EFFECTS OF SUPRATHERMAL FUSION PARTICLES IN TOKAMAK PLASMAS

BY

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THESIS

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EFFECTS OF SUPRATHERMAL FUSION PARTICLES IN TOKAMAK PLASMAS

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Several crucial properties of suprathermal (>500 keV) fusion-products are explored, both in their initial phase and during their slowing-down period. A guiding center drift theory, which predicts the effect of energy loss on the motion of these suprathermals, is derived for a low- β , symmetric (non-ripple) tokamak. Velocity-space scattering is ignored. Among the important implications of this theory are: (1) the net inward drift of fusion particles during their slow-down phase, and (2) the importance of the plasma density and temperature in determining this drift.

The effect the inward drifting has on the spatial profile for the suprathermals approaching thermal energies, on the energy distribution, and on the plasma heating profile is demonstrated for five reactor cases, ranging from near-term low-current devices to conceptual power reactors. It is found that "in situ" energy deposition is a reasonable assumption for the higher current machines, like UWMAC I, differing in the plasma heating evaluation by only a few percent from our more exact treatment (SYMALF) over most of the plasma. However, for lower current tokamaks, the discrepancies between "in situ" and SYMALF can become quite sizeable ($\sim 25\%$). It is also shown that, due to the slowing-down drifts, the fusion-product energy distribution near the plasma edge can have a positive slope with increasing energy, suggesting a possible driving mechanism for microinstabilities.

In the case of magnetic ripple, a probabilistic density function is employed to determine drift losses associated with ripple trapped, 3.52-MeV alpha particles. When used to determine 3.52-MeV alpha particle wall-loadings, code RIPALF, which is based on this probability function, predicts the position of local "hot spots" along the first wall.

Either of the codes, SYMALF or RIPALF, can be used to calculate the net spatial charge build-up arising from rapid fusion particle losses. A fluid model which incorporates this information is developed to predict toroidal plasma rotation induced by these losses. The rotation resulting from "first bounce" alpha particles losses for a corresponding D-T system may yield measurable results under certain conditions. Toroidal rotation generated in a TFTR-like tokamak is most likely to be undetectable for deuterium-injection into a ^3He thermal background.

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CHAPTER I: INTRODUCTION AND BACKGROUND

I.1 Definition and Review of the Problem

The purpose of this thesis is four-fold: (1) to derive a theory which accurately describes the motion of highly energetic (≥ 1 MeV) fusion particles (FP's) during their slowing-down phase in a low- β , axisymmetric toroidal field (β is the ratio of plasma pressure to toroidal magnetic field pressure); (2) to study the implications of this slowing-down theory on plasma heating, thermalization of fusion-products, and their distribution functions; (3) to derive a method for assessing the effects of energetic FP's trapped in a toroidal magnetic field with ripple; and (4) to study the implications of first bounce fusion product losses on toroidal plasma rotations.

1.1a Motivation and Assumptions for the Slowing-Down Model

As one can see from the diverse list of topics in the previous paragraph, highly-energetic FP's can make their presence felt in a variety of ways. However, one unifying aspect is that all of these phenomena are associated with or derived from the large radial drifts that the FP's undergo while thermalizing. Although thermal ions have only a relatively small displacement compared with the minor radius of the torus, the fusion-products themselves can have displacements comparable to or even greater than the minor radius especially in near term tokamaks.

The trajectories for a low- β , axisymmetric* tokamak are a superposition of the gradient-centrifugal drifts and the motion of the FP along the magnetic-field lines. By applying general "guiding-center" theory⁽¹⁻³⁾, it was shown⁽⁴⁻¹¹⁾ that the orbits for mono-energetic particles can be expressed in closed form in terms of magnetic moment (μ), canonical angular momentum (P_ϕ), and total particle energy (E), all of which are considered constants of motion.

*By "axisymmetric" one means that the applied toroidal magnetic field does not vary along the toroidal direction.

Even though FP's with energies exceeding 1 MeV generally interact very little with the background plasma over one bounce period, the cumulative effect of such interaction is that the FP's eventually transfer their energies to the plasma. During this process, neither μ nor P_ϕ remain constant so that the conventional guiding center approach is inadequate, and the orbits are not longer expressible in closed form.

A new method (SYMALF) of treating fusion products during the slowing-down period has been developed by the author.⁽¹²⁻¹⁴⁾ The analysis, which is presented in Chapter II, shows that the evolution of the FP-orbit is essentially determined by (a) the thermal plasma density and temperature profiles and (b) the magnetic field configuration.

In addition to the assumptions of a low- β , circular cross-section plasma with no magnetic ripple, it is assumed that the pitch-angle scattering is negligible. In fact, to some degree the highly-energetic FP's do have collisions with thermal background ions and thus velocity-space scattering occurs (the electron contribution to this scattering is much smaller than that of the ion). The greater the pitch-angle dispersion, the more unreliable the method becomes.

Fortunately, the assumption that the velocity-space scattering is small holds up well until the ratio of the FP's energy to the background electron temperature is approximately 50 (See Appendix D). For alpha particles with energies initially at 3.52 MeV slowing down in a 10 KeV D-T plasma, this means that SYMALF will yield reliable results for energies >500 KeV. By then, however, many of the important fusion-product effects have already occurred. For example, such D-T produced alpha particles will have lost more than 85% of their energy before arriving at 500 KeV. In addition, the "first bounce" losses which occur during the first 10 μ s after the alpha's "birth" are not affected at all.

1.1b. Consequences of Slowing-Down Theory

One of the original applications of FP-orbital theory was to evaluate 3.52 MeV alpha particle heating profiles in tokamak. Stringer⁽⁷⁾ first

estimated the heating profile by approximating the FP-orbits by circles, since the implicit nature of his orbit equation did not allow a more exact treatment. McAlees⁽⁸⁾ has also studied the energy deposition profiles using a more explicit formulation of the orbit equation. Both Stringer and McAlees assumed that the FP's energy is deposited (i.e., lost to the background plasma) along its initial orbit.

Fusion particle drifts, however, were not accounted for in either of the above models. This effect was first considered by the author who showed⁽¹²⁻¹⁵⁾ that due to the net inward drifting of the fusion products, the heating is generally greater at the plasma center than predicted by either Stringer or McAlees, although for higher current tokamaks (i.e., UWMAK) this discrepancy becomes insignificant. Differences are small in high current cases because the energetic particles are better confined to the magnetic flux surface of their birth when the ratio of poloidal magnetic field to toroidal field is large.

Just as slowing-down drift motion affects plasma heating, it likewise has an effect in determining where in the plasma the FP's will thermalize. (This is especially true for the lower current plasmas, like TFTR.) The thermalization profile can be important; for example, alpha particle "ash" build-up is quite sensitive to the current density profile.⁽¹⁶⁾

The actual shape of the fusion-product energy distribution during thermalization is of interest for predicting the possible occurrence of microinstabilities. If the slope of the particle distribution is positive with respect to energy, for instance, general plasma theory⁽¹⁷⁾ suggests that free energy is available which can be used as a driving mechanism for microinstabilities, like electro-magnetic or Alfvén waves.⁽¹⁸⁾

1.1c Magnetic Ripple Model

The forgoing discussion presupposes that the magnetic field possesses toroidal symmetry. However, the discrete character of the toroidal field coils partly destroys this symmetry, with the result that "ripples" form in the magnetic field. Unlike the axisymmetric case, the

individual guiding center trajectories in the ripple field are not well-understood, although the overall net drift is in the $\vec{B} \times \nabla B$ direction.

Nevertheless, interesting consequences and limiting conditions involving ripple effects on plasma transport have been examined by various authors.⁽¹⁹⁻²¹⁾ Methods for reducing the seriousness of ripple loss have been proposed⁽¹⁹⁾ or, alternatively, schemes for utilizing ripple fields for injection⁽²²⁾ or temperature control⁽²³⁾ have been suggested.

Yang and Emert⁽²⁴⁾ have derived an approximate analytic expression for 3.5 MeV alpha particle loss fractions, although a computer is required when more realistic alpha particle loss functions are used. More recently, Uckan et al.⁽²⁵⁾ have applied Fokker-Planck theory to appraise additional alpha particle losses during the thermalization process. Although their results imply that losses via scattering into the ripple loss-cone may be several times the immediate 3.5 MeV loss, much of the scattering losses occurs after the alpha particle is already degraded in energy. Consequently, these losses are not as important to the plasma heating and wall bombardment distributions as are the 3.5 MeV losses and initial trajectories of the high-energy alphas.

In Chapter IV, a model for dealing with several aspects of the 3.5 MeV ripple problem is discussed. Instead of following the individual trajectories of each phase-space group, the author shows that the problem is much more tractable using the kinematic probability approach. This makes it possible, using the code RIPALF, to calculate the 3.5 MeV alpha particle wall loading as a function of both poloidal and toroidal positions. Loading information is very important to reactor design since alpha particle bombardment can produce severe wall erosion via blistering.⁽²⁶⁾ The overall loss fraction, which has been the main concern of previous investigations cited earlier, is also easily obtained.

I.1d Plasma Rotation

When a radial electric field initially acts on a tokamak plasma, toroidal and poloidal flows result. Such rotations of the plasma can greatly affect stability and transport⁽²⁷⁻³⁷⁾ both during start-up and

steady-state operation. When rotational speeds build high enough ($\sim$ ion thermal speeds), Kelvin-Helmholtz instabilities arising from large shear in fluid velocities⁽³⁸⁻³⁹⁾ or centrifugal instabilities⁽⁴⁰⁾ resulting from the curvature in the system are possible. More favorable consequences of high rotation could be a method of enhanced plasma heating, much analogous in principle to turbulence induced by centrifugal forces in mirrors.⁽⁴¹⁾

One way to generate the fields necessary to cause toroidal rotation (the poloidal contribution is small) is by the fast FP losses. As mentioned earlier, highly-energetic FP's can have initial trajectories lying outside the plasma. Consequently, these particles can be permanently lost to the system (e.g. in collisions with the limiter or first wall). Since the confinement times for such particles are typically in microseconds, and since neither the background electrons nor ions can cross magnetic surfaces fast enough to neutralize the charge left by the escaping fusion products, a radial electric field results. (The thermal electron motion along the magnetic flux surfaces is sufficiently rapid to damp any poloidal component in the electric field.) It is the interaction between this radial electrostatic field and the toroidal magnetic field that results in the characteristic toroidal rotation the author will evaluate. Poloidal rotation in the plasmas of interest will be shown to be small.

A "steady-state" solution for toroidal rotation is found and applications to a possible TFTR (Toroidal Fusion Test Reactor) experiment is suggested. In order to evaluate the toroidal rotation, the FP loss current as a function of radial position must be known, since it is this process which is vital in the free charge build-up inside the plasma. The code SYMALF supplies us with this information. Of equal importance are those mechanisms which inhibit net charge accumulation inside the plasma. These include charge-exchange, magnetic ripple, and perpendicular viscosity, which will be discussed in detail in Chapter V. "Steady-state" toroidal rotation speeds are then calculated via a simple fluid model.

I.2 Motivation and Importance

It was the author's original intention to examine the effects that neoclassical "bootstrap" currents have on plasma transport during reactor start-up and steady-state operation. It soon became clear that in order to study the problem in depth, detailed information on the motion of the high-energy fusion products was necessary. For example, where the FP's deposit their energy is very important in ascertaining the plasma behavior both during the initial start-up phase and long pulsed operation. While solving this FP motion problem, the author became increasingly aware of the influence of fusion particles on a wide variety of plasma-related problems, and ultimately decided to pursue the subject full-time.

Among the important problems in the fusion area addressed by this thesis (in addition to the plasma heating problem) the question of where in the plasma the FP's eventually thermalize (at least their position when their energy is about 500 KeV). This knowledge can aid in predicting the fusion-product ash build-up⁽¹⁶⁾ in steady-state operation.

This thesis will also focus on the effect of slowing down on FP distributions functions. Knowledge of these distribution functions are important in predicting possible disruptive microinstabilities. The conditions that can bring these instabilities about will be discussed in section (III-4).

Large radial motion of the FP's can result in their striking the first wall. When this happens, various types of wall-erosion occur, like blistering and sputtering.* Erosion resulting from continual FP bombardment will seriously shorten first wall lifetime. When the "blister" breaks, moreover, high-Z wall material may enter the plasma, with premature quenching of the plasma burn the likely outcome. A method for predicting the intensity of these FP wall loadings as a function of position has been developed by the author for a tokamak with high magnetic ripple.

*Blistering occurs when the pressure of neutral gases trapped near the surface of a solid forces a deformation of that surface (e.g., bubble formation). Sputtering is the ablation of the surface due directly to bombardment of high energy particles.

The rapid loss of FP's from the plasma causes a net negative charge build-up inside the plasma. The plasma rotation that results could excite instabilities if the rotation speed grows too large ($\sim$ thermal ion speed). The author estimates these probable rotation speeds and discusses the possibility of detecting them in a near-term TFTR experiment.

I.3 Original Contributions

The original contribution of this thesis are:

- 1) First derivation of a theory to evaluate the slowing down orbits of energetic fusion particles.
- 2) Implementation of this theory in a code (SYMALF). Important results of the SYMALF source code include: (a) plasma heating profile; (b) FP thermalization source functions; and (c) FP energy distribution functions as a function of position. With (c), it was first found that under certain circumstances, the drift may distort the FP distribution function and cause "bump on the tail" type instabilities.
- 3) A derivation of a method (RIPALF) to evaluate the wall-loading due to magnetic ripple effects and the prediction of high damage to localized regions of the first wall.
- 4) Development of a fluid model to approximate the toroidal rotation resulting from first-bounce fusion product losses.

The results predict detectable toroidal rotation in a TFTR D-T plasma, and, under certain circumstances, rotational speeds which might lead to gross disruptions in the plasma.

CHAPTER II. FUSION PARTICLES ORBITAL THEORY

II.1 FP Slowing-Down: Theory

As we have already noted in the introduction, the actual path of the suprathermal in slowing-down is not expressible in closed form. As done by P. R. Meka,⁽⁴²⁾ the trajectory can be found by solving a system of first-order, differential equations involving drift and energy losses. However, in this approach, the timesteps must be much smaller than the characteristic orbital time ($\sim 1 \mu s$). Consequently, an inordinate amount of effort is required to follow the evolution of even a single orbit. For example, 40 minutes of computation time (IBM-360) is consumed in tracking the movement of one alpha from 3.5-MeV down to about 1.1-MeV. This procedure, then, is clearly not suitable for practical situations where large numbers of superthermal trajectories must be calculated.

In this chapter, we describe a method of handling a large number of changing alpha orbits. The code SYMALF reduces computational demands to reasonable proportions while simultaneously retaining good accuracy. The theoretical basis for SYMALF is presented in this section. Its applications are reserved for the succeeding chapter.

The toroidal geometry is shown in Fig. II.1, where a "typical" orbit is projected on the (r, θ) -plane. {In this figure and succeeding discussion, untrapped alphas are assumed. The same logic, however, applies to trapped particles governed by symmetric (non-ripple) trapping. This is discussed in Appendix A.} The track in this plane is divided up into $2N$, equal segments, arranged such that segments 1 and $N+1$ correspond to position P_{MAX} and P_{MIN} , respectively. The time, τ_i , which the particle spends in the i^{th} segment is approximately $L/(2Nv_{Di})$, where L is the total orbital path length projected on the (r, θ) -plane and v_{Di} is the drift speed of the alpha in segment P_i ; that is:

$$v_{Di} = \sqrt{v_{ri}^2 + v_{\theta i}^2}, \quad (II-1a)$$

Fig. (II.1). Tokamak geometry is shown, as is the projection of a sample orbit (denoted by arrows) on the r - θ plane. The maximum and minimum points (actually, segments) of the orbit are denoted by P_{MAX} and P_{MIN} , respectively, as well as an arbitrary segment along the orbit, P_i .

with

$$v_{r_i} = \frac{-2\mathcal{E}}{(Ze)R_o B_o} \left(1 - \frac{KB_o}{2h_i}\right) \sin\theta_i, \quad (\text{II-1b})$$

$$v_{\theta_i} = \frac{-2\mathcal{E}}{(Ze)R_o B_o} \left(1 - \frac{KB_o}{2h_i}\right) \cos\theta_i + \sqrt{\frac{2}{M}} \frac{B_{\theta}}{B_T} \cos\psi_i. \quad (\text{II-1c})$$

In the above, $Ze \equiv$ alpha charge, $M \equiv$ alpha mass, $h_i \equiv 1 + r_i \cos\theta_i/R_o$, $\mathcal{E} \equiv$ particle energy, $K \equiv \mu/\mathcal{E}$ and $\psi_i \equiv$ angle between the velocity vector, B_{θ} is the poloidal field, and B_T is the toroidal field, with B_o the value of B_T on the plasma centerline.

Suppose the particle starts at P_{MAX} with energy $\mathcal{E}_o$. As the supra-thermal alpha passes through the i^{th} orbital segment, P_i , it loses energy ϵ_i , in addition to the energy $\sum_{j=1}^{i-1} \epsilon_j$ it has already lost passing through the $i-1$ previous segments. The radial change, δD , in P_{MAX} as a result of one complete orbit, illustrated as Case III in Fig. II-2, is given by

$$\delta D = \sum_{i=1}^{2N} v_{r_i} \tau_i = -\frac{L\Gamma}{2N} \sum_{i=1}^{2N} \sqrt{\frac{\sin\theta_i}{\Gamma^2 - \epsilon_i/\sqrt{\mathcal{E}} + \Omega_i/\mathcal{E}}}, \quad (\text{II-2})$$

where

$$\Gamma = \frac{2\mathcal{E}}{(Ze)R_o B_o}, \quad (\text{II-2a})$$

$$\epsilon_i = \frac{\sqrt{\frac{8}{M}} \Gamma \cos\theta_i \cos\psi_i}{(1 - KB_o/(2h_i))} \frac{B_{\theta_i}}{B_T}, \quad (\text{II-2b})$$

$$\Omega_i = \frac{\left(\frac{2}{M}\right) \cos^2\psi_i}{(1 - KB_o/(2h_i))^2} \left(\frac{B_{\theta_i}}{B_T}\right)^2. \quad (\text{II-2c})$$

As inferred from Spitzer⁽⁴³⁾ (see Appendix B) or demonstrated by Meka,⁽⁴²⁾ K is a constant of motion in the slowing down process, as long as pitch angle scattering is negligible. It is clear from symmetry

Fig. (II.2). Various slowing-down schemes resulting from an initial orbit (solid line) passing through P_{MAX} and P_{MIN} . Case I (alternating dashed curve) assumes particle energy loss to occur only in the immediate vicinity of P_{MAX} . Case II (dotted curve) assumes the energy is lost only in the immediate vicinity of P_{MIN} ; δD_{MAX} represents the displacement for a Case II orbit. Case III (dashed line) assumes uniform energy loss along the orbit, δD_S is the displacement in P_{MAX} after one orbit.

considerations that $\sin\theta_i = -\sin\theta_{2N+2-i}$ and $\cos\theta_i = \cos\theta_{2N+2-i}$ in the lossless case. This implies $\delta D_{\text{lossless}} = 0$, as expected. In fact, for $\sum_{i=1}^{2N} \frac{\epsilon_i}{\epsilon_0} < 1$ and $\epsilon_i \approx \epsilon_{2N+2-i}$, follows that $\sin\theta_i \approx -\sin\theta_{2N+2-i}$ and $\cos\theta_i = \cos\theta_{2N+2-i}$. Using these facts and letting $\delta E \equiv \sum_{i=1}^{2N} \epsilon_i$, we can reduce Eq. (II-2) to the form (see Appendix C).

$$\begin{aligned} \delta D &\approx \frac{L\Gamma}{2N} \sum_{i=1}^{2N} \frac{\sin\theta_i}{\sqrt{\Gamma^2 - \frac{\epsilon_i}{\sqrt{\epsilon_0}} + \frac{\Omega_i}{\epsilon_0}}} \\ &+ \frac{1}{2N} \left\{ \sum_{i=1}^{2N} (f_i - g_i) \sum_{j=1}^i \epsilon_j \right\} \\ &\approx -\frac{1}{2N} \left\{ (\delta E - \epsilon_1) \sum_{i=1}^{N+1} (f_i - g_i) - \sum_{i=3}^{N+1} (f_i - g_i) (\epsilon_i + 2 \sum_{j=2}^{i-1} \epsilon_j) \right\}, \end{aligned} \quad (\text{II-3})$$

where

$$f_i \equiv \frac{L\Gamma}{2\epsilon_0^2} \frac{\Omega_i \sin\theta_i}{(\Gamma^2 - \frac{\epsilon_i}{\sqrt{\epsilon_0}} + \frac{\Omega_i}{\epsilon_0})^{3/2}}, \quad (\text{II-3a})$$

and

$$g_i \equiv \frac{L\Gamma}{4} \frac{\epsilon_i \sin\theta_i}{(\Gamma^2 - \frac{\epsilon_i}{\sqrt{\epsilon_0}} + \frac{\Omega_i}{\epsilon_0})^{3/2}}. \quad (\text{II-3b})$$

Since the f_i -contribution can be shown to almost always dominate the g_i -effects (Appendix C), we will simplify Eq. (II-3) by neglecting the latter in subsequent discussion.

Even so, due to the complicated nature of the f_i 's, the δD expression does not represent a marked simplification over the brute force technique mentioned earlier. It is therefore, advantageous to simplify Eq. (II-3) further. In order to do this, it is necessary to first examine some limiting cases. There are three situations of importance:

- Case I: Assuming that alpha energy losses were only to occur in segment P_{MAX} , then $\delta D_I \equiv \delta D \approx 0$;
- Case II: Conversely, if the energy loss were only to occur in segment P_{MIN} , then $\delta D_{II} \equiv \delta D \approx -\frac{\delta E}{2N} \sum_{i=1}^{N+1} f_i$;
- Case III: If the energy loss occurs uniformly along the orbit, then⁽¹⁴⁾ $\delta D_{III} = \frac{1}{2} \delta D_{II}$. Since δD_{II} represents the maximum displacement, we will label it δD_{MAX} for future purposes. (See Fig. II-2 and Appendix C.)

Under typical plasma conditions in low safety-factor Tokamaks (i.e., $q \equiv \frac{r}{R_0} \frac{B_T}{B_\theta} \approx 2$), energy loss rates will not vary radically along the trajectory, and Case III will fairly well describe the deviation over an energy decrement of δE . However, in low-current devices where large radial drifts are possible, the alpha can experience a much wider range of plasma density and temperature conditions, and hence, energy loss rates. These potentially large variations in plasma and magnetic properties along the orbit (in which Cases I and II are extremes) imply that a case III approximation to δD above may be an oversimplification in some instances.

In seeking a realistic alternative to the δD of Eq. (5), then, we can re-write Eq. (5) as the sum of two contributions: (1) a "zero order" term which is characterized by δD_{III} above and (2) a "first order" term, which represents the effect of the above variations as a linear function between the orbital extremes P_{max} and P_{min} . Thus, Eq. (II-3) simplifies to

$$\delta D_s = \frac{1}{2} \delta D_{max} (1 + (\epsilon_{P_{min}} - \epsilon_{P_{max}}) / \delta E), \quad (II-4)$$

where $\epsilon_{P_{\max}}$ and $\epsilon_{P_{\min}}$ are the energies lost in $P_{\max}$ and $P_{\min}$, respectively. (The practical determination of $\epsilon_{P_{\max}}$ and $\epsilon_{P_{\min}}$ is left for later in this chapter - see Eqs. (II-9a), (II-9b), and (II-9c).) One notes that Eq. (II-4) is consistent with the three limiting cases mentioned above (e.g., if $\epsilon_{P_{\max}} \approx \delta\epsilon \gg \epsilon_{P_{\min}}$, then $\delta D_s \approx 0$, as in Case I). Thus far, $P_{\max}$ and $P_{\min}$ have not necessarily been restricted to a particular point on the orbit. Consequently, the P's could, if desired, be defined as an infinitesimal segment or as even half the orbit. The question of how to deal with $P_{\max}$ and $P_{\min}$ will be left to the following section, when the plasma ring method of tabulating information will be discussed.

