CONF-850672--14

RANDOM PHASE APPROXIMATION TREATMENT OF THE PAIRING PHASE

TRANSITION IN STRONGLY ROTATING NUCLEI

CONF-850672--14

DE86 008983

R. A. BRUGLIA

The Niels Bohr Institute, University of Copenhagen
Copenhagen, Denmark

and

Department of Physics and Astronomy, University of Tennessee
Knoxville, Tennessee 37996-0154 U.S.A.

and

The Joint Institute for Heavy Ion Research*, Holifield Heavy Ion Research Facility
Oak Ridge, Tennessee 37831 U.S.A.

and

M. GALLARDO

The Niels Bohr Institute, University of Copenhagen
Copenhagen, Denmark

to be presented at

Second International Conference on
Nucleus-Nucleus Collisions

Visby, Sweden
June 10-14, 1985

DISCLAIMER

By acceptance of this article, the publisher or recipient acknowledges the U.S. Government's right to retain a nonexclusive, royalty-free license in and to any copyright covering the article.

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

*The Joint Institute for Heavy Ion Research has as member institutions the University of Tennessee, Vanderbilt University, and the Oak Ridge National Laboratory; it is supported by the members and by the Department of Energy through Contract No. DE-AS05-76ER0-4936 with the University of Tennessee. This work is supported partially by a National Science Foundation grant under Contract No. PHY-8406676.

MASTER

RANDOM PHASE APPROXIMATION TREATMENT OF THE PAIRING PHASE TRANSITION IN STRONGLY ROTATING NUCLEI

R.A. BROGLIA

The Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark
and
University of Tennessee, Department of Physics and Astronomy, Knoxville,
Tennessee 37996-0154, USA

and
The Joint Institute for Heavy Ion Research*, Holifield Heavy Ion Research
Facility, Oak Ridge, Tennessee 37831

and

M. GALLARDO

The Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark

The rotational frequency for which the energy of pairing vibrations becomes zero provides a useful definition of ω_{cr} . The fluctuations associated with these modes induce ground state correlations which renormalize in an important way the energy and alignment of rotational bands over a broad range of frequencies around ω_{cr} . Based on the random phase approximation a simple prescription to take these effects into account is discussed.

1. INTRODUCTION

The random phase approximation treatment of pairing correlations^{1,2,3} leads to a model displaying near closed shells, collective vibrations which change the number of particles in two. For systems progressively removed from the magic nucleon numbers, these excitations become correspondingly softer. The system turns superfluid when $W(+2)=W(-2)=0$, $W(\pm 2)$ being the energy of the pair addition and pair removal modes respectively. This condition for the appearance of a pairing gap in the system coincides with the one provided by the BCS-model⁴ of superconductivity (cf. also ref. 5)

For superfluid systems, pairing vibrations can be viewed as a bound pair of states with an energy lying at the top of the gap. In this case there is another collective mode which has zero energy and corresponds to a free rotation of the system in gauge space (Goldstone boson). The proper inclusion of the

*The Joint Institute for Heavy Ion Research has as member institutions the University of Tennessee, Vanderbilt University, and the Oak Ridge National Laboratory; it is supported by the members and by the Department of Energy through Contract No. DE-AS05-76ER0-4936 with the University of Tennessee. This work is supported partially by a National Science Foundation grant under Contract No. PHY-8406676.

associated fluctuations repairs the violation of gauge invariance to an adequate degree, and projects the different solutions into the space of fixed particle number.

For superfluid and spatially deformed nuclei, a pairing phase transition can be induced by strongly spinning the system.

It was suggested^{6,7,8} that the inclusion of the zero-point fluctuations associated with pairing vibrations of both normal and superfluid systems provide an adequate continuity to the value of the different observables across the phase transition as a function of the rotational frequency (cf. also ref. 9). Further evidence for this contention has been obtained by comparing the RPA quantities with exact solutions in the case of an exactly solvable schematic model.¹⁰

In the present paper we study some of the consequences pairing fluctuations have on a variety of nuclear properties close to the pairing phase transition induced by the Coriolis force associated with strong rotations.

2. SPONTANEOUS SYMMETRY BREAKING AND COLLECTIVE MODES

The fact that many-body Hamiltonians can have ground states which do not have the same symmetry as the Hamiltonian itself is the broken symmetry phenomenon.

2.1 Cooper condensation

The standard theory of superconductivity is due to Bardeen, Cooper and Schrieffer. The BCS Hamiltonian contains a single-particle term and a residual interaction which scatters pairs of fermions moving in time reversal states.

Linearizing the pairing Hamiltonian in a quasiparticle basis, one obtains the gap and number equations

$$N = 2 \sum_{\nu > 0} V^2(\nu), \quad (1a)$$

and

$$\frac{2}{G} = \sum_{\nu > 0} \frac{1}{E(\nu)}, \quad (1b)$$

where N is the average number of particles, while $V^2(\nu)$ is the occupation probability of the single-particle orbital labeled by the quantum numbers ν . The quasiparticle excitation energy is denoted by $E(\nu)$, while G is the pairing coupling constant. The BCS Hamiltonian reads, in the quasiparticle basis,

$$H = (U)_{\text{BCS}} + (H_{11})_{\text{BCS}} + H_{\text{int}}, \quad (2)$$

where

$$(U)_{\text{BCS}} = 2 \sum_{\nu > 0} V^2(\nu) \epsilon(\nu) - \frac{\Delta^2}{G}, \quad (2a)$$

and

$$(H_{11})_{BCS} = \sum_{\nu} E(\nu) \alpha^+(\nu) \alpha(\nu), \quad (2b)$$

H_{int} being the residual interaction acting among quasiparticles. The quasiparticle vacuum is

$$|BCS\rangle = \prod_{\nu>0} (U(\nu) + V(\nu) \alpha^+(\nu) \alpha^+(\bar{\nu})) |0\rangle, \quad (3)$$

where $\alpha^+(\nu)$ creates a particle in the state ν and $\alpha^+(\bar{\nu})$ in its time reversal state. The occupation parameters fulfill the normalization condition $U^2(\nu) + V^2(\nu) = 1$.

The main effect of the pairing interaction in the quasiparticle approximation is to depress the ground state and to change the occupation number of the levels around the Fermi surface. There are no collective excitations in the present model.

There are two parameters which characterize pairing distortions, namely $U(\nu)$ and $V(\nu)$. Out of them one can create an even and an odd field with respect to the Fermi surface, that is $U^2(\nu) + V^2(\nu)$ and $U^2(\nu) - V^2(\nu)$. These fields induce two-quasiparticle excitations in the system generated by the operators $(\alpha^+(\nu) \alpha^+(\bar{\nu}) + \alpha(\bar{\nu}) \alpha(\nu))$ and $(\alpha^+(\nu) \alpha^+(\bar{\nu}) - \alpha(\bar{\nu}) \alpha(\nu))$ respectively. In the first case the action of the field leads to a breathing of the Fermi surface, and thus to a change of the number of particles. One of these modes has zero energy and is the Goldstone boson associated with the violation of the number of particles.

The excitations generated by the "antisymmetric" combination of two quasiparticles can be viewed as fluctuations of the pairing gap around its equilibrium position, and thus to a change of the Fermi surface diffusivity without altering λ .

