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On Line Computer Recalibration of Instruments

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Final Report

For the Period
October 1, 1978 - September 30, 1979

R.W. Gilchrist, W.J. Barnett

July 1979

MASTER

Work Performed Under Contract EY-77-S-09-1011-Mod. 1

Form 189 Number 860.1

Clemson University
Clemson, South Carolina 29631

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On Line Computer Recalibration of Instruments

R.W. Gilchrist, W.J. Barnett

ABSTRACT

This report presents the results of work on the development of a system for on-line computer recalibration of instruments. The system consists of two computer programs; an off-line data analysis program and an on-line error reduction program.

The off-line program accepts extensive calibration data, analyzes it and prepares it for use in the on-line program. The on-line program accepts the instrument observations and corrects for errors according to the current system state.

The on-line program is appropriate for use with a microprocessor.

Corrections are made for such deterministic errors as bias, linearity, and hysteresis and errors due to system state parameters such as temperature and humidity.

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1.0 Introduction:

1.1 Need for Improved Instrument Accuracy:

With the increased emphasis on accountability in processing nuclear materials, improved accuracy in instrumentation becomes important. In previous work* methods were developed and demonstrated for improving the accuracy of quantities measured or estimated in the presence of random error or noise. In the current work, methods of reducing certain deterministic errors are developed. The deterministic errors considered are those due to bias, nonlinearity, hysteresis and environmental effects such as temperature and humidity or other parameters such as viscosity or density as appropriate.

Deterministic errors exist in meter observations because of imperfect instruments, and the observations are made for ambient conditions or measurand states other than the calibration conditions, and a manual correction of each observation using calibration data is impractical.

It is common practice to consider all instrument errors as random errors. A one percent of full scale specification for error may be considered as one standard deviation on a normally distributed error curve. Statistical methods are then used to define the expected values and the confidence interval. It is clear that if deterministic errors are removed before the statistical treatment of data, then better accuracies can be obtained and the statistical methods are more valid in treating the residual errors. In this work, representation of data using regression equations tends to suppress any random errors, and corrections are made only for deterministic errors.

*"Improving Process Monitoring Data for Nuclear Fuel Reprocessing Plants Using State Estimation and Pattern Recognition," R.W. Cilchrist, et al, Report SR-1011-5. Final Report.

With the current cost and availability of microprocessors or other dedicated instrumentation computers, it is now practical to consider on-line computer recalibration of instruments.

This report presents the results of the development of a computer system for on-line calibration or correction for certain deterministic errors.

1.2 Objectives:

The objective of the work reported on here is to develop a computer based system which is capable of storing large amounts of calibration data and operating in an on-line mode to give a corrected value for each instrument observation. Corrections are to be made for errors due to bias, nonlinearity, hysteresis and two parameters such as temperature and humidity or viscosity or density as may be appropriate.

The on-line computer program is to be compatible with micro-processor use and the system should be applicable to a large class of instruments.

Documentation of the computer programs will be included in this final report.

The objective is accomplished through the following tasks:

1. Collect calibration data on an instrument which exhibits typical bias, non-linearity and hysteresis characteristics. Include calibration data over a range for two parameters such as temperature and humidity.
2. Develop algorithms for analyzing and storing calibration data. Develop algorithms for correcting instrument observations.

3. Programming of algorithms and testing the effectiveness of the calibration.
4. Documentation and reporting.

1.3 Definition of Terms:

In this section the various deterministic errors are discussed. The classical definitions of these errors are given. These error definitions give a means of specifying the quality of an instrument. In the design and manufacture of an instrument, limiting these errors gives a more accurate or better quality instrument. However, having a number which is a classical measure of the non-linearity of an instrument does not give a way to correct an observation for non-linearity error. Similar statements apply to bias, temperature and humidity errors.

INSTRUMENT HYSTERESIS
Figure 1.1

In general one corrects an observation for these deterministic errors by having calibration data available (maybe in the form of curves) for the specific conditions for which the observation was made.

Figure 1.1 illustrates the characteristic of the instrument that is designated as hysteresis error. The plot of the observation as a function of input (measurand) does not follow the same curve for increasing values of the measurand as for decreasing values of the measurand. The hysteresis is defined as the maximum difference between the increasing and decreasing curves shown in the figure as the measurand is varied from 0 to 100% to 0. It should be noted that these effects are greatly exaggerated in the figure.

INSTRUMENT NONLINEARITY
AND BIAS
Figure 1.2

Figure 1.2 illustrates the nonlinear characteristic of an instrument. The hysteresis which is also included is incidental to the discussion here. To obtain a measure of the nonlinearity, a "best" straight line is constructed and the maximum distance of any data point above and below the

"best" straight line determined. There are several ways of obtaining "best" straight line. A number specifying nonlinearity is only meaningful if the method of defining the line is specified. In this work the "best" straight line is the line obtained by linear regression using data for both the increasing and decreasing curves.

Figure 1.2 can also be used to define bias. The zero off-set of the observation when the measurand is zero is the bias. The zero off-set of the "best" straight line is used here. It should be noted again that the characteristics in the figure are exaggerated for the purpose of illustration.

CALIBRATION VARIATION
WITH TEMPERATURE
Figure 1.3

Figure 1.3 illustrates how the calibration curve or data may vary with some parameter such as temperature or humidity. The figure shows the calibration curve for three different temperatures. In this work a different set of calibration data is proposed for each of a set values of temperatures

over the range of interest. The parameter could just as well be humidity or some other factor effecting the instrument observation.

1.4 Overview of Calibration Technique:

When an instrument is constructed it has certain inherent inaccuracies. If these errors are too large for a given application, observed values can be improved using calibration curves or tables. The calibration data are obtained by comparing a given instrument with a standard. For each true value of the measurand the observed value and hence the error for that value of the measurand are noted. In general the error will be different for each value of the measurand thus there is the need for a calibration curve or table rather than a single correction factor. Further, if the instrument error is sensitive to some system parameter such as temperature a new calibration curve is needed for each temperature over the expected range of that parameter. If the error is sensitive to a second parameter such as humidity then there is a need for a family of calibration curves for a range of humidities for each temperature. If hysteresis effects are significant then the problem is somewhat more complicated.

The research reported on here is the development of a computer based system which will analyze and store the calibration data and make appropriate corrections to instrument observations in an on-line mode. The system developed consists of off-line data analysis programs and on-line error corrections programs.

The off-line program performs a statistical analysis of the calibration data and also computes regression coefficients for storing the calibration data in equation form. The on-line program accepts regression coefficients, instrument observations and system state data and outputs the corrected observation.

The following cases were considered separately:

1. Calibration neglecting the effects of temperature, humidity, and hysteresis.
2. Calibration for a range of temperatures and a range of humidities. Neglect hysteresis.
3. Calibration for a range of temperatures and a range of humidities. Include a correction for hysteresis.

Finally the effectiveness of the error correction program was analyzed and the computer requirements of the on-line system were estimated.

1.5 Side Benefits of Digital System:

An instrumentation computer can be assigned other tasks in addition to the recalibration tasks. The computer can be made to control sampling rates, perform data logging functions and perform statistical or other analysis on data. In particular, many instruments require some preconversion of measured values. Software can be included for making the appropriate conversion between the variable measured and the desired variable for display. For example, when a fixed orifice plate and differential pressure transducer is used to measure flow, the measured variable is pressure; therefore, this pressure measurement has to be converted to flow indication using the pre-determined transfer function of the orifice plate.

2.0 Reduction of Simple, Deterministic Errors:

The errors to be discussed in this section are those caused by an imperfect instrument response and are not related to some parameter change (e.g. temperature) or to hysteresis. The most common of this type error are non-linearity and bias. These simple errors are the most prevalent inaccuracies in instrumentation and also the easiest to reduce. Techniques for reducing these simple errors are presented in this section. Additional techniques for handling more complicated situations will be presented in later sections.

The underlying concept for making the improvement is to (1) determine the actual or true response characteristics of the instrument, (2) analyze that data and store in a computer sufficient information to represent the true response, and (3) make improvement to on-line measurements by inputting instrument indications into the computer and having it calculate the "true" value based on the previously stored data.

The primary method used in this research to represent instrument response characteristics was based on polynomial equations derived from least square error analysis which is called regression analysis. This will be discussed below in detail. Another method based on the use of data tables was also considered and will be briefly discussed.

2.1 Calibration Data Needed for Error Correction:

A fundamental part of these error correction methods is the availability of data which indicates how the instrument responds to all signal values expected during operation. This data is obtained by applying an accurately known "true" signal to an instrument and recording the indicated or measured value. There should be many values of the true signal applied across the expected range of the instrument, and the associated

pairs of true and measured data should be recorded. This table of values, commonly called a calibration table, is the input to the off-line analysis program to be described below.

The amount, type, and complexity of the calibration data will depend on the extent of error correction. The simplest data will be needed for the correction of simple deterministic response errors as discussed in this section. This data consists of a table of pairs of numbers representing the true (or calibration) value and measured (or indicated) value from the instrument. The table is usually generated by varying the true signal and obtaining the measured value from the instrument. The number of points across the range of the instrument depends to some degree on the particular instrument being tested. If the calibration data points fall on a straight line then only a few points would be needed. If the data points form a more curved line, additional data points would be required to provide the information about the response. For example, an instrument which exhibits a "lower-noise", "more-linear" response might require only ten points whereas an instrument which has a "higher-noise", "less-linear" response might require fifty points across the measurand range.

There are two typical arrangements for performing calibration tests as shown in Figure 2.1. In one case a very accurate, variable signal source provides the input signal to the instrument to be calibrated. In the other case an inaccurate signal is measured by both the instrument to be calibrated and a very accurate instrument that provides the "true" value. Both arrangements can provide the information needed by this error reduction technique.

The accuracy of any instrument is dependent on the accuracy of the techniques used for calibration. Likewise, the error reduction obtainable

Figure 2.1. TWO TYPES OF CALIBRATION APPARATUS

by the techniques developed in the report is dependent on the accuracy of the calibration data. In this report, all error measurements are made with respect to the "true" signal as measured by the calibration instrument. Therefore, if the absolute accuracy is needed, the tolerance of the standard calibration instrument must be added.

2.1 Regression Analysis:

Regression analysis refers to the method of representing data points by a polynomial equations (e.g. $T = a_2m^2 + a_1m + a_0$) with the requirement that the equation minimize the square of the error (i.e., leastsquares).

One important parameter of regression analysis is the order of the polynomial equation to be used in the data representation. This work considered three possible forms:

- a. First order ($T = a_0 + a_1m$) which is a straight line
- b. Second order ($T = a_0 + a_1m + a_2m^2$) which in general yields a parabolic shaped curve.
- c. Third order ($T = a_0 + a_1m + a_2m^2 + a_3m^3$) which in general yields an "S" shaped curve.

High order equations could have been used, but for the type of instrument generally considered in this study, the third order curve should be sufficiently accurate, and the high order equation tend to add complications, increase computer storage requirements, and reduce speed of operation.

After the calibration data points are recorded as described previously, the coefficients of the polynomial equations can be calculated and an analysis of the best fitting curve can be made. The equation for finding the coefficients of the least square methods are presented below:

a. First Order

For first order equation, of the form $T = a_0 + a_1 m$ has the following equations

$$\begin{bmatrix} N & \sum_{i=1}^N M_i \\ \sum_{i=1}^N M_i & \sum_{i=1}^N M_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^N T_i \\ \sum_{i=1}^N M_i T_i \end{bmatrix}$$

N is number of data points (M_i, T_i) where M_i are measured values and T_i are true values. Using matrix inversion to solve for a_0 and a_1 ,

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} N & \sum_{i=1}^N M_i \\ \sum_{i=1}^N M_i & \sum_{i=1}^N M_i^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^N T_i \\ \sum_{i=1}^N M_i T_i \end{bmatrix}$$

b. Second Order

Similarly, for the second order equation of the form

$T = a_0 + a_1 m + a_2 m^2$, has the following equations

$$\begin{bmatrix} N & \sum_{i=1}^N M_i & \sum_{i=1}^N M_i^2 \\ \sum_{i=1}^N M_i & \sum_{i=1}^N M_i^2 & \sum_{i=1}^N M_i^3 \\ \sum_{i=1}^N M_i^2 & \sum_{i=1}^N M_i^3 & \sum_{i=1}^N M_i^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^N T_i \\ \sum_{i=1}^N M_i T_i \\ \sum_{i=1}^N M_i^2 T_i \end{bmatrix}$$

Using matrix inversion to solve for a_0, a_1, a_2

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} N & N \sum_{i=1}^N M_i & N \sum_{i=1}^N M_i^2 \\ \sum_{i=1}^N M_i & \sum_{i=1}^N M_i^2 & \sum_{i=1}^N M_i^3 \\ \sum_{i=1}^N M_i^2 & \sum_{i=1}^N M_i^3 & \sum_{i=1}^N M_i^4 \end{bmatrix}^{-1} \begin{bmatrix} N \sum_{i=1}^N T_i \\ N \sum_{i=1}^N M_i T_i \\ N \sum_{i=1}^N M_i^2 T_i \end{bmatrix}$$

c. Third Order

Similarly for third order equation of the form $T = a_0 + a_1 m +$

$a_2 m^2 + a_3 m^3$, one has the following equation.

$$\begin{bmatrix} N & N \sum_{i=1}^N M_i & N \sum_{i=1}^N M_i^2 & N \sum_{i=1}^N M_i^3 \\ \sum_{i=1}^N M_i & \sum_{i=1}^N M_i^2 & \sum_{i=1}^N M_i^3 & \sum_{i=1}^N M_i^4 \\ \sum_{i=1}^N M_i^2 & \sum_{i=1}^N M_i^3 & \sum_{i=1}^N M_i^4 & \sum_{i=1}^N M_i^5 \\ \sum_{i=1}^N M_i^3 & \sum_{i=1}^N M_i^4 & \sum_{i=1}^N M_i^5 & \sum_{i=1}^N M_i^6 \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \begin{bmatrix} N \sum_{i=1}^N T_i \\ N \sum_{i=1}^N M_i T_i \\ N \sum_{i=1}^N M_i^2 T_i \\ N \sum_{i=1}^N M_i^3 T_i \end{bmatrix}$$

Using matrix inversion to solve for a_0, a_1, a_2, a_3

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} N & N \sum_{i=1}^N M_i & N \sum_{i=1}^N M_i^2 & N \sum_{i=1}^N M_i^3 \\ \sum_{i=1}^N M_i & \sum_{i=1}^N M_i^2 & \sum_{i=1}^N M_i^3 & \sum_{i=1}^N M_i^4 \\ \sum_{i=1}^N M_i^2 & \sum_{i=1}^N M_i^3 & \sum_{i=1}^N M_i^4 & \sum_{i=1}^N M_i^5 \\ \sum_{i=1}^N M_i^3 & \sum_{i=1}^N M_i^4 & \sum_{i=1}^N M_i^5 & \sum_{i=1}^N M_i^6 \end{bmatrix}^{-1} \begin{bmatrix} N \sum_{i=1}^N T_i \\ N \sum_{i=1}^N M_i T_i \\ N \sum_{i=1}^N M_i^2 T_i \\ N \sum_{i=1}^N M_i^3 T_i \end{bmatrix}$$

2.2 Off-Line Program Description:

A computer program was written to make all three sets of computation (for first, second, and third order equations) and to determine how well the equation represented the data. These measures for the "quality of fit" of the equations were (1) mean of the error, (2) standard deviation of the error, and (3) maximum absolute error.

This program was written in FORTRAN IV and runs in batch mode. The program is based around matrix inversion and multiplication algorithms typically found in scientific subroutine packages. The primary input to the program are the pairs of data representing the values for true and measured quantities at each point. The outputs are the coefficients of the first, second, and third order equations and the quality of fit measures just discussed.

This program performs the off-line analysis for the simple error reduction methods as well as the more complicated error reduction methods described later. Because it is applicable to all three techniques, it is described in detail in Appendix I.