After evaluating the effect of energy loss for a single orbit, one next generalizes to a multiple orbit situation. In the time required for an energy loss of Δ ($\delta\epsilon < \Delta\epsilon \ll \epsilon_0$), the alpha experiences little relative change in either μ and P_ϕ . Consequently, the amount $P_{\max}(\epsilon_0)$ drifts after each successive orbit is approximately constant over this energy interval $\epsilon_0, \epsilon_0 - \Delta\epsilon$. The total shift in $P_{\max}$ resulting from an energy loss Δ is, therefore, δD_s times the number of orbits completed ($\approx \Delta\epsilon/\delta\epsilon$). Likewise, the most $P_{\max}$ may be displaced (i.e., Case II) after a $\Delta\epsilon$ loss is $\delta D_{\max}$ times $(\Delta\epsilon/\delta\epsilon)$. Equation (II-4) can then be rearranged to obtain

$$P_{\max}(\epsilon_0 - \Delta\epsilon) = P_{\max}(\epsilon_0) + \frac{1}{2} (\tilde{P}_{\max}(\epsilon_0 - \Delta) - P_{\max}(\epsilon_0)) \left(1 + \frac{\epsilon_{P_{\min}} - \epsilon_{P_{\max}}}{\delta\epsilon}\right). \quad (\text{II-5a})$$

$P_{\max}(\epsilon_0 - \Delta)$ is the position of segment $P_{\max}$ under the Case II energy loss assumption. It is, in fact, the maximum radial position determined by the orbital equation. (12,42)

$$\cos\theta = \frac{R_0}{r} \left\{ \frac{B_0}{2} \left(K + \sqrt{K^2 + \frac{2M}{\epsilon} (P_\phi - \Psi(r))^2} \right) - 1 \right\}, \quad (\text{II-5b})$$

with the conditions that $E \rightarrow E_0 - \Delta E$, $P_\phi \rightarrow P_\phi(E_0 - \Delta E)$, and $P_{\text{MIN}}(E_0) = P_{\text{MIN}}(E_0 - \Delta E)$. Here, $\Psi(r)$ is the poloidal flux function given by $\frac{e}{MB} \int_0^r B_\theta dr$.

Since $P_{\text{MAX}}(E_0 - \Delta E)$ lies on the toroidal plane, the values that $\cos\theta$ can assume are ± 1 , depending on whether $P_{\text{MAX}}(E_0 - \Delta E)$ falls on the inside ($\theta=180^\circ$) or the outside ($\theta=0^\circ$) of the torus. From these, r , which represents in this case $\tilde{P}_{\text{MAX}}(E_0 - \Delta E)$, is calculated. Using these results, one evaluates $P_{\text{MAX}}(E_0 - \Delta E)$ via Eq. (II-5a). In accordance with $K = (\sin \psi)^2/B$, a new canonical angular momentum value, $P_\phi(E_0 - \Delta E)$, is found. Knowledge of $P_{\text{MAX}}(E_0 - \Delta E)$ and $P_\phi(E_0 - \Delta E)$ is sufficient to determine the new orbit (via Eq. (II-5b)). This entire process (see Fig. II-3) can then be repeated over subsequent ΔE loss intervals.

As a check of the above slowing-down theory, this approximate orbit description has been compared to the more exacting drift model of Meka cited at the beginning of this section. Typical results are shown in Fig. II-4 for a low current, Tokamak device. Although a small discrepancy between the two calculations is observed, the speed of SYMALF is well worth the cost in accuracy. In this particular example, the SYMALF theory ran about 4000 times faster than the drift model.

As discussed in previous work,^(12,15) a net inward drift of P_{MAX} occurs during the slowing down of high-energy alphas, this being a consequence of the interplay between the suprathermal's tendency to remain on a magnetic flux surface and the energy-loss mechanism along the trajectory. This slowing-down drift continues until the pitch-angle scattering of alphas off the background plasma (mainly ions) becomes the dominant transport mechanism.

II.2 Velocity Space Scattering

One expects this transition to significant velocity-space scattering also to impair the reliability of the SYMALF approach, which essentially neglects this effect. In turn, we now estimate how this

Fig. (II.3). The initial orbit is denoted by a (r, θ) -trajectory passing through segments $P_{\text{MIN}}(\epsilon_0)$, (X) and $P_{\text{MAX}}(\epsilon_0)$, (W), (i.e., solid arrows). The dashed line represents the intermediate orbit, which passes through $P_{\text{MIN}}(\epsilon_0)$ and $P_{\text{MAX}}(\epsilon_0 - \Delta)$, (Y). The final orbit (open arrow) shows a new $P_{\text{MAX}}(\epsilon_0 - \Delta)$, (Z).

Fig. (II.4). A SYMALF orbit is compared with the more detailed drift model after an energy loss of 2.4-MeV from the initial 3.5-MeV. The Tokamak has TFTR-like parameters: $a=56$ cm, $R_0=1.0$ MA, $I_{\text{plasma}}=1.0$ MA, $n=5 \times 10^{13}/\text{cm}^3 (1-(r/a)^2)$, $T_{\text{e}}=6$ keV $(1-(r/a)^2)$, current density $j_{\text{plasma}} \propto (1-(r/a)^2)$, with initial conditions $\psi_0=70^\circ$, $r_0=29.12$ cm, and $\theta_0=0$.

scattering may affect SYMALF's accuracy. We first calculate the total rms deviation of pitch-angle in velocity space, $\Delta\psi$, by employing the standard Fokker-Planck approximation (see Appendix D). For a thermal D-T plasma, this gives:

$$\Delta\psi \approx 2 \sin^{-1} \left\{ \sqrt{0.26 \ln \left[\left(\frac{x_0}{x} \right)^{3/2} \left(\frac{x^{3/2} + 296 \langle Z \rangle}{x_0^{3/2} + 296 \langle Z \rangle} \right) \right]} \right\}, \quad (\text{II-6a})$$

where $x_0 \equiv \mathcal{E}_0/T_e$, $x \equiv \mathcal{E}/T_e$, and $\langle Z \rangle \equiv$ effective Z of the plasma.

For background electron temperatures of about 5 to 10 keV, $\Delta\psi$ ranges only from 8 to 12 degrees as the alpha's energy drops from 3.5 MeV down to about 2 MeV. As the particle energy approaches 500 keV, however, the $\Delta\psi$ spread extends out 15 to 30 degrees, and serious dispersion in velocity space starts. To facilitate comparison, these $\Delta\psi$ -values can be translated into a dispersion λ in radial drift. By approximating the particle's maximum radial displacement, ρ , over an orbit by $\frac{1}{2} \Sigma V_{r_i} \tau_i$ and by reducing $\Delta\psi$ to a first order, linear function in E over the range of interest (i.e., $T_e \lesssim 10$ keV and $E \gtrsim 500$ keV), one finds (see Appendix E).

$$\lambda_D = \int_{E_0}^E \left(\frac{\partial \rho}{\partial \Psi} \right)^2 \frac{\partial}{\partial E} \left(\frac{\Delta\psi}{2} \right) dE. \quad (\text{II-6b})$$

The brackets in Eq. (II-6b) refer to r -, θ -, and Ψ -averaging over the alpha trajectory. Taking $\mathcal{E}_f = 500$ keV and $T_e = 6$ keV, we estimate an approximate dispersion in the drift distance of $\lesssim 0.01m$ for a TFTR-like Tokamak. Since typical drifts in P_{MAX} are $\sim 0.1m$ for such a device, λ_D is roughly one-tenth the drift distance (see Appendix E). On the other hand, for alpha energies lower than 500 keV, λ_D increases rapidly, approaching or even exceeding the slowing-down drift value. Consequently, to extend this model to energies less than 500 keV (or $\mathcal{E}/T_e \lesssim 50$) is of questionable value.

II.3 FP Slowing Down: Code SYMALF

Section II.1 covered the theoretical considerations of the thermalization process. This section is concerned with the application of these concepts into a working code (SYMALF). Special emphasis will be given to both the plasma heating and thermalization source profiles calculated via SYMALF.

The mechanics of SYMALF are relatively straightforward. The background particle and current densities, as well as the plasma temperatures, are assigned spatial profiles of the form $(1-\xi(r/a)^\omega)^\nu$, where ω , ξ , and ν are input constants. This background (Maxwellian) plasma is divided into 50 equally spaced, concentric rings. Thus, for typical plasma radii (i.e., $a \sim 1.0$ m), the width of each ring is in order of or larger than the 3.5-MeV alpha gyroradii, but still narrow enough to discern orbital effects. A characteristic energy deposition rate, $\frac{d(r_i, \epsilon_k)}{dt}$, is calculated for each of these rings. (The selection of the energy stepsize, Δ , will determine the ϵ_k 's). The general Sivukhin⁽⁴⁴⁾ energy-loss formula is used.

As the alpha moves along its orbit, it deposits a certain amount of its energy in each ring it crosses. The time, Δt_i , spent in, say, the i^{th} ring is approximately the ring width divided by the representative radial drift velocity given by Eq. (II-1b). Then the energy lost to that particular ring in one orbit is

$$\epsilon_i \approx \frac{d(r_i, \epsilon_k)}{dt} \times 2\Delta t_i, \quad (\text{II-7})$$

where the 2 accounts for double-crossings of the rings. This process is repeated for each of the rings crossed by the orbit. Thus, the energy which the i^{th} ring gains, Δ_i , after the alpha drops Δ can be written:

$$\Delta_i \approx \Delta \epsilon \times (\epsilon_i / \delta) \quad (\text{II-8})$$

The Δ_i value can then be stored in the program, a new orbit determined, and the above steps repeated until cutoff at 500 keV.

In order to ascertain the new orbit after an energy loss of ΔE , $P_{MAX}(E - \Delta E)$ must be evaluated using Eqs. (II-5a) and (II-5b). The maximum and minimum rings through which the orbit passes serve as P_{max} and P_{min} , respectively. Using this to define $\epsilon_{P_{max}}$ and $\epsilon_{P_{min}}$ gives results which compare well with the time-consuming drift solution (recall Fig. II.4). This technique is a particularly convenient method when large numbers of orbit computations are involved. The total energy lost per orbit, δE , is still, of course, the sum of each of the ring contributions; that is:

$$\epsilon_{max} \approx \frac{dE}{dt}(r_{max}, E_k) \times 2\Delta t_{max}, \quad (II-9a)$$

$$\epsilon_{min} \approx \frac{dE}{dt}(r_{min}, E_k) \times 2\Delta t_{min}, \quad (II-9b)$$

and

$$\delta E \approx \sum_{i_{min}}^{i_{max}} \frac{dE}{dt}(r_i, E_k) \times 2\Delta t_i. \quad (II-9c)$$

In order to calculate the alpha deposition curve for a D-T plasma, we use a phase-space grouping method. Each group is comprised of alphas born within a given spatial-velocity-space volume. Alphas are assumed to be born with an isotropic angular distribution at a rate consistent with the temperature and density at the birth point. Although the precision increases as the number of groups increases, the cost in time and money correspondingly grows. The author's experience has shown that 3.52-MeV alpha sources represented by 13-radial, 18-azimuthal, and 18-velocity space-coordinates give reasonable resolution for calculations involving TFTR-(45) and EPR-type(46) plasmas. This is equivalent to requiring that each computer particle represent roughly 10^{13} "real" alphas.

A schematic for the heating calculation is shown in Fig. (II-5). As before, an initial orbit along which the particles move is found and energy deposition for that group is evaluated; a new orbit is then

Fig. (II.5). Flow chart of SYMALF. Energy loss rates per ring are determined beforehand for each E_k , which represents the characteristic energy at each slowing-down energy level. The number of particles for a given phase space group (characterized by r_0, θ_0, ψ_0) are calculated. At A, the orbit is determined by the method outlined in Section II; at B, the energy lost in each ring crossed by the orbit is found; at C, this information is stored. If $E_k > 500 \text{ keV}$, the sequence is repeated as shown; if not, a new phase space group is chosen. This process is continued until all radial, azimuthal, and velocity space possibilities are exhausted.

calculated as is energy deposition along it, etc. As mentioned above, the number of groups in our code is approximately 4000.

Five energy groups are sufficient for determining both the orbit during the thermalization phase and the energy deposition (i.e., the slowing-down drift occurring during an energy interval will not exceed the radial width of the spatial group it moves across). Each of these intervals (say, the k^{th}) is represented by the time-averaged energy, $\bar{\epsilon}^{(k)}$. For D-T produced alphas, the slowing down interval ranges from 3500 keV to 500 keV. Since five energy intervals were used in the present calculations, the width, $\Delta\epsilon$, of each interval is 600 keV---with $\bar{\epsilon}^{(1)} \cong 3200$ keV, $\bar{\epsilon}^{(2)} \cong 2600$ keV, etc.).

In Chapter III, the effect of the slowing down drifts on plasma heating will be discussed in detail.

CHAPTER III. APPLICATIONS OF SLOWING DOWN THEORY

III.1 Introduction

In the last chapter, the slowing down theory for highly energetic FP's was developed. With the use of a phase-space grouping technique, the theory was incorporated into a working code (SYMALF). In this chapter, application of the theory is made to key problems in FP physics: (1) plasma heating; (2) FP thermalization source functions; and (3) FP energy distribution functions.

To facilitate comparisons between variously-sized Tokamaks, we will use the reference designs of three specific reactors: the "small" TFTR,⁽⁴⁵⁾ the intermediate ANL-I,⁽⁴⁶⁾ and the large UWMAK-I.⁽⁴⁷⁾ This scaling also corresponds to the orbit width parameter ρ_θ/a , where ρ_θ is the alpha's poloidal gyroradius. The $\frac{\rho_\theta}{a}$ for TFTR for example is ~20 times that of UWMAK-I, implying FP are much better confined in the latter. The functional forms for background temperature, density, and current density for each of the Tokamaks are indicated in Table I. In a majority of the cases, parabolic profiles are assumed.

Along with the above, two additional cases are considered for the TFTR configuration. The first is a neutral blanket design,⁽⁴⁸⁻⁵⁰⁾ which will be designated TFTR-(GB). Designed to alleviate such problems as plasma contamination during Tokamak start-up, the idea is to allow the nuclear burn to propagate from the central regions outward to the rest of the plasma. With a high concentration of neutral fuel surrounding the plasma present during this phase, the plasma density is expected to increase with radial distance from the central regions before rapidly falling off at the boundary. Since this results in parts of the plasma having a positive radial gradient, classical transport theory suggests that serious inward diffusion of impurities from the wall could be retarded, at least in the critical stages of start-up.⁽⁴⁹⁾ For this case, we assume that the plasma density can be expressed as

$$n_{i,e}(r) = 8 \times 10^{19}/m^3 \cdot (1 + \exp \{-\frac{r-0.7}{0.2}\}^2) \text{ for } \hat{r} \leq 0.7, \text{ and} \quad (\text{III.1.a})$$

TABLE I. TOKAMAK PARAMETERS

Device	TFTR-I	TFTR-(GB)	TFTR-(³ He)	ANL-I	UWMAK-I
R_0 (m)	2.5	2.5	2.5	6.25	13.0
a (m)	0.5	0.5	0.5	2.1	5.0
I_p (MA)	1	1	1	7.6	21
B_0 (KG)	45	45	45	44.7	38.2
T_i (keV)	$10(1-0.8\hat{r}^2)$	$10 \exp(\frac{\hat{r}^2}{0.214})$ for $\hat{r} \leq 0.7$ ----- 1 for $\hat{r} \geq 0.7$	$10(1-0.8\hat{r}^2)$	$10(1-\hat{r}^2)$	$11.1(1-\hat{r}^3)$
$n_i (\frac{1}{3} \times 10^{19})$	$8(1-0.8\hat{r}^2)$	$8(1-\exp(\frac{\hat{r}-0.7^2}{0.4}))$ for $\hat{r} \leq 0.7$ ----- $16(1-\frac{\hat{r}-0.7^2}{0.09})$ for $\hat{r} > 0.7$	$5(1-0.8\hat{r}^2)$	$9.36(1-\hat{r}^2)^{0.8}$	$8(1-\hat{r}^3)$
Current density profile	$(1-\hat{r}^2)$	$(1-\hat{r}^2)$	$(1-\hat{r}^2)$	$(1-\hat{r}^5)$	$(1-\hat{r}^2)$
Driving mechanism	Beam	Beam	Beam	Thermonuclear	Thermonuclear

$$n_{i,e}(r) = 1.6 \times 10^{20}/m^3 \left(1 - \left(\frac{\hat{r}-0.7}{0.3}\right)^2\right) \text{ for } \hat{r} > 0.7 \quad (\text{III.1.b})$$

where $\hat{r} \equiv r/a$. The plasma temperature is assumed gaussian for $\hat{r} \lesssim 0.7$ and flat for $\hat{r} > 0.7$, i.e.,

$$T_{i,e}(r) = 10 \text{ keV} \exp\left(-\left(\frac{\hat{r}^2}{0.214}\right)\right) \text{ for } \hat{r} \lesssim 0.7 \quad (\text{III.1.c})$$

$$T_{i,e}(r) = 1.0 \text{ keV for } \hat{r} \gtrsim 0.7 \quad (\text{III.1.d})$$

An alternate TFTR plan uses deuterium neutral beam injection into a ^3He background plasma, ⁽⁵¹⁻⁵²⁾ instead of a tritium background. Since all the fusion products here are charged particles (i.e., $D + ^3\text{He} \rightarrow \alpha + p + 18300 \text{ keV}$), presumably all the energy of fusion can go into heating the plasma. However, poor confinement of these products in small, near-term devices, combined with very low production rates (about 1% of the D-T reaction rate), eliminate D- ^3He as a candidate for energy breakeven experiments in the 1980's. This fuel, nevertheless, may well serve as a neutron- and tritium-clean alternative to D-T in early tests.

All of the TFTR designs referred to above are beam-driven. For the cases used in this paper, we assume that the neutral beam is deuterium and that the spatial profile of the beam injection is gaussian in $\hat{r}$. The F-value data for both D-T and D- ^3He have been taken from Towner ⁽⁵³⁾ and fitted to an analytic expression. The number of fusion particles of a given specie, $\dot{W}(r, E_{IN})$, produced per unit volume per unit time, resulting from deuterium injection at energy E_{IN} (=120 keV), is

$$\dot{W}(r, E_D) = \left(\begin{array}{l} \text{probability} \\ \text{of a deuterium} \\ \text{ion fusing during} \\ \text{slowing-down} \end{array} \right) \times \left(\begin{array}{l} \text{neutral deuterium} \\ \text{injected per second} \\ \text{per m}^3 \text{ of plasma} \end{array} \right) \\ \times \left(\begin{array}{l} \text{fraction of the} \\ \text{neutral beam} \\ \text{becoming ionized} \end{array} \right) \quad (\text{III-2})$$

$$= \frac{E_{IN}}{Q_f} F(r, E_D) \times \frac{I_o \exp \left\{ -\left(\frac{r}{\gamma} \right)^2 \right\}}{(2\pi R_o) a^2 \gamma^2 (1 - \exp(-\frac{1}{\gamma^2}))}$$

where Q_f is the total fusion energy liberated per fusion reaction, I_o is the total neutral beam current⁽⁴⁵⁾ (≈ 600 amps), R_o is the major radius ($= 2.5$ m), γ is the "standard deviation" of the Gaussian ($= 0.5$), and h is the fraction of the neutral beam that is actually trapped (≈ 0.8). The discussion of the F-values and neutral injection characteristics is deferred to a later time (Section V-2.).

On the other hand, the ANL-I and UWMAK-I Tokamaks are thermonuclear in nature. This means that the production rates for fusion reactions are equal to $n_1^2 \langle \sigma v \rangle$, where $\langle \sigma v \rangle$ is the Maxwellian-averaged reactivity parameter.⁽⁵⁴⁻⁵⁵⁾ These reactivities are generally one or two orders of magnitude less than comparable designs for the beam-target method.

III.2 Radial Heating Profile

One very important aspect in designing a fusion reactor system is the effectiveness with which the fusion product energy is utilized. Although the possibilities for converting their energy directly to electric power are being studied,⁽⁵⁶⁾ the primary function of suprathreshold fusion particles still rests in heating the background Tokamak plasma, both during start-up and steady-state operation. The energy deposition profile has considerable influence on particle and energy transport in the background plasma.

In transport calculations published to date (e.g., Refs. (46) and (57)) have assumed that these highly-energetic particles deposit their energy at their point of birth; this is often referred to as "in situ" deposition. This assumption makes general transport calculations considerably more manageable but ignores the effect of the actual alpha trajectories. Stringer⁽⁷⁾ and McAlees⁽⁸⁾ have considered the effect of 3.52-MeV alpha-particle orbits on plasma heating in steady-state. Assuming that the initial orbit remained unchanged in slowing-down (this will be called the single orbit approximation, SOA), they found that the heating profile was considerably broader than that predicted by the "in situ" model. It was also noted that, as plasma current (actually, poloidal magnetic flux) increased, radial displacements of the alpha orbits decreased so that the SOA distribution gradually reverted to the "in situ" case.

The effect which energy loss has on the alpha orbit (and hence, the energy deposition function) has been examined by Petrie and Miley.⁽¹²⁻¹³⁾ There is an average inward drift of alphas as they slow down in systems with parabolic-temperature and density-profiles; consequently, more energy is lost in the central regions than predicted by SOA. This increased central deposition occurs at the expense of the outer plasma regions. Although both SOA and the detailed slowing-down method (hereafter referred to as SYMALF) include both first bounce losses and the effects of the displacement of the FP drift-orbit surfaces from the initial magnetic flux surfaces, only SYMALF accounts for the energy-loss particle drifts. It is, therefore, of interest to compare the SOA and SYMALF results for selected cases in order to get a feeling for the magnitude of the effects associated with orbit drifts.

For certain cases we will also compare the SYMALF results with what we will call "modified in situ" (MIS). The MIS approach multiplies the usual "in situ" results by $(1 - \gamma_L)$, where γ_L is the total fusion product loss fraction which occur during first bounce escapes. This quantity is not difficult to evaluate numerically, and has been done so by several authors (e.g., Refs. (8), (58) and (59)). As we

will show, this tends to reduce the discrepancy between "in situ" and the other techniques by accounting for suprathreshold product losses. For situations where this method gives results consistent with the more exact SYMALF, substantial simplification in various fusion product calculations can result.

Figure III.1 represents a "standard" D-T plasma for TFTR (we will refer to this model as TFTR-I). Both SOA (dotted curve) and SYMALF (dashed curve) have considerably different heating profiles than "in situ" (solid curve). Initial alpha particle losses account for this large discrepancy. When these losses are accounted for by MIS (with $\gamma_L \approx 0.3$), the comparison between MIS and SYMALF is very good for $\hat{r} < 0.7$, however, the predictions between the two models diverge. This behavior will be discussed further at the end of this section.

The difference between SOA and SYMALF is less than 10% for the central plasma regions, although the variation is more marked in the outer plasma. Evidently, as the alpha particles drift sizeable distances as energy is lost (15-20% of the minor radius is not uncommon), the energy lost by those alphas drifting into a given plasma region is roughly equal to the energy that would have been deposited by those alphas which in the meantime have drifted out of the same region. The exceptions are the center (where there is a net energy influx) and the edge (from which many energetic alphas drift before depositing all their energy).

As with the SOA model, SYMALF becomes indistinguishable from "in situ" for Tokamaks of higher poloidal flux. As before, the reason is that the fusion products are better confined to their magnetic flux surfaces as poloidal flux is increased. Figure III-2 shows the heating profile difference between "in situ" and SYMALF for ANL-I is only 2 or 3% in the central plasma; for $\hat{r} \approx 0.8$, however, these differences can become much larger, although the contribution at this region to overall plasma heating is relatively small. With a UWMK-I power reactor even these differences at the edge are no longer very noticeable (see Fig.

Fig. (III.1). Alpha power deposition density $\hat{r}$ is plotted versus radial plasma position $\hat{r} (\equiv r/a)$ for TFTR-I. In this and in the figures to follow, "in situ", MIS, SYMALF, and SOA are represented by the solid, hatched solid, dashed, and dotted curves, respectively.

Fig. (III.2). Alpha power deposition density is plotted versus radial position $\hat{r}$ for ANL-1.

III-3). Apparently, for these larger power reactor designs, the "in situ" approximation suffices very well.

With all the above, it was assumed that density and temperature profiles were more or less "standard" (i.e., monotonically decreasing to the plasma boundary). The effect of a neutral gas blanket, however, can increase the plasma density with increasing $\hat{r}$ up to a maximum before falling off at the edge. Furthermore, the electron temperature falls off much faster than for the TFTR-I case. Because the alpha slowing-down rate is proportional to $n_e/T_e^{3/2}$, where n_e and T_e are the electron density and temperature, respectively, relatively more energy is lost to the outside than for the TFTR-I case. Figure III.4 shows what this does to plasma heating (Compare with Fig. III.1). Both SYMALF and SOA differ appreciably from "in situ." Further, in this case, even when the first bounce losses are considered (i.e., MIS with $\gamma_L \approx 0.26$), the MIS result is still differs considerably from the SYMALF model. (Note that this is a semilog plot.) For example, the MIS prediction at $\hat{r} = 0$ differs by $> 30\%$ from that of SYMALF. One also notes that the inward energy-loss drift results in a peak in the SYMALF profile of about 16% larger than that corresponding to SOA. In compensation, the SOA heating profile is about twice the SYMALF deposition in the outer plasma regions.

The D-³He case for TFTR is shown in Fig. III.5. The "in situ" heating profile is 40% above the SYMALF model predictions near $\hat{r} = 0$. Note, also, that while most of the 14.7-MeV protons are lost during the initial orbit ($\sim 67\%$ for this case), they still provide the bulk of the heating for energies above 500 keV. Though not shown in Fig. III.5, the MIS method compares well with SYMALF for alphas (if $\hat{r} \lesssim 0.6$).