The residual interaction associated with the even and odd fields are

$$H_{int} = H_p^{\sim} + H_p^{\sim'}, \quad (4)$$

$$H_p^{\sim} = -\frac{G}{4} \left(\sum_{\nu>0} (U^2(\nu) - V^2(\nu)) (\alpha^+(\nu) \alpha^+(\bar{\nu}) + \alpha(\bar{\nu}) \alpha(\nu)) \right)^2, \quad (5a)$$

and

$$H_p^{\sim'} = \frac{G}{4} \left(\sum_{\nu>0} (\alpha^+(\nu) \alpha^+(\bar{\nu}) - \alpha(\bar{\nu}) \alpha(\nu)) \right)^2. \quad (5b)$$

The wavefunction (3) does not conserve the number of particles. The proper inclusion of the zero-point motion (orientation in gauge space) of the $W_1^{\sim}=0$ solution of the Hamiltonian (2) + (5b) repairs this situation by projecting the

product solution onto the space of fixed number of particles. In fact, an eigenfunction of the $W_1^z=0$ rotation about the z-axis in gauge space is just

$$\int d\phi |BCS(\phi)\rangle \exp(iN\phi), \quad (6)$$

where $|BCS(\phi)\rangle = \mathcal{G}(\phi) |BCS(\phi=0)\rangle$ is the product wavefunction rotated by an angle ϕ induced by the operator

$$\mathcal{G}(\phi) = \exp\left(-\frac{i}{2} N\phi\right). \quad (7)$$

From (3) and (7), one can write

$$|N\rangle = \int d\phi e^{iN\phi} \prod_{\nu>0} (U(\nu)e^{-i\phi/2} + e^{i\phi/2} V(\nu)a^{\dagger}(\nu)a^{\dagger}(\bar{\nu})) |0\rangle, \quad (8)$$

which corresponds to the projected wavefunction. That is, the BCS wavefunction can be viewed as a wavepacket in the number of particles. The different states (8) obtained by projecting from this wavepacket are the members of a pairing rotational band which display enhanced two-nucleon transfer matrix elements (cf. fig. 1). In fact

$$\langle N' | \sum_{\nu>0} a^{\dagger}(\nu)a^{\dagger}(\bar{\nu}) | N \rangle \quad (9)$$

$$\begin{aligned} &= \int d\phi e^{-i(N+2-N')\phi} \langle BCS(N') | a^{\dagger}(\nu)a^{\dagger}(\bar{\nu}) | BCS(N) \rangle \\ &= \delta(N', N+2) \frac{\Delta}{G}, \end{aligned} \quad (10)$$

the associated two-nucleon transfer cross section being proportional to $(\Delta/G)^2$.

2.2 Pairing vibrations

Pairing rotations exhaust the collectivity associated with the gauge angle. They are generated by the field $U^2(\nu)+V^2(\nu)$ which is even with respect to the Fermi surface.

Pairing vibrations are associated with collective fluctuations of the pairing gap, generated by the field $U^2(\nu)-V^2(\nu)$ which is odd with respect to λ . The Hamiltonian describing this excitation is

$$H = (U)_{BCS} + (H_{11})_{BCS} + H_p'. \quad (11)$$

The associated linearized equations of motion lead to

$$A = \sum_{\nu>0} \frac{(U^2(\nu)-V^2(\nu))^2}{(2E(\nu))^2 - \frac{W^2}{n}} - \frac{1}{G} = 0. \quad (12)$$

It can be shown that the lowest root this dispersion relation can have lies at the top of the gap. That is, the pairing phonons have a finite mass which is controlled by Δ .

The X and Y amplitude defining the phonon are given by

FIGURE 1

Experimental energies of the $J^\pi=0^+$ states of the even Sn isotopes. The heavy drawn lines represent the values of the expression $E = -B(\text{Sn}) + E_{\text{exc}} + 8.50N + 45.3$ (MeV), where $B(A)$ denote the binding energies. The dashed line represents the parabola $0.10 (A-65.4)^2$, which corresponds to a rotational energy parameter $\hbar^2/2\mathcal{I} = 0.040$ MeV. Also displayed is the excited rotational band associated with the pairing vibrational mode. In all cases where more than one $J^\pi=0^+$ state has been excited below 3 MeV in two-neutron transfer processes, the energy $\sum_i \sigma(0_i^-) E(0_i^-) / \sum_i \sigma(0_i^+)$ of the centroid is quoted, as well as the corresponding cross-section $\sum_i \sigma(0_i^+)$. The quantity $\sigma(0_i^+)$ is the relative cross-sections with respect to the ground-state cross-sections. The numbers along the abscissa are the ground-state (p,t) and (t,p) cross-sections normalized to the $^{116}\text{Sn} \leftrightarrow ^{118}\text{Sn}(\text{gs})$ cross-section.

$$\left. \begin{array}{l} X_n^-(\nu) \\ Y_n^-(\nu) \end{array} \right\} = \frac{\Lambda_n^-(U^2(\nu) - V^2(\nu))}{2E(\nu) \mp W_n^-}, \quad (13)$$

while the zero point amplitude is equal to

$$\frac{\Lambda_n^-}{G} = \frac{1}{2G} \left(\sum_{\nu>0} \frac{(U(\nu)-V(\nu))^2}{[(2E(\nu))^2 - W_n^2]^2} 2E(\nu) W_n^- \right)^{-1/2}. \quad (14)$$

The field specific to excite pairing vibrations and thus test its collectivity is

$$F = \frac{1}{2} \sum_{\nu > 0} (U^2(\nu) - V^2(\nu)) (r^+(\nu) + r(\nu))$$

$$= \frac{1}{2} \sum_{\nu > 0} (a^+(\nu)a^+(\nu) + a(\nu)a(\nu)), \quad (15)$$

where terms conserving the number of quasiparticles have been neglected.

The enhancement in the matrix element $F'_n = \langle n | F | \bar{0}' \rangle$ (where $|\bar{0}'\rangle$ is the correlated ground state associated with the Hamiltonian (11) and where $|n'\rangle = r^+_n |\bar{0}'\rangle$ is a one-phonon state) with respect to the single-particle matrix element, measures the degree of collectivity which the state $|n'\rangle$ has acquired. This matrix element is equal to

$$F'_n = \Lambda'_n \sum_{\nu} (U(\nu) - V(\nu))^2 \frac{4E^2(\nu)}{4E^2(\nu) - W_n'^2}$$

$$= \left(\frac{\Lambda'_n}{G} \right), \quad (15a)$$

which is the zero-point fluctuation associated with the pairing vibration.

We now make a rough estimate of the matrix element (15a) for the lowest root $W_1 = 2\Delta$, obtaining

$$F'_1 = \frac{D}{\pi G}. \quad (16)$$

In the above estimate we have assumed that the nucleons move in a set of uniformly spaced levels, where D is the distance between them. The two-particle cross section is proportional to the square of this quantity. Normalized with respect to the ground state transition, one obtains, using (10),

$$R = \frac{\sigma(\text{gs} \rightarrow \text{pv})}{\sigma(\text{gs} \rightarrow \text{gs})} = \left(\frac{D}{2\Delta} \right)^2. \quad (16a)$$

For values of $D=0.3$ MeV and $\Delta=1.5$ MeV typical of deformed nuclei, this ratio is $R=10^{-2}$, in overall agreement with the data (cf. fig. 1).