2.3 Results of Regression Analysis:

In order to test this approach, experiments were run on a Weston 341 voltmeter. This is a portable, multi-range, analog voltmeter with a mirror on the meter face for improved readability. Nominal accuracy is 1%, but the voltmeter had not been calibrated in many years and had seen considerable use. The "true" voltage was measured with an HP 3465 voltmeter. This is a 5 place, digital voltmeter which had been recently calibrated and has a specified accuracy of 0.2% of reading + 0.1% range.

The data shown in Table 2.1 was one set of data taken at various temperatures and humidities and hysteresis conditions. The meter response was dependent on all these conditions, but to experiment with simple error correction the other factors must be negligible. Therefore, this data represents the typical response of an instrument, with negligible hysteresis characteristics, and at a constant temperature and humidity.

The results of the regression analysis on this data is as follows:

First Order regression:

$$T = 0.3617418 + 0.9156691 M$$

Second Order regression:

$$T = 0.3234549 + 0.9450922 M - 0.0036781 M^2$$

Third Order regression:

$$T = 0.2923775 + 0.9936295 M - 0.018961 M^2 + 0.0012723 M^3$$

A tabulation of the results of using these equations to calculate the true value given the measured value is shown in Table 2.2. These are also plotted as per cent of full scale in Figure 2.2. Compare the quality of measurement indicators (i.e. mean, standard deviation, and max. absolute error) for the original measurements and the improved ones. Note that with each higher ordered equation, the error indicators are improved.

If the maximum absolute error, as a percentage of full scale, is used to indicate meter accuracy, then the following data is found.

<u>Method</u>	<u>Max. Abs. Error % F.S. (8 volts)</u>
incorrect measurement	3.7500%
first order regression	0.9435%
second order regression	0.6400%
third order regression	0.2583%

TABLE 2.1. Calibration Data for Simple Error Correction

<u>True Value</u>	<u>Measured Value</u>	<u>Error</u>
7.70000	8.00000	0.30000
7.50000	7.81000	0.31000
7.30000	7.60000	0.30000
7.10000	7.37000	0.27000
6.90000	7.15500	0.25500
6.70000	6.94000	0.24000
6.50000	6.73500	0.23500
6.30000	6.50500	0.20500
6.10000	6.27000	0.17000
5.90000	6.05500	0.15500
5.70000	5.84000	0.14000
5.50000	5.62500	0.12500
5.30000	5.39500	0.09500
5.10000	5.17000	0.07000
4.90000	4.95000	0.05000
4.70000	4.73500	0.03500
4.70000	4.73500	0.03500
4.50000	4.49000	-0.01000
4.30000	4.26500	-0.03500
4.10000	4.05000	-0.05000
3.90000	3.84000	-0.06000
3.70000	3.63000	-0.07000
3.50000	3.40000	-0.10000
3.30000	3.17000	-0.13000
3.10000	2.96500	-0.13500
2.90000	2.75000	-0.15000
2.70000	2.53500	-0.16500
2.50000	2.32000	-0.18000
2.30000	2.09500	-0.19500
2.10000	1.87000	-0.23000
1.90000	1.66000	-0.24000
1.70000	1.45000	-0.25000
1.50000	1.25400	-0.25500
1.30000	1.02500	-0.27500
1.10000	0.82500	-0.27500
0.90000	0.61500	-0.28500
0.70000	0.41000	-0.29000
0.50000	0.21000	-0.29000
0.20000	0.01500	-0.28500

MEAN = 0.02667
 STANDARD DEVIATION = 0.20075
 MAX. ABS. ERROR = 0.3000

TABLE 2.2. Results of Regression Analysis for Simple Error Correction.

First Order Regression
 $T = 0.36174 + 0.91567 M$

Second Order Regression
 $T = 0.32345 + 0.94509 M$
 $- 0.003678 M^{**2}$

Third Order Regression
 $T = 0.29238 + 0.99363 M$
 $- 0.01896 M^{**2} + 0.001272 M^{**3}$

Calculated Value	Error	Calculated Value	Error	Calculated Value	Error
7.68709	-0.01291	7.64880	-0.05120	7.67934	-0.02066
7.51312	0.01312	7.48028	-0.01972	7.50219	0.00219
7.32083	0.02083	7.29371	-0.00629	7.30729	0.00730
7.11022	0.01022	7.08900	-0.01100	7.09486	-0.00514
6.91335	0.01335	6.89729	-0.00271	6.89715	-0.00285
6.71649	0.01649	6.70524	0.00524	6.70022	0.00022
6.52877	0.02877	6.52181	0.02181	6.51309	0.01309
6.31817	0.01817	6.31564	0.01564	6.30382	0.00382
6.10299	0.00299	6.10459	0.00459	6.09064	-0.00936
5.90612	0.00612	5.91114	0.01114	5.89609	-0.00391
5.70925	0.00925	5.71735	0.01735	5.70192	0.00192
5.51238	0.01238	5.52322	0.02322	5.50805	0.00805
5.30178	0.00178	5.31517	0.01517	5.30092	0.00092
5.09575	-0.00425	5.11127	0.01127	5.09846	-0.00154
4.89430	-0.00570	4.91154	0.01154	4.90057	0.00057
4.69743	-0.00257	4.71600	0.01600	4.70717	0.00717
4.69743	-0.00257	4.49277	-0.00723	4.48669	-0.01331
4.47310	-0.02690	4.28737	-0.01263	4.28401	-0.01599
4.26707	-0.03293	4.09075	-0.00925	4.09009	-0.00991
4.07020	-0.02980	3.89837	-0.00163	3.90037	0.00037
3.87791	-0.02209	3.70567	0.00567	3.71026	0.01026
3.68562	-0.01438	3.49325	-0.00575	3.50154	0.00154
3.47502	-0.02498	3.28244	-0.01756	3.29218	-0.00782
3.26441	-0.03559	3.09332	-0.09332	3.10496	0.00496
3.07670	-0.02330	2.89464	-0.00536	2.90793	0.00793
2.87983	-0.02017	2.69563	-0.00437	2.71011	0.01011
2.68296	-0.01704	2.49627	-0.00373	2.51143	0.01143
2.48609	-0.01391	2.28728	-0.01272	2.30251	0.00251
2.38079	-0.01923	2.07792	-0.02208	2.09248	-0.00752
2.07404	-0.02596	1.88217	-0.01783	1.89537	-0.00463
1.88175	-0.01825	1.68611	-0.01389	1.69715	-0.00285
1.68946	-0.01054	1.49439	-0.00561	1.50251	0.00251
1.50175	0.00175	1.28831	-0.01169	1.29230	-0.00770
1.30030	0.00030	1.10065	0.00065	1.09993	-0.00007
1.11717	0.01717	0.90330	0.00330	0.89658	-0.00342
0.92488	0.02488	0.71032	0.01032	0.69667	-0.00333
0.73717	0.03717	0.52176	0.02176	0.50022	0.00022
0.55403	0.05403	0.33763	0.03763	0.30728	0.00728
0.37548	0.07548				
MEAN	= 0.00003	MEAN	= - .00001	MEAN	= .00000
S.D.	= 0.02349	S.D.	= .01624	S.D.	= 0.01488
MAX. ABS.	= 0.07480	MAX. ABS.	= 0.05120	MAX. ABS.	= 0.02066

Figure 2.2. PLOT OF ERROR REDUCTION USING REGRESSION EQUATIONS.

2.4 On-Line Program for Simple Error Correction:

A program was written to run in an on-line mode to make the error correction using one of the regression equations. The purpose was to measure the amount of memory required and the speed of operation when implemented on this type of hardware.

The flow chart of the program is shown below:

The program was implemented on a Cromemco Z-2D microcomputer. This is an S-100 bus, 8 bit computer built around the Z80 microprocessor. The program was written in a high level language (FORTRAN) instead of assembler language. A copy of the program is shown in Figure 2.3.

2.5 Table Look-up and Interpolation Method:

Another method, that could be used instead of the regression equations, for making simple error corrections involves saving the information about the instrument response characteristics by the set of data points

Simple Error Correction Program (2nd Order Regression)

```
WRITE(5,100)
DO 15 I= 1,1000
X=INP(25)/12.5
Y=0. 3234549 + 0.9450922*X - 0.0036781*X*X
WRITE(5,101)X,Y
15 CONTINUE
100 FORMAT(1H0,5X,4HVMES,3X,6HCORVAL)
101 FORMAT(1H0,5X,F7,3,3X,F7,3)
END
```

Example of Program Run

A
[1047 27DC 39]
[BEGIN EXECUTION]

VMES	CORVAL
0.160	.475
0.960	1.227
1.600	1.826
2.000	2.199
2.160	2.348
2.400	2.570
2.480	2.645
2.800	2.941
3.040	3.163
3.120	3.236
3.440	3.531

Figure 2.3. SIMPLE ERROR CORRECTION PROGRAM

(measured, true) taken during the calibration testing. The correction to a measured value is then made by finding the two points in the table that bound the measured value and using linear interpolation to find the true value for the actual measure values.

The major advantages of this method are associated with the fewer arithmetic operations required as compared to the second and third order regression equations. Interpolation requires only one division and one multiplication which is faster to perform and does not have as many digits to be carried for accuracy. In addition, interpolation can be used with any response curve including ones which have very complex variations; whereas; the third order regression will at most handle "S" shaped curves.

The disadvantages with this method is that the calibration data points must be taken extremely careful because the corrected value is based on only the two bounding points and any error in those two points will show up directly. Another way of stating this is the many calibration data points must be on, or very close to, a smooth curve. In the regression analysis method, the calibration data points can have some random error without adversely effecting the final regression equations.

For this work, the regression methods were found to be much preferred because calibration data was found to contain some random error. In addition, the use of a FORTRAN compiler greatly simplified all arithmetic operations thereby removing the need for simpler mathematics. Because of these factors, the complete development of the table look-up and interpolation method was not made in this research.

3.0 Error Reduction Including Measurement Parameters:

As discussed in the introductory section, many instruments have errors due to changes in quantities other than the quantity being measured. For example, if voltage is being measured, the accuracy of the voltmeter sometimes depends on the temperature and humidity of the environment. Other examples of these error dependent quantities are viscosity of a fluid in some flow measurements and excitation voltage in some pressure measurements. For lack of a better term, these non-measurand, error dependent quantities will be called measurement parameters.

For any given instrument and operating condition, there can be any number of measurement parameters from none to many. Usually there are only one or two which have significant effect. This work has concentrated on developing a system which can handle one or two parameters, but the technique could be easily extended to three or more.

The general method to be utilized in correcting these errors is to

- (1) gather data about the operation or response of the instrument (i.e. calibration curves) for many different values of the measurement parameters.
- (2) Analyze the sets of data with an off-line, regression analysis program that provides the best set of equations to represent the calibration curves at the various parameters values.
- (3) Use an on-line program which stores the coefficients of the regression equations, accepts as input a measured value plus the then current values of the measurement parameters, and determines the improved ("true") value for the measurement.

The following section will explain each of these steps and will give results of experiments made on the developed system.

3.1 Calibration Data Needed for Measurement Parameter Corrections:

The data needed to make corrections to instruments under different parameter conditions are essentially the same type of calibration data needed for simple error corrections described previously but a complete calibration test is needed for many sets of parameter conditions.

As an example, consider the case where the measurement parameters are temperature and humidity of a voltmeter. It was determined that the range of these parameters under expected operating conditions would be as shown below:

Temperature Range 10° - 50°C

Humidity Range 30% - 60% Rel. Hum.

Data points (i.e. true and measured) for a calibration curve will therefore be needed for sets of conditions within this range. Possible sets are shown below.

<u>Calibration</u> <u>Curve #</u>	<u>Temp.</u>	<u>Relative</u> <u>Humidity</u>	<u>Calibration</u> <u>Curve #</u>	<u>Temp.</u>	<u>Relative</u> <u>Humidity</u>
1	10	30	11	30	50
2	10	40	12	30	60
3	10	50	13	40	30
4	10	60	14	40	40
5	20	30	15	40	50
6	20	40	16	40	60
7	20	50	17	50	30
8	20	60	18	50	40
9	30	30	19	50	50
10	30	40	20	50	60

In order to take this data, the voltmeter would have to be placed in a temperature and humidity controlled chamber and the proper calibration instrumentation, as described in a previous section, would have to be connected. After the temperature and humidity settings of the chamber are made, some time is to be allowed while the chamber and voltmeter stablized.

(A calibration curve across the full range of the instrument would then be run.) These calibration data points, and the temperature and humidity setting would then be inputted to the off-line analysis program.

It should be noted that this data taking could be largely automated, although no attempt to do so in this research. A computer could be interfaced to the environmental chamber as well as the calibration instruments so as to significantly reduce the labor required.

3.2 Off-Line Regression Analysis Program:

The analysis needed for these calibration data sets is performed by the off-line regression analysis program described in Appendix I. This program performs a first, second, and third order regression analysis for each data set (i.e. each parameter condition). It outputs the coefficients of the regression equation as well as the measures of quality of fit for each data set. For example, in the case of the voltmeter tested at 20 different combinations of temperature and relative humidity, there would be 20 complete analyses performed.

The user could then decide which equations to use in the on-line system. This decision is based on the same factors as in the simple error correction case. That is, a trade off between better error reduction with higher order regression equations, versus more computer memory and slower speed of operation.

The coefficients of the chosen regression equations are then put into the on-line program for error reduction computations.

3.3 Error Correction Technique:

The technique for correcting errors due to changes in measurement parameter is based on storing regression equation representing the instrument response at certain given parameter values, and then interpolating between these equations for other values of the measurement parameters.

There are two types of data needed to make the error corrections involving measurement parameters

TYPE 1.

Regression coefficients for calibration curves and the associated parameter values. (These come from the off-line regression analysis program.)

TYPE 2.

Measurement to be corrected and the then current values of parameters. (These come from the instruments/sensors in the plant.)

After the Type 1 data are loaded into the on-line program, measured values of the then current parameters along with the measurement to be corrected are fed into the computer. The correction to the measurand is then made using the following algorithm.

Step 1. The two bounding values (i.e. higher and lower) for each of the measurement parameters are found by searching a table of the Type 1 parameter values stored in the computer. This identifies the regression equations which are closest to the current conditions. As an example consider the following case.

Assuming the previous table of parameters (relative humidity and temperature) values and the following measured quantities have been determined:

Instrument measurement (m) = 6.21 volts

Current Measurement Parameter #1 ($P1_c$) = 40%

Current Measurement Parameter #2 ($P2_c$) = 12.5°C

The bounding parameter values, and their associated regression equation as found to be:

Higher Parameter #1 ($P1_H$) = 50%

Lower Parameter #1 ($P1_L$) = 40%

Higher Parameter #2 ($P2_H$) = 20°C

Lower Parameter #2 ($P2_L$) = 10°C

Figure 3.1 illustrates these four bounding regression equations.

Step 2. For each of the bounding regression equations, the "true" value corresponding to the measured value is calculated. These represent the corrected measurand if the parameters had been exactly the value used during calibration testing.

In our example (Figure 3.1), these are found as following:

<u>Bounding parameter conditions</u>		<u>Calculated from regression equations</u>
$T(P1_H, P2_H) = T(50\%, 20^\circ)$	=	6.76 volts
$T(P1_H, P2_L) = T(50\%, 10^\circ)$	=	6.44 volts
$T(P1_L, P2_H) = T(40\%, 20^\circ)$	=	6.68 volts
$T(P1_L, P2_L) = T(40\%, 10^\circ)$	=	6.32 volts

Step 3. In order to find the corrected value at the then current value of the parameters, a series of linear interpolations are performed. If only one parameter were involved, only one interpolation would be needed. If two parameters are used, two sets of interpolations are performed. The interpolations for two parameters are as follows:

Figure 3.1. ERROR CORRECTION INCLUDING MEASUREMENT PARAMETERS.