Before concluding this section, we wish to make a few additional comments. In Table II, each of the four models (i.e., "in situ," MIS, SOA, and SYMALF) has been used to calculate the average power deposition density, $\bar{P}$, in the outer 1/6 of the plasma (call this Region II) for various of the reactor cases mentioned above. Interestingly enough, as Table II shows, $P_{in} \approx P_{SOA}$ for TFTR-I and TFTR-(GB), where the

Fig. (III.3). Alpha power deposition density is plotted versus plasma position $\hat{r}$ for UWMAK-I.

Fig. (III.4). Alpha power deposition density is plotted versus plasma positive $\hat{r}$ for the TFTR neutral gas blanket design.

Fig. (III.5). Fusion product power deposition density is plotted versus radial plasma position $\hat{r}$ for the D- ^3He design. The alpha particle and proton contributions are shown individually, as well as their sum for both SYMALF and "in situ."

subscript designates the model (e.g., $\bar{P}_{in}$ is the average "in situ" power depositive density). Alpha particles which are born in Region II but which have orbits lying outside the plasma are quickly lost, and, therefore, do not make any contribution to heating Region II. However, this resulting loss in heating in Region II is roughly compensated for by the heating power derived from alphas born in the inner 5/6 of the plasma (Region I), but having parts of their orbits lying out in Region II.

Although the differences in the heating predictions of SYMALF and SOA are in the 10% to 20% range in the central plasma, the discrepancy can become quite sizeable near the plasma boundary. As Table II again shows, $\bar{P}_{SOA}$ differs from $\bar{P}_{SYMALF}$ by about a factor of 2 in TFTR-I and TFTR-(GB). The energy loss drift, which is included in the SYMALF calculations, has depleted Region II of half the heating power predicted by SOA (and the "in situ" model). Although a better estimate in average power deposition can be made for Region II by using MIS in place of "in situ," the difference between the MIS and SYMALF results remains considerable.

For the larger current reactors (ANL-I and UWMAK-I), the ratios of $\bar{P}_{in}/\bar{P}_{SYMALF}$ and $\bar{P}_{MIS}/\bar{P}_{SYMALF}$ are again noticeably different from unity, the values being less than 1. For these devices, fast particle losses in Region II are much less serious than for lower current reactor designs. On the other hand, there are still the contributions from Region I mentioned previously. In fact, this latter effect more than compensates for Region II particle losses. The same can be argued for the ratios $\bar{P}_{in}/\bar{P}_{SOA}$ and $\bar{P}_{MIS}/\bar{P}_{SOA}$.

Since the temperature of the background ions and electrons in Region II depends on the thermal energy transport from the hotter core region and plasma-wall physics (as well as from direct alpha heating), the implications of the previous discussion to plasma heating near the boundary are not clear-cut at present. The efficiency of neutral beam⁽⁵⁰⁾ and pellet injection⁽⁶⁰⁾ is sensitive to Region II temperatures, especially with respect to charge-exchange reactions. Wall erosion via sputtering

Table II: Average Power Deposition Density for the Outer One-Sixth of the Plasma

Device	$\bar{P}_{in}$ (watts/m ³)	$\bar{P}_{MIS}$ (watts/m ³)	$\bar{P}_{SYMALF}$ (watts/m ³)	$\bar{P}_{SOA}$ (watts/m ³)	$\frac{\bar{P}_{in}}{\bar{P}_{SYMALF}}$	$\frac{\bar{P}_{in}}{\bar{P}_{SOA}}$	$\frac{\bar{P}_{MIS}}{\bar{P}_{SYMALF}}$	$\frac{\bar{P}_{MIS}}{\bar{P}_{SOA}}$
TRTR-I	1.04×10^5	0.75×10^5	0.493×10^5	0.962×10^5	2.1	1.1	1.52	0.78
TRTR-(GB)	0.47×10^5	0.34×10^5	0.24×10^5	0.50×10^5	2.0	0.94	1.42	0.68
ANL-I	1.10×10^2	1.10×10^2	4.89×10^2	6.00×10^2	0.22	0.18	0.22	0.18
UWMAK-I	1.22×10^2	1.22×10^2	2.02×10^2	2.02×10^2	0.60	0.60	0.60	0.60

by charge-exchange neutrals is also strongly dependent on the plasma temperature of the outer region.

III.3. Thermalization Source Function

The presence of thermal FP's (like alpha particles) decreases the plasma reactivity, and this may even quench the burn if the alpha density is allowed to increase appreciably (due to both increased Bremsstrahlung losses and decreased plasma reactivity since FP's do not fuse). The question of where these energetic particles thermalize in the plasma is then of interest from the standpoint of fusion, product "ash" build-up. By knowing the rate and position at which alpha particles appear just before slowing-down is complete, one has, in effect, a thermalized alpha source function. For example, Downum et al.⁽¹⁶⁾ have used this source function for evaluating the thermal alpha-particle density. Since SYMALF is generally valid only for particle energies greater than 500 keV, we have somewhat liberally defined "thermalization." Strictly speaking, the present results serve as the source function for the intermediate ($T_e < E < 500$ keV) energy region.

Since this source function depends on the alpha orbits at thermalization rather than the particle paths at birth (unlike energy deposition), one expects the SYMALF results to differ much more markedly from SOA and "in situ" than the corresponding SYMALF curves for plasma heating. Figure III.6 demonstrates this for TFTR-I. Not only does SYMALF differ quantitatively, but also qualitatively, being considerably flatter around the origin. The reason for this is as follows: the number of alphas which are born in the central region but which leave the core before thermalization is greater than the number which are born for $r > 0.5$ but thermalize in the core, thus flattening the distribution; whether or not this occurs depends on how the orbits evolve, which, in turn, is largely influenced by the thermal plasma temperature and density. The SOA model has a lower peak value than SYMALF and larger values for the source function in the outer plasma region; this is a consequence of energy loss drifting. Once again, because of

initially lost alphas, the "in situ" model is still considerably greater than the other two. The MIS method (Fig. III.6, hatched solid curve) yields fairly good results over much of the plasma.

For ANL-I (see Fig. III.7) and UWMAK-I (see Fig. III.8), the comparison between "in situ" and SYMALF is, if not very large, at least easily observable. The difference between SYMALF and "in situ" is about 10% at the center of ANL-I, though decreasing to about 2% for UWMAK-I. Since the fusion product loss fraction during the first bounce is almost negligible for both of these Tokamaks, the "in situ" and MIS models can be taken as equal.

The alpha source functions for the TFTR-GB are seen in Fig. III.9. Here, SYMALF and SOA have roughly the same distribution, with the former about 8% larger in the center. The "in situ" model is about 30% larger than the other two predictions. The MIS approach reduces this discrepancy to about 10% from SYMALF, although the slope of the former curve is still notably steeper than SYMALF's.

Figure III.10 illustrates the proton and alpha particle thermalization profiles for the $D-^3\text{He}$ case in TFTR. The solid line represents the "in situ" thermalization source for both the protons or alphas. The SYMALF alpha curve is flatter near the origin for reasons similar to those given for TFTR-I. The proton distribution, due to much higher losses, lies considerably beneath the alpha particle curve. The MIS method seems to give good results for the alpha particle component of the reaction, though there are larger differences between SYMALF and MIS for protons, especially near $\hat{r} > 0.7$. This is a consequence of large inward drifts as the proton loses its 14^+ MeV.

III.4. Suprathermal Particle Density

The shape of the fusion product velocity distribution function in the non-thermal energy range is of interest for predicting the occurrence of microinstabilities. If the slope of the velocity distribution is positive, plasma kinetic theory suggests that free energy is available to be used as a driving mechanism for microinstabilities. In this

Fig. (III.6). Alpha thermalization source function is plotted against $\hat{r}$ for TFTR-I.

Fig. (III-7). Alpha thermalization source function is plotted against $\hat{r}$ for ANL-I.

Fig. (III.8). Alpha thermalization source function is plotted against $\hat{r}$ for UWMK-I.

Fig. (III.9). Alpha thermalization source function is plotted against $\hat{r}$ for TFTR gas blanket design.

Fig. (III.10). Fusion product thermalization source function is plotted against $\hat{r}$ for the TFTR D-³He case.

section, we will examine the spatially dependent slowing-down distribution of fusion products from the point of view of our drift model.

Before doing this, however, we mention for the sake of later comparison a second method which gives a well-known and useful approximation to slowing down distribution functions.⁽⁶¹⁾ By assuming that the spatial and velocity components are separable, that the energetic ion moves in an infinite homogeneous medium, and that $V_i \ll V \ll V_e$, one can write the (normalized) speed distribution function as

$$f_{\alpha}(r, V) = \frac{S(r)}{4\pi v_{\alpha e}} \frac{V_{\alpha 0}^3 + V_c^3}{V^3 + V_c^3} \quad \text{for } V_c < V < V_{\alpha 0}, \quad (\text{III.3})$$

where $V_{\alpha 0}$ is the initial alpha speed at 3.52 MeV, V_c is the alpha speed at which its energy losses to background ions and electrons are equal, V_i is the ion thermal speed, V_e is the electron thermal speed, $S(\hat{r})$ is the number density of alpha particles generated at position $\hat{r}$ per unit time, and $v_{\alpha e}$ is the alpha particle-electron collision rate. From Eq. (III.3) the suprathermal alpha density, $n(\hat{r}, k)$, for particles in the k^{th} slowing down energy group is⁽⁶¹⁾

$$n_{\alpha}(r, k) = \frac{S(\hat{r})}{3v_{\alpha e}} \ln \frac{\epsilon_u^{(k)3/2} + \epsilon_c^{3/2}}{\epsilon_L^{(k)3/2} + \epsilon_c^{3/2}} \quad \text{for } \epsilon_c < \epsilon_L^{(k)} \leq \epsilon_{\alpha 0}. \quad (\text{III.4})$$

Here $\epsilon_L^{(k)}$ and $\epsilon_u^{(k)}$ are the upper and lower energy bounds of the k^{th} energy group, $\epsilon_c \equiv \frac{1}{2} M_{\alpha} V_c^2$, $\epsilon_{\alpha 0} = \frac{1}{2} M_{\alpha} V_{\alpha 0}^2$, and M_{α} is the alpha particle mass. Many of the high energy particles are lost during their first orbit and those that remain can experience drifting while slowing-down. It is consequently not clear whether Eqs. (III.2) and (III.3) are realistic, in particular for low current Tokamaks. Part of the purpose of this present work is to examine this question. The MIS approximation, which is obtained by multiplying the f given in (Eq. (III.2) by $(1 - \gamma_L)$, may be more accurate and it, too, will be discussed in this section.

The SYMALF code described earlier can be used to calculate the

alpha energy distribution as a function of radial position. As before, the change in shape and drift of the orbits are included as the alphas slow down. As mentioned in Chapter II, the general Sivukhin formulation⁽⁴⁴⁾ has been used to compute the energy loss rates along the particle trajectory. Since the time spent by each fusion particle group in a traversed ring is known, the density of suprathermals in a given plasma ring can be deduced. Thus, for any given radial position in the plasma, the number density of suprathermal alphas at $\hat{r}$ and in energy interval k can be found. Furthermore, with $n(\hat{r}, k)$ known, it is not difficult to deduce the normalized suprathermal energy distribution function by (Appendix F)

$$\hat{f}(\hat{r}, \mathcal{E}^{(k)}) = \frac{n(\hat{r}, k)}{\sum_{k=1} n(\hat{r}, k)}, \quad (\text{III.5})$$

where $\mathcal{E}^{(k)}$ is the average energy (= midpoint energy) of the k^{th} energy interval.

Let us first evaluate the suprathermal density as a function of position and energy and compare this result with that predicted by Eq. (III.3). In each of the five energy intervals $\hat{r}$ was held constant. A word of explanation is in order here. Rigorously speaking, one should only plot a histogram of density for each of the labeled intervals. If one assumes, however, this density function displays no pathological behavior during slowing-down and associates a value $n(\hat{r}, k)$ with the respective $\mathcal{E}^{(k)}$, a plot of $n(\hat{r}, k)$ as a function of $\mathcal{E}^{(k)}$ may be made. Points lying on this curve will represent the suprathermal density of particles in the energy interval between $\mathcal{E} - \frac{\Delta\mathcal{E}}{2}$ and $\mathcal{E} + \frac{\Delta\mathcal{E}}{2}$. For the following figures, we have graphed $n(\hat{r}, k)$ as a function of the entire interval. The $\mathcal{E}^{(k)}$ values which "characterize" these energy intervals can be taken as centered between the interval brackets. Also, since to first order in $\Delta\mathcal{E}/\mathcal{E}^{(k)}$, $f(\hat{r}, \mathcal{E}^{(k)})$ is proportional to $n(\hat{r}, k)$, we need only graph one of these--in this case, it will be $n(\hat{r}, k)$. The function $f(\hat{r}, \mathcal{E}^{(k)})$ is obtained via Eq. (III.4).

Comparisons are made in Fig. III.11 for several radial positions. It is quite clear that even at the origin ($\hat{r} = 0$), the suprathermal density calculated via SYMALF (dashed line) is significantly less than that predicted by Eq. (III.2), here designated by the (solid) "in situ" curves. This contrast is again mainly due to first-bounce losses. One can use Eq. (III.4) to estimate $f(r, \epsilon^{(k)})$. We find that $f_{\text{SYM}}(\hat{r}, \epsilon^{(k)}) \approx 0.73 f_{\text{in}}(\hat{r}, \epsilon^{(k)})$ for $\hat{r} \leq 0.7$, where f_{SYM} and f_{in} are the SYMALF and "in situ" distribution functions, respectively. Since losses for TFTR-I are about 30% on the first orbit, the "modified in-situ" model for the distribution function (i.e., $f_{\text{MIS}}(\hat{r}, \epsilon^{(k)}) \equiv (1 - \gamma_L) f_{\text{in}}(\hat{r}, \epsilon^{(k)})$) is consistent with that predicted by SYMALF. For $r \geq 0.7$, the shape of the suprathermal density profile, $n(\hat{r}, k)$, begins to change, first flattening out, and then finally decreasing with decreasing energy (e.g., $\hat{r} = 0.96$). The same trend is true for the energy distribution $f_{\text{SYM}}(\hat{r}, \epsilon^{(k)})$. Neither "in situ" nor MIS can either qualitatively or quantitatively describe this phenomenon. In fact, this behavior is a direct result of the inward orbital drift; as many of the particles with portions of their initial orbits lying in the outer regions of the plasma slow-down, they tend to drift farther to the inside of the plasma. Since there are no alphas of comparable energies drifting into this region $1 \geq \hat{r} \geq 0.8$ from the outside the plasma boundary, the lower energy echelons near the edge are depleted.

Since the drift motion during slowing-down is somewhat restricted in the higher-current Tokamaks, the results for ANL-I (Fig. III.12) and UWMK-I (Fig. III.13) show much less dramatic effects. Although the general shapes of the SYMALF curves do not differ markedly with what the "in situ" and MIS models would conclude (even for $\hat{r} > 0.9$), there is a significant difference in the absolute magnitude of the suprathermal alpha density in the outer regions, especially in the ANL-I case. The reason for this is essentially the same as that given in Section II to explain the enhanced alpha particle heating for ANL-I and UWMK-I near the plasma boundary (i.e., better confinement of FP's in Region I due to increased toroidal current, plus the additional contributions from Region I).

Fig. (III.11). Suprathermal alpha density is plotted versus energy for various values of $\hat{r}$ in TFTR-I.

Fig. (III.12). Suprathermal alpha density is plotted versus energy for various values of $\hat{r}$ in ANL-I.

Fig. (III.13). Suprathermal alpha density is plotted versus energy for various values of $\hat{r}$ in UWMAK-I.

As with the previous TFTR-I case, the "modified in-situ" model (and even "in situ" itself) can serve as a reasonable approximation to $n(r, k)$ and $f(\hat{r}, \epsilon^{(k)})$ for $\hat{r} \lesssim 0.5$ and $\hat{r} \lesssim 0.8$ in the ANL-I and UWMAK-I reactors, respectively. Beyond these points, however, the ratios of $n_{in}(\hat{r}, k)/n_{SYM}(\hat{r}, k)$ begin to change substantially for the various $\hat{r}$ -values. Since the constancy of these ratios is a measure of how good MIS is, Figs. (III.12) and (III.13) imply that this method is not very reliable outside the limits indicated.

The TFTR gas blanket (Fig. III.14) shows a trend similar to that seen in Fig. (III.12) in that the density curve again ultimately changes from a negative to a positive slope as $\hat{r}$ increases. In addition, a new phenomenon arises. Although $n_{SYM}(\hat{r}=0, \epsilon^{(k)}) \sim \frac{1}{2} n_{in}(\hat{r}=0, \epsilon^{(k)})$ we see a complete reversal of this relationship farther out in the plasma (e.g., $n_{SYM}(\hat{r}=0.64, \epsilon^{(k)}) \sim 2n_{in}(\hat{r}=0.64, \epsilon^{(k)})$). Eventually, n_{in} once again becomes much greater than n_{SYM} (e.g., $r=0.96$). There are two effects at work here. First, the usual slowing-down drift still accounts for the positive slopes in the suprathermal densities in the boundary region. Second, as a result of the peculiar background density and temperature profiles, the energy loss rate around $\hat{r} \sim 0.7$ is much greater than either farther in or farther out. Particles confined in this region slow-down much faster than usual. Consequently alphas born in the surrounding plasma rings, but which spend only part of their time in the high-energy loss-rate region raises the SYMALF density over that of "in situ." Needless to say, the MIS approximation will be a very poor representation for either $n(\hat{r}, k)$ or $f(\hat{r}, \epsilon^{(k)})$.

For the D-³He case, we consider both the alpha and proton components (Fig. III.15 and Fig. III.16, respectively). The trends in the alpha density profile are not very different from the TFTR-I results, except for D-³He, the SYMALF prediction is slightly greater than "in situ" for $r \gtrsim 0.8$ at the higher energies. This is due to the fact that the alpha particle production profiles (which here are essentially determined by the F-value Profile) is relatively broader for D-T than for D-³He.

(Fig. (III.14). Suprathermal alpha density is plotted versus energy for various values of $\hat{r}$ in the TFTR gas blanket.

Fig. (III.15). Suprathermal alpha particle density is plotted versus energy for various values of $\hat{r}$ in the D- ^3He TFTR.

Fig. (III.16). Suprathermal proton particle density is plotted versus energy for various values of $\hat{r}$ in the D- ^3He TFTR.

Thus, the inner plasma region makes a relatively greater contribution to the suprathermal density in the outer plasma for the D-³He case than for that of D-T. With the protons the trends discussed previously are clearly seen. A new feature, however, appears. Since the proton is initially at 14.7 MeV, $V \sim V_e$, and Eqs. (III.2) and (III.3) do not strictly apply. In this "transition regime," we have used the general Sivukhin⁽⁴⁴⁾ slowing-down model for the "in situ" case (as we have all along for SYMALF). The result is that the suprathermal density curves are somewhat flatter than what could occur if Eq. (III.3) were used.

III.5 Summary and Conclusions

In section III.1 it was shown that the "in situ" assumption for plasma heating is generally very good for high current Tokamaks, like ANL-I and UWMAK-I, although sizeable differences from the SYMALF model can exist in the outer regions of the plasma. Although Region II contributions have little effect on the overall energy deposition, the resulting implications for fusion product heating of the outer plasma region are not at present clear. Likewise, these boundary heating problems exist for the lower current TFTR-devices, and are, in fact, aggravated by relatively high fusion product losses. It was also found that the "modified in-situ" (MIS) model gave fairly good agreement with SYMALF over most of the plasma, although, like "in situ," it failed in Region II. The effect of the slowing-down drift on plasma heating, as manifested in the comparison of the SOA and SYMALF models, was found to be relatively small, except for TFTR-(GB). Here the unusual plasma density and temperature profiles coupled with the slowing-down drifting produced a marked effect.

The thermalization source functions predicted by SYMALF differ noticeably from that of "in situ" for all five cases. This is again especially true for the TFTR designs. Although the MIS model alleviates the problem somewhat, notable exceptions exist (e.g., the protons in the D-³He and the alphas in TFTR-(GB)). In addition, the thermalization function given by SYMALF for TFTR-I is almost flat near the origin,

unlike anything the other three methods predicted. The implications of these results on fusion particle build-up are presently under study. (62)

Perhaps the most interesting application of SYMALF lies in evaluating the suprathermal density and distribution functions both as a function of radial position and energy. The most intriguing consequence of the orbital drifts is clearly the transition from negative to positive slope in the distribution functions for increasing $\hat{r}$ near the plasma boundary. Neither the "in situ" or MIS approximations predict this effect. Although the distribution functions for several of the reactor cases (like TFTR-I) have large positive slopes (implying kinetic micro-instabilities), the actual suprathermal density may be too small to trigger serious turbulence. This area is also presently under investigation. (63)

CHAPTER IV: MAGNETIC RIPPLE TRAPPING

IV.1. Introduction

Ideally, magnetic ripple effects can be continually reduced by adding greater numbers of field coils. This, however, can become increasingly costly and can result in uneconomical capital costs.⁽⁶⁴⁾ Consequently, in any future Tokamak system, periodic variations in the toroidal field may be unavoidable. In this section, we develop a theory and subsequent code (RIPALF) which handle several facets of the ripple problem.

As discussed in Chapter I, the discrete character of the main field coils partly destroys this symmetry, resulting in the formation of ripples in the magnetic field. One may approximate this modified field by superimposing the ripple contribution on the standard magnetic (Knorr) configuration.⁽⁶⁵⁾ This yields:⁽¹⁹⁾

$$\vec{B} = \frac{B_0}{h} \{ \theta(r) \hat{\theta} + (1 + \delta(r, \theta) \cos(S\phi)) \hat{\phi} \} , \quad (IV-1)$$

where, as usual, θ is the poloidal angle, ϕ is the toroidal angle, S is the number of field coils, $R_0 \equiv$ major radius, $h \equiv 1 + r \cos\theta/R_0$ and $\theta \equiv B_\theta/B_T$. The well-depth parameter, $\delta(r, \theta)$, may be written in the form $g_a \left(\frac{r}{a}\right)^w \exp\{-\left(\frac{\alpha\theta}{\pi}\right)^2\}$, with g_a , w , and α determined by the nature of the field coils.⁽²⁴⁾ Here a is the plasma minor radius. A radial component of $\vec{B}$ required to satisfy $\nabla \cdot \vec{B} = 0$ has been omitted⁽²¹⁾ under the assumption $\frac{r}{R_0} \frac{S}{w+2} < 1$.

Localized magnetic wells are produced by the ripple field, provided the perturbation $\delta(r, \theta)$ is sufficiently large compared with the normal trapping produced by the rotational transform. This condition requires that⁽¹⁹⁻²⁰⁾ $\delta(r, \theta) > |\sin\theta|/S$. If particles trapped in these regions have a drift confinement time less than the time required to be scattered out of the well ("effective" scattering time), their drift can carry them out of the plasma in micro-seconds. For this to happen for

alphas, it is necessary that (19,25)

$$\delta(r, \theta) > \frac{6 \times 10^{-19} a R_0 B_0 n}{E^{5/2}}, \quad (\text{IV-2})$$

where $B_0 \equiv B_T(r=0)$, $n \equiv$ ion density, a , and E are in units of Tesla, m^{-3} , m , and keV. For example, if one assumes $g_a = 0.03$, $\alpha = 3.2$, and $w = 3$, which are representative of a UWMAK-type device, (47) the above relation is satisfied over almost the entire plasma, indicating significant ripple trapping. A method of estimating FP ripple losses and the distribution of such losses on the first wall will now be discussed.

IV.2 The Model

The distance along $\vec{B}$ between coils is divided into $2N$ equally-spaced divisions (note that N has been redefined in this section; see Fig. IV-1). In the following, it will again be convenient to divide the class of 3.5-MeV alphas into representative phase-space groups. As before, radial and aximuthal coordinates represent two of a particle's birthpoint characteristics. With ripple present, however, toroidal symmetry is destroyed and a third spatial coordinate is necessary. We, thus, introduce a toroidal coordinate depicting any of the $2N$ divisions between magnets. Each r, θ group is, thus, characterized by $2N$ additional subgroups. Velocity-space is divided into groups, each born within a given spatial volume along $\vec{B}$ and having a common "reflection" interval along $\vec{B}$. This latter point will be discussed shortly.

One can, in principle, follow the trajectories of each trapped phase-space group as was done in the non-ripple trapped problem. However, now due to the much more complicated orbital equation, even this approach would consume very large amounts of computer time. Thus, in this section an alternate approach is proposed that employs the probability distribution of trapped 3.5-MeV alphas in phase-space. While this method involves some approximations, it is thought to provide a reasonable compromise between speed and accuracy.