2.5 Coupling of the spurious state to pairing vibrations

The overlap between the Goldstone boson and pairing vibrations is given by

$$\langle \bar{0}'''' | r_1^- r_n^+ | \bar{0}'''' \rangle = \langle \bar{0}'''' | [r_1^-, r_n^+] | \bar{0}'''' \rangle = 2\Lambda_n' E$$

where

$$E = \sum_{\nu > 0} \frac{U^2(\nu) - V^2(\nu)}{(2E(\nu))^2 - W_n'^2}, \quad (17)$$

and $r_n^- |\bar{0}''''\rangle = r_n^- |\bar{0}''''\rangle = 0$.

To make the spurious state orthogonal to the pairing vibrations, one has to simultaneously linearize H_p^+ and H_p^- . In this case, one obtains instead of a single dispersion relation, a set of coupled equations. The excitation energies are given by the roots of the dispersion relation

$$\begin{vmatrix} A & W_n E \\ W_n E & B \end{vmatrix} = 0 \quad (18)$$

$$= W_n \left\{ (W_n^2 - 4\Delta^2) \left(\sum_{\nu > 0} \frac{1}{2E(\nu) [(2E(\nu))^2 - W_n^2]} \right)^2 - \left(\sum_{\nu > 0} \frac{(U^2(\nu) - V^2(\nu))^2}{(2E(\nu))^2 - W_n^2} \right)^2 \right\} = 0.$$

There is always a root $W_1=0$. In the case of a symmetric distribution of levels around the Fermi surface, the last term vanishes and the next root comes at twice the pairing gap. Otherwise $W_2 > 2\Delta$.

When expressed in terms of the collective operators r_n^+ , the BCS Hamiltonian reads

$$(H)_{\text{RPA}} = (U)_{\text{BCS}} + (U)_{\text{RPA}} + (H_{11})_{\text{RPA}} \quad (19)$$

where

$$(U)_{\text{RPA}} = \frac{1}{2} \sum_n W_n - \frac{1}{2} \sum_{\nu > 0} 2E(\nu), \quad (20a)$$

and

$$(H_{11})_{\text{RPA}} = \sum_n W_n r_n^+ r_n. \quad (20b)$$

The term (20a) is the contribution to the ground state energy of the zero point motion associated with the collective modes (cf. Fig. 2(a)).

FIGURE 2

Lowest order contribution, to the zero point energy of the two-quasiparticle pairing vibrations in superfluid (a) and in normal nuclei (b).

3. PAIRING FLUCTUATIONS IN NORMAL SYSTEMS

At zero temperature and zero rotational frequency the parameter which controls whether the BCS equations (1a)-(1b) display a solution or not is the ratio $G\Omega/D$. The quantity G is the pairing coupling constant, Ω the effective degeneracy allowed to the particles and D the typical distance between the single-particle levels. While $G\Omega > D$ for open shell nuclei, $G\Omega \ll D$ at closed shells. In this case $\Delta = 0$, and the system becomes normal. Nonetheless, it can display important pairing fluctuations.

In this case one can distinguish between particles and holes. This is the reason why one finds two types of pair vibrations; namely pair addition and pair subtraction modes. The first is labeled by the transfer quantum number $\alpha=+2$ and essentially corresponds to an excitation which adds two particles to the system. The pair subtraction mode is a vibration which reduces by two the number of particles, and carries a transfer quantum number $\alpha = -2$.

The creation operators of the corresponding phonon excitations are

$$\begin{aligned} \Gamma_n^{+}(\alpha=+2) &= \sum_{\omega>0} \chi_n(+2;\omega) \Gamma^{+}(\omega) + \sum_{\gamma>0} \gamma_n(+2;\gamma) \Gamma(\gamma) \\ \Gamma_n^{+}(\alpha=-2) &= \sum_{\gamma>0} \chi_n(-2;\gamma) \Gamma^{+}(\gamma) + \sum_{\omega>0} \gamma_n(-2;\omega) \Gamma(\omega), \end{aligned} \quad (21)$$

where

$$\Gamma^{+}(\nu) = \begin{cases} a^{+}(\omega) a^{+}(\bar{\omega}) & \epsilon(\omega) > \epsilon_F \\ a(\gamma) a(\bar{\gamma}) & \epsilon(\gamma) < \epsilon_F. \end{cases} \quad (22)$$

The pairing Hamiltonian reads, in the new variables,

$$\begin{aligned} H &= \sum_{\gamma>0} e(\gamma) V^2(\gamma) \Gamma^{+}(\gamma) \Gamma(\gamma) + \sum_{\omega>0} U^2(\omega) e(\omega) \Gamma^{+}(\omega) \Gamma(\omega) \\ &- G \sum_{\gamma, \omega>0} (\Gamma(\gamma) + \Gamma^{+}(\omega)) \sum_{\gamma', \omega'>0} (\Gamma^{+}(\gamma') + \Gamma(\omega')). \end{aligned} \quad (23)$$

Linearizing these equations, one obtains

$$\begin{aligned} \frac{1}{G} &= \sum_{\omega>0} \frac{U^2(\omega)}{2e(\omega) - W_n(+2)} + \sum_{\gamma>0} \frac{V^2(\gamma)}{2e(\gamma) + W_n(+2)}, \\ \frac{1}{G} &= \sum_{\omega>0} \frac{U^2(\omega)}{2e(\omega) + W_n(-2)} + \sum_{\gamma>0} \frac{V^2(\gamma)}{2e(\gamma) - W_n(+2)}. \end{aligned} \quad (24)$$

When expressed in terms of the operators (21), the Hamiltonian (23) reads

$$H_{\text{RPA}} = (U)_{\text{RPA}} + (H_{11})_{\text{RPA}}, \quad (25)$$

where

$$(U)_{\text{RPA}} = \frac{1}{2} \sum_{n\alpha=\pm 2} W_n(\alpha) - \frac{1}{2} \left(\sum_{\omega>0} 2e(\omega) U^2(\omega) - \sum_{\gamma>0} 2e(\gamma) V^2(\gamma) \right), \quad (26)$$

and

$$(H_{11})_{\text{RPA}} = \sum_{n\alpha=\pm 2} W_n(\alpha) \Gamma_n^+(\alpha) \Gamma_n(\alpha). \quad (27)$$

The factor 1/2 appearing in (26) indicates that this ground state energy is associated with zero point fluctuations, arising because of the uncertainty principle which sets limits of localization of the nucleons in the nucleus. Particles moving independently in an average potential display zero point fluctuations associated with changes of the number of particles in two, which implies a breathing of the Fermi sphere without change in density. The associated zero-point energy given by the last term in eq. (26) was obtained expressing the single-particle Hamiltonian in terms of the normal modes associated with the pair addition and pair removal modes.

It is noted that the quantity (26) is second order in the particle-vibration coupling vertex (cf. fig. 2(b)). The difference appearing in (26) between the energy of the collective states and the unperturbed poles gives the desired answer which coincides with the nuclear field theory expression of graphs 2(b).