For constant Parameter #1 at the "high" value:

$$T(P1_H) = T(P1_H, P2_L) + \frac{(P2_C - P2_L)}{(P2_H - P2_L)} (T(P1_H, P2_H) - T(P1_H, P2_L))$$

for example:

$$\begin{aligned} T(50\%) &= T(50\%, 10^\circ) + \frac{(12.5^\circ - 10^\circ)}{(20^\circ - 10^\circ)} (T(50\%, 20^\circ) - T(50\%, 10^\circ)) \\ &= 6.44 + \frac{2.5}{10} (6.76 - 6.44) \\ &= 6.52 \text{ volts} \end{aligned}$$

Likewise of Low Parameter #1:

$$T(P1_L) = T(P1_L, P2_L) + \frac{(P2_C - P2_L)}{(P2_H - P2_L)} (T(P1_L, P2_H) - T(P1_L, P2_L))$$

for example:

$$\begin{aligned} T(40\%) &= T(40\%, 10^\circ) + \frac{(12.5^\circ - 10^\circ)}{(20^\circ - 10^\circ)} (T(40\%, 20^\circ) - T(40\%, 10^\circ)) \\ &= 6.32 + \frac{2.5}{10} (6.68 - 6.32) \end{aligned}$$

Then interpolate between these two points using Parameter #1 to find final corrected (true) value.

$$T = T(P1_L) + \frac{(P1_C - P1_L)}{(P1_H - P1_L)} (T(P1_H) - T(P1_L))$$

For example:

$$\begin{aligned} T &= T(40\%) + \frac{(40\% - 40\%)}{(50\% - 40\%)} (T(50\%) - T(40\%)) \\ &= 6.41 + \frac{6}{10} (6.52 - 6.41) \\ &= 6.476 \text{ volts} \end{aligned}$$

3.4 Description of On-line Program:

Software to implement the measurement parameter error correction technique was written in FORTRAN to run on the Cromemco Z-2D microcomputer. A simplified flow chart is shown in Figure 3.2. A listing and description of the program and a complete description of input and output is contained in Appendix II.

This program was written so that all data was entered from a keyboard terminal, but for on-line operation, the input device would more likely be an analog to digital converter connected to sensors or instruments measuring the needed quantities. There are other modifications that could be made to the program depending on the specific application. For example, the parameter measurements would not necessarily be made as frequently as the corrected quantity because they might not change as rapidly. Modification to fit this specific circumstance can easily be made, once the conditions are specified.

An example of the printout for this program is shown in Table 3.1.

3.5 Experimental Results

To test the system, the Weston voltmeter, Model 341 was placed inside a temperature and humidity controllable environmental chamber. Calibration tests* were run for various pairs of temperature and relative humidity. The off-line program was used to analyze this data and produced the results shown in Table 3.2.

To test the error correction involving measurement parameters the following test was performed.

*These are all the "up" portion of the calibration curve. This was taken so that the hysteresis effect would be eliminated.

Figure 3.2. SIMPLIFIED FLOW CHART OF THE ON-LINE MEASUREMENT PARAMETER ERROR REDUCTION PROGRAM

TABLE 3.1. Print-out of On-line Program for Measurement Error Reduction.

----- SET OF COEFFICIENTS -----

TEMP (C)	HUMID (%)	REG ORDER	A	B	C	D
10.0	30.0	3	.0023520	-.0393295	1.1319580	-.0598450
10.0	40.0	3	.0010762	-.0228920	1.0779870	-.0120773
10.0	50.0	3	.0012228	-.0252228	1.0892020	-.0278397
10.0	60.0	3	.0009954	-.0223160	1.0816950	-.0073853
20.0	30.0	3	.0018966	-.0322456	1.1004180	-.0162964
20.0	40.0	3	.0009954	-.0223160	1.0816950	.0073853
20.0	50.0	3	.0007584	-.0198326	1.0776060	.0098419
20.0	60.0	3	.0002139	-.0111847	1.0396570	.0289383
30.0	30.0	3	.0021663	-.0360031	1.1154630	.0445175
30.0	40.0	3	.0010762	-.0228920	1.0779870	.0120773
30.0	50.0	3	.0010021	-.0221367	1.0778190	-.0214539
30.0	60.0	3	.0018966	-.0322456	1.1004180	.0162964

----- RESULTS -----

MEASURED VALUE	CORRECTED VALUE	TEMPERATURE	HUMIDITY
0.000	.002	25.00	45.00
.800	.852	25.00	45.00
1.600	1.676	25.00	45.00
2.400	2.479	25.00	45.00
3.200	3.262	25.00	45.00
4.000	4.030	25.00	45.00
4.800	4.784	25.00	45.00
5.600	5.528	25.00	45.00
6.400	6.265	25.00	45.00
7.200	6.997	25.00	45.00
8.000	7.728	25.00	45.00

TABLE 3.2 Off-Line Analysis of Parameter Dependent Data

Parameter Conditions		Error in Calibration Data			Result of Regression Analysis Regression Equations (Listed in order, 1st, 2nd, 3rd)	Quality of Fit of Regression Equation (in Terms of Error)		
Temp. (°C)	Hum. (%)	Mean	S.D.	Max.		Mean	S.D.	Max.
10	30	-.1030	.2097	.38	T = 0.4597 + 9170M T = 0.4332 + .9355M + .00215M ² T = 0.3913 + .9931M - .01883M ² + .00129M ³	.0000 .0000 .0001	.0268 .0241 .0192	.0454 .0384 .0327
10	40	-.0890	.2144	.38	T = 0.4545 + .9152M T = .4538 + .9157M - .00006M ² T = .3883 + 1.0056M - .02608M ² + .00201M ³	.0000 .0000 .0001	.0258 .0258 .0122	.0407 .0384 .0200
10	50	-.1110	.2367	.37	T = .5101 + 9070M T = .4871 + .9233M - .00190M ² T = .4414 + .9878M - .02070M ² + .00145M ³	.0000 .0000 .0001	.0210 .0182 .0061	.0456 .0285 .0128
10	60	-.145	.2205	.23	T = .5123 + .9137M T = .4934 + .9272M - .00159M ² T = .4611 + .9730M - .01495M ² + .001038M ³	.0000 .0000 .00016	.0386 .0376 .03554	.0512 .0533 .0394
20	30	-.022	.166	.20	T = .3045 + .9355M T = .3680 + .8924M + .00496M ² T = .2707 + 1.0193M - .03147M ² + .00282M ³	.0000 .0000 -.00003	.0494 .0422 .0300	.0786 .0740 .0397
20	40	-.031	.1688	.20	T = .3173 + .9344M T = .3658 + .9015M + .0038M ² T = .2624 + 1.0365M - .0350M ² + .0030M ³	.0000 .0000 -.00009	.0503 .0463 .03385	.0848 .0812 .0444

TABLE 3.2 Off-Line Analysis of Parameter Dependent Data

(Continued-Page 2)

Parameter Conditions		Error in Calibration Data			Result of Regression Analysis Regression Equations (Listed in order, 1st, 2nd, 3rd)	Quality of Fit of Regression Equation (in Terms of Error)		
Temp. (°C)	Hum. (%)	Mean	S.D.	Max.		Mean	S.D.	Max.
20	50	-.042	.1664	.20	$T = .3218 + .9358M$	.0000	.0534	.0980
					$T = .3612 + .9090M + .0030M^2$	.0000	.0510	.0953
					$T = .2545 + 1.0481M - .0369M^2 + .0031$	-.00015	.0391	.0575
20	60	-.047	.1664	.16	$T = .3312 + .9347M$	.0000	.0440	.0535
					$T = .3325 + .9338M + .00009M^2$	.0000	.0440	.0534
					$T = .2537 + 1.0348M - .0286M^2 + .0022M^3$	.00008	.0370	.0526
30	30	.076	.2156	.40	$T = .2859 + .9191M$	.0000	.0752	.1088
					$T = .3734 + .8611M + .0066M^2$	.0000	.0668	.0965
					$T = .1945 + 1.0847M - .0559M^2 + .0047M^3$	.00005	.0438	.0904
30	40	.089	.2212	.42	$T = .2787 + .9180M$	.0000	.0832	.1137
					$T = .3844 + .8479M + .00793M^2$	.0000	.0719	.1057
					$T = .2032 + 1.0747M - .0554M^2 + .0048M^3$	.00012	.0507	.1011
30	50	.05	.2028	.38	$T = .2943 + .9226M$	.0000	.0634	.0878
					$T = .3341 + .8963M + .0029M^2$	.00001	.06142	.0990
					$T = .2042 + 1.0591M - .0426M^2 + .0035M^3$	.00016	.0494	.0930
30	60	.059	.1817	.36	$T = .2260 + .9360M$	.0000	.0918	.1987
					$T = .222 + .9387M - .00029M^2$	-.00001	.0918	.1982
					$T = .1723 + .9994M - .0173M^2 + .00129M^3$	-.00009	.0908	.2087

TABLE 3.2 Off-Line Analysis of Parameter Dependent Data

(Continued-Page 3)

Parameter Conditions		Error in Calibration Data			Result of Regression Analysis Regression Equations (Listed in order, 1st, 2nd, 3rd)	Quality of Fit of Regression Equation (in Terms of Error)		
Temp. (°C)	Hum. (%)	Mean	S.D.	Max.		Mean	S.D.	Max.
40	30	.186	.271	.6	$T = .2632 + .902M$	.0000	.1061	.1657
					$T = .4066 + .8082M + .0104M^2$	.0000	.0899	.1452
					$T = .1785 + 1.0878M - .06625M^2 + .00572M^3$	-.00001	.0642	.1478
40	40	.178	.2655	.6	$T = .2621 + .9038M$	.0000	.1043	.1576
					$T = .3974 + .8155M + .0098M^2$	.0000	.0897	.1560
					$T = .1949 + 1.0631M - .05799M^2 + .00505M^3$	.00004	.0702	.1582
40	50	.171	.2579	.58	$T = .2597 + .9057M$	.0000	.0974	.1396
					$T = .3848 + .8241M + .00907M^2$	.0000	.0842	.1462
					$T = .1803 + 1.0729 - .05897M^2 + .00507M^3$	.00002	.0631	.1465
40	60	.16	.2559	.55	$T = .2775 + .904M$	.0000	.0780	.1018
					$T = .3759 + .8398M + .00712M^2$	.0000	.0676	.1132
					$T = .2023 + 1.0512M^2 + .0043M^3$	-.00001	.0474	.107
50	30	.226	.2756	.61	$T = .2442 + .8984M$	.0000	.0920	.1242
					$T = .3930 + .8023M + .0106M^2$	.0000	.0714	.1044
					$T = .1868 + 1.050M - .0563M^2 + .0049M^3$	-.00007	.0441	.0998
50	40	.219	.2918	.6	$T = .2765 + .8927M$	.0000	.0951	.1221
					$T = .4400 + .7856M + .0118M^2$	.00001	.0687	.0887
					$T = .2381 + 1.036M - .0566M^2 + .0050M^3$	-.00003	.0355	.0838

TABLE 3.2 Off-Line Analysis of Parameter Dependent Data

(Continued-Page 4)

Parameter Conditions		Error in Calibration Data			Result of Regression Analysis Regression Equations (Listed in order, 1st, 2nd, 3rd)	Quality of Fit of Regression Equation (in Terms of Error)		
Temp. (°C)	Hum. (%)	Mean	S.D.	Max.		Mean	S.D.	Max.
50	50	.185	.2917	.57	$T = .3250 + .8888M$	.0000	.0494	.0577
					$T = .4048 + .8366M + .0057M^2$	.0000	.0367	.0525
					$T = .3018 + .9653M - .0294M^2 + .00259M^3$	-.00014	.0188	.0437
50	60	.146	.2923	.6	$T = .3668 + .8872M$	.0000	.0097	.0205
					$T = .3680 + .8864M + .000088M^2$	-.00001	.0097	.01995
					$T = .3540 + .9038M - .0047M^2 + .000348M^3$	-.00017	.0086	.0179

1. The on-line program was loaded with third order regression coefficients for pairs of parameter conditions which bounded another condition. These sets are shown below:

<u>Test Set 3</u>	<u>Bounding Parameter Conditions</u>				<u>Bounded Conditions</u>	
	<u>Temp</u>	<u>Rel. Hum.</u>	<u>Temp</u>	<u>Rel.Hum.</u>	<u>Temp</u>	<u>Rel. Hum.</u>
A	10	30%	30	50%	20	40%
B	10	40%	30	60%	20	50%
C	20	30%	40	50%	30	40%
D	20	40%	40	60%	30	50%
E	30	30%	50	50%	40	40%
F	30	40%	50	60%	40	50%

2. Measurand and parameter values from the bounded condition data were then utilized as inputs to the on-line program.

3. The outputs of the on-line program would then be analyzed to measure the effectiveness of the correction technique. Comparisons were made between (a) the corrected value including parameters and the true value from the original data, (b) four sets of corrected values without parameter correction based on regression equations of bounding conditions and the true values from the original data. The results (see Table 3.3) show that the mean error with parameter corrections is considerably less than the mean error without parameter corrections except at two of the possible 24 bounding conditions.

Note that if no parameter corrections were included for a real-world application, there would be only one temperature and humidity condition for making comparisons (the condition existing when the calibration data was taken) which would not always be the closest condition to the test conditions. In that case the errors would be expected to increase as the parameter values become further away from the calibration condition. The test conditions used in this report were used to evaluate the performance for the worst case conditions.

TABLE 3.3. Experimental Results of Measurement Parameter Error Reduction Techniques

Parameter Conditions		Results of Para. Error Reduction	Results of Simple Error Reduction Program for Closest Parameter Conditions			
Temp.	Rel. Hum.	Error Mean (volts)	Higher Temp. Higher Rel. Hum.	Lower Temp. Higher Rel. Hum.	Higher Temp. Lower Rel. Hum.	Lower Temp. Lower Rel. Hum.
			Error Mean (volts)	Error Mean (volts)	Error Mean (volts)	Error Mean (volts)
20°	40%	-.010	-.077	.073	-.102	.067
20°	50%	-.021	-.096	.095	-.125	.043
30°	40%	.015	-.078	.125	-.093	.107
30°	50%	-.013	-.103	.092	-.120	.078
40°	40%	.041	-.009	.121	-.046	.098
40°	50%	.040	.022	.015	-.046	.080

4.0 Reduction of Hysteresis Error:

4.1 Description of Hysteresis Error:

Instrument errors due to the hysteresis effects are caused by friction or damping of stressed parts internal to the instruments. Sometimes external friction and dead space effects are also included as part of the hysteresis.

The hysteresis characteristic of an instrument being calibrated or tested is illustrated by Figure 1.1.

In this calibration test, the measurand is increased from 0% to 100% of full scale value and the output or observations are expressed as % of full scale at each observation point. Care must be taken not to decrease the measurand at any point during this increasing portion of the test procedure. Also, if external friction effects are not to be included a gentle mechanical vibration or "dithering" should be applied before each observation. Next the measurand is decreased from its 100% value down to 0%. Care must be taken not to increase the measurand at any point. The hysteresis effect is evidenced by the fact that the curve for the increasing measurand is different from the curve for the decreasing measurand.

In specifying the hysteresis error of a given instrument the maximum width of the hysteresis loop is given as indicated on Figure 1.1. This may give an indication of the instrument quality but does not give the error for a particular observation. This is a maximum error.

Note the partial-range hysteresis loop of Figure 1.1. In this small loop the true value excursion was from 0 to 30% to 0. The maximum hysteresis error here is much smaller than for 100% excursion. Figure 4.1 shows partial range hysteresis loops for the instrument under test in this work, the Weston Model 341. Here the measurand was varied from 0 to 30% to 0

for one data set and from 0 to 55% to 0 in the other data set. Note that the curve is not an input-output curve but an input-% error curve.

Figure 4.2 shows partial range hysteresis loops for the same instrument but the measurand was varied from 100% - 62.5% - 100% - 25% - 100%.

Figure 4.3 shows partial range hysteresis loops for the same instrument where the measurand was varied from 40% - 60% - 40% and 25% - 75% - 25%.

Clearly, from Figures 4.1, 4.2, and 4.3 the hysteresis error in an observation depends on whether the measurand is increasing or decreasing and on the values of the measurand in the period prior to the observation. In Figure 4.3 in particular, for a value of the measurand M_1 , the observation error depends on whether one is on the increasing or decreasing portion of the curve and also whether one is on partial loop A, B, or C. It should be noted that the partial loops shown here represent symmetric excursions about the 50% full scale value. Excursions about other points have similar characteristics. That is, small excursions give narrow loops or small errors.