Fig. (IV.1). Top and cross-sectional (side) views of trapped particle orbits lying between adjacent coil magnets. Line segments AC and A'C' represent trapped particle orbits for $q \rightarrow \infty$ and q finite cases, respectively. The shaded sections are those segments where there is finite probability of finding the given trapped particle group along its orbit. In the side view, those shaded portions lying along A'C' project to the first-wall, constituting the wall-loading contribution of that group. Also shown is the side view of orbit AC, which is a point in the (r, θ) -plane.

Let us assume for the moment that our Tokamak has a safety factor value much greater than one. A 3.5-MeV particle born into such a system would have two types of motion: parallel to the toroidal magnetic field and vertical to the toroidal plane. (Electrostatic drifts due to gradient-centrifugal charge separation are neglected.) We superimpose the ripple field onto the toroidal field, B_T , as in Eq. (IV-1). The resulting magnetic barriers for this case are of equal height.

The force acting on a trapped particle in the V_{11} -direction as it oscillates between reflection points A and C (Fig. IV-1) is $-\mu \nabla_{11} B$. The resulting equation of motion is solved along lines similar to that of the classical harmonic oscillator problem by assuming that the trapping field does not change radically during the particle's vertical drift. When one knows the equation of motion of the alpha between A and C, the problem reduces to a calculation of the time it spends within a given segment of the orbit, say between X_m and X_{m-1} ($\equiv \Delta X_m$) (see Fig. IV-2). The probability $P(\Delta X_m; X_T)$ of finding a suprathermal in ΔX_m whose turning point in the ripple will lie at X_T is directly proportional to the time it spends in this segment; hence, we have (Appendix H)

$$P(X_m; X_T) = \frac{\int_{\psi_{m-1}}^{\psi_m} \frac{d\psi}{\sqrt{1 - \sin^2 \theta_D \sin^2 \psi}}}{2 \int_0^{\pi/2} \frac{d\psi}{\sqrt{1 - \sin^2 \theta_D \sin^2 \psi}}}, \quad X_T \geq X_m \quad (\text{IV-3})$$

where

$$\sin \psi_m = \frac{\sin(\pi X_m / 2L)}{\sin(\pi X_T / 2L)} \quad (\text{IV-3a})$$

$$\sin \psi_{m-1} = \frac{\sin(\pi X_{m-1} / 2L)}{\sin(\pi X_T / 2L)} \quad (\text{IV-3b})$$

Fig. (IV.2). The magnetic ripple well is shown for a $q \rightarrow \infty$ (symmetric) case. X_N is the position of one of the coils; $-X_N$ is the position of the adjacent coil. ΔX_m is the segment lying between X_m and X_{m-1} . The horizontal dashed line represents a ripple-trapped group with turning points in ΔX_m (or $-\Delta X_m$).

and

$$\sin\theta_D = \frac{X_T S}{2R_0 h} \quad (IV-3c)$$

Here, S is the number of the magnet coils. Equation (IV-3) contains elliptic integrals of the first kind which can be evaluated from tables. Care must be taken to avoid singularities at $X_T = X_m = \pi R_0 h/S$. We set $X_m = X_T - \lambda_1$ where λ_1 is taken as approximately half of the length ΔX_m . Since there are a large number of segments, and this only represents a small class of particles, the choice of λ_1 is not crucial.

While the particle is executing its oscillatory motion along B_T , the vertical drift carries it out of the plasma. Suppose this drift is up (depends on toroidal field direction). The probability of the trapped alpha impacting at a given region H along the first wall (Fig. IV-1) is equal to the probability of finding that particle inside the vertical projection of H (i.e., the sectioned segments along $\overline{AC}$ in Fig. IV-1).

Since each segment lying along $\overline{AC}$ between magnets can contribute, the rate, $\dot{N}(r, \theta; \Delta X_m)$, at which trapped 3.5-MeV alphas strike the region H on the first wall directly above ΔX_m for suprathermals with a common r, θ is (Appendix G)

$$\dot{N}(r, \theta; \Delta X_m) = 2n\Delta x \left\{ m \sum_{j=m}^N \Delta B_j P(\Delta X_m; X_j) + \sum_{j=m+1}^N (j-m) \Delta B_j P(\Delta X_m; X_j) \right\}, \quad (IV-4)$$

where

$$\Delta B_j \equiv \sqrt{1 - B(X_{j-1})/B_{TM}} - \sqrt{1 - B(X_j)/B_{TM}}.$$

B_{TM} is the toroidal field at the magnets for (r, θ) , ΔX is the length of one of the $2N$ segments ($= \pi R_0 h / (S \times N)$), and n is the number of alphas born per unit length along (r, θ) . Finally, the total rate of 3.5-MeV alpha impacts at H is found by summing the contributions from equivalent

ΔX_m segments in all rings above the midplane lying vertically below region II. In doing this care must be exercised since r and θ depend in the ring involved.

Up to this point it has been assumed that $q \gg 1$. For realistic cases, the preceding analysis is generalized to account for rotational transform. Since q is known on any magnetic surface, the path followed by an alpha born at some (r, θ) value along the flux surface can be found, sectioned into $2N$ segments, and the magnetic field intensity at both coils evaluated. This corresponds to A'C' in Fig. IV-1. Trapping occurs if the particle is reflected by the lower of these fields. As before, the probability density is proportional to the time spent within a certain segment or approximately inversely proportional to the average value of V_{11} in that region. Thus, V_{11} is calculated in each of the segments along the path where $V_{11} (= \sqrt{2/m} (\mathcal{E} - \mu B)^{1/2})$ is not complex, and $P(\Delta X_i; X_T)$ determined (Appendix H). If it happens that the period of an oscillation is greater than four times the drift confinement time

$(\approx |\sqrt{a^2 - r^2 \cos^2 \theta} - r \sin \theta| / v_D)$, that group is assumed to strike the first wall in a region directly below its birth segment. (In other words, a particle that is unable to complete at least a quarter of an oscillation is assumed to be lost instantaneously.)

IV.3 Discussion

The above presupposes that once trapped, a 3.5-MeV particle will remain trapped for a sufficient time to escape. As we have already remarked, scattering out of the ripple loss cone does not occur fast enough to affect the confinement. Another possibility of the 3.5-MeV alpha becoming untrapped involves any decrease in well depth as the particle moves vertically toward the surface. Were this to be the case, the alphas could "leak" out of the trapped region. However, with the field configuration used here (Eq. (IV-1)), in order for alphas in either half-plane to remain trapped, w must be $\lesssim 2$ if $\alpha \approx 3.2$. The w values most often cited in the literature⁽²⁴⁾ are typically > 3 . Thus, if we assume a vertically upward drift, we consider particles to be trapped

only in the upper half-plane. Alphas that drift from the lower-half plane upward must move through regions where the ripple field may no longer be sufficient for localized trapping. Our particle loss rates, consequently, are lower by about a factor of one-half than those reported by other investigators,⁽²⁴⁾ who assume equal losses from both half-planes. We believe that our assumption is reasonably close to reality in many cases and should satisfy present purposes while avoiding the considerable complexity of treating the untrapping problem (Appendix H.)

Implicit in the wall loading calculation is the assumption that the trapping field does not increase too much during the drift to the surface. With $\alpha \approx 3.2$, and $w = 3$, a "worst case" analysis (Appendix H) indicates that the turning point along the path can vary by as much as 25% of the length A'C'. For most other cases we studied, however, a variation of 5 to 15% was typical. A partial correction is possible by averaging the trapping field over the vertical drift distance.

In code RIPALF, we take $N=10$ (i.e., 20 segments), radial groups = 13, azimuthal groups = 18, and the number of magnets (S) = 12. Current density, particle density, and temperatures are arbitrary. Figure (IV-3) shows the effect on wall-loading of raising B_T (i.e., higher q) in a UWMAK-type device (poloidal range is 60-80 degrees). With segments 10 and 11 representing the approximate midpoint between coils, a definite skewing effect is detected in cases with lower (realistic) q -values. Also the number of particles lost by trapping decreases markedly in the lower q -case.

Figure (IV-4) shows the sharp peaking in the poloidal direction (taking segment 10 as constant). As might be expected the extreme upper and lower azimuthal range of the first wall get very little alpha bombardment, while the 60-80 degree segment experiences most of the punishment.

IV.4. Conclusions

As one sees in Fig. (IV.3) and Fig. (IV.4), the effect of magnetic

Fig. (IV.3). The wall-loading distribution along the first wall for poloidal angles (measured from the plasma center to the first-wall) between 60° and 80° . Segments 1 and 20 represent the position of the adjacent coils. The parameters chosen for this UWMK Tokamak are: $n = 10^{14}/\text{cm}^3(1-(r/a)^2)$, $T_e = 10 \text{ keV}(1-(r/a)^2)$, current density $\propto (1-(r/a)^2)$, $a = 5 \text{ m}$, $R_0 = 13 \text{ m}$, and $I_{\text{PLASMA}} = 21 \text{ MA}$. For the case $q \rightarrow \infty$, there is the expected symmetry in the wall-loading characteristics (i.e., segments 10 and 11 have the maximum values). For decreasing B_T , the number of trapped particles drops precipitously and the maximum clearly shifts toward one of the magnets.

Fig. (IV.4). The first-wall loading distribution (along segment 10) as a function of poloidal angle. The parameters are unchanged from Fig. (IV.3). There is a very sharp angular dependency in the first wall current. A perceptible shift in the maximum is also seen for various toroidal field strengths.

ripple in large reactors results in a highly non-uniform wall-loading. With UWMAK-I as a model, the peak values of FP wall loading appears to be located at 50° in poloidal angle and skewed toward one of the magnets by about 15% (off center). The peak value is $\sim 10^{16}/(\text{m}^2 \cdot \text{sec})$, which is roughly a factor of 30 above that predicted by Hively for a non-ripple case.⁽⁵⁹⁾ In fact, magnetic ripple losses will generally dominate "axisymmetric" (i.e., non-ripple) losses for large, high current Tokamaks, as implied by Eq. (IV.2). However, for smaller low current devices, like TFTR, it is the axisymmetric losses which determine 3.5 MeV FP losses.

It is found experimentally⁽⁶⁶⁾ that blistering begins occurring for an accumulated FP dose $\sim 10^{22}/\text{m}^2$. If one assumes continuous machine operation, this only corresponds to a running time of order two weeks in TFTR and 1 year in UWMAK. Based on an analysis by Hively,⁽⁵⁹⁾ this wall-loading leads to an erosion rate for stainless steel of about 180 $\mu\text{m}/\text{year}$ for UWMAK-I. Consequently, on the strength of the magnetic ripple wall-loading calculations, blistering* could be a serious problem for the larger Tokamak devices from the economics of first wall repair. Furthermore, the impurity effect of the first wall blistering on plasma dynamics. Even small amounts ($\sim 0.1\%$) of seriously hampering the successful operation of a tokamak reactor.

*Ref. (59) finds sputtering effects of secondary importance.

CHAPTER V: TOROIDAL PLASMA ROTATION

I. Introduction

There are two principal ways to induce rotation in a tokamak: (1) electrostatically-induced drifts and (2) beam-induced momentum transfer.⁽⁸⁾ Since the TFTR (Toroidal Fusion Test Reactor) is to serve as the model, the second mechanism is mentioned, because it is by far the major contributor to plasma rotation in this beam-driven device. For example, if one assumes that all twelve of the injectors are used to inject neutral deuterium tangentially into a thermal tritium background, the rate of momentum transfer from the beam to the plasma would be about two orders of magnitude more than secondary driving mechanisms arising from poor energetic ion confinement (e.g., fusion products). However, if the injectors are arranged so that the net neutral current in the toroidal direction is zero (i.e., opposing beams), or if the injection is perpendicular to the toroidal direction, this secondary process could become important. (See Fig. V-1).

Chief among these secondary processes are the electrostatically-driven drifts. When significant charge separation builds up, large radial electric fields result. This chapter is concerned with the fields generated by fast FP losses. As shown earlier highly-energetic FPs have initial trajectories lying outside the plasma, causing these particles to be lost from the system (e.g., in collisions with the limiter or first wall). Since the confinement times for such particles are typically in microseconds, and since the timescale for background electrons or ions to cross magnetic surfaces ($\approx 10^{-1} - 10^{-2}$ sec) is not fast enough to neutralize the charge left by the escaping fusion products, a radial electric field results. (The thermal electron motion along the magnetic-flux surfaces is sufficiently rapid to damp any poloidal component in the electric field.) It is the interaction between this radial electrostatic field and the toroidal and poloidal magnetic fields which result in the characteristic poloidal and toroidal drifts, respectively [Again refer to Fig. (V-1)].

Other mechanisms can also cause charge build-up, resulting in at

Fig. (V.1). Schematic of the Tokamak geometry, which features the toroidal and poloidal rotations driven by $\hat{E}_r \times \hat{B}$ - and $\hat{E}_\theta \times \hat{B}$ - drifts, respectively. Also shown are the possible injector configurations: (1) injectors 1 and 2 are aligned for tangential injection, and (2) injector 3 illustrates perpendicular injection. Not shown is the counter-beam scheme, where the neutral beams of injectors 1 and 2 for example, are opposed.

least a transient rotational phase. These include (1) trapping of energetic ions due to inhomogeneities in the magnetic field,^(19,25) (2) large radial variations in the orbits of ions, (arising from ionization of neutral injected particles)⁽⁶⁷⁾ and (3) loss of thermal ions with large "banana" orbits near the plasma boundary.⁽⁵¹⁾ Process (3) will be neglected since it appears as a higher order effect compared with the rotation resulting from either beam or FP loss mechanisms. The rotation due to (1) and (2) will not appear directly in this analysis. The losses in small, low current tokamaks due to the inhomogeneities in the magnetic field are small compared with the nearly instantaneous "banana orbit" losses. The contribution to the radial electric field by FPs which are not lost on the first bounce is negligible compared with field established by the FP losses. (See Appendix I.)

Although neutral beam injection plays the dominant role in the TFTR scheme, present conceptual power reactor designs, like ANL⁽⁴⁶⁾ and UWMAK,⁽⁴⁷⁾ rely on the thermonuclear burn to sustain operation. Hence, in the absence of appreciable beam contributions, the fast fusion-product losses should play a determining role in the rotation dynamics for these reactors. Since preliminary results⁽⁶⁸⁾ are that sizeable rotational speeds may be possible in certain situations for a UWMAK-type reactor, it is highly desirable to test the predictions of the fluid theory to gain confidence in the UWMAK result. TFTR offers a near-term opportunity to do this. By eliminating the net toroidal velocity component of the beam, it is possible, as mentioned above, to eliminate the neutral beam contribution to rotation, and hence measure the fusion loss effects directly. This point will be discussed in Section (V-5).

In this section a steady-state analysis is used to study rotation. This implicitly assumes that the beam-induced fusion is continuous, as opposed to the 500 ms pulse designed for TFTR.⁽⁴⁵⁾ Because the toroidal rotation reaches its maximum in steady state, setting an upper bound on rotational velocity will indicate whether or not introducing a considerably more complicated time-dependent treatment is worthwhile. Nevertheless, an estimate of the build-up time for rotation however, will be discussed later.

The reference fuels will be deuterium injected into tritium (i.e., $D + T \rightarrow \alpha + \text{Neutron}$) and deuterium injected into ^3He (i.e., $D + ^3\text{He} \rightarrow \alpha + P$). The former has been chosen because of its anticipated use in "break-even" experiments in TFTR, while the latter has been proposed as a tritium- and neutron-clean test-fuel for early TFTR experiments. (51,52) A neutral deuterium injection energy of 120 KeV is anticipated for both cases. (45)

In the following subsections, a qualitative description will be given of the processes involved in our plasma rotation model. In sections (V-2), (V-3) and (V-4), these mechanisms will be quantitatively. Section V-5 is reserved for general discussion and conclusions.

V.1a Alpha Current Effects

Typical of most fusion reactors, in the plasma designed for TFTR, the poloidal component of rotation damps quickly. (33,69) This phenomenon is basically due to flow equilibrium being established on a very fast time scale (32) (τ_{ii} ion-ion collision time) over a given magnetic surface, which is assumed to be a function of r only. This being the case, the plasma flow then becomes essentially toroidal in nature. (The situation is somewhat similar to the topological observation that a donut is resistant to twisting in the poloidal direction, but quite easily rotates in the toroidal direction.) In subsequent analysis, poloidal rotation effects are ignored.

To create toroidal rotation, a net force must act on the plasma in the toroidal direction. The origin of this force is ultimately traced to the flow of charge carried out of the plasma by fusion-product losses. To balance this charge loss (e.g. of 3.5 MeV alpha particles produced in D-T reactions), the plasma responds by establishing an inwardly-directed radial current, J_r^{PL} , which, by interacting with the poloidal magnetic field, produces a $J_r \times B_\theta$ -type toroidal force on the plasma.

The radial plasma current has both a real and (during transient phases of operation) a polarization component. This may be written

$$J_r^{PL} = J_r^R + J_r^{POL} \quad (V-1)$$

where J_r^R is the real plasma current flowing across the magnetic surfaces and J_r^{POL} is the polarization current given by $\epsilon_r \frac{\partial E_r}{\partial t}$ (in which ϵ_r is the radial permittivity).

These currents exactly balance the radial FP loss current in steady-state, which is labeled J_r^α . To show this, we use the continuity equation and the differential form of Gauss' law to obtain

$$\nabla \cdot \vec{J} = \nabla \cdot (J_r^R + J_r^\alpha) = - \frac{\partial \rho_e}{\partial t} = - \nabla \cdot \epsilon_r \frac{\partial E_r}{\partial t}, \quad (V-2)$$

$$J_r^R + J_r^\alpha = - \epsilon_r \frac{\partial E_r}{\partial t}, \quad (V-3)$$

or

$$J_r^\alpha = - J_r^R - \epsilon_r \frac{\partial E_r}{\partial t}, \quad (V-4)$$

where ρ_e is the charge density. For steady-state operation, $J_r^\alpha = - J_r^R$, as expected. In Section II we will evaluate J_r^α .

The real component of the radial plasma current, J_r^R , can be expressed in terms of the radial electric field⁽⁷⁰⁾ as

$$J_r^R = \sigma_\perp (E_r - E_r^0), \quad (V-5a)$$

where E_r^0 is the "ambipolar" radial electric field for the plasma if there were no fusion losses, and $\sigma_\perp$ is the perpendicular conductivity calculated by Boozer⁽⁷⁰⁾ under the conditions of high poloidal rotation damping, that is

$$\sigma_\perp \cong \sum_k \frac{\rho_m}{\tau_k B_\theta^2}, \quad (V-5b)$$

where ρ_m is the plasma mass density, τ_k is a phenomenological time constant for the k^{th} damping process, and B_θ is the poloidal magnetic field. Interestingly, this expression shows that collisions enhance perpendicular conductivity, contrary to the case of parallel conductivity, where

collisions increase the resistivity. The situation is analogous to that for a partially ionized gas where neutral-ion collisions enhance cross-field currents in the direction of an applied electric field. Substituting Eq. (V-5) into Eq. (V-4), one finds

$$E_r = E_r^0 - \frac{J_\alpha r}{\sigma_\perp} \left(1 - \exp \left\{ - \sum_k \frac{t}{\tau_k} \right\} \right), \quad (V-6)$$

where it is assumed, for simplicity, $E_r = E_r^0$ at $t = 0$ (i.e., the alpha current is turned on at $t = 0$). The important statement of Eq. (V-6) is that the characteristic build-up time (or, conversely, decay time) for the radial field and, by implication, the toroidal rotation velocity, is given by the sum of the inverse damping times.

If there were no toroidal damping, the rotation would in principle continue to increase until the plasma met with destructive rotational instabilities. Fortunately, damping does occur and is found to generally be strong enough to prevent excessive rotation speeds. In the following subsection, the damping mechanisms that will be considered in our analysis will be discussed. These are (1) charge-exchange, (2) elastic collisions between ions and neutrals, (3) magnetic trapping of plasma ions, and (4) perpendicular viscosity.

V.1b Charge-Exchange Damping

The charge-exchange (C-X) of hot, ionized plasma ions in the 1- to 10- KeV range with cold neutrals is a particularly effective means of damping rotation.⁽⁷¹⁾ When a hot ion moving with a toroidal flow, V_ϕ , charge exchanges with a cold neutral (which is assumed stationary) the net effect (as far as the plasma is concerned) is a net loss in toroidal momentum, as the hot neutral carries its toroidal momentum from the plasma (higher order generations of C-X are not considered). Thus, the C-X process corresponds to a frictional force that acts to damp net momentum along the toroidal direction. [An alternate explanation, as discussed in Ref. 72, is to view this frictional force as inducing a radial drift in the ions (i.e., in the $\vec{F}_f \times \hat{B}_\theta$ direction, where $\vec{F}_f$

represents this frictional force). Since this ion drift is non-ambipolar in nature (electrons do not participate significantly in this radial plasma drifting current), the radial electric field then, is decreased and subsequently the toroidal flow is damped (i.e., $v_\phi \propto \frac{E_r - E_r^0}{B_\theta}$.)

This damping force is strongly dependent on radius. The number of cold neutrals rapidly decreases toward the central regions of the plasma, owing to the higher plasma temperatures and densities found there. Therefore, C-X is most efficient in damping near the outer edge of the plasma.

V.1c Ion-Neutral Scattering

Ion-neutral elastic collisions can also contribute as neutrals build-up inside the thermonuclear plasma. The same principle that applies to C-X also applies here (i.e., when an ion collides with a cold neutral, the ion experiences what amounts to a frictional force opposite to its net flow direction with the same effect as described above). Electrons can participate in this process, too, although their contribution to damping is much less than that of the ions ($\propto \sqrt{\frac{M_e}{M_i}}$, where M_e and M_i are the electron and ion masses, respectively). Ion-neutral elastic collisions are generally less than 20% of the damping efficiency of C-X (i.e., $\frac{1}{\tau_{in}} \propto \frac{1}{5\tau_{cx}}$, where τ_{in} is the ion-neutral damping time and τ_{cx} is the C-X damping time).

For future discussion, it will be convenient to refer to the combination of C-X and ion-neutral damping simply as Neutral-Induced Damping (NID).

V.1d Magnetic-Ripple Trapping

Another process which reduces plasma rotation involves the enhanced radial transport of the thermal plasma ions which are trapped in the periodic inhomogeneities of the toroidal field. [The electrons also experience this effect, but it can be shown⁽²¹⁾ that the electron transport is considerably

smaller $\left(\sim \sqrt{\frac{M_e}{M_i}}\right)$ than that of the ions.] Consequently, such transport is non-ambipolar and acts to reduce any free charge build-up induced by the FP losses. This process, which is commonly referred to as magnetic-ripple damping, is especially important when the neutral-particle density is small because the C-X and NID effects become negligible. However, since the ripple field is created by the external toroidal-field asymmetries, ripple effects decrease rapidly towards the center of the plasma.^(20,21) Consequently, magnetic ripple damping (MRD), like C-X and NID, is most effective near the outer edge of the plasma.

It should be noted that in the case of highly energetic ions (e.g., fusion products with energies >1 MeV), the magnetic ripple can also enhance the free charge build-up. The reason for this is that, although these ions are little restrained by the typical radial ambipolar electric fields found in the plasmas ($\lesssim 10^4$ V/m, generally, they can, however, be trapped in the ripple wells and quickly (~ 10 - 100 μ s) drift out of the system. In doing this, the electrons "associated" with these energetic particles are left behind in the plasma. However, as discussed in Chapters 2 and 4, for a small device, such as TFTR, the added alpha losses resulting from ripple are significantly less than the losses which occur from the large neoclassical drifts and are ignored in this analysis.

V.1e Perpendicular-Viscosity Damping

The final damping mechanism that will be actively considered is based on perpendicular-viscosity damping (PVD). This process arises when the thermal ions characterized as belonging to one magnetic surface exchange momentum with plasma ions on adjacent surfaces; this opposes the build-up of radial velocity shear in the toroidal direction. The ion with the greater toroidal drift speed feels the net frictional force, $\vec{F}_f$, which resists its toroidal motion; the ion on the second flux surface (which is assumed to have a lower toroidal drift speed) experiences the equal but opposite force, $-\vec{F}_f$, which accelerates its toroidal drift. (This interaction distance between flux surfaces is of the order of the ion poloidal gyroradius for the neoclassical "banana" regime.

Because the PVD coefficient, $\chi_{\perp}$, is proportional to the square root of the mass, the electron contribution once again turns out to be negligible. Since the wall is stationary, the momentum will flow ultimately out of the plasma (i.e., plasma-wall effects).