The specific operator to excite pairing modes corresponds to the transfer of two identical particles. It reads

$$\begin{aligned} T &= \sum_{\omega>0} \Gamma^+(\omega) + \sum_{\gamma>0} \Gamma(\gamma) \\ &= \sum_n \left(\sum_{\omega>0} \chi_n(+2;\omega) - \sum_{\gamma>0} \gamma_n(+2;\gamma) \right) \Gamma_n^+(+2) \\ &\quad + \sum_n \left(\sum_{\gamma>0} \chi_n(-2;\gamma) - \sum_{\omega>0} \gamma_n(-2;\omega) \right) \Gamma_n(-2). \end{aligned} \quad (28)$$

All the quantities $e(\gamma), e(\omega), E_n(\pm 2)$ appearing in the energy denominators are positive and are measured from the common minimum of the curves representing the righthand side of eq. (24). This minimum coincides with the value of the Fermi surface λ entering in the BCS equations 1(a) and 1(b) in the limit in which

$$W_1(+2) = W(-2) = 0. \quad (29)$$

This is the situation for which the gap and the number equation accept a solution with Δ exactly equal to 0. The condition (29) then gives a dynamical definition of the pairing phase transition, and corresponds to the situation in which the addition or the subtraction of a pair of particles from the system costs no energy. Particles and holes lose their identity, and the fluctuation associated with the pairing mode can grow indefinitely, polarizing the system, that is, aligning all the quasispins in a definite direction in gauge space such as to lead to a spontaneous breaking of the gauge symmetry.

The matrix elements corresponding to the transfer of two nucleons to the

ground state and to a two-phonon state starting from the state with one removal mode are given by (c.f., e.g., fig. 3)

$$\begin{aligned} \langle \bar{0} | T r_1^+(-2) | \bar{0} \rangle &= \Lambda_1(-2)/G, \\ \langle \bar{0} | r_1(-2)r_1(+2) T r_1^+(-2) | \bar{0} \rangle &= \Lambda_1(+2)/G. \end{aligned} \quad (30)$$

These quantities are also equal to the zero-point fluctuations of the two modes. In terms of the X and Y amplitudes, they can be written as

$$\begin{aligned} \Lambda_n(+2)/G &= \sum_{\omega>0} X_n(+2,\omega)U^2(\omega) - \sum_{\gamma>0} Y_n(+2,\gamma)V^2(\gamma) \\ \Lambda_n(-2)/G &= \sum_{\gamma>0} X_n(-2,\gamma)V^2(\gamma) - \sum_{\omega>0} Y_n(-2,\omega)U^2(\omega). \end{aligned} \quad (31)$$

Typical values for these quantities are 5-8, indicating the number of particles involved in the zero-point fluctuations of the pairing modes. Note that this quantity has the same order of magnitude as $\Delta/G \approx 10$, which is a measure of the

FIGURE 3

Pairing vibrational band based on the ^{208}Pb ground state. The long horizontal bars indicate the experimental energies of the $\text{Pb } 0^+$ states. Except for a term linear in the excess of neutrons relative to ^{208}Pb , added for convenience, the energies are the difference between the ^{208}Pb and the ^APb binding energies. The coefficient in the linear term was chosen to make $E(206) = E(210)$. The broken level lines indicate the energies predicted from the harmonic pairing vibrational model, using a phonon energy of 2.49 MeV, i.e. equal to $E(206)$. The pair of numbers above the levels counts the number of removal phonons (hole-phonons) N_r (left) and the number of addition phonons (particle-phonons) N_a (right). The quantity x, which symbolizes the basic $^{208}\text{Pb}(t,p)^{210}\text{Pb}$ (g.s.) cross section and other (t,p) intensities where an addition phonon is added, are given relative to the basic intensity. The quantity y has a similar meaning for pickup. The short level bars indicate predictions with a simple two-body anharmonic term included.

zero-point fluctuation associated with the rotational motion of pairing deformed systems. It is noted that the quantities (31) are also equal to the derivative of the dispersion relations (24) at the point at which they intercept the constant $1/G$ (cf. inset fig. 10).

The ratio between the cross sections associated with the transitions (30) is

$$R = \frac{\sigma((A-2)(gs) \rightarrow pv(A))}{\sigma((A-2)(gs) \rightarrow gs(A))} \sim \left(\frac{\Lambda_1(+2)}{\Lambda_1(-2)} \right)^2. \quad (32)$$

Assuming $w_1(+2)=w_1(-2)$ and that the particle and hole subspace have comparable degeneracies, an ansatz which is reasonable for the case of heavy nuclei, one obtains

$$R \approx 1. \quad (33)$$

Note that in the case of superfluid nuclei this ratio is $\approx 10^{-2}$ (cf. eq. (16a)). A typical example of a pairing vibrational band is given in fig. 3.

4. SIMPLE ESTIMATES OF THE ROLE OF PAIRING FLUCTUATIONS IN BOTH NORMAL AND SUPERFLUID SYSTEMS

In the case of superfluid nuclei, the energy of the ground state is proportional to the square of the number of particles, that is,

$$E_0 \sim GN^2. \quad (34)$$

typical of a pairing rotational band (cf. fig. 1).

The contribution to this quantity, due to the zero-point fluctuations of the pairing modes in the RPA, is given by (cf. fig. 2(a))

$$(U)_{\text{RPA}} \sim \frac{\Lambda^2}{\text{DEN}} = \frac{(G \sqrt{\Omega})^2 \Omega}{G\Omega} \approx G\Omega \approx GN. \quad (35)$$

The quantity Λ is proportional to the zero-point fluctuations associated with the pairing vibration (cf. eq. (14)) and gives the strength with which the pairing mode couples to the two-quasiparticle excitations. The quantity Ω is the effective degeneracy of the subspace where the particles are allowed to correlate.

One thus obtains

$$\frac{(U)_{\text{RPA}}}{E_0} \sim N^{-1} \text{ (superfluid)}. \quad (36)$$

In the case of normal systems

$$E_0 \sim GN, \quad (37)$$

typical of pairing vibrational bands (cf. fig. 3).

The quantity (35) is still a valid estimate of the processes described in

fig. 2(b). Thus

$$\frac{(U)_{\text{RPA}}}{E_0} = 1 \text{ (normal)}. \quad (38)$$

It is noted that in the case of infinite systems the corrections (36) should be negligibly small (cf. also ref. 5). However, for magnetic fields or temperatures larger than the corresponding critical quantities, there should also be, in this case, a finite effect (cf. eq. (38)). This seems, in fact, supported by experiment (cf. ref. 11 and references therein).

5. PAIRING PHASE TRANSITION IN STRONGLY ROTATING NUCLEI

From the estimates of the previous section, one expects the effects of the zero-point fluctuations associated with pairing vibrations to be more important at rotational frequencies larger than the critical frequency.

The success enjoyed by the BCS cranked shell model (CSM)¹² at $\omega < \omega_c$, and the conspicuous discrepancies found between the experimental findings and the normal CSM predictions at $\omega > \omega_c$ may reflect that fact.

In what follows we apply the concepts developed in the previous sections to the study of the pairing phase transition in strongly rotating nuclei. This is done first in the case of a schematic model and later for realistic situations, which are then compared with experiment.