The maximum hysteresis error would occur when the observation is made after the measurand has made an excursion from the zero value, or the 100% value, to the current state. This is the least probable measurement condition. Figure 4.4 illustrates more typical measurand variations. The normal measurand fluctuations around some nominal values are shown in the range from a to c and d to e. In these ranges the hysteresis error is limited to that defined by the small partial loops similar to those shown in Figure 4.3. If, however, an observation is made at point d of Figure 4.4 after an excursion from point c, then the error is limited by a larger loop.

Hysteresis Curves of
Weston Model 341 Measurand
Range

A 0 - 100 - 0%

B 0 - 55 - 0%

C 0 - 30 - 0%

Figure 4.1. PARTIAL HYSTERESIS LOOPS

Hysteresis Curves of
Weston Model 341 Measurand
Range

- A 100 - 0 - 100%
- B 100 - 62.5 - 100%
- C 100 - 25 - 100%

Figure 4.2. PARTIAL HYSTERESIS LOOPS

Figure 4.3. PARTIAL HYSTERESIS LOOPS

Figure 4.4. MEASURAND FLUCTUATION

When observations are made manually, care in timing of the observations to points such as b or e and not d may eliminate most of the hysteresis error. If there are large measurand fluctuations and observations are needed at points such as d of Figure 4.4, then it may be desirable to correct for hysteresis error.

It may be better to try to minimize or avoid hysteresis error in measurements than to try to make a correction for it. If it becomes necessary to make corrections for hysteresis error, care must be taken to assure that correction of calibration data are taken under conditions that closely match the measurand conditions expected when observations are to be made.

In a later section of this report, a method is outlined for correcting for hysteresis error.

4.2 An Intuitive but Infeasible Method for Complete Hysteresis Error Correction:

One method that might be considered for correcting hysteresis errors is to develop, in some way, equations that relate the measured value to the true value for all possible paths within a hysteresis loop. Then, correction could be performed by inputting the starting measured value, and every subsequent measured value, and keeping track of the paths followed within the hysteresis loop. An example of this proposed technique is outlined below and shown in Figure 4.5.

- a. Start at a known point (M_1, T_1).
- b. As measured value increases from M_1 to M_2 , the true value could be found from the equation of the outside hysteresis curve.

Figure 4.5. INFEASIBLE HYSTERESIS CORRECTION METHOD

- c. As the measured value decreases above from M_3 to M_4 , the true value could be found if the equation of the curve segment "A" is known.
- d. As the measured value increases above from M_3 to M_4 , the true values can be determined if the equation curve segment "B" is known.

This process could continue indefinitely by following the same general procedure.

The major problem with this method is developing the equations which represent the theoretically infinite, and practically very large, number of possible paths that could be traversed. In addition, the problems associated with error accumulation would have to be solved. These problems made it impossible to use this method for complete hysteresis correction.

4.3 General Approach for Partial Correction of Hysteresis Error:

A partial correction of hysteresis errors is, however, possible. This partial correction is based on the following characteristics of hysteresis curves.

1. The measured value always lags behind the true value. (i.e. when the true signal is increasing, the measured value is always lower than when the true signal is decreasing, the true signal is higher).
2. The amount of lag (error) is dependent on how much the signal has changed.

These two characteristics lead to the approach used in this research for partially correcting hysteresis errors. The approach is, first, to determine a corrected value for a given measured value using the techniques discussed previously which do not consider hysteresis, and then make an

improvement on that value based on the direction of change and amount of change from the last measured point.

4.4 Calibration Data Needed to Implement Approach:

In order to implement this approach, information about the hysteresis characteristics of the particular instrument and information about the previous measured values are required.

The instrument characteristic information is obtained from the calibration data used in the techniques discussed previously with the additional requirement that two separate data sets are taken for each parameter condition. One set for the "UP" hysteresis curve and one for the "DOWN" hysteresis curve. For example, in the case where there were 20 parameter conditions, there would be 20 sets of calibration data with each set consisting of a group of points taken while increasing the signal (i.e. UP) and a group of points taken while decreasing the signal (i.e. DOWN).

During the collection of the calibration data, the true signal value is begun at the lowest value of the instrument scale and then steadily increased to the full scale value. The test signal is then steadily decreased to the lowest value. There should be no reversal of direction of signal change during these tests.

4.5 Off-line Calibration Data Analysis:

As in the previous cases involving other error correction methods, the calibration data is used as input to the off-line regression analysis program as described in Appendix I. In fact, this off-line program requires both UP and DOWN calibration data even though no hysteresis is expected. This inputting for both UP and DOWN data prevents hysteresis errors from being overlooked.

The results of the off-line regression analysis is similar to that described previously but with additional information. For each parameter condition, a complete analysis is performed consisting of first, second, and third order regression equation for the UP data, DOWN data, and COMBINED data (i.e. UP and DOWN together). The combined equation represents the curve most closely fitting all points considered together. It is very near the average of the UP and DOWN curves.

In addition to the quality measures described previously, information about the amount of hysteresis for each curve is printed. This information consists of the largest hysteresis error, as measured by the maximum difference between UP and DOWN curves, and the point at which this occurs.

The user can decide whether or not to utilize the more complicated on-line hysteresis reduction program depending on the size of the hysteresis error. If the hysteresis error is small, then the additional hardware and slower computation speed would not be required. In this case, the coefficients of the COMBINED equation would be used in the on-line programs described in the previous section. If the hysteresis error is significant, the coefficients for all three UP, DOWN, and COMBINED equation will be needed in the on-line partial hysteresis correction program described later.

4.6 Algorithm for Partial Hysteresis Correction:

The data needed for partial hysteresis correction concerns the immediate past history of the instrument operation which can be determined from (1) the previous measured value and (2) the direction of movement at the previous value. This information is sufficient for a partial correction if the input signal has changed only slightly since the last data point; that is, the measurement sampling frequency is fast enough.

Before discussing the algorithm, the following definitions are needed.

UP: The UP curve is the regression analysis equation representing data points taken with an increasing signal. The notation UP_i will be used to represent a particular point on the curve.

DN: The DN curve is the regression analysis equation representing data points taken with a decreasing signal. The notation DN_i will represent a particular point on the curve.

CM: The CM (combined) curve is the regression analysis equation representing the combination of up and down data point, CM_i will represent a particular point on the curve.

M_i : The most recent (or current) measured value.

M_{i-1} : The measured value prior to M_i .

M_{i-2} : The measured value prior to M_{i-1} .

F_i : The first (or initial) estimate of the true signal for the M_i value.

B_i : The improved or "best" estimate of the true signal for the M_i value*.

*Note that the term "best" does not carry the mathematically rigorous meaning as in the Estimation Theory field. Instead, it means the final answer to be found using the techniques used in this research.

K: A constant nominally set to the Full Scale value of the instrument. However, this value can sometimes be modified to provide better results.

These curves and points are shown on a typical hysteresis curve in Figure 4.6.

Assuming the needed information is available, the algorithm is as follows:

<u>Step Number</u>	<u>Action</u>
1.	The algorithm is initialized (or reset) (a) when the first measurement is made (e.g. after turning on the power) or more often, whenever the direction of change at the current point is different from the direction of change at the previous point. That is, whenever the sign of $(M_i - M_{i-1})$ is different from $(M_{i-1} - M_{i-2})$. At this starting point, the "best" estimated for the true signal is the value determined from the COMBINED curve for the measured value. That is, $B_0 = CM_0$, for the value M_0 .

Figure 4.6. PARTIAL HYSTERESIS CORRECTION ALGORITHM

This has the effect of resetting the algorithm everytime the measurand changes direction so that the initial corrected value lies on the combined curve. This was done because the algorithm is not exact and, therefore, must be frequently reset. The combined curve is the best choice because (1) it would be the equation utilized if no attempt was being made to reduce hysteresis error and (2) on line instruments in most industrial applications typically fluctuate around some nominal value which means the hysteresis curves are generally small loops centered about the combined curve.

2. If the direction of change at a point (i) is the same as the direction of change at the previous point (i-1), the first value F_i is based on the previous value B_{i-1} .

$$F_i = UP_i - \frac{(UP_i - DN_i)}{(UP_{i-1} - DN_{i-1})} (UP_{i-1} - B_{i-1})$$

or equivalently,

$$F_i = DN_i + \frac{(UP_i - DN_i)}{(UP_{i-1} - DN_{i-1})} (B_{i-1} - DN_{i-1})$$

This step has the effect of maintaining the relative position across the hysteresis loop while moving the level up or down.

3. An improvement is then made based on the distance change since the last measurement.

For upward movement:

$$\frac{F_i - B_i}{UP_i - F_i} = \frac{M_i - M_{i-1}}{K}$$

Solving:

$$B_i = F_i + (UP_i - F_i) \frac{(M_i - M_{i-1})}{K}$$

(under no case can $F_i > UP_i$)

For downward movement:

$$\frac{B_i - F_i}{B_i - DN_i} = \frac{M_{i-1} - M_i}{K}$$

solving

$$B_i = F_i - (F_i - DN_i) \frac{(M_{i-1} - M_i)}{K}$$

(under no case can $F_i < DN_i$)

This step has the effect of improving the estimated "best" value based on the general characteristics of hysteresis curves. This is for increasing signals, the "best" value is shifted to the right and for decreasing signals, the "best" value is shifted to the left on a hysteresis curve.

An illustration of the algorithm is shown in Figure 4.7. The following discussion refers to that figure.

Measurement Number 1. Assuming the slope of the measurand has just changed, B_1 is found from the combined curve for input M_1 .

Measurement Number 2. The slope is the same as the previous step, so F_2 is calculated using algorithm equations which implement the following graphical proportionality: $\frac{C_2}{W_2} = \frac{D_1}{W_1}$

B_2 is then calculated: $B_2 = F_2 + I_2$

where: $\frac{I_2}{C_2} = \frac{M_2 - M_1}{K}$

Measurement Number 3. The slope is unchanged so F_3 is

calculated based on the graphical proportionality: $\frac{C_3}{W_3} = \frac{D_2}{W_2}$

B_3 is then calculated: $B_3 = F_3 + I_3$

where: $\frac{I_3}{C_3} = \frac{M_3 - M_2}{K}$

MEASUREMENT NUMBER	$\underline{m_i}$	$\underline{m_{i-1}}$	$\underline{F_i}$	$\underline{B_i}$
1	m_1	-	-	B_1
2	m_2	m_1	F_2	B_2
3	m_3	m_2	F_3	B_3

Figure 4.7. ILLUSTRATION OF PARTIAL HYSTERESIS CORRECTION

One of the elements of the algorithm is the constant K. This constant is nominally specified as the full scale value of the measurand, and has the same unit dimension as the measurand. However, the value of K can be adjusted for a given instrument so that, on the average, the corrected values from the algorithm more closely matches the instrument hysteresis characteristics. A discussion of the effect of different values of K will be presented in a later section on experimental results.

4.7 Description of Partial Hysteresis Correction On-Line Program:

This program should have a name like "On-line Program for Reducing Errors associated with Bias and Non-Linearity, Changes in Measurement Parameter, and Hysteresis" because it performs all three functions. It is referred to just as the "Partial Hysteresis Correction Program" because the previous programs did not perform that correction.

As with the other on-line programs, this one was written in FORTRAN to run on the Cromemco microcomputer.

A simplified flow chart of the program is shown in Figure 4.8 with a complete listing and description in Appendix II. The program stores all the regression equation coefficients for the three equations (UP, DOWN, COMBINED) representing the instrument response with hysteresis for all parameter conditions. The routine begins its normal operation by inputting the values for the measurand and two parameters. The exact method for inputting these will depend on the particular application. For normal on-line operation, they would be entered by way of an analog to digital converter. The next three steps in the program then calculate a "true" value from the three curves (UP, DOWN, COMBINED) representing the hysteresis response of the instrument. This is performed using the routine for interpolating between

Figure 4.8 A SIMPLIFIED FLOW CHART OF THE ON-LINE PARTIAL HYSTERESIS CORRECTION PROGRAM.

parameter conditions described in the measurement parameter error correction program. At this point in the program, the partial hysteresis error reduction routine begins. If the direction of change in the measurand for the last point was different than at the previous point, or if this is the first time the program has been executed since power up, the value calculated from the COMBINED equation is used as the best "TRUE" value. If, however, the direction of change in the measurand is the same as before, the partial hysteresis algorithm is performed. Finally the best "TRUE" value is outputted and a number of variables are saved for use at the next input point.

An example of a program run is shown in Figure 4.9.

4.8 Experimental Results:

To test the system, experimental data were collected for the Weston voltmeter, model 341. The temperature and humidity of the voltmeter were varied by placing the meter inside a temperature and humidity controllable chamber. Calibration curves representing the increasing and decreasing signal condition were obtained. The off-line program was run on the data and yielded the first, second and third order regression equation coefficients for the UP, DOWN, and COMBINED calibration data.

The third order regression equation coefficients were loaded into the on-line program and experimental tests were made. The results of these experiments are discussed below.

As a demonstration of the response of the partial hysteresis correction algorithm, the input signal was cycled between 40% and 60% and then between 25% and 75%. The results of these tests are shown in Figures 4.10.

A0 SET OF COEFFICIENTS

UP CURVE

TEMP (C)	HUMID (%)	REG ORDER	A ₀	A ₁	A ₂	A ₃
10.0	30.0	3	.0012918	-.0188274	.9930954	.3913116
10.0	40.0	3	.0020134	-.0260792	1.0056300	.3882675
10.0	50.0	3	.0014548	-.0206985	.9878693	.4413567
20.0	30.0	3	.0057892	-.0711690	1.1089000	.3974500
20.0	40.0	3	.0030096	-.0349884	1.0364990	.2624359
20.0	50.0	3	.0031083	-.0369244	1.0481570	.2544937
30.0	30.0	3	.0047464	-.0559578	1.0847850	.1945267
30.0	40.0	3	.0048039	-.0554008	1.0746920	.2032852
30.0	50.0	3	.0034707	-.0426559	1.0591200	.2042084

COMBINED CURVE

TEMP (C)	HUMID (%)	REG ORDER	A ₀	A ₁	A ₂	A ₃
10.0	30.0	3	.0014362	-.0176430	.9797058	.3280792
10.0	40.0	3	.0017457	-.0213223	.9903870	.3248291
10.0	50.0	3	.0015342	-.0175686	.9641190	.3589478
20.0	30.0	3	.0034794	-.0429290	1.0331000	.3878700
20.0	40.0	3	.0035357	-.0392227	1.0365450	.2171097
20.0	50.0	3	.0037262	-.0411453	1.0399780	.2163696
30.0	30.0	3	.0063326	-.0746078	1.1373600	.1164474
30.0	40.0	3	.0067375	-.0794601	1.1485140	.1077347
30.0	50.0	3	.0058968	-.0704155	1.1275940	.1195908

DOWN CURVE

TEMP (C)	HUMID (%)	REG ORDER	A ₀	A ₁	A ₂	A ₃
10.0	30.0	3	.0019355	-.0217247	.9896240	.2371597
10.0	40.0	3	.0018823	-.0225410	1.0009770	.2324600
10.0	50.0	3	.0022848	-.0244904	.9851456	.2231598
20.0	30.0	3	.0011210	-.0138070	.9542900	.3762000
20.0	40.0	3	.0043004	-.0465775	1.0487210	.1577072
20.0	50.0	3	.0045137	-.0474472	1.0398560	.1671800
30.0	30.0	3	.0081394	-.0961647	1.2015280	.0242538
30.0	40.0	3	.0090652	-.1085472	1.2408900	-.0078201
30.0	50.0	3	.0084927	-.0998840	1.2015230	.0255966

Coefficient Print-Out Section

Figure 4.9 EXAMPLE RUN OF ON-LINE PARTIAL HYSTERESIS PROGRAM

RESULTS

	MEASURED VALUE	CORRECTED VALUE	TEMPERATURE	HUMIDITY
K=2.				
VMES=3.6				
	3.600	3.710	20.00	30.00
VMES=3.680				
	3.680	3.785	20.00	30.00
VMES=3.76				
	3.760	3.861	20.00	30.00
VMES3.84				
	3.840	3.936	20.00	30.00
VMES=3.92				
	3.920	4.011	20.00	30.00
VMES=4.				
	4.000	4.086	20.00	30.00
VMES=4.08				
	4.080	4.161	20.00	30.00
VMES=4.16				
	4.160	4.236	20.00	30.00
VMES=4.24				
	4.240	4.310	20.00	30.00
VMES=4.32				
	4.320	4.385	20.00	30.00
VMES=4.4				
	4.400	4.459	20.00	30.00
VMES=4.48				
	4.480	4.533	20.00	30.00
VMES=4.56				
	4.560	4.607	20.00	30.00
VMES=4.64				

Measurement Error Correction Section

Figure 4.9 (cont.) EXAMPLE RUN OF ON-LINE PARTIAL HYSTERESIS PROGRAM

This plot shows that the outputs of the partial hysteresis corrections program have the general characteristics of hysteresis curves. Additional tests were then performed to appraise the effectiveness of the technique.

