1,2),it=1,4)
450  continue
      write (50,1016)
      do 460 jpp=1,njpp
        write (50,1019) (stress(jpp,i),i=1,5)
460  continue
      write (50,1017)
      jps=njpp-1
      write (50,1018) (djinc(jpp),dl(jpp),jpp=1,jps)
c
c
480  continue
      write (50,1007) valu
      write (50,1020) symj
      write (50,1021) asymj
      goto 900
c
c
c tty messages
c
4998  write (59,5015)
      goto 900
4999  write (59,5008)
      goto 900
5000  write(59,5001)
      goto 900
5004  write(59,5006)
      goto 80
900   call exit(1)
c
c
997   format("poisson's ratio=",f10.2,2x,"young's modulus=",e10.3/)
998   format(2f10.2)
999   format("time state for calculation=",i5/)
1000  format(12a6)
1001  format(f5.1,2i5)
1002  format(10(i5,2x,i1))
1003  format("crack angle in degrees=",f5.1,3x/

```

```

1"number of j-integral path points=",15/)
1004 format(" numnp=",15/2x,"numelv=",15/2x,"nummat=",15/)
1005 format(/"def of j integral path(element,gausspoint,con array,
1y and z coordinates)"/)
1007 format(/"the value of the j integral=",e10.3/)
1006 format(615,2x,e10.3,2x,e10.3)
1010 format(12a6,/)
1012 format(315,/)
1013 format(/"y and z coordinates")
1014 format(8(e10.3,2x))
1015 format(/"displacements u and v")
1016 format(/"stresses")
1017 format(/"incremental values of j and line segment length")
1018 format(2e10.3)
1019 format(5e10.3)
1020 format(/"the half-value of the symmetric j integral=",e10.3/)
1021 format(/"the half-value of the asymmetric j integral=",e10.3/)
5001 format("error from abs diskfile read"/)
5006 format("oops j-integral path in plastic region"/)
5007 format("1st nike2d plot file opened",110/)
5008 format("jacobian has zero value"/)
5009 format("2nd nike2d plot file opened",110/)
5010 format("type name of input and output files(r8,2x,r8)")
5011 format(r8,2x,r8)
5012 format("type name of plot files from nike2d")
5013 format("sym and asym j calculated?1=yes,0=no(11)"/)
5014 format(11)
5015 format("even number of j integral path points required")
5016 format("long output desired?1=yes,0=no(11)"/)
end

```

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