The model Hamiltonian to be treated in all cases is

$$H^\omega = H_{\text{sp}}^\omega + H_p. \quad (39)$$

The first term describes the average rotating potential

$$H_{\text{sp}}^\omega = H_{\text{sp}}^N - \omega j_x, \quad (40)$$

the term H_{sp}^N being the Nilsson Hamiltonian, while j_x is the x-component of the single-particle angular momentum and ω the rotational frequency. The second term in (39) is the residual pairing force.

It can be shown that in the presence of even-multipole deformations

$$[H_{\text{sp}}^\omega, \tau] \neq 0, \quad (41)$$

and

$$[H_{\text{sp}}^\omega, \pi] = [H_{\text{sp}}^\omega, R_x] = 0 \quad (42)$$

where τ and π are the time reversal and parity operators. The signature operator

$$R_x = \exp(-i\pi j_x), \quad (43)$$

induces rotations through an angle π around the x-axis. Consequently the eigen-

states of H_{Sp}^{ω} can be labeled by the parity and the signature quantum numbers $r = \pm i$, eigenvalues of the operator (43). The state with $r = +i$ is said to have signature $+1/2$, while that with $r = -i$, signature $-1/2$.

5.1 The $i_{13/2}$ model

In this section we consider some of the consequences of the relation (41) for the matrix elements of particles interacting through a pairing force and moving in a cranked $i_{13/2}$ orbital, displaying axial symmetry.

The eigenstates of this Hamiltonian can be written as

$$|j\rangle = |\pi, 1/2\rangle = \sum_K G_K^j(\omega) |K\rangle, \quad (44)$$

and

$$|\hat{j}\rangle = |\pi, -1/2\rangle = \sum_K H_K^{\hat{j}}(\omega) |\bar{K}\rangle, \quad (45)$$

where the states $|K\rangle$ and $\tau|K\rangle = |\bar{K}\rangle$ are linear combinations of the two-fold degenerated Nilsson states.

The eigenstates (44) and (45) of the cranked shell model Hamiltonian are labeled by the parity quantum numbers π and by the signature $\pm 1/2$. Note that the states $|j\rangle$ and $|\hat{j}\rangle$ are related by the operation of time reversal only at $\omega = 0$. The violation of this symmetry is measured by the deviation from 1 of the pairing matrix element

$$\langle j\hat{j} | P^+ | 0 \rangle = \sum_{KK'} G_K^j H_{K'}^{\hat{j}} \langle KK' | P^+ | 0 \rangle, \quad (46)$$

where

$$P^+ = \sum_{\substack{v_\alpha > 0 \\ v_\beta > 0}} \langle v_\beta | \tau | v_\alpha \rangle a^+(v_\alpha) a^+(v_\beta), \quad (47)$$

and

$$\langle KK' | P^+ | 0 \rangle = \langle K' | \tau | K \rangle = \delta(K', \bar{K}), \quad (48)$$

the time reversal operator being denoted by τ . It is noted that the pair field (47) coincides with the transfer operator which creates two particles in time reversal states. From the relation (48), one obtains

$$M_{j\hat{j}} \equiv \langle j\hat{j} | P^+ | 0 \rangle = \sum_K G_K^j H_K^{\hat{j}}, \quad (49)$$

which is the pairing overlap between the orbital j and $\hat{j}$. It gives a direct measure of the amplitude of the state $\tau|j\rangle$ to be found in the state $|\hat{j}\rangle$.

The fact that the pairing field only connects states of different signature can be understood by the fact that at $\omega = 0$, $|j\rangle \rightarrow |K\rangle$ and $|\hat{j}\rangle \rightarrow |\bar{K}\rangle = \tau|K\rangle$, which are time-reversal states.

The square of some of the pairing overlaps (49) are displayed in fig. 4 for a variety of configurations. The most conspicuous features displayed by these quantities can be summarized as follows:

a) Matrix elements that start being one at $\omega = 0$ decrease with increasing values of ω , the opposite being true for those matrix elements which are zero at $\omega = 0$. Many of them become zero again at $\omega \rightarrow \infty$ displaying a maximum for finite values of ω .

b) Aside from the matrix elements $\langle 1, j | P_2^+ | 0 \rangle$, all other matrix elements oscillate.

The first property can be understood making use of the sum rule

$$\sum_{jj'} \langle jj' | P^+ | 0 \rangle^2 = \Omega, \quad (50)$$

where Ω is the number of pairs one can place in the shell, and from the fact that at $\omega = 0$ the two-fold degenerate Nilsson states are time reversal partners, while for $\omega j_x \gg H_{sp}^N$, where m_x is a good quantum number, the time reversal

FIGURE 4

Square of the pairing matrix elements as defined in eq. (49) connecting different eigenstates of the cranking Hamiltonian as a function of the rotational frequency.

states are those associated with the values $\pm m_x$ of the magnetic quantum number. The effect of the violation of time reversal invariance measured by the overlaps (49) is illustrated also in fig. 5.

The oscillations displayed by M_{jj}^2 are associated with states exchanging character by crossings. The points at which this occurs seem to correspond to the loci where the separatrix is about halfway between signature partners (cf. ref. 6).

Making use of the language of the linear response function, we can view the operator P^+ as an external field which adds to the system under study a pair of particles in time reversal states. Probes of this type are found in nature, and correspond to two-nucleon transfer processes induced by both light and heavy ions (cf. refs. 13-14 and refs. therein). In particular in a (t,p) reaction the two transferred nucleons which essentially move in a (0s)-state in the triton can be viewed as a Cooper pair. It is noted that this possibility of transferring one Cooper pair at a time is quite unique of nuclear physics. In general, the experimental probe is coupled to some physical quantity A (density, current, spin, pair-density, etc.); one wishes to measure the linear response of

FIGURE 5

Contour plots for different values of ω of the two dimensional 7×7 pairing matrices M_{jj}^2 connecting all the negative signature j^- -states with those of positive signature j . The open circles represent the square of matrix elements larger than 0.2. Consequently, the dashed zones indicate the region where pairing correlations are about one-half of the value displayed by the diagonal configurations at $\omega = 0$.

some other quantity B to the probe. The response is conveniently described in terms of the "B-A response function"; the first letter denotes the measured quantity; the second determines the form of the probe-system coupling. In the present case, eq. (24) denotes the (pair density)-(pair density) linear response function.

In fig. 6 we display the time reversal response function for six particles moving in the lowest single-particle levels for $\omega > \omega_{cr}$. The pairing vibrational pattern which emerges from these results is quite different from that observed in nuclei like ^{208}Pb (cf. fig. 3), where a large single-particle gap exists between empty and occupied states. In fact, in ^{208}Pb , the lowest pair addition and pair removal modes essentially concentrate all the collectivity. These modes can thus be used as building blocks of the nuclear spectrum, their identity not being associated with a particular isotope.

In the present situation, although one has a system displaying important pairing fluctuations, this collectivity is distributed over many states, and there is not a single mode which is really collective. The detailed microscopy which is at the basis of these results is discussed below.