A determination of the effectiveness of the partial correction of hysteresis error is more difficult than with the previous error reduction methods. For the non-hysteresis type errors, there was only one true value for every measured value; therefore, the error associated with each measured value, was easily found. When an instrument has hysteresis, there are an infinite number of true values for every measured value. That is, for any given measured value, the true value can be any value equal to or greater than the associated point on the DOWN system curve and equal to or less than the associated point on the UP hysteresis curve. The exact true value depends on the past history of operation of the instrument as well as the measured value. The net result of this problem is that a measure of the effectiveness of the partial hysteresis correction must be done on a statistical basis over a large number of occurrences.

The statistical method utilized in making this effectiveness measure involved the following.

1. Choosing a set of operating conditions that represent typical operating conditions of an instrument. Which included
 - (a) starting measured values,
 - (b) The step size for changing the measured value,
 - (c) The stopping value.

Figure 4.10. ILLUSTRATION OF PARTIAL HYSTERESIS RESPONSE

2. Estimating the probability density function of the true values for each starting measured value.
3. Determining the probability density function of the true values at the end of the prescribed change in measured value.

In order to determine the effectiveness of the overall technique, this analysis was performed three times: first, to find the actual response of the experimental instrument; second, to find the response if no hysteresis correction were made, and third, to find the response when the partial hysteresis correction program was utilized.

The condition chosen to represent typical operating conditions were to assume the algorithm had just initialized (i.e. either power up or the measured value had just reversed direction) and that the direction of increase was UP and then to specify the following starting and stopping values.

<u>Starting measured value in % F.S.</u>	<u>Ending measured value in % F.S.</u>
10%	20%
20%	30%
30%	40%
40%	50%
10%	50%
20%	50%
30%	50%

The distribution of the true values for each of the starting measured values was assumed to follow the normal probability function between the UP and DOWN hysteresis curves. The mean of the distribution was taken as

the center point between the two curves and the standard deviation was chosen so that the 3-sigma points were on the curves.

The actual distribution of true values for a given measured value depends to a large extent on the application being considered. For example, if the instrument were installed on a process that continuously cycled between zero to full scale, the distribution would contain two peaks. One peak on the DOWN curve and the other peak on the UP curve. In most operations, such wide excursions do not occur. Rather, the measured quantity fluctuates around some nominal values. In such cases, the mean of the true value would be very near the center of the hysteresis curve with a low value near the edges (i.e. UP and DOWN curves). Under these conditions, the normal distribution is a good approximation.

4.8.1 Determination of "Average" Response for the Experimental Instrument:

The average response is found by making many test under the same conditions and performing a statistical analysis.

Such statistical techniques are required because for every measured value there are infinite number of true values. As illustrated in Figure 4.11, if M_{i-1} is the indicated reading of the instrument, then the true value corresponding to M_{i-1} lies in the range A to B. Suppose that the input changes so that the instrument now indicates a reading M_i . The true value of the input for the new test indicated reading M_i is in the range C to D. The experiment is to trace the response from a specific starting point in the range A to B, to a corresponding final point in the range C to D. The experiment is performed a large number of times and statistical analyses are performed on the set of starting values, and on the set of corresponding final values.

Figure 4.11. HYSTERESIS CURVE OF INSTRUMENT RESPONSE

The first step then, is to determine what conditions exist for the starting values; that is a distribution of the true values in the range A to B. Typically this will not be known exactly as it depends on the application as well the instrument. A normal distribution of the starting values was assumed. The assumption is valid because in most applications there will be small fluctuations around the mid-point of the range A to B, that will tend to produce this type of distribution. The experiment is to determine the actual response of the instrument from a starting value to a change in the input. The "path" followed for a change in the input to yield a final value is to be determined. The experiment is to be repeated a large number of times so that all starting, and all final values are covered. This would be very difficult if not impossible to do with an actual instrument because the starting point can not be controlled. Therefore the tests were simulated using Monte Carlo techniques.

The responses used in the simulation were based on data obtained from laboratory tests on the Weston voltmeter. The laboratory results outlined the characteristic response of the instrument, and "typical" curves were filled in at other parts. The data used in the simulation is shown in Table 4.1.

The mechanics of the Monte Carlo simulation are illustrated in Figure 4.12. A random number that is normally distributed in the range A to B is generated. It is assumed that the mean is at the mid-point of the range, F_{i-1} , and that the limits of the range are the 3σ points, where σ is the standard deviation of the normal distribution. To the randomly chosen value in the range A to B, a value in the range C to D is assigned. The assignment is done in the following manner. The laboratory data of Table 4.1 is used to divide the ranges A to B, and C to D, into a number of

TABLE 4.1 HYSTERESIS DATA GENERATED FROM LABORATORY RESULTS

INDICATED READING AS% OF FULL-SCALE		TRUE VALUE PATH NUMBER									
		<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>
STARTING VALUE	10%	1.128	1.133	1.139	1.145	1.150	1.157	1.162	1.168	1.174	1.180
ENDING VALUE	50%	4.094	4.097	4.104	4.107	4.109	4.112	4.114	4.116	4.118	4.120
STARTING VALUE	20%	1.864	1.873	1.880	1.890	1.900	1.908	1.917	1.926	1.935	1.940
ENDING VALUE	50%	4.090	4.092	4.094	4.096	4.098	4.104	4.118	4.112	4.116	4.120
STARTING VALUE	30%	2.600	2.608	2.617	2.626	2.635	2.640	2.650	2.660	2.670	2.680
ENDING VALUE	50%	4.060	4.080	4.088	4.092	4.096	4.104	4.108	4.112	4.116	4.120
STARTING VALUE	40%	3.328	3.337	3.345	3.354	3.363	3.372	3.380	3.390	3.400	3.408
ENDING VALUE	50%	4.064	4.072	4.080	4.088	4.092	4.096	4.104	4.112	4.116	4.120
STARTING VALUE	10%	1.128	1.133	1.139	1.145	1.150	1.157	1.162	1.168	1.174	1.180
ENDING VALUE	20%	1.892	1.898	1.905	1.912	1.916	1.920	1.924	1.928	1.935	1.940
STARTING VALUE	20%	1.864	1.873	1.880	1.890	1.900	1.908	1.917	1.926	1.935	1.940
ENDING VALUE	30%	2.628	2.632	2.640	2.647	2.654	2.658	2.668	2.675	2.678	2.680
STARTING VALUE	30%	2.600	2.608	2.617	2.626	2.635	2.640	2.650	2.660	2.670	2.680
ENDING VALUE	40%	3.346	3.352	3.356	3.360	3.369	3.372	3.380	3.390	3.405	3.408

Figure 4.12: ILLUSTRATION OF TECHNIQUE FOR DETERMINING DISTRIBUTION OF ENDING VALUES

corresponding intervals. For a starting value in an interval of the range A to B, a corresponding interval in the range C to D is assigned. After a large number of trials histograms for the starting and final values are plotted. Figure 4.13 is an example histogram showing the distribution of the starting values for 10% full scale while Figure 4.14 shows the distribution of the corresponding final values for a measurand of 20% full scale.

The simulation is repeated for different sets of starting and final values. The results are tabulated in Table 4.2. This table summarizes the statistics for the starting values and ending values for each experiment.

The important points to be made from both the table and from the example histograms is that for increasing signals, the mean of the true value moves toward the UP curve. Note also for the experiments which use the following starting-ending pairs 10%-50%, 20%-50%, 30%-50%, and 40%-50%, that the mean moves further toward the UP curve as the step size increases.

4.8.2 Response with No Hysteresis Correction:

If no method for correcting hysteresis error was applied to the measured value, then the best value to output would be calculated from the COMBINED response curve. This calculation could be done with the measurement parameter error reduction program discussed in the previous section. When that method is used, the probability distribution for a given measured value becomes a discrete function with only one non-zero value. That is, the true value is no longer a random event but is deterministic with the value being found from the combined curve.

Figure 4.13. HISTOGRAM OF STARTING VALUES

Figure 4.14. HISTOGRAM OF ENDING VALUES

TABLE 4.2 RESULTS OF MONTE CARLO SIMULATION EXPERIMENT

<u>MEASURAND RANGE</u>		<u>STARTING VALUES</u>					<u>ENDING VALUES</u>				
<u>FROM</u>	<u>TO</u>	<u>DOWN CURVE</u>	<u>MEDIAN</u>	<u>UP CURVE</u>	<u>MEAN</u>	<u>STANDARD DEVIATION</u>	<u>LOWER END</u>	<u>MEDIAN</u>	<u>UPPER END</u>	<u>MEAN</u>	<u>STANDARD DEVIATION</u>
					μ	σ				μ	σ
10%	50%	1.128	1.154	1.180	1.1563	.0807	4.052	4.086	4.120	4.1093	.0795
20%	50%	1.864	1.904	1.944	1.9054	.0621	4.052	4.086	4.120	4.1021	.0316
30%	50%	2.600	2.640	2.680	2.6389	.0304	4.052	4.086	4.120	4.0935	.0294
40%	50%	3.328	3.368	3.408	3.3637	.0251	4.052	4.086	4.120	4.0908	.0259
10%	20%	1.128	1.154	1.180	1.1546	.0484	1.864	1.904	1.944	1.9176	.0177
20%	30%	1.864	1.904	1.944	1.9034	.0207	2.600	2.640	2.680	2.6784	.0297
30%	40%	2.600	2.640	2.680	2.6385	.0134	3.328	3.368	3.408	3.3718	.0234

For the test data points, this yields the following:

<u>Starting Measured Point</u>		<u>Ending Measured Point</u>		<u>Calculated True Value from COMBINED curve</u>
<u>% FS</u>	<u>Value</u>	<u>% FS</u>	<u>Value</u>	
10%	0.8	20%	1.6	1.9062
20%	1.6	30%	2.4	2.6430
30%	2.4	40%	3.2	3.3683
40%	3.2	50%	4.0	4.0861
10%	0.8	50%	4.0	4.0861
20%	1.6	50%	4.0	4.0861
30%	2.4	50%	4.0	4.0861

4.8.3 Response with Partial Hysteresis Correction:

The representative data for the experimental instrument was run through the off-line and on-line programs. In order to separate the hysteresis effects from the effects of measurement parameters, no change was made in temperature and humidity.

The off-line program calculated the following third order equations to represent the hysteresis loops:

$$\text{UP: } T = .392273 + 1.002532 M - .0241699 M^2 + .0016356 M^3$$

$$\text{COMBINED: } T = .3817291 + .978912 M - .0184174 M^2 + .0010327 M^3$$

$$\text{DOWN: } T = .370636 + .9559784 M - .0132294 M^2 + .0010099 M^3$$

These coefficients were entered into the on-line program and the set of experimental conditions used in the two previous sections were run. There was, however, two additional experimental variables. These were the size of the incremental increases in the measured value and the value of "K" in the correction algorithm. The size of the steps in the measured value is a number that would, in real operations, vary from moment to moment;

therefore, data were gathered to determine the effect of different values. The value of K would be fixed in the program once it was specified by the designer. A more detailed discussion on choosing K will be presented below.

Once these numbers are fixed, the calculated value for the ending measured value as performed by the algorithm is deterministic, in comparison to the random nature of the real instrument. There would be some variation in the ending calculated value, however, because of variations in step sizes. The results of these experiments are shown in Table 4.3. An important result of this experiment is that changes in the size of the incremental steps have very little impact on the output of the partial correction algorithm. This means that for a given value of K , the ending value for a given starting value can be represented as a deterministic value.

There is a variation in the ending calculated "TRUE" value depending on the value of K . Although K is nominally set equal to the full scale value of the instrument, these results indicate there could be a better value utilized. The effect of different values of K is to cause the curves as generated by the correction algorithm to have different shapes as shown in Figure 4.15. The choice of which value to choose will depend on the closeness to the actual response for a particular application. For this instrument and the experiment, a value of $K = 3$ appears to be closest to the actual response as shown in Figure 4.15. However, the value of K should be chosen so that over a wide range of conditions, the correction algorithm response is, on average, closest to the actual instrument response.

TABLE 4.3 Results From On-Line Hysteresis Program

<u>STARTING MEASURED</u>	<u>ENDING MEASURED</u>	<u>STEP SIZE</u>		<u>CALCULATED 'TRUE' VALUE AT END</u>			
		<u>%F.S.</u>	<u>VALUE</u>	<u>K=2</u>	<u>K=4</u>	<u>K=6</u>	<u>K=8</u>
10%	50%	10%	.8	4.116	4.106	4.100	4.097
		5%	.4	4.114	4.105	4.100	4.096
		2%	.16	4.114	4.104	4.099	4.096
20%	50%	10%	.8	4.113	4.102	4.097	4.094
		5%	.4	4.111	4.101	4.097	4.094
		2%	.16	4.110	4.101	4.096	4.094
30%	50%	10%	.8	4.108	4.098	4.094	4.091
		5%	.4	4.106	4.097	4.093	4.091
		2%	.16	4.105	4.097	4.093	4.091
40%	50%	10%	.8	4.099	4.092	4.090	4.089
		5%	.4	4.098	4.092	4.090	4.089
		2%	.16	4.097	4.092	4.090	4.089
10%	20%	10%	.8	1.920	1.913	1.911	1.910
		5%	.4	1.919	1.913	1.911	1.909
		2%	.16	1.918	1.913	1.910	1.909
20%	30%	10%	.8	2.658	2.650	2.648	2.647
		5%	.4	2.657	2.650	2.648	2.646
		2%	.16	2.656	2.650	2.648	2.646
30%	40%	10%	.8	3.383	3.375	3.373	3.372
		5%	.4	3.382	3.375	3.373	3.372
		2%	.16	3.381	3.375	3.373	3.371

Figure 4.15. EFFECT OF K ON RESPONSE OF PARTIAL HYSTERESIS PROGRAM

4.8.4 Analysis of Results:

The previous three sections have presented the results from the experimental data representing the "TRUE" value of an instrument with hysteresis as determined by (1) using a statistical analysis based on response curves of the actual instrument, (2) using the COMBINED regression curve to calculate the best value as is the hysteresis effect was not considered, and (3) using the partial hysteresis correction algorithm to calculate an improved value. A comparison of the results from these three cases is shown in Table 4.4.

In the table the mean of the ending values for the actual instrument is compared to the value determined with no hysteresis correction and with partial hysteresis correction. One most important conclusion is that for every experiment, the partial hysteresis correction is closer to the mean than is the value with no correction indicating that on the average, the algorithm provides more accurate results.

The other conclusion from this table is that there is no single value for K which always provides the best results. For this data, a value of K between 4 and 6 appears to provide better results than the full scale range value of 8. No underlying reason for using and no simpler way of determining such a value was determined. Therefore, unless a set of experiments similar to the ones described above are made, the full scale range is still the value to use on the algorithm. With more experience and data a simple method of determining a better value could very likely be found.