To simplify the treatment, the coupling term [discussed extensively in Refs. (32,37)] between the radial temperature gradient and the toroidal rotation (i.e., $\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{3.5 B_T}{B_\theta} \frac{\partial}{\partial r} \left(r \frac{R}{r} \right)^{\frac{1}{2}} \frac{n \chi_{\perp}}{e B_0} \frac{\partial T}{\partial r} \right)$) will be neglected. This seems justified on the grounds that the ion temperature profile [typically $\sim (1.08(\frac{r}{a})^2)$] is essentially flat in the critical central plasma region, resulting in the "normal" perpendicular viscosity playing the more important role. The MRD and NID processes on the other hand are the determining factors toward the outer plasma regions. Consequently, although the $\frac{\partial T}{\partial r}$ - term is larger than the "normal" contribution there, its impact on rotation is relatively small.

V.1f Other Mechanisms

Other possibilities for damping include the direct shorting out of the electric field due to the formation of magnetic islands⁽⁷³⁾ or the enhancement of the perpendicular viscosity effect resulting from electrostatic fluctuations.⁽⁷⁴⁾ These processes have been suggested by others to help explain the apparent discrepancy between theory and experiment. Such effects will not be considered individually in this analysis.

It is assumed that the transport coefficients (i.e., $\chi_{\perp}$ and τ_R) are neoclassical. Were anomalous effects also to be present, then these damping coefficients might be considerably enhanced. One way to acknowledge this possibility is to allow for an enhanced viscosity by designating $\chi_{\perp} \approx 100 \chi_{\perp \text{ neo}}$ in special cases, where $\chi_{\perp \text{ neo}}$ is the neoclassical coefficient of perpendicular viscosity. Likewise, an enhancement in MRD will also be considered.

of perpendicular viscosity. Likewise, an enhancement in MRD will also be considered.

V.1g The Rotation Model

The driving and damping mechanisms can be related by θ -averaging over the ϕ -component of the standard fluid momentum balance equation, that is,

$$\rho_m \frac{\partial V_\phi}{\partial t}(r, t) = J_r^{PL}(r) b(r) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho_m \chi_{\perp \text{neo}} \frac{\partial V_\phi}{\partial r}(r, t)) - \sum_k \frac{\rho_m V_\phi(r, t)}{\tau_k} + \sum_\ell \frac{\partial P_B}{\partial t}, \quad (V-7)$$

where R_o is the major radius, and $b(r) \equiv \frac{1}{2\pi} \int_0^{2\pi} h B_\theta d\theta$, and the co-ordinate system is defined in Fig. V.1.

The convective term is neglected in Eq. (V-7), and flow equilibrium on the magnetic flux surfaces and a large aspect ratio (R_o/a) are assumed. The first term on the RHS of Eq. (V-7) corresponds to the driving torque exerted on the plasma as it responds to the outward flow of fusion products. The second term on the RHS represents the damping force due to perpendicular viscosity, while the third term is the sum of all other damping contributions, which we have discussed above. It is assumed each τ_k is a function of radial position only.

The final term in Eq. (V-7) accounts for the toroidal momentum, P_B^ℓ , imparted to the plasma by the neutral beams of the ℓ^{th} injector. In keeping with the spirit of the previous discussion, it is assumed, however, that the net momentum given the plasma by the injected beams is zero, and so this term is dropped in subsequent analysis.

Substituting Eqs. (V-1) and (V-4) into Eq. (V-7) the time dependent solution becomes

$$\frac{J_r^\alpha(r) b(r)}{\rho_m \chi_{\perp \text{neo}}} = + \frac{1}{r} \frac{d}{dr} \left(r \frac{dV_\phi(r)}{dr} \right) - \frac{V_\phi(r)}{\chi_{\perp}} \sum_k \frac{1}{\tau_k} \quad (V-8)$$

where it is assumed that $\rho_m \chi_\perp$ does not vary appreciably across the plasma. (For the cases to be examined, this variation is only about 50%--small, for example, compared with the 5 or 6 orders of magnitude change in the other damping mechanisms.)

When the effect of perpendicular viscosity is small, the first term on the RHS of Eq. (V-8) can be neglected and the solution, V_ϕ^A , is written

$$V_\phi^A(r) = - \frac{J_r^\alpha(r) b(r)}{\rho_m \sum_k \frac{1}{\tau_k}} \quad (V-9)$$

This result, which will be referred to as the "asymptotic" solution, is consistent with the solution that is obtainable directly from the drift model of Ref. 70, that is,

$$V_\phi(r) = \frac{(E_r - E_r^0)}{B_\theta} = \frac{J_r^R(r)}{\sigma_\perp B_\theta} = - \frac{J_r^\alpha(r) b(r)}{\rho_m \sum_k \frac{1}{\tau_k}}, \quad (V-10)$$

where we have used Eqs. (V-4), (V-5), and the $\sigma_\perp$ expression mentioned in Section (V-1a).

A second limiting case occurs where the perpendicular viscosity effects are dominant (i.e., where the second term on the RHS of Eq. (V-8) is neglected. The steady state solution then is

$$V_\phi(r) = - \left\{ \int_0^r \frac{dr'}{r'} \int_0^{r'} \frac{r'' J_r^\alpha(r'') b(r'') dr''}{\rho(r'') \chi_\perp(r'')} - \int_0^r \frac{dr'}{r'} \int_0^{r'} \frac{r'' J_r^\alpha(r'') b(r'') dr''}{\rho(r'') \chi_\perp(r'')} \right\}, \quad (V-11)$$

In Eq. (V-11), the boundary conditions $V(r=a) = 0$ and $\frac{dV}{dr}(r=0) = 0$, have been applied, where a is the minor radius. As an example, if the kernel of Eq. (V-11) is constant, V_ϕ reduces to the simple Poiseuille flow, i.e., $V_\phi \sim 1 - (\frac{r}{a})^2$.

Two additional observations should be made before concluding this section. First, this model assumes a toroidal flow which is laminar in nature. A turbulent plasma could significantly alter the above results.

For the plasma conditions under consideration (i.e., ion temperature ~ 10 keV, ion densities $\sim 10^{20}/\text{m}^3$, toroidal and poloidal magnetic fields ~ 4.5 and 0.4 T, respectively), the ratio of Reynolds number to the Hartmann number is several orders of magnitude less than the incipient value for turbulence, generally taken as $\frac{Re}{Ha} \sim 500$.⁽⁵⁶⁾ Consequently, at least for steady state operation, one expects fairly laminar flow.

Second, the boundary conditions applied in this problem deserve mention. Although setting the first radial derivate in $V_\phi(r)$ at the origin to zero is reasonable, imposing $V_\phi(r=a)=0$ is not as straightforward. Clearly, if a limiter or first wall were moved up to the plasma boundary, this latter boundary condition would be appropriate. On the other hand, TFTR has neither limiter nor first wall near enough to the plasma edge for one to a priori use that condition. Since the NID and MRD mechanisms are dominant in the outer plasma, then the "asymptotic" solution to Eq. (9), V_ϕ^A , must represent the upper limit to toroidal velocity at $r = a$. If $V_\phi^A(r=a) \ll V_\phi^P(r=0)$, where V_ϕ^P is the solution to Eq. (V-8) subject to $V_\phi^P(r=a) = 0$, then except for a very small region near the boundary, the discrepancy in toroidal velocity over the remaining plasma is negligible. This result can be demonstrated a posteriori.

V.2 Radial Loss Current

In order to calculate the loss current, $J_r^\alpha(r)$ for the beam-driven TFTR-case, one first evaluates the number of 3.52-MeV alpha particles produced for any given elemental volume in the plasma (just as in Chapter III). Let ϕ be the probability of an injected deuterium ion undergoing a fusion reaction with a background tritium ion at some time during its slowing-down phase. This probability can be written as

$$\phi(r, E_{IN}) = F(r, E_{IN}) \frac{E_{IN}}{Q_f}, \quad (\text{V-12})$$

where E_{IN} is the initial energy of the deuterium beam particle, Q_f is energy released per fusion reaction, and $F(r, E_{IN})$ is the energy multiplication factor, defined as the ratio of the energy released due to fusion reactions

to E_{IN} . These so-called F-values exhibit a complicated dependence on both background plasma temperature and initial beam energy. Since it is assumed that the injected beam particles, are delivered to the plasma at 120 keV, ⁽⁴⁵⁾ the F-values, which have previously been compiled by Towner, ⁽⁷⁵⁾ are analytically fitted to functions which best approximate the data for the temperature range of interest. Thus, for $E_{IN} = 120$ keV, the F-value functions for systems with thermal backgrounds consisting of tritium and ^3He , respectively, are

$$F_{D-T}(r) = 0.017 T_e(r) + 0.47 \ln[1.5 T_e(r) + 0.72] , \quad (V-13a)$$

and

$$F_{D-^3\text{He}}(r) = 0.000445 T_e(r) , \quad (V-13b)$$

where $1 \text{ keV} \lesssim T_e \lesssim 15 \text{ keV}$, for both of the above cases.

For present day experiments, the radial profile density for injected beam trapping is strongly peaked at the center of the plasma, seemingly for both perpendicular and toroidal injection. ⁽¹¹⁾ In that spirit, one assumes that the neutral beam pulse is ionized according to $\exp[-(\frac{r}{\gamma a})]$, where γ is a scaling parameter taken as 0.5 for the cases of interest. The number, $\dot{N}_D(r)$, of injected deuterium atoms ionized at radius r per unit volume per second is

$$\dot{N}_D(r) = \frac{I_0 \exp\left(-\left(\frac{r}{\gamma a}\right)^2\right)}{2\pi R_0 \left[\pi a^2 \gamma^2 \cdot 1 - \exp\left(-\frac{1}{\gamma^2}\right) \right]} \times \Delta , \quad (V-14)$$

where I_0 is the total neutral injected current and Δ is the trapping fraction coefficient for the beam (~ 0.8 and assumed constant here).

The number, $H_j(r, \theta) dV$, of the j^{th} FP specie which is produced per second in the small volume element dV and which instantly escapes the plasma (and provides the necessary torque for a rotating plasma) can be written

$$\begin{aligned}
 H_j(r, \theta) dV = & \left[\begin{array}{l} \text{Probability of injected} \\ \text{ion fusing at } r, \theta \text{ during} \\ \text{slowing down} \end{array} \right] \times \left[\begin{array}{l} \text{Source rate per} \\ \text{unit volume of} \\ \text{injected ions} \end{array} \right] \times \left[\begin{array}{l} \text{Escape fractions} \\ \text{of fusion particles} \\ \text{at } (r, \theta) \end{array} \right] \\
 & \times [\text{elemental volume}] \\
 = & \Phi(r) \times \dot{N}_D(r) \times f_{ES}(r, \theta) \times dV. \quad (V-15)
 \end{aligned}$$

Here $f_{ES}(r, \theta)$ is the escape fraction of FP's born in elemental volume. This loss fraction can be obtained numerically (See Section III.2 for References). With smaller tokamaks, these losses can be estimated by counting the number of fusion products with orbits lying outside the plasma. This is in contrast to the larger reactors, where magnetic ripple trapping of energetic ions is the principal loss mechanism, as mentioned in Section IV.4.

The fusion product current density passing through a magnetic surface at r is given by

$$J_r^\alpha(r) = \sum_j \frac{Z_j e}{2\pi r} \int_0^r \int_0^{2\pi} H_j(r, \theta) r \, d\theta dr, \quad (V-16)$$

where $Z_j e$ is the charge on the j^{th} fusion product specie, while the sum is taken over each of these products. Due to the complicated nature of $H_j(r, \theta)$, $J_r^\alpha(r)$ must be determined numerically. Figure (V.2) shows $J_r^\alpha(r)$ for both the D-T and D- ^3He cases to be illustrated. (See Table I for sample D-T plasma case; the D- ^3He case is identical except that $n_i = 5 \times 10^{19} \text{ m}^{-3}$.) Although the D- ^3He reaction produces twice the number of charged FP's as D-T, the difference in magnitude between the two curves results from the F-values for D-T being two orders of magnitude greater than those of D- ^3He for $T_e \lesssim 10 \text{ keV}$.

The FP escape rate can easily be modified to accommodate thermonuclear-produced particles. For the case of a 50-50 mixture of deuterium and tritium, for example, H_j is re-evaluated as

$$H_{th}(r, \theta) = \frac{n_i^2}{4} (r) \langle \sigma v \rangle f_{ES}(r, \theta) dV,$$

Fig. (V.2). The fusion product loss current is plotted against normalized radial position $\hat{r}$. The selected parameters are listed in Table III for the D-T case. For the ^3He plasma, the only change occurs for the ion density: $n_i = 5.0 \times 10^{14}/\text{m}^3$ ($1.0 - 0.8 \hat{r}^2$).

Table III: Plasma Parameters for Sample D-T Case

R_o	2.5 m	n_i	$= 8.0 \times 10^{19}/m^3 (1-0.8\hat{r}^2)$
a	0.5 m	T_i	$= 10 \text{ keV } (1-0.8\hat{r}^2)$
I_p	1.0 MA	T_e	$= 10 \text{ keV } (1-0.8\hat{r}^2)$
B_o	45 KG	Current Density $\propto (1-\hat{r}^2)$	
G_p	0.01	$n_N(0)$	$= 10^{12}/m^3$
$\chi_{\perp}$	$\chi_{\perp \text{ neo}}$	$n_N(a)$	$= 10^{16}/m^3$
E_{INJ}	120 keV	dueterium injected current	650 amps

where $n_i(r)$ is the background ion density and $\langle\sigma v\rangle$ is the Maxwellian-averaged reaction parameter.

Implicit in the above calculations for $J_r^\alpha(r)$ is that fusion products not lost while completing the first orbit will remain in the plasma during the slowing down phase. This has been shown in Chapter II for negligible ion-ion pitch angle scattering. More recently, using a Monte Carlo method, Ohnishi et al.⁽⁷⁶⁾ demonstrated that for an effective plasma Z of $Z_{\text{eff}} = 1.0$, pitch-angle related losses are indeed insignificant; for a $Z_{\text{eff}} = 7.5$, on the other hand, pitch angle scattering does result in an additional 50% increase total in FP losses. Thus, these secondary losses appear to enhance $J_r^\alpha(r)$ somewhat, depending on the Z_{eff} of the plasma.

The fusion products which are formed when the deuterium beam interacts with the thermalized deuterium in the plasma (i.e., $D + D \rightarrow n + {}^3\text{He}$ and $D + D \rightarrow p + T$) can also make a contribution to $J_r^\alpha(r)$. However, since the F -values for D-D fusion (hence the FP production rates) are much less than those for D-T and D- ${}^3\text{He}$ systems⁽⁵³⁾ for plasma temperatures ≈ 10 keV, this effect will not be considered.

V.3 Damping Mechanisms

In Section (V-1) the damping mechanisms included in the rotation model were briefly discussed. In this section these processes are examined on a much more quantitative basis.

V.3a Neutral-Induced Damping

The density profile of cold neutrals, n_N , in the plasma is modeled by assuming neutral transport according to

$$\frac{dn_N}{dr} = -\frac{n_N}{\lambda_a}, \quad (V-17)$$

where $\lambda_a \ll a$ is the total mean free path for ionization or charge exchange of the cold neutral. If λ_a is assumed to be constant, the solution to this equation with the boundary conditions $n_N(r=0) \equiv n_N(0)$ and $n_N(r=a) \equiv n_N(a)$ is:

$$n_N(r) = n_N(0) \left(\frac{n_N(a)}{n_N(0)} \right)^{r/a} \quad (V-18)$$

Both $n_N(0)$ and $n_N(a)$ are input quantities in this formulation (i.e. this is equivalent to defining λ_a).

Riviere⁽⁷⁸⁾ has fitted the experimental data for C-X by protons in atomic hydrogen with an analytic expression. Since the atomic states of tritium are quite similar to those of hydrogen, Riviere's formula can be modified for charge exchange between hot tritium ions and the cold neutral background by replacing the ion energy $\mathcal{E}$ in his formula by $(M_H/M_T)\mathcal{E}$, where M_H is the mass of the hydrogen ion and M_T the mass of the tritium ion. This substitution is necessary since the C-X process depends on the relative speeds between the interacting ion and neutral. In the following formulas, all energies and temperatures will be expressed in keV, masses in atomic mass units (A.M.U.), and all other dimensions in M.K.S. The charge exchange damping time, τ_{cx}^T , for tritium can be written as:

$$\tau_{cx}^T = \frac{1}{n_N \langle \sigma_{cx}^T \rangle} = \frac{1}{n_N} \left\{ \frac{3 \times 10^5}{\sqrt{M_T}} \int_0^\infty \sigma_{cx}^T(\mathcal{E}) f_T(\mathcal{E}) \mathcal{E} d\mathcal{E} \right\}^{-1}, \quad (V-19)$$

where $f_t(\mathcal{E})$ is the (Maxwellian) energy distribution function for the tritium ions, and σ_{cx}^T is the tritium C-X cross-section given by

$$\sigma_{cx}^T = \frac{0.694 \times 10^{-18} (1 - 0.155 (\log_{10} \frac{\mathcal{E}}{M_T} + 3))^2}{1 - 8.8 \times 10^{-6} \left(\frac{\mathcal{E}}{M_T} \right)^{3.3}} \quad (V-20)$$

This cross-section is thought to be accurate to about 20%.⁽⁷⁹⁾ Since σ_{cx}^T is highly complicated, τ_{cx}^T is determined numerically.

The ^3He C-X cross-section is somewhat more complicated by singly

and doubly electron transfers. For the lower ion energies (<10 keV), however, it is the double electron transfer (i.e., ${}^3\text{He}_{(A)} + {}^3\text{He}_{(B)}^{++} \rightarrow {}^3\text{He}_{(A)}^{++} + {}^3\text{He}_{(B)}$) which is dominant.⁽⁷⁹⁾ The C-X cross-section for this reaction, taken from Ref. 79, can be approximated for low plasma temperatures as

$$\sigma_{\text{cx}}^{\text{He}} \approx 2 \times 10^{-20} \text{ m}^2 \quad (\text{V-21})$$

The associated C-X damping time is then calculated as in Eq. (V-19):

$$\tau_{\text{cx}}^{\text{He}} \approx 1.4 \times 10^{14} \sqrt{\frac{M_{\text{He}}}{T_{\text{He}}}} / n_N(r) \text{ sec} \quad (\text{V-22})$$

where T_{He} (<10 keV) is the ${}^3\text{He}$ background temperature and M_{He} is the mass of the ${}^3\text{He}$.

The momentum transfer cross-section for ion-neutral collisions for tritium (T-T^+) can be pieced together from Refs. 79 and 80. For ion temperatures less than 20 keV, this cross-section can be written as

$$\sigma_{\text{in}}^{\text{T}} \approx 4.6 \times 10^{-21} \sqrt{\frac{M_{\text{T}}}{T_{\text{T}}}} \text{ m}^2 \quad (\text{V-23})$$

Upon averaging $\sigma_{\text{in}}^{\text{T}}$ over a Maxwellian distribution, the damping time is

$$\tau_{\text{in}}^{\text{T}} \approx 2 \times 10^{15} / n_N(r) \quad (\text{V-24})$$

Although elastic scattering cross-section data for He^{++} -He collisions is not available, one can easily argue that this contribution is small compared with that of charge-exchange (e.g., Eq. (V-22)). Because $\tau_{\text{in}}^{\text{T}} \gg \tau_{\text{cx}}^{\text{He}}$, and since one expects $\tau_{\text{in}}^{\text{T}} \approx \tau_{\text{in}}^{\text{He}}$ for the elastic scattering interaction, it follows that $\tau_{\text{in}}^{\text{He}} \gg \tau_{\text{cx}}^{\text{He}}$.

V.3b Magnetic-Ripple Damping (MRD)

The characteristic time for ripple damping has been derived in detail by Tuda.⁽⁸¹⁾ Applying charge conservation (as done in Section V-1), one has

$$\epsilon_r \frac{\partial E_r}{\partial t} = e \Gamma_i, \quad (V-25)$$

where Γ_i is the radial ion flux. Since E_r and Γ_i ^(20,21) are known, Eq. (V-25) can be written in the form

$$\frac{\partial E_r}{\partial t} = - \frac{1}{\tau_R} (E_r - E_r^0), \quad (V-26)$$

where τ_R is the ripple damping time constant. Assuming neoclassical transport scaling, one calculates

$$\tau_R \approx \frac{M R_o^2 v_{ii}}{\delta^{3/2} G(\alpha) T_i} \left(\frac{B_o}{b(r)} \right)^2 \times 10^{-11} \text{ sec}, \quad (V-27)$$

where v_{ii} is the ion-ion collision frequency $\approx \frac{7.7 \times 10^{-17} n_i Z_p^4}{\sqrt{M_i} T_i^{1.5}}$, n_i is ion density, Z_p is the charge number of the target specie, $\delta \equiv \left(\frac{r}{a}\right)^3$, and ϵ_p is the ripple-field well depth selected at $r = a$. [The magnetic ripple field is assumed to be of the form $\epsilon_p \left(\frac{r}{a}\right)^3 \cos S\phi$.] The function $G(\alpha)$ can be approximated⁽²⁵⁾

$$G(\alpha) = \begin{cases} 1 - 3\alpha + 2.8\alpha^2, & \alpha < \frac{1}{2} \\ 0.38 - 0.36\alpha, & \frac{1}{2} \leq \alpha \leq 1 \\ 0.02/\alpha^2, & \alpha > 1 \end{cases} \quad (V-28)$$

where $\alpha \equiv b(r)/(B_o S \delta)$ with the number of magnetic coils denoted by S .

V.3c Perpendicular Viscosity Damping (PVD)

The coefficient of perpendicular viscosity, $\chi_\perp$, in the neoclassical regime is given as⁽³⁷⁾

$$\chi_{\perp \text{neo}} = \frac{1}{10} \rho_i^2 \frac{v_{ii}}{v} \quad (\text{V-29a})$$

where ρ_i is the ion gyroradius and

$$v \equiv \frac{b(r) R_o}{B_o R} \quad (\text{V-29b})$$

As discussed in Section (V-1), the temperature gradient coupling term is not included in this treatment.

V.4 Results

Although Eq. (V-8) has been solved analytically for some interesting special cases, the complicated radial dependencies, especially in the τ_k 's, make a general analytic solution quite formidable. Hence, the results in this section, have been obtained numerically on CYBER-175 by a finite difference method using gaussian elimination. (82)

In Section (V-4a), a sample analysis using "typical" TFTR parameters is carried out. In succeeding sections variations in ϵ_p , $n_N(o)$, $n_N(a)$, T_i , n_i are studied to assess their impact on rotation under differing sets of circumstances. In the final Section (IV-8), an example of the possible radial electric fields that can arise from fusion product losses is calculated, again using the "typical" TFTR case.

V.4a Sample Result

In Fig. (V-3) the inverse of the damping times for MRD NID, and perpendicular viscosity are plotted vs. radial position for a low ripple, (i.e., $\epsilon_p = 0.01$ RMS), low neutral density, tritium plasma (see Table II). The inverse PVD time constant, $\frac{1}{\tau_\chi}$, may be estimated as

$$\frac{1}{\tau_\chi} \approx \frac{\chi_{\perp \text{neo}} \frac{1}{r} \frac{d}{dr} \left(r \frac{dV_\phi^A(\hat{r})}{dr} \right)}{V_\phi^A(\hat{r})}, \quad (\text{V-30})$$

where $V_\phi^A(r)$ is the asymptotic solution referred to previously, c.f. Eq. (V-9). (Since τ_χ depends on derivatives which can oscillate

Fig. (V.3). The inverse damping times are plotted versus $\hat{r}$ for the standard tritium case of Table III.

rapidly from radial point to point, the inverse τ_χ is represented by a curve of finite width.) For regions close to the center of the plasma, it is the perpendicular viscosity contribution that essentially determines the toroidal velocity damping [recall the discussion in Sections (V-I.2), (V-I.3), and (V-I.4).] Beyond $\hat{r} \approx 0.15$, however, NID effects clearly dominate. MRD does not appear to be the determining factor in damping rotation for this example.

Figure (V-4) shows the resulting toroidal rotation for the above plasma. The dashed curve represents the "asymptotic" solution $V_\phi^A(\hat{r})$, which rapidly increases from zero to a peak at about $\hat{r} \approx 0.20$. This increase occurs because the driving current, $J_r^\alpha(\hat{r})$, which is zero at $\hat{r}=0$ [hence $V_\phi^A(\hat{r}=0) = 0$], increases much faster in $\hat{r}$ than either the NID or the MRD processes. For $\hat{r} > 0.2$, the opposite is true, and $V_\phi^A(\hat{r})$ decreases.

The solution to Eq. (V-8), $V_\phi^P(\hat{r})$, shown in Fig. (V-4) as solid lines, experiences the anticipated flattening in the toroidal velocity profile near the origin ($\hat{r} \lesssim 0.1$), due to the relatively strong PVD effects here. In the radial interval $0.1 \lesssim \hat{r} \lesssim 0.2$, the $V_\phi^P(\hat{r})$ is no longer flat, but still is noticeably different from $V_\phi^A(\hat{r})$; this approximately corresponds to the transition region in Fig. V-3, where neither the PVD nor the other damping mechanisms are clearly dominant. Beyond $\hat{r} \approx 0.2$, however, V_ϕ^A and V_ϕ^P differ little; here, the PVD effect is relatively small. (The imposed boundary condition $V_\phi(\hat{r}=1) = 0$ does produce a small discrepancy between $V_\phi^A(\hat{r})$ and $V_\phi^P(\hat{r})$ at the very edge of the plasma; however, this difference affects the result very little, since the ratio $\frac{V_\phi^A(\hat{r}=1)}{V_\phi^P(\hat{r}=0)}$ here is $\sim 10^{-2}$.)