In any case, the physics is rather simple. In fact, the pattern shown in fig. 6 resembles that of collective modes in the presence of strong Landau damping. Due to the absence of a gap in the spectrum, the fluctuations of the pairing gap are trapped between closely spread poles (unperturbed response function) and almost on-the-energy shell transition can occur to uncorrelated two-particle or two-hole states. A situation similar to the one encountered here, although it is in the particle-hole channel, is found in the case of paramagnons (spin-vibrations) in liquid ^3He . The paramagnon, like the zero-sound, is a particle-hole pair, but with the particle and the hole coupled to spin one, rather than zero. However, because the interaction is attractive, these vibrations have to propagate within the continuum of uncorrelated particle-hole excitations and consequently suffer a very strong Landau damping (cf. fig. 7). In fact, the paramagnon is not a real excitation with a real velocity, but is a virtual excitation with a kind of diffusive motion. Although not directly observable,* it has an important consequence on the specific heat (effective mass).

One would similarly expect that even if the different states in which the pairing fluctuations split cannot be directly observed, one could still detect

*This mode has been shown¹⁵ to explain the deviations of the ^3He -response observed in neutron scattering from the unperturbed fermion response at low values of k . The paramagnon have been directly observed¹⁶ in the presence of a magnetic field, which provides a finite mass to the phonon.

FIGURE 6

Time reversal response associated with the pair addition ($\alpha=+2$) and the pair removal ($\alpha=-2$) modes for the frequency $\omega = 0.4 \text{ MeV} \gg \omega_{cr}$ and for a number of particles $N = 6$. The quantities $\langle j|\tau|\hat{j}\rangle^2$ are proportional to the two-nucleon transfer cross section in a reaction proceeding from the yrast state of the system with $N-2(=4)$ particles. Also displayed is the value of the energy-weighted sum rule (EWSR) associated with the different pair addition ($\alpha=+2$) and pair subtraction ($\alpha=-2$) modes. What is denoted as "unperturbed" corresponds to the values $(\epsilon(j) + \epsilon(j))|\langle j|\tau|\hat{j}\rangle|^2$, while "collective" denotes the quantity $W_\alpha(n)|\langle n(\alpha)|\tau|0\rangle|^2$. It is noted that $\sum_{j,j} (\epsilon(j) + \epsilon(j))|\langle j|\tau|\hat{j}\rangle|^2 = \sum_{n,\alpha} W_\alpha(n)|\langle n(\alpha)|\tau|0\rangle|^2$, that is, the RPA conserves the time-reversal energy-weighted sum rule.

FIGURE 7

Schematic representation of the dispersion relation associated with the paramagnon (left) and with the plasmon (right). The hatched area corresponds to the unperturbed particle-hole excitations. The spin-spin interaction in ^3He giving rise to the spin vibration is attractive and the corresponding phonon encounters from the outset a continuous background. This is responsible for the existence of strong Landau damping. Conversely, in the plasma oscillations the residual interaction is repulsive. This, combined with the fact that the screening of the Coulomb field produced by the ions gives a finite mass (gap) to the plasmon, has the consequence that this mode moves away from the continuum of particle-hole states, and the only damping it has in the corresponding region of k -values is due to the coupling to $2p$ - $2h$ states. This coupling is very small because of the strong correlation existing between the particle and the hole in the plasmon.

their presence through renormalization effects of, e.g., ω_{cr} , eventually having a different influence on signature partners.

In fig. 8 the ground state correlations for a system of 5 and 6 particles are displayed. Pairing vibrations distort dynamically the nucleus in gauge space in much the same way as surface vibrations lead to virtual distortions of the nuclear surface. The polarization becomes so large that for $\omega < \omega_{cr}$ the effective pairing gap becomes static.

Pairing vibrations lead also to a dealignment of the system as a whole, as ω approaches ω_{cr} which can hardly be explained in terms of an effective pairing gap (cf. fig. 13). Its magnitude is given by

$$\frac{d(U)_{RPA}}{d\omega} = -i_{gsc} \quad (51)$$

and is also displayed in fig. 8. The total spin of the system is given by $j_x^N + i_{gsc}$.

It is noted that a similar effect of dealignment (expulsion of the magnetic

FIGURE 8

Ground-state correlations ($E_{gsc} = \langle U \rangle_{RPA}$) and dealignment (cf. eqs. (26) and (51)) associated with the pairing modes for a system of 5 and 6 particles moving in the lowest states of the $i_{13/2}$ orbital as a function of ω and for a prolate quadrupole deformation $\beta = 0.3$. The pairing coupling constant used was $G = 0.7$ MeV.

field) is found in condensed matter in the normal to superfluid phase transition observed changing the temperature from values larger than the critical temperature.¹¹ In fact, for weak magnetic fields, it has been observed¹⁸ that the onset of perfect diamagnetism as T tends to T_c is a smooth function of T . Theories which do not take into account pairing fluctuations predict a discontinuous drop of the magnetic field at the transition point.

It is noted that the description of pair vibrations discussed above is equivalent to the description of pairing fluctuations provided by the Ginsburg-Landau phenomenological theory of superconductivity when the term proportional to Δ^4 is neglected.¹¹

In connection with the discussion above concerning the existence of an effective pairing gap, let us mention the studies of the pairing collapse done

making use of particle number projection techniques.⁹ In this scheme the quantity $(\Delta_{\text{eff}}/G)^2 = \langle 0 | PP^\dagger | 0 \rangle$ was calculated, as now $\langle 0 | P^\dagger | 0 \rangle = 0$. It was found that $(\Delta_{\text{eff}}/G)^2$ behaves continuously with ω , and approaches zero asymptotically only for $\omega \gg \omega_{\text{cr}}$. This result was interpreted as showing that the pairing gap is much more stable than that predicted by BCS calculations. We will see that the quantity $(\Delta_{\text{eff}}/G)^2$ does not measure the pairing gap but the pairing correlations. In fact

$$\left(\frac{\Delta_{\text{eff}}}{G}\right)^2 = \langle 0 | PP^\dagger | 0 \rangle = \begin{cases} |\langle \text{gs} | P^\dagger | \text{gs} \rangle|^2 = \left(\frac{\Delta}{G}\right)^2 = \left(\frac{1.65}{0.7}\right)^2 \approx 5.6 & (\omega < \omega_{\text{cr}}) \\ \sum_n |\langle n | P^\dagger | \text{gs} \rangle|^2 \approx \sum_n \left(\frac{\Lambda_n(+2)}{G}\right)^2 \approx 3.4 & (\omega > \omega_{\text{cr}}), \end{cases} \quad (52a)$$

(52b)

where the numbers correspond to the results of the $i_{13/2}$ shell calculated with $B = 0.3$ and $G = 0.7$ MeV, which leads to a pairing gap $\Delta = 1.65$ MeV at $\omega = 0$. The numerical result (52) simply indicates that the zero-point fluctuations (cf. eq. (31)) associated with pairing vibrations for frequencies larger but not much different than ω_{crit} are very big.