4.9 Sampling Frequency Needed for Hysteresis Error Improvements:

The frequency at which measurements are made and inputted into the computer for error reduction must exceed a minimum value if the hysteresis program is to work properly. Because the algorithm must detect each change in slope,

TABLE 4.4 Comparison of Results of Hysteresis Error Experiments

MEASURAND RANGE		RESULTS OF SIMULATION (AVERAGE)	VALUE WITHOUT HYSTERESIS CORRECTION	VALUE CALCULATED WITH PARTIAL HYSTERESIS CORRECTION			
Start (%FS)	Stop (%FS)			K=2	K=4	K=6	K=8
10	50	4.1093	4.086	4.114	<u>4.105</u>	4.100	4.096
20	50	4.1021	4.086	4.111	<u>4.101</u>	4.097	4.094
30	50	4.0935	4.086	4.106	4.097	<u>4.093</u>	4.091
40	50	4.0908	4.086	4.098	4.092	<u>4.090</u>	4.089
10	20	1.9176	1.906	<u>1.919</u>	1.913	1.911	1.909
20	30	2.6784	2.643	<u>2.657</u>	2.650	2.648	2.646
30	40	3.3718	3.368	3.382	3.375	3.373	<u>3.372</u>

the minimum sampling frequency is twice the highest frequency component of the input signal. In practice, however, the sampling frequency should be higher than this minimum.

Even when this minimum rate is used, there will still be some errors made by the hysteresis algorithm when the direction of movement changes, but the number of times for which the change in slope is not detected and the magnitude of the error will depend on the sampling rate as shown in Figure 4.16 (It should be noted from the figure that the magnitude of error being considered here is relatively small.). However, the percentage of these occurrences and the overall errors can be reduced with higher sampling frequencies. There are two changes of slope in each cycle of a sine wave, and both of these can be missed (i.e. delayed detection) for one sample point; therefore, as the number of samples per cycle goes up the percentages of missed changes goes down as shown below:

<u>Number of Samples per cycle of Highest frequency</u>	<u>% of Times Slope Changes Improperly Detected</u>
3	67%
4	50%
6	33%
8	25%
10	20%

Based on this consideration, it would appear that sample frequencies of the order of 10 times the highest frequency are required; however, it should be remembered that this is for the highest frequency component of the measured signal, and the number of missed slope changes will generally be much lower. Also, the size of the hysteresis error is generally small for the small amplitude changes which generally accompany the higher frequency

Figure 4.16. EFFECT OF SAMPLING FREQUENCY ON HYSTERESIS CORRECTION

components. Based on these and other considerations, sampling rates of 4 times the highest frequency could be sufficient. Except for the minimum frequency rule, there are no firm guidelines to easily determine the best sampling rate. Such determination will have to be made based for each particular application.

5.0 Reduction of Time Dependent Errors:

In addition to all the errors types previously discussed, there is another type of error caused by changes in the instrument response characteristics as time passes. These changes in characteristics necessitates periodic recalibration of the instrument or sensor. The reasons for these changes are sometimes random and unpredictable, therefore, no attempt was made to analyze these type problems. In other cases, however, the change in response characteristics is systematic and predictable. Examples of such systematic changes are the change in a turbine flow meter response due to wear of the bearings, or the change in the characteristic of an electronic sensor inside a nuclear reactor due to accumulated radiation. If there are systematic changes in an instrument or sensor due to some function of time, then errors associated with these changes can be reduced.

It should be noted that a complete system for implementing the reduction of time dependent errors was not developed. However, the techniques for performing this task were explored.

5.1 General Technique for Reducing Time Dependent Errors:

The general technique for reducing time dependent errors is based on (1) gathering data that indicates how the response characteristics change by periodically performing the set of calibration test described previously, (2) analyzing this data to determine equations which predict how the characteristics change as a function of time. (or other time related variable), and (3) modifying, during normal on-line operation, the coefficients which represent the characteristic response based on elapsed time or other accumulated variable.

5.2 Calibration Data Requirements:

The calibration data required for this technique are the same data as described previously, with the addition that a time related variable accompany each data set. This time related variable would be the variable on which the change in instrument response is dependent. The variable can be measured either as an absolute value or a difference value since the prior calibration test. For example, with a watt-hour meter the time related variable could be total accumulated kilowatt-hours or the differences in kilowatt-hours since the last calibration test.

All the calibration data would be collected and stored for at least two and preferably several calibration dates. After the off-line regression program was run on each calibration data set, another off-line program would be run to determine the time dependence of the regression coefficients.

5.3 Off-Line Program for Time Dependent Analysis:

The task to be performed by this program is to determine equations which accurately predict how the coefficients of regression equations change with respect to the time related variable. That is, an equation would be developed for every coefficient of every equation discussed earlier in the report. For example, if third order equations were used for 20 parameters conditions with hysteresis error reduction, then 240 (4 coefs. x 20 conditions x 3 equations) equations would have to be found.

This is certainly a large number of equations, but a computer based analysis system could handle it with little problem.

The major problem is to determine the form of the equation to be used in representing the change in coefficient values. This problem is not as straightforward as with the error correction cases discussed previously.

In those cases, the regression equation would accurately represent the response for many conditions because the instruments response was "close" to a straight line. In the case of representing changes in coefficients as a function of time, the form of the equation cannot accurately be determined until several calibration runs are made.

A simplifying assumption could be made that the coefficients changes are approximated by a linear equation involving just two points, (i.e. the last two values from the off-line analysis). In this case, the equations predicting the future value of each coefficient would be a simple extrapolation formula. As an example, assume two calibration tests were made at an interval of 1500 hours of operational time and the following coefficients of the UP hysteresis curve for a parameter condition of 20°C and 30% relative humidity:

$$\text{Most Recent Calibration: } T = a_0 + a_1 m + a_2 m^2 + a_3 m^3$$

$$\text{Prior Calibration: } T = b_0 + b_1 m + b_2 m^2 + b_3 m^3$$

$$\text{Hours Difference: } 1500.$$

A new equation for the same conditions except for some time (i.e. operational time) in the future would be as follows:

$$c_0 = a_0 + \frac{(a_0 - b_0)H}{1500}$$

$$c_1 = a_1 + \frac{(a_1 - b_1)H}{1500}$$

$$c_2 = a_2 + \frac{(a_2 - b_2)H}{1500}$$

$$c_3 = a_3 + \frac{(a_3 - b_3)H}{1500}$$

H = is lapsed operational time since recent calibration.

Similar equations could be developed for each coefficient required in the on-line error reduction program.

As time passes and many sets of calibration data are collected, more accurate methods for predicting changes in the coefficients could be developed. These techniques could involve higher ordered polynomial equations for curve fitting or it could involve finding equations of other forms such as exponential, logarithmic, or hyperbolic to fit the data. These are all standard curve fitting techniques that are straightforward once the data is available.

5.4 On-Line Programs for Reducing Time Dependent Errors:

The on-line program would be organized very similar to the on-line programs discussed previously, except an additional section would be needed to modify the coefficients based on the elapsed time (or accumulated time dependent variable). There are several ways in which to implement this concept, but the most feasible method for most instrumentation is based on the fact that the changes in instrument response occurs slowly; therefore, the updates to the regression equations coefficients need be made relatively infrequently. For example, the updates could be made once every eight hours or perhaps once every 24 hours.

The modification to the on-line previously discussed programs would be in two parts. The first part would be a subroutine to accumulate the amount of the time dependent variable since the last calibration and to "notify" the main program when an update of the coefficients is needed. In some cases this routine might consist of the inputting from a real-time clock interfaced to the computer and checking to see if the specified amount of time has elapsed since the last update. The second part of the modification would be the update routine that calculates new values for all regression equation coefficients using an extrapolation technique as discussed previously.

This technique for reducing time dependent errors does not significantly increase the size of the program, or the average speed of operation, but it does require more memory, due to the storage of several values of all coefficients, and increases the cost due to the addition of a real time clock or similar hardware. If a fairly simple, electronic timer would be sufficiently accurate for the clock, the system could be reasonably implemented with a microcomputer. However, if a time of day clock were required the technique would probably be best implemented in a multiplex, minicomputer system.

6.0 Computer Systems Considerations:

The objectives of this research was to develop the concepts, techniques, and example programs for reducing deterministic errors; however, it is appropriate to consider the computer systems hardware required to implement this method, so that the feasible of applications can be evaluated.

6.1 Off-Line Calibration System:

The off-line calibration system consists of the equipment needed to run calibration tests on the target instruments/sensors and the computer needed to run the off-line analysis program. The specific calibration equipment will depend very heavily on the type of instrumentation and the extent of error reduction being considered. One of the major costs could be for the equipment needed for the very accurate "standard" in making the calibration tests.

Another major costs consideration is the labor requirements in running the calibration tests. If large numbers of instruments/sensors are to be calibrated, an automated system for controlling the tests and recording the results would be justified.

The off-line regression analysis program is a fairly small program that can run on practically any general purpose computer as well as most minicomputers currently available. The cost of making these computations would be a minor part of the total system costs.

6.2 On-Line Measurement Correction System:

The on-line system inputs the measurements from the process instrumentation and calculates improved values based the algorithms

discussed previously. There are two hardware configurations which generally could be used to implement this system (see Figure 6.1).

The first configuration consists of a general purpose computer (typically a minicomputer) collecting data from a large number of sensors/instruments by using an input channel multiplexer. This system scans through all the input channels one at a time at some rate that allows all measurements to be made within an allowable time interval. The advantage of this system is the system costs can be spread over many instruments so that the cost per instrument is low. In addition, the results from the computer can easily be displayed at one point.

The algorithms developed in this work are readily applicable to this configuration. The amount of memory required for the programs is relatively small for this type system and the data storage requirements for the regression coefficients would be small assuming a mass memory device (i.e. disk) is attached.

The second hardware configuration consists of a special purpose microcomputer dedicated to improving the accuracy of one instrument or sensor. The advantages of this configuration is that it is independent, in both an operational and economic viewpoint, from other instruments. Therefore, the system can be applied to one or a few devices which might be remotely located. The disadvantage is that all costs are related to one unit which could generally lead to higher per unit cost.

6.3 Experimental Results Microcomputer System:

In an effort to determine the feasibility of using a microcomputer to implement the on-line algorithm, experiments were made on a typical example of such equipment.

MULTIPLEXED, GENERAL PURPOSE COMPUTER

DEDICATED, SPECIAL PURPOSE COMPUTER

Figure 6.1. TWO POSSIBLE ON-LINE HARDWARE CONFIGURATIONS

The microcomputer was a model Z2-D manufactured by Cromemco, Inc. This is an S-100 bus, microcomputer built around a Z-80, 8 bit microprocessor. Although it is a general purpose machine, the operating characteristics would be essentially the same as if a special purpose system was built of the same components.

The two major concerns with a microcomputer system are the memory requirements and the speed of operations. The speed must be fast enough to handle the measurements in real-time, and the memory requirements must be small enough to be within the address space of a microprocessor, which in this case was 64K words. In addition, the memory is a major cost element.

As described previously, the programs were written in FORTRAN so as to reduce the time and manpower required for software development. Although programs written in a high-level language require more memory and run slower than those written in assembler language, these experiments did provide "worst-case" types of indications of time and memory.

The results of the microcomputer experiments are shown in Table 6.1. Note that while the memory requirements are well within the 64K word limitations of most 8-bit microprocess, they changed only from 10K to 13K words. This is largely due to the use of a high level language and the large memory overhead associated with a complicated floating point package. Note also that while speed of operation varied significantly over the range of experiments, that the worst case speed of 250 milliseconds is still fast enough for most applications.

TABLE 6.1 Microcomputer Time and Memory Requirements

<u>PROGRAM</u>	<u>TIME OF OPERATION</u>		<u>MEMORY REQUIREMENT (8-bit words)</u>
	<u>With output to terminal (ms.)</u>	<u>Without output to terminal (ms.)</u>	
SIMPLE CORRECTION			
1st order	31	7	10K
2nd order	43	12	
3rd order	45	15	
CORRECTION INCLUDING PARAMETERS			
1st order	143	28	12K
2nd order	150	40	
3rd order	155	43	
CORRECTION WITH PARAMETERS AND HYSTERESIS			
1st order	165	62	13K
2nd order	200	95	
3rd order	250	140	

Appendix I

Documentation of Off-Line Data Analysis Program.

Figure I.1 gives the flow chart of the off-line program.

General Program Functions:

The off-line program accepts calibration data and computes the regression coefficients for representation of the data in equation form. First, second, and third order regression coefficients are computed.

A set of true values is calculated for the observed values of the calibration data for each of the three representations.

Four sets of errors are computed. One set is computed as the differences between the observed and true values from the calibration data. The other sets are computed as the differences between the calculated true values and the true values, one set for each of the three representations.

The mean, standard deviation and maximum is computed for each of the error sets.

A "best" representation is selected from the first, second, and third order regression.

The regression coefficients, the error statistics and "best" representation are printed out.

The above detailed calculations are first made on a combination of "up" and "down" calibration data. The calculations are then repeated using only the "up" data and then repeated again using only the "down" data.

Hysteresis error is computed as the maximum error in comparing the "best" up and "best" down representation.

This completes the analysis on a set of data taken at one set of temperature and humidity values. If ranges of temperature and humidity are to be considered a new set of data is required for each new combination of temperature and humidity values.

OFF-LINE DATA ANALYSIS PROGRAM

FIGURE I.1

Data Input and Format:

The first card requires the user to assign unit numbers to the printers and reader that apply for the user's installation. The next 8 data cards contain information identifying the instrument and the particular data set being analyzed.

The first card of this group allows up to a 9 character expression for identifying the particular problem. The card is formatted according to the statement

FORMAT (A5, A4).

The second card is for recording the date of the calibration run. It allows up to five character statement such as DATE and two digits each for month, day, and year. The card is formatted according to the statement

FORMAT (A5, T30, I2, I2, I2).

The third card is for identifying the instrument involved in the calibration. It allows up to ten characters and a two digit number to identify the instrument. The card is formatted according to the statement

FORMAT (2A5, T30, I2).

The fourth card is for identifying the temperature or other parametric variable for a particular set of a sequence of data sets. It allows a 25 character description of the parameter and a numerical value for the variable. The card is formatted according to the statement:

FORMAT (5A5, T30, F6.2).

The fifth card is for identifying the humidity or another parametric variable. It allows a 25 character description of the parameter and a numerical value for the variable. The card is formatted according to the statement

FORMAT (5A5, T30, F6.2).

The sixth card is for recording the time of the calibration run. It allows for up to a five character statement such as TIME and a numerical

listing of the value. The card is formatted according to the statement

FORMAT (A5, T30, F7.4).

The seventh card is for recording N the total number of data points in a calibration data set. The card allows up to a 25 character statement describing the data set and a three digit integer listing the number of points. The card is formatted according to the statement:

FORMAT (5A5, T30, I3).

The eighth card is for recording the range of instrument calibration data. It allows up to a 5 character statement such as RANGE and numerical values for upper and lower range values. The card is formatted according to the statement

FORMAT (A5, T30, F4.2, T40, F4.2).

The next set of cards contains the calibration data set. The first card in the data set gives N the total number of points in the set, NU the number of these points which are taken on the "UP" curve and ND the number of these points which are on the "DOWN" curve. This card is formatted

FORMAT (I3, T10, I3, T20, I3).

The next N cards contain the calibration data points in pairs. The observed value X(I) first and the true value Y(I) second. The "UP" points are listed first.

The read statement READ (IR,*) X(I),Y(I) allows a pair of observation and true values on each data card. The numbers can be located any place on the card and are separated by a comma.

This completes the data input for a single set of temperature and humidity parameters. Other data sets for different pairs of temperature and humidity are treated as a separate analysis or run.

Program Output:

The program first outputs the information on cards 2-9 of the input deck. This identifies the instrument and the particular data set being analyzed. Refer to data input section.

The next output is for combined "UP" and "DOWN" data and includes:

1. The regression coefficients for first, second, and third order in equation form.
2. A column of the true values input, $Y(I)$.
3. A column of the observed values input $X(I)$.
4. A column of the errors implied in the calibration data

$$E(I) = X(I) - Y(I).$$

5. A column of the calculated true value $C1(I)$ using observed data input and first order regression.
6. A column of the errors obtained comparing calculated true value and the true values.
7. The mean error in the calibration data and the mean error in the calculated true values are listed.
8. The standard deviation of the errors in the calibration data and the standard deviation of the error in the calculated true values are listed.
9. The max error of the calibration data and the max error of the calculated true data are listed.
10. The X and Y values at which the above max values occur are listed.
11. The min errors are listed for the calibration data and the calculated true values.