When the $\chi_\perp$ is taken as 100 times the neoclassical value in accordance with the discussion in Section (V-I.5), Fig. (V-4) shows that V_ϕ^P drops about a factor of 2 in peak value. Nevertheless, the toroidal velocity remains roughly constant over a larger plasma volume than for the

Fig. (V.4). The "asymptotic" $v_{\phi}^A(r)$ solution (dashed curve) is compared with $v_{\phi}^p(r)$ for perpendicular viscosity given by $\chi_{\perp neo}$ and $100\chi_{\perp neo}$. The damping parameters used are the same in Fig. (V.3), i.e., Table III.

previous case. (The ratio $\frac{V_{\phi}^A(\hat{r}=1)}{V_{\phi}^P(\hat{r}=0)}$ ($\approx \frac{1}{40}$) is again sufficiently small so that one expects no difficulty resulting from the choice of the outside boundary condition.)

V.4b Light Ripple Damping, Variable $n_N(0)$

Figures (V-5) and (V-6) examine the effect of the neutral gas density by varying $n_N(0)$ in the D-T and D- ^3He systems, respectively. Decreasing $n_N(0)$ increases the peak toroidal velocities, although not as dramatically as $n(0)$ is lowered below $10^9/\text{m}^3$. For these densities MRD effects begin to play a determining damping role, so that NID becomes less important. However, this range of $n_N(0)$ values is over three orders of magnitude lower than neutral densities presently expected for TFTR, and so will not be pursued further. For $n_N(0) > 10^{10}/\text{m}^3$, the NID mechanisms are by far the most important (except near the plasma center). For this situation, one can write approximately

$$V_{\phi}^P(r) \approx V_{\phi}^A(r) = \frac{J_r^{\alpha}(r) b(r)}{\rho_m} \tau_{cx} \propto \frac{1}{n_N(r)} \quad (V-31)$$

Equation (V-31) generally applies to the region just outside the plasma center where larger $n_N(\hat{r})$ values occur and where NID dominates PVD.

In Fig. (V-6), one notes that $V_{\phi}^P(r)$ for the D- ^3He operation of TFTR are generally one or two orders of magnitude lower than for comparable values of D-T (c.f., Fig. V-5). The much higher fusion rate, hence product loss currents, for D-T at temperatures < 10 keV basically accounts for this difference [c.f., Fig. (V-2)], although the larger τ_{cx} for the ^3He plasma (20 times that corresponding to tritium) tends to reduce this difference (c.f., $n_N(0) = 10^{13}/\text{m}^3$ cases in Fig. (V-5) and Fig. (V-6)).

When $n_N(0)$ is varied for the ^3He -target plasma case, $V_{\phi}^P(r)$ does not change as dramatically as with the tritium target plasma. The reasons are twofold: (1) the strength of the PVD is greater in the ^3He plasma, as $\chi_{\perp\text{neo}} \propto (Z_p e)^2$, where $Z_p e$ is the charge of the background plasma ions,

Fig. (V.5). Toroidal velocity $V_{\phi}^D(\hat{r})$ is plotted vs. $\hat{r}$ for the standard tritium plasma case with light MRD ($\epsilon_{\perp} = 0.01$), $n_N(a) = 10^{16}/\text{m}^3$, and $\chi_{\perp} = \chi_{\text{neo}}$. The central neutral density, $n_N(r)$ is varied.

Fig. (V.6). Toroidal velocity $V_{\theta}(\hat{r})$ is plotted vs. $\hat{r}$ for the standard ^3He plasma case with light MRD ($\epsilon_p = 0.01$), $n_N(a) = 10^{16}/\text{m}^3$, and $X_{\perp} = X_{\perp\text{neo}}$. The central neutral density, $n_N(0)$ is varied.

and (2) as already noted, the damping effectiveness of NID (i.e., C-X) is 20 times less for the ^3He plasma than for that of tritium.

V.4c High MRD, Variable $n_N(0)$

Here $n_N(0)$, is again varied but high MRD (i.e., $\epsilon_p = 0.25$) is introduced for both plasmas. Since in this case MRD effects dominate, changing $n_N(0)$ is not expected to alter $V_\phi^P(\hat{r})$ appreciably, except when very large values of $n_N(0)$ are employed. Figure V-7 shows this for both tritium (solid curve) and ^3He plasmas.

V.4d Low $n_N(0)$, Variable ϵ_p

In Figure (V-8), the MRD parameter ϵ_p is varied. The neutral density at $\hat{r} = 0$ (i.e., $n_N(0)$ is $10^{10}/\text{m}^3$). Even in the heavy MRD case for tritium, speeds in excess of 10^4 m/sec are possible. On the other hand, for no ripple at all, the ^3He plasma barely exceeds $5 \times 10^3 \text{ m/sec}$.

If $n_N(0)$ should be raised, for example, to $10^{12}/\text{m}^3$, the peak toroidal speeds for the D-T system ranges only from $\sim 8 \times 10^3 \text{ m/sec}$ (for $\epsilon_p = 0.0$) to $\sim 5 \times 10^3 \text{ m/sec}$ (for $\epsilon_p = 0.25$), this due to the diminished importance of ripple damping against NID.

V.4e Light Ripple, Low $n_N(0)$, Variable $n_N(a)$

Figure (V-9) shows the effect of varying the plasma boundary neutral density, $n_N(a)$, while assuming light MRD ($\epsilon_p = 0.01$), low $n_N(0)$ ($= 10^{10}/\text{m}^3$) parameter. For these cases, only the tritium results are plotted. The toroidal velocity at $\hat{r} = 0$, $V_\phi^P(\hat{r} = 0)$, only varies about a factor of 4 over a four magnitude change in $n_N(a)$, although the $V_\phi^P(\hat{r})$ profile does fall off much faster for higher $n_N(a)$, as one would expect.

Likewise, when the central neutral density is raised to $10^{12}/\text{m}^3$, the toroidal speed still varies less than an order of magnitude for very large changes in $n_N(a)$. The rotation does not appear to be overly sensitive to even large changes in $n_N(a)$.

Fig. (V.7). Toroidal velocity $v_{\phi}^D(\hat{r})$ is plotted vs. $\hat{r}$ for both the standard tritium and ^3He plasma cases with heavy MRD ($\epsilon_D = 0.25$), heavy $n_N(a) = 10^{16}/\text{m}^3$, and $\chi_{\perp} = \chi_{\perp, \text{neo}}$. The central neutral density, $n_N(r)$, is varied. The tritium results (solid curves) are read off the left axis, while the ^3He cases are read off the right axis.

Fig. (V.8). Toroidal velocity, $v_{\phi}^D(\hat{r})$, is plotted vs. $\hat{r}$ for both the standard tritium and ^3He plasma cases with light NID ($n_N(0) = 10^{10}/\text{m}^3$ and $n_N(a) = 10^{16}/\text{m}^3$) and $\chi_{\perp} = \chi_{\perp\text{neo}}$. The magnetic ripple parameter ϵ_p , is varied. As in Fig. (V.7), the tritium results (solid curve) are read to the left, while the ^3He cases are read to the right.

Fig. (V.9). Toroidal velocity, $v_{\phi}^0(r)$, is plotted vs. $\hat{r}$ for the standard tritium case with low MRD ($\epsilon_r = 0.01$) and $n_N(0) = 10^{10}/\text{m}^3$ (solid curve) and $n_N(0) = 10^{12}/\text{m}^3$ (dashed curve, which is referred to the RHS ordinate). The effect of varying the density of neutrals at the plasma boundary (i.e., $n_N(a)$) is shown.

V.4f Light Ripple, Low $n_N(0)$, Variable Ion Density

When ion density, $n_i(\hat{r})$, decreases, so do $\tau_R(\propto n_i(\hat{r}))$ and $\chi_{\perp\text{neo}}$ ($\propto n_i(\hat{r})$), although τ_{NID} is independent of $n_i(r)$. (Since beam driven fusion is used, $J^\alpha(r)$ also does not depend on $n_i(r)$ either.) One then expects the LHS and the second term on the RHS of Eq. (V-I.8) to increase in value relative to the first term on the RHS (i.e., the PVD term). For small $\chi_{\perp\text{neo}}$, one finds

$$v_\phi^p \sim v_\phi^A \propto \frac{1}{n_i} \left(\frac{1}{\tau_{\text{NID}}} + \frac{1}{\tau_R} \right)^{-1} \quad (\text{V-32})$$

For $\tau_{\text{NID}} < \frac{1}{10} \tau_R$ (as it is in this light ripple case), changing n_i even one order of magnitude does not alter the $\left(\frac{1}{\tau_{\text{NID}}} + \frac{1}{\tau_R} \right)$ contribution appreciably; hence $v_\phi^p \sim \frac{1}{n_i}$. Figure V.10 demonstrates this for both tritium and ^3He plasma. Lowering the ion density evidently can increase $v_\phi^p(\hat{r})$ dramatically.

However, where magnetic ripple is the prevalent damping mechanism, varying $n_i(r)$ will change v_ϕ^p very little. Repeating the above analysis for $\tau_{\text{NID}} \gg \tau_R$, one finds

$$v_\phi^p \sim \frac{1}{n_i} \left(\frac{1}{\tau_{\text{NID}}} + \frac{1}{\tau_R} \right)^{-1} \sim 1 \quad (\text{V-33})$$

For such a case, toroidal speed becomes essentially independent of n_i .

V.4g Light ripple, Low $n_N(0)$, Variable Ion Temperature

Decreasing the target ion temperature, T_i , results in an increase in $(\propto \frac{1}{T_i})$, $\tau_R(\propto \frac{1}{T_i^{5/2}})$, and $\tau_{\text{cx}}(\propto \frac{1}{T_i})$, where $\tau_{\text{NID}} \approx \tau_{\text{cx}}$,

Fig. (V.10). Toroidal velocity, $v_{\phi}^D(\hat{r})$, is plotted vs. $\hat{r}$ for the standard tritium and ^3He plasma cases, except that the central ion densities are now varied. We also assume light NID ($n_N(0) = 10^{10}/\text{m}^3$ and $n_N(a) = 10^{16}/\text{m}^3$), MRD ($\epsilon_D = 0.01$), and $\chi_L = \chi_{L\text{neq}}$. The tritium cases (solid curve) are read to the left, while the He cases are read at the right.

$(J_r^\alpha(\hat{r}))$ is not affected significantly⁽⁵³⁾ by changing $T_i(\hat{r})$ for the beam-driven case (vs. Maxwellian fusion)). Proceeding as in Section (IV-6),

one finds for $\tau_{cx} \leq \frac{1}{10} \tau_R$ that $V_\phi^P \propto \frac{1}{T_i}$. Figure V-11 shows the increase in the $V_\phi^P(\hat{r}=0)$ for decreasing ion temperature in the tritium case. For the temperature range of interest, varying T_i does not strongly affect toroidal rotation.

For heavy ripple ($\tau_{cx} \gg \tau_R$), however, $V_\phi^P \propto \frac{1}{T_i^{5/2}}$. Although the dependence of V_ϕ^P on T_i is strong, the conditions for $\tau_{cx} \gg \tau_R$ are not of immediate interest and so will not be pursued.

V.4h Radial Electric Fields

An estimate of the steady state radial electric field due to the FP losses can be made via Eq. (V-6) which gives:

$$E_r - E_r^0 = - \frac{J_r^\alpha(r) b^2(r)}{\rho_m \sum_k \frac{1}{\tau_k}} \quad (V-34)$$

The dashed curve in Fig. (V-12) plots $(E_r - E_r^0)$ vs. $\hat{r}$ for the D-T case given in Table III and Fig. (V-2). This approximation does not include perpendicular viscosity effects. If, however, $V_\phi(\hat{r})$ is known from solving Eq. (V-8) and the rotation is strictly toroidal, one can estimate $(E_r - E_r^0)$ with perpendicular viscosity included by

$$E_r - E_r^0 = - V_\phi^P(r) \cdot b(r) \quad (V-35)$$

The solid line in Fig. (V-12) shows $(E_r - E_r^0)$ with perpendicular viscosity included. As with the toroidal velocity result, the effect of including perpendicular viscosity is to lower the peak value and broaden the profile of $(E_r - E_r^0)$ given by Eq. (V-34).

Fig. (V.11). Toroidal velocity, $V_\phi^0(\hat{r})$ is plotted vs. $\hat{r}$ for the standard tritium plasma case, except that the central ion temperatures are now varied. The damping parameters are the same as in Fig. (V.10).

Fig. (V.12). The electrostatic field due to rapid fusion product escape, $(E_r - E_r^0)$, is plotted vs. $\hat{r}$ for the standard tritium plasma case. The damping parameters are given in Table III. The $(E_r - E_r^0)$ values are plotted with (solid curve) and without (dashed curve) the perpendicular viscosity contribution.

V.5 Discussion and Conclusions

In Sections (V.1), (V.2) and (V.3), the theory of FP induced rotation was developed and the assumptions underlying it discussed. In Section (IV), the toroidal rotation, given a set of steady-state conditions was predicted. In this section, the possibilities of confirming these predictions will be examined. To do this, one must address three questions: (1) Are the damping mechanisms used here consistent with experimentally determined plasma damping?; (2) Can the predicted plasma rotation be measured?; and (3) What are the most reasonable plasma parameters to use in the TFTR experiment in order to study rotation?

V.5a Plasma Damping: Theory vs. Experiment

There have been several experiments performed to date which have measured plasma rotation. The results, however, have been inconclusive in ascertaining whether the "standard" damping mechanisms (i.e., NID and MRD are adequate to describe the observed rotations.

Sigmar and Clarke⁽⁷¹⁾ have calculated the theoretical poloidal and toroidal rotation in ORMAK, assuming the primary damping mechanism as C-X and the applied electric field in the ϕ direction as the driving force. The experiment⁽⁷²⁾ was found to agree with the theory. The results obtained from ATC⁽⁸³⁾ and TFR⁽⁷⁴⁾ experiments, however, imply that none of the listed damping processes explain the observed rotation, which are about an order of magnitude less than expected.

On the other hand, more recent experimental measurements with ORMAK⁽⁸⁴⁾ and PLT⁽⁸⁵⁾ plasmas appear to indicate that the "classical" damping mechanisms discussed above are sufficient in explaining the observed results. Thus, because of whatever microscopic and macroscopic transport processes which may vary from tokamak to tokamak in a not altogether predictable way, one cannot say with certainty that these "classical" processes will explain future TFTR behavior. However, they do appear to currently be the most likely candidates.

V.5b Detectability Limit of Rotation

The method used in measuring the rotational speeds in the ORMAK experiment referred to above involved the detection of Doppler shifts

in neutral hydrogen H_α radiation. With this technique, the minimum measurable toroidal velocity^(84,85) is $\sim 10^4$ m/sec.

In the ATC experiment rotation was measured by observing the Doppler shift of the low-frequency fluid modes via X-ray emission.⁽⁸³⁾ The limit of detectability here also appears to be in the 10^3 to 10^4 m/sec range.

V.5c Selection of Parameters

Educated guesses based on the TFTR conceptual design studies⁽⁴⁵⁾ are reflected in the selection of values given earlier in Table II. Anticipated values for neutral density and magnetic ripple are $n_N(0) \approx 10^{13}/\text{m}^3$, $n_N(a) \approx 10^{16}/\text{m}^3$, and $\epsilon_p \lesssim 1\%$.

V.5d Toroidal Rotation for Selected Case

The choice of parameters made in Section (V-V.3) lead to predictions of toroidal rotational speeds of $\sim 10^4$ m/sec and $\sim 10^3$ m/sec for the tritium and ^3He cases, respectively (see Figs. (V-5) and (V-6). These results, it should be iterated, do not include the anomalous effects and so must be thought of as probable upper limits in V_ϕ^p . Although the detection of rotation in the ^3He plasma is somewhat improbable, there is a chance of detecting FP-driven toroidal rotation in the tritium case if beam-driven effects cancel out.

Figure (V-13) shows the sensitivity of toroidal rotation to changes in the parameters discussed in Section (V-IV). The quantities $\frac{\Delta X}{X_0}$ and $\frac{\Delta V}{V_0}$ are defined as $\frac{X_0 - X}{X_0}$ and $\frac{V_0 - V}{V_0}$, respectively, where V_0 is the peak value of the plasma rotation for the Table III case, and X_0 is the corresponding parameter being varied. As expected from the previous discussion, n_i and $n_N(0)$ appear to be the most important factors affecting toroidal speed; decreasing the central ion density, for example, by $\frac{1}{2}$ almost doubles V_0 . The central ion temperature has a much smaller effect on V_0 ; furthermore, the limitations in the allowed temperature range restricts its usefulness

Fig. (V.13). Sensitivity of peak toroidal velocity, V_0 , to changes in a given parameter x_0 , which represents n_i , $n_N(0)$, T_i , $n_N(a)$, $\chi_{\perp}$, and ϵ_p . $\Delta X \equiv X - X_0$ and $\Delta V = V - V_0$. The base case is for that given in Table III.

in studying rotation. The edge neutral density is also shown to be of relative unimportance in that, for instance, an order of magnitude decrease in $n_N(a)$ increases rotation only about 50%. Finally, both ϵ_p and $\chi_{\perp}$ variations have very little impact on rotation, mainly because the damping is so heavily controlled by NID.

From the above, lowering n_i and $n_N(0)$ appear to increase the chances of detecting rotation. Although $n_N(0)$ may for the present be beyond the ability of the experimenter to control, controlling n_i offers a good opportunity in aiding the detection of rotation. For example, lowering n_i by an order of magnitude can result in a rotation increase of 10.

V.5e Electrostatic Radial Fields

Figure (V.12) shows that sizeable radial electric fields can build up from FP losses. This result, however, only applies after a steady state situation is achieved. One may now estimate the field build-up time. From the discussion in Section (V-I.1), it was found that this time is of order

$(\sum_k \frac{1}{\tau_k})^{-1}$. The peak value of $E_r - E_r^0$ in Fig. (V-12) occurs at $\hat{r} \approx 0.25$; from

Fig. (V-3) (which uses identical plasma parameters as Fig. (V-12)), one estimates the build-up time ~ 1 second. The injection time⁽⁴⁵⁾ for TFTR ~ 500 ms.

Hence, since $\sum_k \frac{t}{\tau_k} < 1$, Eq. (V-I.6) can be approximated to lowest order in

$\sum_k \frac{t}{\tau_k}$ by

$$E_r - E_r^0 \approx - \frac{J_r^\alpha}{\sigma_{\perp}} \sum_k \frac{t}{\tau_k} \quad (V-36)$$

At $t \sim 500$ ms, $E_r - E_r^0 \approx -\frac{1}{2} \frac{J_r^\alpha}{\sigma_{\perp}}$. Hence, when the beam is turned

off, $(E_r - E_r^0) \approx 750$ V/m.

As a consequence of the flow equilibrium assumption, the build-up time V_{ϕ}^p will also correspond to that of $(E_r - E_r^0)$, c.f., Eq. (V-IV.6).

In the Fig. V.4 case, for example, we estimate the value of V_{ϕ}^p at $t \approx 500$ ms to be about a factor of 2 less than the steady state value (i.e., V_{ϕ}^p now is $\approx 4 \times 10^3$ m/sec).

V.5f Final Comments

In light of previous discussion concerning anomalous damping and finite injection times, our steady-state solutions serve as an upper limit to toroidal rotation. Since the D- ^3He system does not give especially high toroidal rotation (even for steady-state), it appears that the ^3He option is not a good way to observe this fusion-product induced effect.

On the other hand, the D-T system must be regarded much more favorably for generating detectable toroidal rotation. Help in this respect may come from operating at lower plasma ion densities.

Additional support may be given by injecting the deuterium perpendicularly, as opposed to tangentially. Since hot deuterium injected ions injected across $\vec{B}_T$ generally have much larger "banana" orbits, ^(11,86) some of them can be lost to the plasma almost as fast as the escaping fusion products themselves. The losses of these unconfined deuterium ions can be evaluated in a manner similar to the fusion products and may serve as pseudofusion product losses, thereby augmenting J_r^{α} , the driving mechanism for the rotation. A second plus for this injection scheme is that the toroidal momentum given by the beam to the plasma is much less than that imparted by tangential injection. Hence, the masking effect of direct toroidal momentum transfer may be reduced to manageable proportions.

CHAPTER VI: SUMMARY AND CONCLUSIONS

This thesis has dealt with a variety of problems related to high-energetic fusion products. In this section, the main points are summarized, and suggestions are made for future study.

VI.1 Slowing-Down Model (SYMALF)

The slowing down theory developed in Chapter II accounted for the FP energy loss drifts by evaluating the contributions to the displacement in P_{MAX} of each of the "segments" along the orbit. To facilitate this computation, this displacement was written in terms of a "zero order" contribution (arising by representing each of the orbital segments by an average $\bar{e}$ and $\bar{f}$ and a "first order" contribution (accounting, in part, for variations in the energy losses). Comparisons of this theory (embodied in SYMALF) with TRAKEM,⁽⁴²⁾ a code which solves the full drift equations numerically, show good agreement, justifying the approximations involved.

Even so, worthwhile improvements in SYMALF can be made. FP's that have much of their orbits lying outside the plasma will not be represented well by the zero order assumption because the ϵ_i 's, for example, will differ greatly over the orbit. To remedy this, more "energy loss" points lying along the trajectory (in addition to P_{MAX} and P_{MIN}) can be applied. Although there may be some technical hurdles to consider, this extension, in principle, offers no serious difficulties.

The quantities calculated (e.g., heating deposition) are effectively averaged over the poloidal coordinate. Since it is sometimes of interest to know the poloidal variation, SYMALF can be extended to also keep track of the poloidal position, as well as the radial position. Although, this step is, in principle, straightforward, care must be taken to properly weight each of the poloidal "bins."

VI.2 Slowing Down Effects

The heating profiles, which are necessary in predicting the plasma heating efficiency of the FP's (and hence in large part the success or

failure of a reactor design), were found to be flatter than "in situ" would have predicted. The reasons are (a) the "first bounce" losses of FP's, and (b) the large initial drift surfaces of the FP's. However, it was also shown in addition that the slowing down drifts themselves can affect the heating profile, especially in reactors of the TFTR-design. As it was explained in Chapter III, the inward energy loss drifts tend to result in the FP energy deposition being larger at the plasma center than the SOA model predicts this being done at the expense of the outer portions of the plasma. On this last point it was suggested that this reduced heating of the background plasma near the edge may have repercussions on the effectiveness of neutral beam or pellet refueling. Certainly this deserves closer examination.

The slowing down drift in the larger reactor designs, ANL I and UWMAK I, does not result in a noticeable difference in the heating profiles as compared to the "in situ" model. Here the "in situ" model appears to be a very good approximation. However, like TFTR, the predictions of "in situ" and SYMALF in the FP heating of Region II are at variance. SYMALF predicts much more energy deposition occurring there than "in situ." Again, this heating problem should be examined from the standpoint of refueling.

The thermalization source function was also shown to be affected by slowing down drifts. This drift process, as with heating profile, depletes the outer plasma regions of suprathermal FP's. However, now the effect is even more pronounced in that the lower energy fusion products, which is of concern here, are much more likely to have drifted away from Region II than higher energy fusion products. Consequently, the "thermalized" particles become more numerous farther into the plasma. In cases where the drifts are large enough, a flattening of the thermalization profile is expected, as the P_{MIN} of the orbit, which has been approaching the plasma center passes through and begins receding somewhat.

Unlike the heating profile case, the thermalization function is more affected by velocity space scattering between 3500 KeV and 500 KeV occurs in the range of 500 KeV to 1 Mev. On the other hand, at these lower

energies the radial displacements along the drift surfaces of the FP's are not as large as they were at birth ($\approx 1/2$ times) so that the dispersion in velocity space at these lower energies may not seriously affect the overall SYMALF thermalization curves. Velocity-space scattering below 500 keV, however, should be considered in a full Fokker-Planck treatment to obtain maximum accuracy, especially for cases where Z_{eff} is large.

The energy distribution functions generated by SYMALF for the five reactors considered here can be used in two ways. First, in more complicated transport problems, where a distribution function must be assumed, SYMALF's results can provide an indication of just how good that assumption is. As with UWMAK I, an "in situ" distribution function [as given in Section (III-3)] is very good over most of the plasma, except for a small region near the plasma boundary. On the other hand, due to the slowing down drift, small tokamaks, like TFTR, can have a $\frac{\partial f}{\partial E} > 0$ for the Region II plasma. For cases where this condition held the suprathermal FP densities are apparently sufficient for E.M. cyclotron or ion Alfvén waves to be found. Additional study in this area is highly recommended.