It is noted, nonetheless, that something quite dramatic takes place around $\omega \approx \omega_{\text{cr}}$ as far as the coupling of the particles is concerned. In fact, studying the system through two-particle transfer reactions which is the specific tool to measure pairing correlations, we obtain a situation like the one depicted in fig. 9. For $\omega < \omega_{\text{crit}}$ only the transitions between yrast states would carry

FIGURE 9

Schematic representation of the two-nucleon transfer processes exciting the one- and two-phonon pairing modes in the N system starting from the corresponding yrast state of the $N-2$ system. The yrast $\rightarrow$ yrast transition corresponds to the distinction of a pair removal mode, while the yrast $\rightarrow$ (two-phonon) state corresponds to the creation of a pair addition mode (cf. eqs. (16a) ($\omega < \omega_{\text{cr}}$) and (33) ($\omega > \omega_{\text{cr}}$)).

strength, while a strong fraction of the strength is expected to go into pairing vibrational states for $\omega > \omega_{\text{crit}}$. This pattern is, of course, typical of a transition between pairing rotational and pairing vibrational schemes.¹⁷

In concluding this section, it is fair to say that although one has the specific probe to test pairing collapse in nuclei, namely two-nucleon transfer processes, kinematical limitations make it difficult to observe the transfer of two nucleons between yrast and pairing vibrational states at $\omega > \omega_{\text{cr}}$ (cf., e.g., ref. 14).

A detailed picture of the pairing phase transition in nuclei is likely to emerge from the study of a variety of more accessible even if less specific quantities. Among these, the renormalization effects induced by pairing fluctuations seem to be a good candidate.

One can conclude that the coupling of nucleons typical of the BCS state, as well as of one- and two-quasiparticle states is present also at $\omega > \omega_{\text{cr}}$. This does not mean that $\Delta \neq 0$, but that the fluctuations, i.e. the dynamical pairing correlations, are large even at very high values of ω and are profited to different degrees by the variety of configurations of the CSM.

6. APPLICATIONS

In this section we apply the ideas developed above to the calculation of a variety of properties of the $^{167,168,169}\text{Yb}$ -isotopes (cf. refs. 6 and 7).

In fig. 10 we display the quantity (cf. eq. (24))

FIGURE 10

Critical frequencies associated with the pairing phase transition corresponding to the condition $W(+2) = W(-2) = 0$ at $\omega = \omega_{\text{c}}$. The transfer quantum number is denoted by $\beta = \pm 2$.

$$\delta = \left(\sum_{\omega > 0} \frac{1}{2e(\omega) - W} + \sum_{\gamma > 0} \frac{1}{2e(\gamma) + W} - \frac{1}{G} \right)$$

$$= \left(\sum_{\gamma > 0} \frac{1}{2e(\gamma) - W} + \sum_{\omega > 0} \frac{1}{2e(\omega) + W} - \frac{1}{G} \right)_{\lambda}, \quad (53)$$

which measures the distance of the minimum of the pairing dispersion relation to the value $1/G$, as a function of the rotational frequency ω . The condition $\delta = 0$ implies $W_1(+2) = W_1(-2) = 0$ and determines the critical frequency for which a phase transition between normal to superfluid phases takes place (cf. eq. (29) and fig. 10).

In fig. 11(a) we show the quantity (26) for a variety of configurations corresponding to neutron number $N = 97, 98$, and 99 , i.e. situations typical of the nuclei $^{167,168,169}\text{Yb}$, as a function of the rotational frequency. Only the neutron degrees of freedom were included in the calculations of $(U)_{\text{RPA}}$. This is because at the frequencies under consideration it is expected that the proton pairing gap has essentially the same value as for $\omega = 0$.

FIGURE 11
Ground-state correlations associated with different configurations in nuclei around neutron number $N = 96$. The quantity (26) is plotted for the neutron degrees of freedom in (a) as a function of the rotational frequency and for the configurations indicated in (b).

The calculations were carried out⁷ making use of a cranked Nilsson Hamiltonian in whose orbitals neutrons are allowed to scatter under the influence of a monopole pairing force (cf. eq. (39)). A fixed deformation of $\epsilon = 0.26$, $\epsilon_4 = \gamma = 0$ was used to generate the single-particle levels, which comprise the $i_{13/2}$ orbital and the levels associated with the $N=5$ harmonic oscillator shell. The particles were distributed in these orbits according to the different configurations (cf. fig. 11(b)), that is, setting the occupation numbers U^2 and V^2 in eqs. (23)-(25) equal to 1 or 0 according to the occupancy of the states. It is noted that for the configuration $(-,1)$ there are two terms less in the summation over n in eq. (26) as compared to $(+,0)$, as an orbital above and another below the Fermi energy become blocked. In the calculations the pairing coupling constant was set equal to $G = 0.185$ MeV, which leads to a pairing gap $\Delta = 1.1$ MeV at $\omega = 0$ for the ground state configuration.

The curves displayed in fig. 11 were discontinued at the phase transition frequency (cf. fig. 10). It is noted that one can use the RPA throughout the phase transition by solving the RPA also for the superfluid case and interpolating between the two results (cf. ref. 10).

An outstanding feature of the results shown in fig. 11, is the easiness with which the even parity configurations react to pairing correlations. In fact, there is a distinctive clustering of the curves according to parity. The configuration $(+,0)$ is the analogue of the BCS vacuum at $\omega > \omega_{cr}$, in the sense that the lowest single-particle levels of both even- and odd-parity are filled with an even number of particles (cf. also refs. 8 and 12). This is the reason why pairing is optimally exploited in this configuration at any ω . A consequence of this fact is that the $(+,0)$ -configuration, which can be viewed as the pairing ground state, displays the largest value of ω_{cr} .

The configurations $(-,0)$ or $(-,1)$ are associated with situations where one has an odd number of particles in both the even- and the odd-parity levels. They thus correspond to two-quasiparticle configurations at $\omega < \omega_{cr}$, relative to the $(+,0)$ configuration. This implies that the $(-,0)$ and $(-,1)$ configurations are affected by a high degree of blocking. Consequently, pairing vibrations typical of a normal system can develop at a lower frequency than in the case of the $(+,0)$ configuration. That is, one needs only moderate values of ω to achieve the situation in which the fluctuations of the pairing gap are as large as its average value. The ground state correlations associated with the odd- N system lie in between the two limiting cases discussed above. This is because these configurations evolve into 1-quasiparticle states at $\omega < \omega_{cr}$.

Making use of the above results the Routhians

$$\langle H^N - \omega j_x \rangle + (U(\pi, \alpha))_{RPA} + \frac{47.3}{2} \omega^2 \text{ (MeV)}, \quad (54)$$

associated with the configurations of the $^{167,168,169}\text{Yb}$ isotopes shown in fig. 11(b) have been calculated. They are displayed in Fig. 12 in comparison with the experimental data referred to a configuration with a constant moment of inertia of 66 MeV^{-1} i.e.

$$\langle H^N - \omega j_x \rangle + \frac{66}{2} \omega^2 \text{ (MeV)}. \quad (55)$$

The important role played by the dynamic pairing correlations in the normal phase of the rotating nucleus is illustrated in fig. 13 for the case of the three configurations $(+,0)$, $(-,0)$, and $(-,1)$ of ^{168}Yb (cf. also fig. 14). Here the RPA and the cranked shell model (HFBC) results (54) and (55) are compared to the experimental data. The effect of dynamical pairing correlations is to make more positive the slope of the positive signature bands as indicated by the

FIGURE 12

Comparison of experimental (points) and calculated (curves) routhians. The data are taken from ref. 20 and are referred to a configuration with a constant moment of inertia equal to 66 MeV^{-1} . The theoretical curves were calculated in the RPA as indicated in the text. They are referred to the calculated moment of inertia for the yrast configuration of ^{168}Yb of 47.3 MeV^{-1} . The difference between the references of the experimental and calculated routhians accounts for the protons which were not included in the calculations.