12. The X and Y values at which the above min values occur are listed.
 13. The sum of the squared errors are listed for the two error columns.
 14. Items 2 through 13 are repeated for calculated true values obtained with second order regression.
 15. Items 2 through 13 are repeated for calculated true values obtained with third order regression.
 16. The next output is a statement that either first, second, or third order regression gives a "best" representation.
 17. The bias and nonlinearity with the "best" representation are given.
 18. Outputs 1 through 17 are given using only "UP" data.
 19. Outputs 1 through 17 are given using only "DOWN" data.
 20. The following equations are listed
 - (a) best curve for combined data
 - (b) best curve for UP data
 - (c) best curve for DOWN data.
 21. The maximum distance between the UP and DOWN curves is listed as the hysteresis error.
 22. Items 1 through 21 are repeated for each data set.
- There are separate data sets for each temperature, humidity pair considered.

Subroutines:

This section gives a brief description of the off-line program subroutines. Further detail is contained in the comment statements of the program listing.

RDCD - Reads in information identifying the instrument and the particular conditions of the data set.

PRNT - Prints out information input by RDCD.

RDCD1 - Reads in calibration data set.

SUM - Forms the entries for the coefficient matrix in the equations for finding the regression coefficients.

MATINV - Inverts the coefficient matrix formed in SUM as required in the solution for regression coefficients.

LINEAR - Calls MATINV for the linear regression case.

SOLVE - Performs a multiplication of the inverse matrix and the SXY column vector to yield the regression coefficients.

QUAD - Calls MATINV for the second order regression case.

CUBE - Calls MATINV for the third order regression case.

ERROR - Calculates a set of errors from two input sets. For example, the errors ER are computed from the true values and the calculated true using first order regression.

STAT - Calculates the mean, standard deviation, max, min, and sum of the squared errors for the error set under consideration.

FIT - Computes calculated true values using regression coefficients (first, second, or third) and the instrument observations from calibration data.

PRNT1 - Prints out the regression equation with computed coefficients.

PRNT2 - Prints out the statistical information computed in STAT.

GFIT - Determines best representation as the simplest of the first, second, or third order that meets a criterion of the RMS error less than some arbitrary constant E.

BIASS - Identifies the bias as the zero intercept of the "best" representation.

NONLN - Computes the nonlinearity by comparing "best" to the linear regression representation.

PRNT3 - Prints out best representation, bias and nonlinearity.

STORE - Identifies the regression coefficients for the best representation for the up data and for the down data.

Calls HYSTER.

HYSTER - Computes the max hysteresis error and prints this out.

Also prints out best regression equation for combined, up and down data.

Program Listing:

```

1      REAL MUL,MUQ,MUS,MU,MAX,MAX1,MAX2,MAX3,
      1 NONLIN,MIN,MIN1,MIN2,MIN3
2      INTEGER BEST,DATE(3),HC
3      COMMON /READ,IPRT/VALT/Y(150)/VALM/X(150)
4      COMMON /CHOICE/FM1,FM2,FM3
5      DIMENSION SX(5,5),AINV(5,5),SXY(5),ER(150),ERR(150),ERRD(150),
      1 E(150),ALPHA(5),BETA(5),GAMMA(5),C1(150),C2(150),C3(150),
      2 COEF(3,5),RANGE(2),LABEL(8,7)

C
C THIS PROGRAM ASSIGNS CARD READER AS UNIT NO. IREAD
C THE LINE PRINTER IS ASSIGNED UNIT NO. IPRT
C SO THE FIRST INFORMATION TO BE READ IS IPRT FOR THE USER'S INSTALLATION
6      READ(1,*)IREAD,IPRT

C
C SUBROUTINE REDCD READS THE FOLLOWING
C DATE, TIME, INSTRUMENT DESIGNATED BY 2 DIGIT NUMBER, TEMPERATURE,
C HUMIDITY, RANGE, AND NUMBER OF DATA POINTS
7      CALL REDCD(N,LABEL,DATE,TIME,INST,TEMP,HUM,RANGE)

C
C NOW READ THE TRUE AND INDICATED READINGS
8      CALL REDCD1(N,NU,ND)
9      HC=3
C CODE HC IS AS FOLLOWS: HC=1 DATA FOR UP MOVEMENT ONLY
C                          HC=2 DATA FOR DOWN MOVEMENT ONLY
C                          HC=3 DATA FOR COMBINED DIRECTIONS

C
C OBTAIN COEFFICIENTS FOR LINEAR REGRESSION
10     20 CALL ZERO(SX,SXY)
11         CALL SUM(N,SX,SXY)
12         CALL LINEAR(SX,AINV)
13         CALL SOLVE(2,AINV,SXY,ALPHA)

C
C OBTAIN COEFFICIENTS FOR QUADRATIC REGRESSION
14         CALL QUAD(SX,AINV)
15         CALL SOLVE(3,AINV,SXY,BETA)

C
C OBTAIN COEFFICIENTS FOR CUBIC REGRESSION
16         CALL CUBE(SX,AINV)
17         CALL SOLVE(4,AINV,SXY,GAMMA)

C
C OBTAIN MEAN AND STANDARD DEVIATION OF INPUT ERROR
C INPUT ERROR = INDICATED - TRUE READING
18         CALL ERROR(N,X,E)
19         CALL STAT(N,MU,SIGMA,E,MAX,MIN,LO,LL,SSE)

20         CALL FIT(N,ALPHA,C1,2)

C
C OBTAIN LINEAR FIT AND COMPUTE ERROR IF THIS FIT IS USED
C DEFINE, ERROR = FITTED VALUE - TRUE READING
21         CALL ERROR(N,C1,ER)
22         CALL STAT(N,MUL,SIGMAL,ER,MAX1,MIN1,LO1,LL1,SSER)
23         CALL PRNT1(ALPHA,1)
24         CALL PRNT2(N,E,C1,ER,MU,SIGMA,MUL,SIGMAL,MAX,MAX1,LO,LO1,SSE,SSER,
      1 MIN,MIN1,LL,LL1)

C
C OBTAIN QUADRATIC FIT AND REPEAT ERROR ANALYSIS.
C DEFINE, ERROR = FITTED VALUE - TRUE READING
25         CALL FIT(N,BETA,C2,3)
26         CALL ERROR(N,C2,ERR)
27         CALL STAT(N,MUQ,SIGMAQ,ERR,MAX2,MIN2,LO2,LL2,SSERR)
28         CALL PRNT1(BETA,2)
29         CALL PRNT2(N,E,C2,ERR,MU,SIGMA,MUQ,SIGMAQ,MAX,MAX2,LO,LO2,SSE,SSER,
      1 R,MIN,MIN2,LL,LL2)

C
C OBTAIN CUBIC FIT AND REPEAT ERROR ANALYSIS.
C DEFINE, ERROR = FITTED VALUE - TRUE READING
30         CALL FIT(N,GAMMA,C3,4)
31         CALL ERROR(N,C3,ERRD)
32         CALL STAT(N,MUS,SIGMAS,ERRD,MAX3,MIN3,LO3,LL3,SSERRR)
33         CALL PRNT1(GAMMA,3)
34         CALL PRNT2(N,E,C3,ERRD,MU,SIGMA,MUS,SIGMAS,MAX,MAX3,LO,LO3,SSE,SSE,
      1 RRD,MIN,MIN3,LL,LL3)

C
C
35         CALL GFIT(C1,C2,C3,BEST,NONLIN,N,SSER,SSERR,SSERRD,RANGE)
36         CALL BIASS(BEST,BIAS,ALPHA,BETA,GAMMA)
37         CALL PRNT3(BEST,BIAS,NONLIN)
38         CALL STORE(BEST,COEF,ALPHA,BETA,GAMMA,HC,RANGE)
39         IF(HC.EQ.2) GOTO 50
40         IF(ND.LE.2) GOTO 50
41         IF(HC.EQ.1) GOTO 30
42         IF(NU.LE.2) GOTO 30
43         HC=1
44         N=NU
45         GO TO 20
46     30 HC=2
47         DO 40 I=1,ND
48             Y(I)=Y(I+NU)
49         40 X(I)=X(I+NU)
50         GO TO 20
51     50 STOP
52     END