VI.3 Magnetic Ripple

The magnetic ripple formulation (RIPALF) made use of an harmonic oscillator idea in predicting the wall loadings for the large reactors like UWMAK-I (vs. smaller TFTR-like devices where "first bounce," non-ripple processes dominate).

The wall-loadings predicted by RIPALF are highly dependent on position [Recall Figs. (IV.3) and (IV.4)]. Local "hot-spots" along the wall appear susceptible to various materials problems, like blistering. On the other hand, large segments along the first wall appear to be relatively free from 3.5-MeV alpha bombardment. (e.g. the "inboard" part of the wall) This implies that there may be a large disparity in the erosion of various sections of the wall. Although other problems

enter into consideration (e.g., thermal wall loading), the spatial effects of high energy alphas on the first wall should be examined carefully in a tokamak reactor design.

IV.4 Plasma Rotation

The "steady state" toroidal rotation caused by "first bounce" axisymmetric FP losses was examined from a perspective of a TFTR-device. The results showed that a D-³He fuel will not in all likelihood give a measurable rotation. The D-T case appears borderline.

Charge exchange supplies most of the damping when the anticipated plasma parameters are considered, although experiments in other devices suggest that potentially a stronger damping mechanism may also be involved that has yet to be identified. As long as the value of this process is not yet understood, the implications of toroidal rotation on the conceptual power reactors, like UWMK-I, remains unclear.

APPENDIX A: TRAPPED PARTICLE CASE

The analysis, which was used in Section (II-1) for untrapped particles, follows through exactly for trapped particles. Figure (A-1), which is analogous to Fig. (II-3), represents Segment P_{MIN} as the "anchor" segment X. The final orbit, shown as the dashed curve passing through segment Z, will fall at or between segments X and W of the initial trajectory, depending on the energy loss along the orbit and magnetic field configuration (as it did with the untrapped particles, c.f., cases I, II, and III examined in Section (II-1)).

Figure A-2 plots a trapped particle orbit for an alpha particle born at 3.5 MeV and slowing down to 1.1 MeV. The initial conditions are the same as in Fig. (II-4), except that now $r_o = 4$ cm, $\psi_o = 108^\circ$, and $I_{plasma} = 1.8$ MA. SYMALF's results are compared with the drift model's predictions.⁽⁴²⁾ During this slowing down process, both models predict a P_{MIN} drift of 15 cm and a P_{MAX} drift of 10 cm, amounting to 27% and 18%, respectively, of the minor radius (56 cm).

It is interesting to note that the slowing down theory allows the possibilities of initially untrapped particles becoming trapped and initially trapped particles becoming untrapped. An example of the latter is shown in Fig. (A-3A) where r_o represents the initial P_{MIN} position and s_o the X-axis distance from the plasma center to the point of reflection. Suppose the particle energy were lost at P_{MIN} (c.f., Case II of Section II-1), then the final orbit would be a circle of radius r_o , indicating that the particle must ultimately be untrapped. A little additional thought gives an additional necessary condition; that s_o lie inside the plasma center line, i.e., on the LHS of the centerline of Fig. (A-3A).

On the other hand, a necessary condition for untrapped particles to trap is that $t_o > s_o$, where t_o is the distance from the plasma center to P_{MAX} and s_o is again the corresponding distance to the would-be reflection point (See Fig. A-3). With this condition, and supposing the energy were lost only at P_{MAX} , the slowing down trajectory would, sooner or later, encounter a reflection barrier as shown in Fig. (A-3B). Unlike

Fig. (A.1). The initial orbit at $\mathcal{E} = \mathcal{E}_0$ is shown by a (r, θ) -trajectory passing through segments $P_{\text{MIN}}(\mathcal{E}_0)$, (X), and $P_{\text{MAX}}(\mathcal{E}_0)$, (W), (i.e., solid curve is the "intermediate" orbit, while the dashed curve is the final orbit (c.f., Fig. (II.3)).

Fig. (A.2). SYMALF and drift model predictions are compared for a trapped particle case. The Tokamak has TFTR-like parameters: $a = 56$ cm, $R_0 = 250$ cm, $I_{\text{PLASMA}} = 1.8$ MA, $n = 5 \times 10^{13}/\text{cm}^3 (1-\hat{r}^2)$, $T_e = 6$ keV $(1-\hat{r}^2)$, current density $\propto (1-\hat{r}^2)$, with initial conditions $\psi_0 = 108^\circ$, $r_0 = 4$ cm, and $i_{\theta_0} = 0^\circ$.

Fig. (A-3a). Schematic of a trapped particle becoming untrapped during slowing down.

Fig. (A-3b). Schematic of an untrapped particle becoming trapped during slowing down.

the first case, s_0 may lie on either side of the plasma centerline.

Since the slowing down process depends in a very complicated way on energy loss and magnetic field configurations, it does not appear possible to attain a sufficient condition for the above cases. However, examples of trapped particles becoming untrapped and vice-versa have been found using SYMALF.

APPENDIX B: CONSTANT K-CALCULATION

When SYMALF was first being written (1975), it was clear that, since the magnetic moment μ was not a constant as FP energy changed, use of the "banana equation" [Eq. (II-5b)] was awkward. However, since the equation involved terms of μ/E , and since $\frac{\mu}{\epsilon} = \frac{\sin^2 \psi}{B}$ is independent of ϵ , the author postulated that $\frac{\sin^2 \psi}{B} = \text{const.}$ even during slow down.

To understand this, a magnetic mirror trapping analogy is used. Let $F \equiv$ force acting on the particle along the Z-direction as a result of magnetic field inhomogeneities and $B =$ the magnetic field along the Z-direction. Let ∇_Z be the gradient along Z. Then

$$F = - \nabla_Z (\mu B) \quad , \quad (B-1)$$

$$M_T dV_Z = - \frac{d}{dZ} (\mu B) \frac{dV_Z}{V_Z} \quad (B-2)$$

Integrating.

$$\frac{1}{2} M_T V_Z^2 \Big|_{V_{Z_0}}^{V_Z} = - \mu B \Big|_{Z_0}^Z \quad , \quad (B-3)$$

where V_{Z_0} and Z_0 are the initial speed and position of the particle, respectively. Let ψ be the pitch angle (i.e., $\cos \psi = \vec{V} \cdot \vec{B}$):

$$\mathcal{E} (\cos^2 \psi - \cos^2 \psi_0) = - \mu B + \mu_0 B \quad , \quad (B-4)$$

Upon simplifying

$$\frac{\sin^2 \psi}{B} = \frac{\sin^2 \psi_0}{B_0} \quad (B-5)$$

Equation (B-5) should hold no matter what the particle's energy might be at any given instant (as long as velocity space scattering is

negligible). Thus,

$$\frac{\sin^2 \psi'}{B'} = \frac{\sin^2 \psi_0}{B_0} = \frac{\sin^2 \psi}{B}, \quad (\text{B-6})$$

where the primed and unprimed notation refer to different times in the FP slowing down. Then

$$\frac{\mathcal{E}' \sin^2 \psi'}{\mathcal{E}' B'} = \frac{\mathcal{E} \sin^2 \psi}{\mathcal{E} B}, \quad (\text{B-7a})$$

or

$$\frac{\mu'}{\mathcal{E}'} = \frac{\mu}{\mathcal{E}} \quad (\text{B-7b})$$

It was assumed that Eq. (7b) could be applied to tokamaks also, since the mirror principle is at work there also. Thus this was incorporated into SYMALF.

Subsequently later Meka⁽¹⁰⁾ proved Eq. (B-7b) rigorously with his general slowing down approach. Meka has named the quantity involved "kappa," and the author is grateful to him for helping put SYMALF on more solid footing.

APPENDIX C: δD -APPROXIMATION

We first Taylor series expand the δD in Eq. (II-2) about $\mathcal{E}_0$, the energy at P_{MAX} at the beginning of the orbit:

$$\delta D = -\frac{L\Gamma}{2N} \sum_{i=1}^{2N} \left\{ \frac{\sin \theta_i}{\left(\Gamma^2 - \frac{\Xi_i}{\sqrt{\mathcal{E}_0}} + \frac{\Omega_i}{\mathcal{E}_0} \right)^{1/2}} - \frac{\frac{1}{2} \left(\frac{\Xi_i}{\mathcal{E}_0^{3/2}} - \frac{\Omega_i}{\mathcal{E}_0^2} \right) \sin \theta_i}{\left(\Gamma^2 - \frac{\Xi_i}{\sqrt{\mathcal{E}_0}} + \frac{\Omega_i}{\mathcal{E}_0} \right)^{3/2}} (\mathcal{E} - \mathcal{E}_0) + \dots \right\} \quad (C-1)$$

Rewriting Eq. (C-1):

$$\delta D \approx -\frac{L\Gamma}{2N} \sum_{i=1}^{2N} \frac{\sin \theta_i}{\left(\Gamma^2 - \frac{\Xi_i}{\sqrt{\mathcal{E}_0}} + \frac{\Omega_i}{\mathcal{E}_0} \right)^{1/2}} - \frac{L}{2N} \sum_{i=1}^{2N} \frac{\sin \theta_i \sum_{j=1}^i j}{\left(\Gamma^2 - \frac{\Xi_i}{\sqrt{\mathcal{E}_0}} + \frac{\Omega_i}{\mathcal{E}_0} \right)^{3/2}} \left\{ \frac{\Xi_i}{4\mathcal{E}_0^{3/2}} - \frac{\Omega_i}{2\mathcal{E}_0^2} \right\}, \quad (C-2)$$

where $(\mathcal{E} - \mathcal{E}_0) (\mathcal{E}_0 - \sum_{j=1}^i (\epsilon_j - \mathcal{E}_0)) = -\sum_{j=1}^i \epsilon_j$. The first term on the RHS of

(C-2) vanishes over the sum (due to symmetry). Thus, Eq. (C-2) becomes

$$\delta D = \frac{1}{2N} \sum_{i=1}^{2N} (f_i - g_i) \sum_{j=1}^i \epsilon_j. \quad (C-3)$$

Equation (C-3) represents the first order (in $\mathcal{E} - \mathcal{E}_0$) approximation to δD . Higher orders are neglected consistent with the assumption that

$$\sum_{i=1}^{2N} \frac{\epsilon_i}{\mathcal{E}_0} \ll 1.$$

The ratio of f_i to g_i is:

$$\text{Ratio} \equiv \frac{f_i}{g_i} = \frac{R_o}{\rho_\theta} \frac{\cos \psi_o}{\cos \theta_i} \frac{1}{\left(1 - \frac{KB_o}{2h_i}\right)} \quad (C-4)$$

where ρ_θ is the poloidal gyroradius.

Hence, for $f_i \geq g_i$, we require

$$\frac{R_o}{\rho_\theta} \frac{\cos \psi_i}{\cos \theta_i} \frac{1}{\left(1 - \frac{KB_o}{2h_i}\right)} \geq 1 \quad (C-5)$$

On the other hand, for orbits to be closed, the condition

$$\begin{aligned} v_\theta = & - \frac{2\mathcal{E}}{ZeR_o B_o} \left(1 - \frac{KB_o}{2h_i}\right) \cos \theta_i \\ & + \left(\frac{2\mathcal{E}}{M_T}\right)^{1/2} \frac{B_{\theta_i}}{B_o} \cos \psi_i \geq 0, \end{aligned} \quad (C-6)$$

must hold at one point of contact with the x-axis.

Thus

$$\left(\frac{2\mathcal{E}}{M_T}\right)^{\frac{1}{2}} \frac{B_{\theta i}}{B_o} \cos \psi_i \geq \frac{2\mathcal{E}}{ZcR_o B_o} \left(1 - \frac{KB_o}{2h_i}\right) \cos \theta, \quad (C-7)$$

$$\frac{R_o}{\rho_\theta} \frac{\cos \psi_i}{\cos \theta_i} \frac{1}{1 - \frac{KB_o}{2h_i}} \geq 1, \quad (C-8)$$

which is identical with Eq. (C-5). Consequently, it follows that $f_i > g_i$ for all cases of interest.

Thus, we rewrite Eq. (C-3) as:

$$\begin{aligned} \delta D &= \frac{1}{2N} \sum_{i=1}^{2N} f_i \sum_{j=1}^i \epsilon_j \\ &= \frac{1}{2N} \{f_1 \epsilon_1 + f_2 (\epsilon_1 + \epsilon_2) + f_3 (\epsilon_1 + \epsilon_2 + \epsilon_3) \\ &\quad + \dots + f_{N+1} (\epsilon_1 + \epsilon_2 + \dots + \epsilon_{N+1}) \\ &\quad + \dots + f_{2N} (\epsilon_1 + \epsilon_2 + \dots + \epsilon_{2N})\} \end{aligned} \quad (C-9)$$

But $f_i \approx f_{2N+2-i}$ and $\epsilon_i \approx \epsilon_{2N+2-i}$ (by symmetry); therefore, we write:

$$\begin{aligned} \delta D &= \frac{1}{2N} \{f_1 \epsilon_1 + f_2 (\epsilon_2 + 2\epsilon_3 + \dots + 2\epsilon_N + \epsilon_{N+1}) \\ &\quad - f_3 (\epsilon_3 + 2\epsilon_4 + 2\epsilon_5 + \dots + 2\epsilon_N + \epsilon_{N+1}) \\ &\quad - \dots - f_N (\epsilon_N + \epsilon_{N+1}) + f_{N+1} (\epsilon_1 + \dots + \epsilon_{N+1})\} \end{aligned} \quad (C-10)$$

Equation (10) can be rearranged into a more suitable form:

$$\begin{aligned}
\delta D = & - \frac{1}{2N} \{ f_1(\epsilon_1 + 2\epsilon_2 + \dots + 2\epsilon_N + \epsilon_{N+1}) \\
& + f_2(\epsilon_1 + 2\epsilon_2 + \dots + 2\epsilon_N + \epsilon_{N+1}) \\
& + \dots + f_{N+1}(\epsilon_1 + 2\epsilon_2 + \dots + 2\epsilon_N + \epsilon_{N+1}) \\
& - f_1(2\epsilon_1 + 2\epsilon_2 + \dots + 2\epsilon_N + \epsilon_{N+1}) \\
& - f_2(\epsilon_1 + \epsilon_2) - f_3(\epsilon_1 + 2\epsilon_2 + \epsilon_3) \\
& - \dots - f_N(\epsilon_1 + 2\epsilon_2 + \dots + 2\epsilon_{N-1} + \epsilon_N) \\
& - f_{N+1}(2\epsilon_1 + 3\epsilon_2 + \dots + 3\epsilon_N + 2\epsilon_{N+1}) \} \\
= & - \frac{1}{2N} \left\{ (\delta\epsilon - \epsilon_1) \sum_{i=1}^{N+1} f_i - \sum_{i=3}^{N+1} f_i (\epsilon_i + 2 \sum_{j=2}^{i-1} \epsilon_j) \right. \\
& \left. - f_2 \epsilon_2 - f_1 \delta\epsilon - f_{N+1}(2\epsilon_1 + \epsilon_2 + \dots + \epsilon_{N+1}) \right\} \quad (C-11)
\end{aligned}$$

where

But $\sum f_i$'s scale as N (i.e., $\sum f_i \sim N\bar{f}$, where $\bar{f}$ is an average $\bar{f}_i$ -value) thus, as for N very large; the individual f_1 , f_2 , and f_{N+1} terms vanish:

$$\begin{aligned}
\delta D = & - \frac{1}{2N} \left\{ (\delta\epsilon - \epsilon_1) \sum_{i=1}^{N+1} f_i \right. \\
& \left. - \sum_{i=3}^{N+1} f_i (\epsilon_i + 2 \sum_{j=2}^{i-1} \epsilon_j) \right\} \quad (C-12)
\end{aligned}$$

We now will cast Eq. (C-12) into the form of Eq. (II-4). We first make the following simplifications:

$$\sum_{i=1}^{N+1} f_i (\epsilon_i + 2 \sum_{j=2}^{i-1} \epsilon_j) = \sum_{j=1}^N f_j (\epsilon_j + 2 \sum_{i=2}^{j-1} \epsilon_i) + f_{N+1} (\epsilon_{N+1} + 2 \sum_{j=2}^N \epsilon_j), \quad (C-13a)$$

$$\bar{f} = \frac{1}{N} \sum_{i=1}^N f_i, \quad (C-13b)$$

$$\bar{\epsilon} = \frac{1}{2N} \sum_{i=2}^{2N} \epsilon_i \equiv \frac{\delta \mathcal{E}}{2N}. \quad (C-13c)$$

Thus, Eq. (C-12) can be rearranged as

$$\begin{aligned} \delta D &\cong - \frac{1}{2N} \{ (\delta \mathcal{E} - \epsilon_1) N \bar{f} - \bar{f} \epsilon (3 + 5 + \dots + (2N+1)) \\ &\quad - f_{N+1} (\delta \mathcal{E} - \epsilon_1) \} \\ &= - \frac{1}{2N} \{ (\delta \mathcal{E} - \epsilon_1) N \bar{f} - \bar{f} \epsilon (N+2)N - f_{N+1} (\delta \mathcal{E} - \epsilon_1) \} \\ &= - \frac{1}{2N} \{ (\delta \mathcal{E} - \epsilon_1) N \bar{f} - \frac{\delta \mathcal{E}}{2} N \bar{f} - f_{N+1} (\delta \mathcal{E} - \epsilon_1) \}. \end{aligned} \quad (C-14c)$$

Letting N get very large and noting that $\delta \mathcal{E} = \delta \mathcal{E} \epsilon_1 - \epsilon_{N+1}$, we have

$$\begin{aligned} \delta D &= \delta D_{\text{MAX}} \left\{ \left(1 - \frac{\epsilon_1}{\delta \mathcal{E}} \right) - \frac{1}{2} \left(1 - \frac{(\epsilon_1 + \epsilon_{N+1})}{\delta \mathcal{E}} \right) \right\} \\ &= \frac{\delta D_{\text{MAX}}}{2} \left\{ 1 - \frac{(\epsilon_1 - \epsilon_{N+1})}{\delta \mathcal{E}} \right\}, \end{aligned} \quad (C-15)$$

where $\delta D_{\text{MAX}} \equiv -\frac{\overline{f}\delta E}{2}$. This proves the assertion of Section II.

APPENDIX D: CUMULATIVE PITCH-ANGLE SCATTERING OF FP'S WHILE SLOWING-DOWN

D.1. Pitch-Angle Scattering at Energy

The inverse 90° scattering time, $\frac{1}{\tau_{Dj}}$, an FP test particle off plasma specie j can be written⁽¹⁷⁾

$$\frac{1}{\tau_{Dj}} = A_j \frac{\partial}{\partial V} \left\{ \frac{2}{\sqrt{\pi}} v_j \exp\left(-\frac{U^2}{v_j^2}\right) + \frac{v_j^2}{U} \left(1 + \frac{2U^2}{v_j^2}\right) \Phi\left(\frac{U}{v_j}\right) \right\}. \quad (D-1)$$

where

$$A_j = \frac{4\pi n_j (Z_j e)^2 q_T^2 \ln \Lambda}{M_T^2 U^3}, \quad (D-1a)$$

$$v_j = (2T_j/M_j)^{1/2}, \quad (D-1b)$$

$$\Phi = \frac{2}{\sqrt{\pi}} \int_0^{\frac{U}{v_j}} \exp(-x^2) dx, \quad (D-1c)$$

and n_j , $Z_j e$, v_j , and M_j are the density, charge, thermal speed and mass of the j^{th} specie, respectively. The test particle's charge, mass, and energy are represented by q_T , M_T , U , and E . (In this appendix, all units are in the CGS system.)

Equation D-1 can be simplified:

$$\frac{1}{\tau_{Dj}} = A_j \left\{ \frac{2}{\sqrt{\pi}} \frac{v_j}{U} \exp\left(-\frac{U^2}{v_j^2}\right) + \left(2 - \frac{v_j^2}{U^2}\right) \Phi\left(\frac{U}{v_j}\right) \right\}. \quad (D-2)$$

Suppose $v_e \equiv \frac{2T_e}{M_e} \gg U \gg v_j$, where T_e and M_e are the electron

temperature and mass, it follows that the total inverse 90° deflection time is

$$\frac{1}{\tau_D} \approx \frac{4\pi q_T^2 n}{M_T^2 U^3} \left\{ \underbrace{\frac{8e^2 n_e U}{3\sqrt{\pi} v_e}}_{(1)} + 2 \underbrace{\sum_j (Z_j e)^2 n_j}_{(2)} \right\}. \quad (D-3)$$

Term (1) in Eq. (D-3) is the electron contribution to pitch angle scattering, while term (2) is that of the j ion species. (The electron

density $\equiv n_e = \sum_j Z_j n_j$). Is term (1) $\ll$ term (2)? Suppose $T_e =$

10 keV, $Z_j = 1$, $= 3500$ keV, and $M = 4.0$ AMU:

$$\frac{\text{Term (1)}}{\text{Term (2)}} = \frac{4}{3\sqrt{\pi}} \sqrt{\frac{\mathcal{E}}{T_e}} \sqrt{\frac{M_e}{M_T}} \approx \frac{1}{8}. \quad (D-4)$$

Thus, at 3500 keV, the electrons comprise $\lesssim 13\%$ of the pitch-angle scattering contribution. At lower FP energies, the background ions will demonate even more.

Equation (D-3) can be rewritten:

$$\tau_D \approx \frac{\sqrt{2M_T} \mathcal{E}^{3/2}}{4\pi q_T^2 e^2 \sum_j Z_j^2 n_j \ln \Lambda \left(1 + \frac{3}{4\sqrt{\pi}} \sqrt{\frac{M_e}{M_T}} \frac{\mathcal{E}}{T_e} \frac{n_e}{\sum_j Z_j^2 n_j} \right)} \quad (D-5)$$

For scattering through an angle $\Delta\psi$ (instead of 90°), one may write

$$\tau_D = \tau_D \sin^2 \Delta\psi, \quad (D-6)$$

whereupon

$$\tau_{D\psi} = \mathcal{E}^{3/2} Y^D \sin^2 \left(\frac{\Delta\psi}{2} \right) \quad (D-7)$$

In Eq. (D-7), we let

$$Y^D \equiv \frac{\sqrt{2M_T}}{4\pi q_T^2 e^2 \sum_j Z_j^2 n_j \ln \Lambda} \quad (D-7)$$

D.2 Slowing-Down Time

The slowing down time, τ_s , for an FP is now calculated. From page 302 of Ref. 17:

$$\tau_s = -U \left(\frac{\partial U}{\partial t} \right)^{-1} \quad (D-8)$$

One first finds $\frac{\partial U}{\partial t}$:

$$\begin{aligned} \frac{\partial U}{\partial t} = C_e & \left\{ - \frac{\left(1 + \frac{M_T}{M_e}\right)}{U^2} \phi\left(\frac{U}{V_e}\right) + \frac{2}{\sqrt{\pi}} \frac{\left(1 + \frac{M_T}{M_e}\right)}{UV_e} \exp\left(-\frac{U^2}{V_e^2}\right) \right\} \\ & + \sum_j C_j \left\{ - \frac{\left(1 + \frac{M_T}{M_j}\right)}{U^2} \phi\left(\frac{U}{V_j}\right) + \frac{2}{\sqrt{\pi}} \frac{\left(1 + \frac{M_T}{M_j}\right)}{UV_j} \exp\left(-\frac{U^2}{V_j^2}\right) \right\}, \quad (D-9) \end{aligned}$$

where $C_j \equiv 4\pi(Z_j e)^2 q_T^2 n_j \ln \Lambda / M_T^2$. When $V_i \ll U \ll V_e$, it can be shown that

$$\begin{aligned} \frac{\partial U}{\partial t} = - \frac{4\pi e^2 q_T^2 \ln \Lambda}{M_T^2} & \left\{ \frac{4}{3} \frac{\left(1 + \frac{M_T}{M_e}\right)}{\sqrt{\pi} V_e^3} n_e U \right. \\ & \left. + \sum_j \frac{\left(1 + \frac{M_T}{M_j}\right)}{U^2} Z_j^2 n_j \right\}. \quad (D-10) \end{aligned}$$

Substituting Eq. (D-10) into Eq. (D-8), one has:

$$\frac{1}{\tau_s} = \frac{16\sqrt{\pi} e^2 q_T^2 \ln \Lambda}{3 M_T^2 v_e^3} n_e \left(1 + \frac{M_T}{M_e}\right) \left(1 + \frac{3}{4} \sqrt{\pi} \frac{v_e^3}{U^3} \sum_j \left(\frac{1 + \frac{M_T}{M_j}}{1 + \frac{M_T}{M_e}} \right) z_j^2 \frac{n_j}{n_e} \right) \quad (D-11)$$

Since $\frac{\partial U}{\partial t} = -\frac{U}{\tau_s}$, then $\frac{\partial}{\partial t} \left(\frac{1}{2} M_T U^2 \right) = \frac{M_T U^2}{\tau_s} = -\frac{2\mathcal{E}}{\tau_s}$.