FIGURE 13

Comparison of experimental routhians for the lowest $(+,0)$, $(-,1)$, and $(-,0)$ configurations in ^{168}Yb with three different theoretical calculations: (top) RPA calculations based on single-neutron excitations in a rotating deformed potential; (middle) HFBC calculations without pair correlations; and (bottom) cranking model calculations with $\Delta = .30$ MeV, or about 30% of the odd-even mass difference. All the calculations and the data have been referred to a constant moment of inertia reference that makes the $(-,1)$ configuration independent of rotational frequency at the highest frequencies.

data^{19,20} (cf. also fig. 14). The overshooting observed in connection with the $(+,0)$ configuration can easily be corrected by making use of the interpolation prescription discussed in ref. 10.

As mentioned in connection with the discussion carried out in connection with eq. (51), the dynamical renormalization effects can hardly be taken into account making use of the standard cranked shell model with a static pairing gap $\Delta = \Delta_{\text{eff}}$. This is shown at the bottom of fig. 13.

7. CONCLUSIONS

To obtain a proper description of the pairing phase transition, fluctuations of the pairing field have to be included for all values of ω , but in particular for $\omega > \omega_{\text{cr}}$.

FIGURE 14

Calculated routhians for the lowest $(+,0)$ and $(-,1)$ configurations in ^{168}Yb . The solid (and dashed) curves indicate calculations with (and without) the effects of pair fluctuations. The correlation energy associated with pair fluctuations is indicated as is the critical frequency (stars), above which a static pairing gap does not exist. The lowest $(+,0)$ state is the most strongly correlated configuration; therefore, pair correlations survive to larger rotational frequencies in this configuration. Thus, at large frequencies, the routhian for the $(+,0)$ configuration is predicted to have a larger slope (i.e. smaller alignment) than that of other frequencies. This characteristic feature is observed experimentally for ^{168}Yb and for several other even-even nuclei for which the data extend to sufficiently large rotational frequencies (cf. fig. 12).

At frequencies smaller than the critical frequency, the inclusion of the fluctuation associated with different orientations in gauge space restore, to an adequate degree, the gauge invariance violated by the BCS theory, leading to pairing rotational bands.

A convenient mathematical framework to include the effect of fluctuations is provided by the random phase treatment of the pairing Hamiltonian. This description is analogous to the phenomenological Ginsburg-Landau theory of superconductivity when the term proportional to the quartic power in the order parameter is neglected.

At rotational frequencies $\omega > \omega_c$ the inclusion of fluctuations of the pairing field realign the system in a continuous way such that for ω approaching ω_c from above the value of $\mathcal{J}^{(1)}$ becomes essentially equal to that of $\mathcal{J}^{(2)}$. This is similar to what is found in the case of the normal to superconducting phase transition of metals in the presence of a weak magnetic field. As the temperature decreases towards the critical temperature, pairing fluctuations expel the magnetic field from the interior of the metal in a continuous way (Meissner effect). Theories which do not take fluctuations into account predict that the field in the interior of a large probe would drop discontinuously to zero at the transition point.

The fingerprints of the coupling scheme appropriated to describe the superfluid phase are clearly observed at $\omega > \omega_{cr}$. In fact, positive parity configurations with the lowest values of the signature quantum number are clearly privileged by the dynamical pairing correlations. The configuration $(+,0)$ can be viewed as the dynamical analogue of the BCS pairing vacuum, in the sense that the lowest single-particle levels of both even- and odd-parity are filled with an even number of particles. Configurations of the type $(-,0)$ or $(-,1)$ are less affected by pairing fluctuations. They are associated with situations where one has an odd number of particles in both the even- and the odd-parity levels. They evolve into two-quasiparticle configurations at $\omega < \omega_{cr}$, relative to the $(+,0)$ configuration.

ACKNOWLEDGEMENTS

Many of the ideas presented in this paper are the results of discussions with S. Frauendorf. We want to acknowledge our profound indebtedness for his generous suggestions and comments. We also want to acknowledge discussions with D. R. Bes, M. Diebel, J. Garrett, W. Nazarewicz and Z. Zymanski. We thank J. Garrett for allowing us to use unpublished data and for suggestions in the development of some of the subjects discussed above. One of us (M.G.) wants to acknowledge CAYCIT, Spain.

REFERENCES

- 1) J. Hogaasen, Nucl. Phys. 28 (1961) 258.
- 2) D. Bes and R.A. Broglia, Nucl. Phys. A80 (1966) 289.
- 3) A. Bohr and B.R. Mottelson, Nuclear Structure (Benjamin, Reading, Mass, 1975).
- 4) J. Bardeen, L. Cooper and J.R. Schrieffer, Phys. Rev. 108 (1957) 1175.
- 5) P.W. Anderson, Phys. Rev. 112 (1958) 1900.
- 6) R.A. Broglia, M. Diebel, S. Frauendorf and F. Barranco, Proceedings of the XXIII International Winter Meeting on Nuclear Physics, Bormio, Italy (1985).
- 7) R.A. Broglia, M. Diebel, S. Frauendorf and M. Gallardo, to be published.
- 8) S. Frauendorf, Slide Report of the Workshop on Nuclear Structure with Heavy Ions, The Niels Bohr Institute Tandem Accelerator Laboratory, Riso (1984).
- 9) J.L. Egido, H.J. Mang and P. Ring, Nucl. Phys. A334 (1980) 1; Nucl. Phys. A341 (1980) 229.
- 10) W. Nazarewicz, R.A. Broglia, D.R. Bes, Z. Zymanski and J. Dudek, to be published.

- 11) H. Schmidt, Procs. of the International School of Physics "E. Fermi", ed. H. Morinaga (Academic Press, New York, 1972) p. 144.
- 12) S. Frauendorf and R. Bengtsson, Nucl. Phys. A314 (1979) 27; A327 (1979) 139.
- 13) R.A. Broglia, O. Hansen and C. Riedel, Adv. in Nucl. Physics, Vol. 6 (Plenum Press, New York, 1973) p. 287.
- 14) W. von Oertzen, Nucl. Phys. A409 (1983) 91c.
- 15) C.H. Aldrich, C. Pethick and D. Pines, Phys. Rev. Lett. 37 (1976) 845.
- 16) N. Matsuhora et al., Phys. Rev. Lett. 53 (1984) 1168.
- 17) R.A. Broglia, C. Riedel and B. Sorensen, Nucl. Phys. A107 (1968) 1.
- 18) J.P. Golub et al., Phys. Rev. Lett. 22 (1969) 1288.
- 19) J. Y. Zhang, J.C. Bacelar, J.D. Garrett and S. Frauendorf, to be published.
- 20) J.C. Bacelar, M. Diebel, C. Ellegaard, J.D. Garrett, G.B. Hagemann, B. Herskind, A. Holm, C.X. Yang, P.O. Tjom and J.C. Lisle, Proc. of Fifth Nordic Meeting on Nucl. Phys., Juväskylä, March 1984, p. 201; and NBI-Oslo-Manchester, preprint 1985.