Thus, Eq. (D-11) can be written, after some simplification

$$\frac{\partial}{\partial t} \mathcal{E} = -Y^S (1 + Y^T / \mathcal{E}^{3/2}) \mathcal{E}, \quad (D-12)$$

where

$$Y^S \equiv \frac{8\sqrt{\pi} n_e e^2 q_T^2 \ln \Lambda}{3\sqrt{M_e} T_e^{3/2}} \left(\frac{M_e}{M_T} \right), \quad (D-12a)$$

$$Y^T \equiv \frac{3}{4} \sqrt{\pi} \sum_j \frac{T_e^{3/2}}{n_e} \left(\frac{M_T}{M_j} \right) \sqrt{\frac{M_T}{M_e}} z_j^2 n_j. \quad (D-12b)$$

Solving (D-12), and letting $\mathcal{E}_0 \equiv$ initial FP energy and initial time $t_0 = 0$:

$$t_f = \frac{2}{3} \frac{1}{Y^S} \ln \frac{\mathcal{E}^{3/2} + Y^T}{\mathcal{E}_0^{3/2} + Y^T}. \quad (D-13)$$

D.3. Average $\frac{1}{\tau_{D\psi}}$

$$\left\langle \frac{1}{\tau_{D\psi}} \right\rangle = \frac{\int_0^t \frac{1}{\tau_{D\psi}} dt'}{t} = \frac{\int_{\mathcal{E}_0}^{\mathcal{E}} \frac{1}{\tau_{D\psi}} \frac{d\mathcal{E}'}{\frac{d\mathcal{E}'}{dt}}}{\int \frac{d\mathcal{E}'}{\frac{d\mathcal{E}'}{dt}}} \quad (D-14a)$$

$$\left\langle \frac{1}{\tau_{D\psi}} \right\rangle = \frac{\int_{\epsilon_0}^{\epsilon} \frac{1}{Y^D \epsilon^{3/2} \sin^2 \Delta\psi} \times \frac{d\epsilon}{Y^S (1 + \frac{Y^T}{\epsilon^{3/2}})} }{\frac{2}{3} \frac{1}{Y^S} \ln \left(\frac{\epsilon_0^{3/2} + Y^T}{\epsilon^{3/2} + Y^T} \right)} \quad (D-14b)$$

The integration is straightforward. One finds

$$\left\langle \frac{1}{\tau_{D\psi}} \right\rangle = \frac{1}{Y^D Y^T} \frac{\ln \left\{ \left(\frac{\epsilon_0}{\epsilon} \right)^{3/2} \left(\frac{\epsilon^{3/2} + Y^T}{\epsilon_0^{3/2} + Y^T} \right) \right\}}{\ln \left(\frac{\epsilon_0^{3/2} + Y^T}{\epsilon^{3/2} + Y^T} \right) \sin^2 \left(\frac{\Delta\psi}{2} \right)} \quad (D-15)$$

D.4 Calculation of $\sin^2 \Delta\psi$

To find the RMS deviation in scattering angle $\Delta\psi$ for an $(\epsilon_0 - \epsilon)$ energy loss, one equates the inverse slowing down time with the inverse time necessary to scatter through $\Delta\psi$ that is, from Eq. (D-14) and (D-15),

$$\frac{1}{\tau_f} = \left\langle \frac{1}{\tau_{D\psi}} \right\rangle \quad (D-16)$$

Solving for $\Delta\psi$

$$\Delta\psi = 2 \sin^{-1} \left[\sqrt{\frac{2}{3(1 + \frac{M_T}{M_i})} \ln \left[\left(\frac{x_0}{x} \right)^{3/2} \left(\frac{x^{3/2} + \frac{3\sqrt{\pi}}{4} \langle Z \rangle \sqrt{\frac{M_T}{M_e} \left(1 + \frac{M_T}{M_i} \right)}}{\frac{x_0^{3/2} + \frac{3\sqrt{\pi}}{4} \langle Z \rangle \sqrt{\frac{M_T}{M_e} \left(1 + \frac{M_T}{M_i} \right)}} \right)} \right]} \right] \quad (D-17)$$

where

$$x_o \equiv \frac{\xi_o}{T_e}, \quad x \equiv \frac{\xi}{T_e}, \quad x \equiv \frac{\varepsilon}{T_e}, \quad \text{and} \quad \langle Z \rangle = \sum_j z_j^2 \eta_j$$

APPENDIX E: DISPERSION DUE TO SCATTERING

The dispersion, λ_D , in radial drift along the X-axis resulting from collisions in the energy interval $d\mathcal{E}$ is

$$d\lambda_D = \sqrt{\left\langle \left(\frac{\partial \rho}{\partial \mathcal{E}} \right)^2 \right\rangle} \frac{\partial (\Delta\psi/2)}{\partial \mathcal{E}} d\mathcal{E}, \quad (\text{E-1})$$

where ρ is the "scale length" of the system (closely related to the poloidal gyroradius) and $\frac{\partial (\Delta\psi/2)}{\partial \mathcal{E}}$ is one-half the rate of change in total pitch angle dispersion in interval $d\mathcal{E}$. The total dispersion, λ_D is found by integrating over all energy groups, i.e.,

$$\lambda_D = \int \sqrt{\left\langle \left(\frac{\partial \rho}{\partial \mathcal{E}} \right)^2 \right\rangle} \frac{\partial (\Delta\psi/2)}{\partial \mathcal{E}} d\mathcal{E}. \quad (\text{E-2})$$

As suggested in Section (II-2), we may write ρ as

$$\begin{aligned} \rho &\approx \frac{1}{2} v_r \tau \\ &= \frac{1}{2} \left(\frac{2\mathcal{E}}{Ze R_o B_o} \left(1 - \frac{KB_o}{2h} \right) \sin\theta \right) \left(\frac{2\pi R_o}{v_{11} \frac{R_o B_o}{r B_T}} \right) \\ &\quad \frac{\sqrt{2M_T \mathcal{E}} \pi \left(1 - \frac{KB_o}{2h} \right) r \sin\theta}{Ze B_o R_o \cos\psi}, \end{aligned} \quad (\text{E-3})$$

where $\tau = \frac{2\pi R_o}{v_{11} \left(\frac{R_o B_o}{r B_T} \right)}$, $v_{11} = v \cos$, and $\left(\frac{R_o B_o}{r B_T} \right)$ accounts for the rotational

transform effect. Thus,

$$\frac{\partial \rho}{\partial \psi} \approx - \frac{\sqrt{2M_T \mathcal{E}} \pi \left(1 - \frac{KB_0}{2h}\right) r \sin \theta \sin \psi}{Ze B_\theta R_0 \cos^2 \psi} \quad (E-4)$$

Assuming the test particle is an alpha and writing energy in terms of keV, we have

$$\frac{\partial \rho}{\partial \psi} \approx - 1.4 \times 10^{-2} \frac{\sqrt{\mathcal{E}} \left(1 - \frac{KB_0}{2h}\right) r \sin \theta \sin \psi}{Ze B_\theta R_0 \cos^2 \psi} \quad (E-5)$$

Ultimately, we will integrate the function $\frac{\partial (\frac{\Delta \psi}{2})}{\partial \mathcal{E}} \times \sqrt{\mathcal{E}}$ over the energy interval 3500 keV to 500 keV. This integral is difficult to do analytically without running the chance of the simplifying approximations affecting the accuracy of the result. To avoid risk, we have graphed $(\frac{\Delta \psi}{2})$ as a function of $\mathcal{E}$ for the case of $T_e = 6$ keV (see Fig. E-1). The actual $(\frac{\Delta \psi}{2})$ is shown as a solid curve, while the linear approximation is the dashed line. One finds that with a little algebra, the linear fit can be written

$$\frac{\Delta \psi}{2} \approx 5.8 \times 10^{-5} (3500 - \mathcal{E}) , \quad (E-6)$$

where $\mathcal{E}$ is in keV and $\frac{\Delta \psi}{2}$ in radius. Finally, an expression for $\frac{\partial}{\partial \mathcal{E}} \left(\frac{\Delta \psi}{2} \right)$ is:

$$\frac{\partial}{\partial \mathcal{E}} \left(\frac{\Delta \psi}{2} \right) \approx - 5.8 \times 10^{-5} \quad (E-7)$$

With Eqs. (E-2), (E-5), and (E-7), one may calculate λ_D for a $T_e = 6$ keV background:

$$\lambda_D \approx 8.1 \times 10^{-7} \int_{3500}^{500} \sqrt{\frac{\left(1 - \frac{KB_o}{2h}\right)^2 r^2 \sin^2 \theta \sin^2 \psi}{B_\theta^2 R^2 \cos^4 \psi}} \sqrt{\epsilon} d\epsilon. \quad (E-7)$$

For the orbit in Fig. (II-4), where $\sqrt{\langle \theta^2 \rangle} \approx 18^\circ$ and $\sqrt{\langle \psi^2 \rangle} \approx 73^\circ$, we perform the integration:

$$\lambda_D \approx 0.2 \left\langle \frac{r}{B_\theta R_o} \right\rangle \approx 0.01m,$$

where $r \approx 0.2m$, $B_\theta \approx 0.2 \text{ W/m}^2$, and $R_o = 2.5m$.

The average drift distance, $\left(\equiv \frac{|P_{MAX}(3500 \text{ keV}) - P_{MAX}(500 \text{ keV})|}{2} + \frac{|P_{MIN}(3500 \text{ keV}) - P_{MIN}(500 \text{ keV})|}{2} \right)$, is approximately 7 cm. Thus the scattering contribution to drift is roughly 1/7 that of slowing down.

APPENDIX F: DISTRIBUTION FUNCTION USING ENERGY GROUPS

By definition of distribution function, one has

$$n(\hat{r}) = \int_{\mathcal{E}_A}^{\mathcal{E}_B} f(\hat{r}, \mathcal{E}) d\mathcal{E} , \quad (\text{F-1})$$

where the integral is carried out over the energy range of concern, for example, between $\mathcal{E}_A$ and $\mathcal{E}_B$. The normalized distribution function is then

$$f(\hat{r}, \mathcal{E}) = \frac{1}{n(\hat{r})} f(\hat{r}, \mathcal{E}) . \quad (\text{F-2})$$

In the case at hand, SYMALF has evaluated the number of FP's lying in an energy interval $\Delta \mathcal{E}$ (= 600 keV). Thus, as in Eq. (F-1), one has:

$$\sum_k n(\hat{r}, k) = \sum_k f(\hat{r}, \mathcal{E}^{(k)}) \Delta \mathcal{E} , \quad (\text{F-3})$$

or, by dividing each side of Eq. (F-3) by Δ

$$\sum_k f(\hat{r}, \mathcal{E}^{(k)}) = \sum_k \frac{n(\hat{r}, \mathcal{E}^{(k)})}{\Delta \mathcal{E}} \quad (\text{F-4})$$

For each of the k intervals:

$$f(\hat{r}, \mathcal{E}^{(k)}) = \frac{n(\hat{r}, k)}{\Delta \mathcal{E}} , \quad (\text{F-5})$$

Finally, by normalizing $f(\hat{r}, \mathcal{E}^{(k)})$ as in Eq. (F-2), one arrives at

$$\hat{f}(\hat{r}, \mathcal{E}^{(k)}) = \frac{n(\hat{r}, k)}{\sum_k n(\hat{r}, k)} , \quad (\text{F-6})$$

where the $\Delta \mathcal{E}$'s have cancelled out.

Fig. (E.1). Velocity space dispersion, $\frac{\Delta\psi}{2}$, is plotted against FP energy, .

APPENDIX G: PROBABILITY CALCULATION FOR $q \gg 1$

The mirror field is shown in Fig. (IV-2), and can be represented by

$$B = B_0 \left(1 - \delta \cos \frac{\pi X}{L}\right), \quad (G-1)$$

where $L \equiv |X_N - X_0|$.

The force acting on the guiding center in this magnetic field is

$$F = M_T \frac{d^2 X}{dt^2} = - \nabla_X (\mu B) = \delta \frac{\mu \pi B_0}{L} \sin \frac{\pi X}{L}. \quad (G-2)$$

Let $u = \frac{dx}{dt}$ then

$$\frac{du}{dt} = \frac{du}{dx} \frac{dx}{dt} = u \frac{du}{dx} = - \frac{\mu \pi B_0 \delta}{M_T L} \sin \frac{\pi X}{L}. \quad (G-3)$$

Integrating Eq. (G-3), we have

$$\frac{u^2}{2} = \frac{\mu B_0 \delta}{M_T} \cos \frac{\pi X}{L} + \text{constant}. \quad (G-4)$$

When $X = X_T$, $u = 0$, so that

$$\frac{dX}{dt} = \pm \sqrt{\frac{2\mu B_0 \delta}{M_T}} \left(\cos \frac{\pi X}{L} - \cos \frac{\pi X_T}{L} \right), \quad (G-5)$$

or

$$dt = \sqrt{\frac{2\mu B_0 \delta}{M_T}} \left(\cos \frac{\pi}{L} - \cos \frac{\pi X_T}{L} \right), \quad (G-6)$$

where we have suppressed the minus sign. The time spent by the particle between X_1 and X_2 during one oscillation, where $|x_1|, |x_2| \leq |x_T|$, is

$$\tau_{12} = 2 \int_{x_1}^{x_2} \frac{dx}{\left(\frac{2\mu B_0 \delta}{M_T} \right) \left(\cos \frac{\pi x}{L} - \cos \frac{\pi x_T}{L} \right)} \quad (G-7)$$

The probability, $P((x_2-x_1); x_T)$, of finding the particle with a turning point at x_0 lying between x_1 and x_2 is

$$P(x_2-x_1; x_T) = \frac{\int_{x_1}^{x_2} \frac{dx}{\sqrt{\left(\cos \frac{\pi x}{L} - \cos \frac{\pi x_T}{L} \right)}}}{2 \int_0^{x_T} \frac{dx}{\sqrt{\left(\cos \frac{\pi x}{L} - \cos \frac{\pi x_T}{L} \right)}}} \quad (G-8)$$

But $\cos \frac{\pi x}{L} = -2 \sin^2 \left(\frac{\pi x}{2L} \right) + 1$; thus

$$P((x_1-x_2); x_T) = \frac{\int_{x_1}^{x_2} \frac{dx}{\sqrt{\sin^2 \left(\frac{\pi x_T}{2L} \right) - \sin^2 \left(\frac{\pi x}{2L} \right)}}}{\int_0^{x_T} \frac{dx}{\sqrt{\sin^2 \left(\frac{\pi x_T}{2L} \right) - \sin^2 \left(\frac{\pi x}{2L} \right)}}} \quad (G-9)$$

Let $\sin \left(\frac{x}{2L} \right) = \sin \left(\frac{\pi x_T}{2L} \right) \sin \psi$, then

$$dx = \frac{2L}{\pi} \frac{\sin \left(\frac{\pi x_T}{2L} \right) \cos \psi d\psi}{\sqrt{1 - \sin^2 \left(\frac{\pi x_T}{2L} \right) \sin^2 \psi}} \quad (G-10)$$

Substituting Eq. (G-10) into Eq. (G-9), one finds:

$$P((x_2-x_1); x_T) = \frac{\int_{\psi_{M-1}}^{\psi_M} \frac{d\psi}{(1 - \sin^2 \theta_D \sin^2 \psi)^{1/2}}}{\int_0^{\pi/2} \frac{d\psi}{(1 - \sin^2 \theta_D \sin^2 \psi)^{1/2}}}, \quad \text{for } x_T \geq x, \\ = 0, \quad \text{for } x_T < x_{11} \quad (G-11)$$

where

$$\sin \psi_m = \frac{\sin \left(\frac{x_m S}{2R_o g} \right)}{\sin \theta_D}, \quad (G-11a)$$

$$\sin \psi_{m-1} = \frac{\sin \left(\frac{x_{m-1} S}{2R_o h} \right)}{\sin \theta_D}, \quad (G-11b)$$

$$\sin \theta_D = \sin \left(\frac{x_T S}{2R_o h} \right). \quad (G-11c)$$

In calculating $\dot{N}(r, \theta; \Delta X_m)$, i.e., Eq. (IV-2), one first notes that the contribution of particles born in segment ΔX_m and the segments which have a lower B than ΔX_m (i.e., ΔX_{m-1} , ΔX_{m-2} , etc.) can be written

$$\dot{N}'(r, \theta; \Delta X_m) = 2\dot{n} \Delta X_m \sum_{j=m}^N \Delta B_j P(\Delta X_m; X_j), \quad (G-12)$$

where the prime denotes only the segments mentioned above are considered. The other parameters have already been defined in Section (IV-2).

The contribution from the FP's born in the remaining segments is written as

$$N''(r, \theta; \Delta X_m) = 2\dot{n} \Delta X_m \sum_{j=m+1}^N (j-m) \Delta B_j P(\Delta X_m; X_j). \quad (G-13)$$

The total contribution is the sum of N' and N'' .

APPENDIX H: CODE RIPALF

Code RIPALF is based primarily on probability considerations. The probability of a trapped FP striking a particular segment of the first wall is proportional to the time this FP spends beneath that segment. As with code SYMALF, the plasma is divided into radial and azimuthal groups, with the addition of toroidal groups, since this is a 3-dimensional problem.

The input includes plasma density, temperature, current density, and magnetic field ripple parameters [c.f., Eq. (IV-1)]. The number of FP's produced within each of these groups (we will refer to them as volume or V-groups) is determined. Since the safety factor ($\equiv \frac{rB_o}{R_o B_o}$) is known, the magnetic field line upon which the group moves is extended until it is in the plane defined by magnets #1 and #2, as in Fig. (IV-1). The magnetic fields at these points, B_1 and B_2 , respectively, are compared with the magnetic field, B_b , in the birth segment of the group. If B_1 and B_2 are less than B_b , then there can be no trapping and the next toroidal group is then considered. (See Fig. H-1).

The possibility for trapping between the coils occurs when either B_1 or B_2 is less than B_b . Whether trapping is present can be determined by the following technique. The path along $\vec{B}$ is divided up into 20 equally spaced segments. Each of these segments are represented by an average magnetic field B^i and magnetic fields, $B^{i-1/2}$, and $B^{i+1/2}$, which are the fields at either end of the segment. Turning point segments (TIPS) are those segments where $|B^i| > |B_b|$. Those FP's born in a particular V-group with turning points falling within one of these TIP's is characterized by a magnetic moment given by $\mathcal{E}/B_{TP}^i$ where the subscript TP denotes "turning point." With μ known, the parallel motion of the FP's in the i^{th} TPS is known.

The extent of the orbit along B for the i^{th} TPS group is determined by whether or not $V_{11}^2 > 0$: if yes, then the FP's are allowed to proceed farther along B ; if not, the FP's are assumed to have been reflected.

Fig. (H.1). Schematic of Code RIPALF.

If $V_{11}^2 > 0$ at either of the magnets, then the particles of that TPS group are considered untrapped. The time spent by the i^{th} TPS group along the field line (assuming it is trapped) is determined numerically. Since V is known at all points along the orbit and since the orbit is divided into equal segments, the time spent in each of these segments can be calculated. The probability of finding the i^{th} TPS group in any one of the segments is just proportional to the time spent there (as in the "ideal" case of Section IV-2). The number of FP's in the i^{th} TPS produced per second is then evaluated via

$$\left\{ \begin{array}{c} \text{number} \\ \text{in} \\ i^{\text{th}} \text{ TPS} \end{array} \right\} = \left\{ \begin{array}{c} \text{total number} \\ \text{of particles} \\ \text{produced in} \\ \text{V-group} \end{array} \right\} \times \left\{ \begin{array}{c} \text{Fraction} \\ \text{in } i^{\text{th}} \\ \text{TPS} \end{array} \right\}$$

$$= n_v \times \left(\sqrt{1 - \frac{B_t}{B^{t+\frac{1}{2}}}} - \sqrt{1 - \frac{B_t}{B^{+\frac{1}{2}}}} \right), \quad (\text{H-1})$$

where n_v is the number of FP particles produced per second for the V-group in question.

If it is found that the confinement time, t_{con} , (see Section IV-2) is less than the time, t_{orb} , required to complete a single oscillation along $\vec{B}$, then the position of the FP's is found at t_{con} after birth. From each segment a vertical projection to the first wall is made (e.g., the probability distribution is not used in this case).

When all the TPS's have been treated, a new set of radial, azimuthal, and toroidal are chosen and the process repeated.

APPENDIX I: RADIAL ELECTRIC FIELD

As discussed in Chapter V, a steady-state radial electric field, $\delta E_r^{(1)}$, which results from FP losses can be approximated by Eq. (V.5),

$$\text{i.e., } \delta E_r^{(1)} = \frac{J_r^\alpha}{\sigma_\perp}, \text{ where } J_r \text{ is the FP loss current (or plasma response}$$

current given by Eq. (V.16).

FP's which do not escape the plasma on the first bounce can still have radial displacements which are comparable to the minor radius. Since their "associated" electrons are initially confined by the magnetic field to the immediate radial region of the FP birth surface, a net charge imbalance can result within the plasma. The resulting radial electric field may be written

$$\delta E_r^{(2)} = \frac{J_r^\beta}{\sigma_\perp}, \quad (I.1)$$

where J_r is the plasma response to the electric field established by the confined FP's and can be written (as in the J_r^β case):

$$J_r^\beta = \frac{Z_e}{2\pi r} \int_0^r \int_0^{2\pi} (\dot{n} - \dot{n}_e) r' d\theta dr'. \quad (I.2)$$

Here, $\dot{n}_\alpha$ is the increase in FP density at r per second due to fusion, and $\dot{n}_e$ is the increase in the FP "associated" electrons at r per second. Since we are considering only confined orbits

$$\int_0^a \dot{n}_\alpha r dr = \int_0^a Z \dot{n}_e r dr, \quad (I-3a)$$

and

$$\delta E_r^{(2)} (\hat{r}=1) = 0, \quad (I-3b)$$

since $J_r^\beta(r=a) = 0$. Inside the plasma, however, $\delta E_r^{(2)}$ is not necessarily zero.

The ratio $\delta E_r^{(1)}/\delta E_r^{(2)}$ is

$$\frac{\delta E_r^{(1)}}{\delta E_r^{(2)}} = \frac{J_r^\alpha}{\frac{Ze}{2r} \int_0^r \int_0^{2\pi} (\dot{n}_\alpha - \dot{n}_e) r' d\theta dr'} \quad (I-4)$$

We may estimate this ratio in the following way. From Fig. (III.6), $\dot{n}_e$ may be represented by the MIS curve (which has "filtered" out of the FP first bounce losses). The thermalization time is typically ~ 200 ms, while the damping time for the region near the origin may be from 1 to 10 seconds for TFTR-I [c.f., Fig. (V.3) and Table III]. Thus, the FP has effectively thermalized before the ratio $\delta E_r^{(1)}/\delta E_r^{(2)}$ is

$$\begin{aligned} \frac{\delta E_r^{(1)}}{\delta E_r^{(2)}} &\approx \frac{\int_0^r \gamma_L \dot{n} r' dr'}{\int_0^r (1-\gamma_L) \dot{G} \dot{n} r' dr'} \\ &\approx \frac{\bar{\gamma}_L}{(1-\bar{\gamma}_L) \bar{\gamma}_G} \end{aligned} \quad (I.4)$$

where $\bar{\gamma}_L$ is the averaged loss fraction, $\bar{\gamma}_G$ is the average ratio $\frac{\dot{n}_\alpha - \dot{n}_e}{\dot{n}_e}$,

and n is the total FP's produced at r' .

As an example, let us estimate this ratio for the TFTR case of Table III. $\dot{n}_e$ can be represented by MIS from Fig. (III.6). Using the the alpha particle thermalization function to define where the alphas eventually slow down, one has for $\hat{r} = 0.2$.

$$\frac{\delta E_r^{(1)}}{\delta E_r^{(2)}} \gtrsim 5 \quad (I.5)$$

where $|n-n_e| \sim 3 \times 10^{17}/(\text{m}^2 \text{sec})$, $\dot{n}_e \sim 0.65 \times 10^{18}/(\text{m}^2 \cdot \text{sec})$, and $\bar{\gamma}_L \sim 0.2$.

Hence, the electric field resulting from FP losses is clearly the more important.

In high current axisymmetric tokamaks, the losses near the plasma center are very small (possibly zero if the current is high enough). In such cases the field resulting from the finite orbits of contained particles can be dominant in the central regions. One must keep in mind, however, that, in reality, losses due to magnetic ripple actually determine the J_r^α for these larger devices. On this basis, sweeping generalizations about $\delta E_r^{(1)}/\delta E_r^{(2)}$ are difficult to make. Instead, one must proceed on a case by case basis.

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