```

```

53      SUBROUTINE REDCD(N,LABEL,DATE,TIME,INST,TEMP,HUM,RANGE)
54      INTEGER DATE(3)
55      DIMENSION RANGE(2),LABEL(8,7)
56      COMMON IREAD,IPRT
57      READ(IREAD,110)((LABEL(1,J),J=1,3)
58      READ(IREAD,111)((LABEL(2,J),J=1,2),DATE(1),DATE(2),DATE(3)
59      READ(IREAD,112)((LABEL(3,J),J=1,3),INST
60      READ(IREAD,113)((LABEL(4,J),J=1,7),TEMP
61      READ(IREAD,114)((LABEL(5,J),J=1,7),HUM
62      READ(IREAD,115)((LABEL(6,J),J=1,2),TIME
63      READ(IREAD,117)((LABEL(7,J),J=1,7),N
64      READ(IREAD,118)((LABEL(8,J),J=1,2),RANGE(1),RANGE(2)
65      110  FORMAT(3A4)
66      111  FORMAT(2A4,T30,I2,I2,I2)
67      112  FORMAT(3A4,T30,I2)
68      113  FORMAT(7A4,T30,F6.2)
69      114  FORMAT(7A4,T30,F6.2)
70      115  FORMAT(2A4,T30,F7.4)
71      117  FORMAT(7A4,T30,I3)
72      118  FORMAT(2A4,T30,F4.2,T40,F4.2)
73      CALL PRNT(N,LABEL,DATE,INST,TEMP,HUM,TIME,RANGE)

74      RETURN
75      END

76      SUBROUTINE PRNT(N,LABEL,DATE,INST,TEMP,HUM,TIME,RANGE)
77      INTEGER DATE(3)
78      DIMENSION RANGE(2),LABEL(8,7)
79      COMMON IR,IP
80      WRITE(IP,1000)((LABEL(1,J),J=1,3)
81      WRITE(IP,1001)((LABEL(2,J),J=1,2),DATE(1),DATE(2),DATE(3)
82      WRITE(IP,1002)((LABEL(3,J),J=1,3),INST
83      WRITE(IP,1003)((LABEL(4,J),J=1,7),TEMP
84      WRITE(IP,1004)((LABEL(5,J),J=1,7),HUM
85      WRITE(IP,1005)((LABEL(6,J),J=1,2),TIME
86      WRITE(IP,1007)((LABEL(7,J),J=1,7),N
87      WRITE(IP,1008)((LABEL(8,J),J=1,2),RANGE(1),RANGE(2)
88      1000  FORMAT('1',T5,3A4)
89      1001  FORMAT(' ',T5,2A4,T35,I2,'/',I2,'/',I2)
90      1002  FORMAT(' ',T5,3A4,T35,I2)
91      1003  FORMAT(' ',T5,7A4,T35,F6.2)
92      1004  FORMAT(' ',T5,7A4,T35,F6.2)
93      1005  FORMAT(' ',T5,2A4,T35,F7.4)
94      1007  FORMAT(' ',T5,7A4,T35,I3)
95      1008  FORMAT(' ',T5,'TRUE ',2A4,T35,F4.2,' TO ',F4.2)
96      RETURN
97      END

98      SUBROUTINE REDCD1(N,NU,ND)
99      COMMON IR,IP/VALT/Y(150)/VALM/X(150)
C
C THIS SUBROUTINE READS DATA IN THE FOLLOWING FORMAT
C FIRST CARD: N - TOTAL NUMBER OF READINGS, AT LEAST 4
C             NU - NUMBER OF 'UP' READINGS
C             ND - NUMBER OF 'DOWN' READINGS
C FOLLOWING CARDS: MEASUREMENTS INDICATED AND TRUE VALUES ONE EACH
C                  PER CARD, SEPARATED BY A COMMA.
C
C
C
100      READ(IR,*) N,NU,ND
101      DO 10 I=1,N
102      10  READ (IR,*)X(I),Y(I)
103      RETURN
104      END

105      SUBROUTINE ZERO(SX,SXY)
106      DIMENSION SX(5,5),SXY(5)
C
C THIS SUBROUTINE INITIALIZES THE VAN DER MONDE MATRIX SX(I,J),
C AND THE VECTOR SXY( )
C
C
107      DO 11 I=1,5
108      SXY(I)=0.C
109      DO 11 J=1,5
110      SX(I,J)=0.0
111      11  CONTINUE
112      RETURN
113      END

114      SUBROUTINE SUM(N,SX,SXY)
115      DIMENSION SX(5,5),SXY(5)
116      COMMON/VALM/X(150)
C
C
117      COMMON/VALT/Y(150)
C
C HERE THE VAN DER MONDE MATRIX IS CONSTRUCTED
C SINCE UPTO THIRD ORDER REGRESSION IS TO BE USED
C TERMS UPTO ORDER SIX ARE COMPUTED
C SX(I,J) IS THE VAN DER MONDE MATRIX
C SXY(N) IS THE CROSS PRODUCT VECTOR

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118 C      DO 11 I=1,N
119      SX(1,2)=X(I)+SX(1,2)
120      SX(1,3)=X(I)**2+SX(1,3)
121      SX(1,4)=X(I)**3+SX(1,4)
122      SX(2,4)=X(I)**4+SX(2,4)
123      SX(3,4)=X(I)**5+SX(3,4)
124      SX(4,4)=X(I)**6+SX(4,4)
125      SXY(1)=Y(I)+SXY(1)
126      SXY(2)=Y(I)*X(I)+SXY(2)
127      SXY(3)=Y(I)*X(I)**2+SXY(3)
128      SXY(4)=Y(I)*X(I)**3+SXY(4)
129      11 CONTINUE
130      SX(1,1)=FLOAT(N)
131      SX(2,1)=SX(1,2)
132      SX(2,2)=SX(1,3)
133      SX(2,3)=SX(1,4)
134      SX(3,1)=SX(1,3)
135      SX(3,2)=SX(1,4)
136      SX(3,3)=SX(2,4)
137      SX(4,1)=SX(1,4)
138      SX(4,2)=SX(2,4)
139      SX(4,3)=SX(3,4)
140      RETURN
141      END

142      SUBROUTINE LINEAR(SX,A)
143      DIMENSION SX(5,5),A(5,5)
144      C      SUBROUTINE PREPARES FOR LINEAR REGRESSION
145      C      INPUT IS FIRST 2*2 BLOCK OF MATRIX SX(I,J)
146      C      OUTPUT IS INVERSE OF THIS BLOCK. THE SAME NAME SX IS USED
147      C      FOR THE OUTPUT, HENCE THE STRANGE ARGUMENT LIST.
148      C
149      DO 21 I=1,2
150      DO 21 J=1,2
151      A(I,J)=SX(I,J)
152      21 CONTINUE
153      CALL MATINV(2,A)
154      RETURN
155      END

156      SUBROUTINE SOLVE(NE,AINV,SXY,BETA)
157      DIMENSION AINV(5,5),SXY(5),BETA(5)
158      C      USE THE INVERSE MATRIX TO SOLVE FOR THE COEFFICIENTS OF
159      C      THE FIT EQUATION
160      C
161      DO 31 I=1,NE
162      BETA(I)=0.0
163      DO 31 J=1,NE
164      BETA(I)=AINV(I,J)*SXY(J)+BETA(I)
165      31 RETURN
166      END

167      SUBROUTINE QUAD(SX,A)
168      DIMENSION SX(5,5),A(5,5)
169      DO 21 I=1,3
170      DO 21 J=1,3
171      A(I,J)=SX(I,J)
172      21 CONTINUE
173      CALL MATINV(3,A)
174      RETURN
175      END

176      SUBROUTINE CUOC(SX,A)
177      DIMENSION SX(5,5),A(5,5)
178      DO 21 I=1,4
179      DO 21 J=1,4
180      A(I,J)=SX(I,J)
181      21 CALL MATINV(4,A)
182      RETURN
183      END

184      SUBROUTINE MATINV(N,AB)
185      DIMENSION IK(10),JK(10),AB(5,5)
186      C      SUBROUTINE FOR COMPUTING THE INVERSE OF A MATRIX
187      C      MAXIMUM SIZE HANDLED IS 4*4
188      C      GAUSS-JORDAN REDUCTION WITH COMPLETE PIVOTING IS USED
189      C      INPUT IS ARRAY AB
190      C      OUTPUT IS ARRAY AB ALSO
191      C      ON RETURN INPUT ARRAY HAS BEEN DESTROYED

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179 10 DET=1.
179 DO 100 K=1,N
180 AMAX=0.
181 21 DO 30 I=K,N
182 DO 30 J=K,N
183 23 IF (ABS(AMAX)-ABS(AB(I,J))) 24,24, 30
184 24 AMAX=AB(I,J)
185 IK(K)=I
186 JK(K)=J
187 30 CONTINUE
188 31 IF(AMAX)41,32,41
189 32 DET=0.0
190 GO TO 140
191 41 I=IK(K)
192 IF (I-K) 21, 51, 43
193 43 DO 50 J=1,N
194 SAVE=AB(K,J)
195 AB(K,J)=AB(I,J)
196 50 AB(I,J)=-SAVE
197 51 J=JK(K)
198 IF (J-K) 21, 61, 53
199 53 DO 60 I=1,N
200 SAVE=AB(I,K)
201 60 AB(I,K)=AB(I,J)
202 61 AB(I,J)=-SAVE
203 61 DO 70 I=1,N
204 IF (I-K) 63, 70, 63
205 63 AB(I,K)=-AB(I,K)/AMAX

206 70 CONTINUE
207 71 DO 90 I=1,N
208 DO 80 J=1,N
209 IF (I-K) 74, 80, 74
210 74 IF (J-K) 75, 80, 75
211 75 AB(I,J)=AB(I,J)+AB(I,K)*AB(K,J)
212 80 CONTINUE
213 81 DO 90 J=1,N
214 IF (J-K) 83, 90, 83
215 83 AB(K,J)=AB(K,J)/AMAX
216 90 CONTINUE
217 AB(K,K)=1./AMAX
218 100 DET=DET*AMAX
219 101 DO 130 I=1,N
220 K=N-L+1
221 J=IK(K)
222 IF (J-K) 111, 111, 105
223 105 DO 110 I=1,N
224 SAVE=AB(I,K)
225 AB(I,K)=-AB(I,J)
226 110 AB(I,J)=SAVE
227 111 I=JK(K)
228 IF (I-K) 130, 130, 113
229 113 DO 120 J=1,N
230 SAVE=AB(K,J)
231 AB(K,J)=-AB(I,J)
232 120 AB(I,J)=SAVE
233 130 CONTINUE
234 140 RETURN
235 END

236 SUBROUTINE ERROR(N,X,E)
237 DIMENSION X(150),E(150)
238 COMMON/VALT/Y(150)
239 DO 51 I=1,N
240 E(I)=X(I)-Y(I)
241 51 CONTINUE
242 RETURN
243 END

244 SUBROUTINE STAT(N,MU,SIGMA,ER,MAX,MIN,LO,LL,SSER)
245 REAL MU,MAX,MIN
246 DIMENSION ER(150)
247 XSUM=0.0
248 XSIG=0.0
249 SSER=0.0
250 SCNT=0.0
251 DO 31 I=1,N
252 SCNT=SCNT + 1.0
253 SSER= ER(I)*ER(I)+SSER
254 XSUM=ER(I)+XSUM
255 31 CONTINUE
256 MU=XSUM/SCNT
257 XSIG=ABS(SSER -XSUM*XSUM/SCNT)/(SCNT-1.0)
258 SIGMA=SQRT(XSIG)
259 CALL MAXER(N,MAX,MIN,LO,LL,ER)
260 RETURN
261 END

262 SUBROUTINE MAXER(N,MAX,MIN,LO,LL,EE)

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263      REAL MAX,MIN
264      DIMENSION EE(150)
265      MAX=EE(1)
266      LO=1
267      MIN=EE(1)
268      LL=1
269      DO 31 I=2,N
270      IF(EE(I).LT.MIN) MIN=EE(I)
271 31 CONTINUE
272      DO 32 J=1,N
273      IF(EE(J).EQ.MIN) LL=J
274 32 CONTINUE
275      DO 33 I=2,N
276      IF(EE(I).GT.MAX) MAX=EE(I)
277 33 CONTINUE
278      DO 34 J=1,N
279      IF(EE(J).EQ.MAX) LO=J
280 34 CONTINUE
281      RETURN
282      END

283      SUBROUTINE FIT(N,ALPHA,C1,J)
284      DIMENSION ALPHA(5),C1(150)
285      COMMON/VALM/X(150)
286      K=J+1
287      DO 41 L=K,4
288      ALPHA(L)=0.0
289 41 CONTINUE
290      M=1
291      DO 42 I=1,N
292      C1(I)=ALPHA(M)+ALPHA(M+1)*X(I)+ALPHA(M+2)*X(I)**2+ALPHA(M+3)*X(I)*
293      I**3
294 42 CONTINUE
295      RETURN
296      END

297      SUBROUTINE PRNT1(ALPHA,N)
298      DIMENSION ALPHA(5)
299      COMMON IR,IP
300      M=N+1
301      IF(N.EQ.1) GO TO 51
302      IF(N.EQ.2) GO TO 52
303      IF(N.EQ.3) GO TO 53
304 51 WRITE(IP,5100)
305      WRITE(IP,5200) (ALPHA(I),I=1,M)
306      FORMAT('1',T5,'FIRST ORDER REGRESSION')
307      5200 FORMAT('0',T5,'T=',F10.7,' ',F10.7,'M')
308      GO TO 58
309 52 WRITE(IP,5300)
310      WRITE(IP,5400) (ALPHA(I),I=1,M)
311      5300 FORMAT('1',T5,'SECOND ORDER REGRESSION')
312      5400 FORMAT('0',T5,'T=',F10.7,' ',F10.7,'M ',F10.7,'M**2')
313      GO TO 58
314 53 WRITE(IP,5500)
315      WRITE(IP,5600) (ALPHA(I),I=1,M)
316      5500 FORMAT('1',T5,'THIRD ORDER REGRESSION')
317      5600 FORMAT('0',T5,'T=',F10.7,' ',F10.7,'M ',F10.7,'M**2 ',F10.7,'M**
318      3')
319 58 RETURN
320      END

321      SUBROUTINE PRNT2(N,E,C1,ER,MU,SIGMA,MUL,SIGMAL,MAX,MAX1,LO,LO1,SSE
322      1,SSER,MIN,MIN1,LL,LL1)
323      REAL MU,MUL,MAX,MAX1,MIN,MIN1
324      DIMENSION E(150),C1(150),ER(150)
325      COMMON IR,IP
326      COMMON/VALT/Y(150)
327      COMMON/VALM/X(150)
328      WRITE(IP,6100)
329      DO 61 I=1,N
330      WRITE(IP,6200) Y(I),X(I),E(I),C1(I),ER(I)
331 61 CONTINUE
332      WRITE(IP,6300) MU,MUL
333      WRITE(IP,6400) SIGMA,SIGMAL
334      WRITE(IP,6500) MAX,MAX1
335      WRITE(IP,6600) Y(LO),X(LO),Y(LO1),X(LO1)
336      WRITE(IP,6700) MIN,MIN1
337      WRITE(IP,6800) Y(LL),X(LL),Y(LL1),X(LL1)
338      WRITE(IP,6900) SSE,SSER
339      6100 FORMAT('0',T5,'TRUE VALUE',T20,'MEASURED VALUE',T40,'ERROR',T52,'C
340      1 CALCULATED VALUE',T72,'ERROR')
341      6200 FORMAT(' ',T6,F8.5,T22,F8.5,T38,F8.5,T54,F8.5,T70,F8.5)
342      6300 FORMAT(' ',T33,'MEAN=',F8.5,T65,'MEAN=',F8.5)
343      6400 FORMAT(' ',T19,'STANDARD DEVIATION=',F8.5,T51,'STANDARD DEVIATION=
344      1',F8.5)
345      6500 FORMAT(' ',T24,'MAXIMUM ERROR=',F8.5,T56,'MAXIMUM ERROR=',F8.5)
346      6600 FORMAT(' ',T24,'AT (',F8.5,',',F8.5,')',T56,'AT (',F8.5,',',F8.5,
347      1)')
348      6700 FORMAT(' ',T24,'MINIMUM ERROR=',F9.5,T56,'MINIMUM ERROR=',F9.5)
349      6800 FORMAT(' ',T24,'AT (',F8.5,',',F8.5,')',T56,'AT (',F8.5,',',F8.5,
350      1)')
351      6900 FORMAT(' ',T23,'SUM OF SQUARES=',F8.5,T55,'SUM OF SQUARES=',F8.5)
352      RETURN
353      END

```

```

347 SUBROUTINE GFIT(C1,C2,C3,BEST,NONLIN,N,SSER,SSERR,SSERRO,RANGE)
348 INTEGER BEST
349 REAL NONLIN
350 COMMON /CHOICE/FM1,FM2,FM3
351 DIMENSION C1(150),C2(150),C3(150),DIFF(150),RANGE(2)
352 DATA EPS1,EP2/0.10,0.10/
353 DIV=RANGE(2) - RANGE(1)
354 FM1= SQRT(SSER/FLOAT(N))*100.0/DIV
355 FM2= SQRT(SSERR/FLOAT(N))*100.0/DIV
356 FM3= SQRT(SSERRO/FLOAT(N))*100.0/DIV
357 IF(ABS(FM3-FM2).LE.EPS1) GOTO 20
358 IF(ABS(FM2-FM1).LE.EPS2) GOTO 10
359 BEST=1
360 NONLIN=0.0
361 RETURN
362 10 BEST=2
363 CALL DIF(C2,C1,DIFF,N)
364 CALL MAXER(N,CMAX2,CMIN2,LO,LL,DIFF)
365 IF((CMAX2.GT.0.0).AND.(CMIN2.GT.0.0)) GOTO 15
366 IF((CMAX2.LT.0.0).AND.(CMIN2.LT.0.0)) GOTO 17
367 NONLIN=CMAX2 + ABS(CMIN2)
368 RETURN
369 15 NONLIN=CMAX2
370 RETURN
371 17 NONLIN=ABS(CMIN2)
372 RETURN

373 20 BEST=3
374 CALL DIF(C3,C1,DIFF,N)
375 CALL MAXER(N,CMAX3,CMIN3,LO,LL,DIFF)
376 IF((CMAX3.GT.0.0).AND.(CMIN3.GT.0.0)) GOTO 25
377 IF((CMAX3.LT.0.0).AND.(CMIN3.LT.0.0)) GOTO 27
378 NONLIN=CMAX3 + ABS(CMIN3)
379 RETURN
380 25 NONLIN=CMAX3
381 RETURN
382 27 NONLIN=ABS(CMIN3)
383 RETURN
384 END

385 SUBROUTINE BIAS(BEST,BIAS,ALPHA,BETA,GAMMA)
386 INTEGER BEST
387 DIMENSION ALPHA(5),BETA(5),GAMMA(5)
388 IF(BEST.EQ.1) BIAS=ALPHA(1)
389 IF(BEST.EQ.2) BIAS=BETA(1)
390 IF(BEST.EQ.3) BIAS=GAMMA(1)
391 RETURN
392 END

393 SUBROUTINE PRNT3(BEST,BIAS,NONLIN)
394 INTEGER BEST
395 REAL NONLIN
396 COMMON IR,IP
397 COMMON /CHOICE/FM1,FM2,FM3
398 IF(BEST.EQ.1) GO TO 11
399 IF(BEST.EQ.2) GO TO 12
400 IF(BEST.EQ.3) GO TO 13
401 11 WRITE(IP,1000)
402 FM=FM1
403 GO TO 14
404 12 WRITE(IP,1001)
405 FM=FM2
406 GO TO 14
407 13 WRITE(IP,1002)
408 FM=FM3
409 14 WRITE(IP,1003) BIAS
410 WRITE(IP,1004) NONLIN
411 WRITE(IP,1005) FM
412 1000 FORMAT('0',T5,'DATA IS BEST FITTED BY A FIRST ORDER REGRESSION CUR
413 *VE')
414 1001 FORMAT('0',T5,'DATA IS BEST FITTED BY A SECOND ORDER REGRESSION CU
415 *RVE')
416 1002 FORMAT('0',T5,'DATA IS BEST FITTED BY A THIRD ORDER REGRESSION CUR
417 *VE')
418 1003 FORMAT(' ',T5,'BIAS=',F10.7)
419 1004 FORMAT(' ',T5,'NONLINEARITY=',F10.7)
420 1005 FORMAT(' ',T5,'FIGURE OF MERIT=',F10.7)
421 RETURN
422 END

423 SUBROUTINE DIF(C1,C2,DIFF,N)
424 DIMENSION C1(N),C2(N),DIFF(N)
425 DO 11 I=1,N
426 DIFF(I)=C1(I)-C2(I)
427 11 CONTINUE
428 RETURN
429 END

```

```

427      SUBROUTINE STORE(BEST,COEF,ALPHA,BETA,GAMMA,HC,RANGE)
428      INTEGER BEST,HC
429      DIMENSION RANGE(2)
430      DIMENSION ALPHA(5),BETA(5),GAMMA(5),COEF(3,5)
431      COMMON IR,IP
432      IF(HC.NE.3) GO TO 40
433      9    DO 10 I=1,3
434          DO 10 I=1,5
435      10    CDEFIL(I)=0.
436      40    CONTINUE
437      IF(HC.EQ.3) J=1
438      IF(HC.EQ.1) J=2
439      IF(HC.EQ.2) J=3
440      N=BEST+1
441      IF(BEST.EQ.1) GO TO 11
442      IF(BEST.EQ.2) GO TO 13
443      IF(BEST.EQ.3) GO TO 15
444      11    DO 12 I=1,N
445          COEF(J,I)=ALPHA(I)
446      12    CONTINUE
447      GO TO 17
448      13    DO 14 I=1,N
449          COEF(J,I)=BETA(I)
450      14    CONTINUE
451      GO TO 17
452      15    DO 16 I=1,N
453          COEF(J,I)=GAMMA(I)
454      16    CONTINUE
455      17    IF(J.EQ.3) CALL HYSTER(RANGE,COEF)
456      RETURN
457      END

458      SUBROUTINE HYSTER(RANGE,COEF)
459      REAL MAX,MIN
460      DIMENSION RANGE(2),COEF(3,5),U(30),D(30)
461      DIMENSION DIFF (150)
462      COMMON IR,IP
463      SPAN=(RANGE(2)-RANGE(1))/25
464      X=RANGE(1)
465      DO 11 I=1,26
466          U(I)=COEF(2,1)+COEF(2,2)*X+COEF(2,3)*X**2+COEF(2,4)*X**3
467          D(I)=COEF(3,1)+COEF(3,2)*X+COEF(3,3)*X**2+COEF(3,4)*X**3
468          X=X+SPAN
469      11    CONTINUE
470      L=26
471      CALL DIFF(U,D,DIFF,L)
472      CALL MAXER(L,MAX,MIN,LD,LL,DIFF)
473      HYST=ABS(MAX)
474      WRITE(IP,100) (COEF(I,I),I=1,4)
475      WRITE(IP,101) (COEF(2,I),I=1,4)
476      WRITE(IP,102) (COEF(3,I),I=1,4)
477      WRITE(IP,103) HYST
478      100  FORMAT('C',T5,'THE COMBINED CURVE EQUATION IS',F10.7,'+',F10.7,'M+
479          *',F10.7,'M**2+',F10.7,'M**3')
480      101  FORMAT(' ',T5,' THE UP CURVE EQUATION IS',F10.7,'+',F10.7,'M+
481          *',F10.7,'M**2+',F10.7,'M**3')
482      102  FORMAT(' ',T5,' THE DOWN CURVE EQUATION IS',F10.7,'+',F10.7,'M+
483          *',F10.7,'M**2+',F10.7,'M**3')
484      103  FORMAT(' ',T5,'THE MAXIMUM DISTANCE BETWEEN THE UP CURVE AND THE D
485          *OWN CURVE IS',F8.5)

486      RETURN
487      END

```

Example Problem Output:

```

APRIL 101
DATE 4/17/79
INSTRUMENT 1
TEMPERATURE CELSIUS 50.00
PER CENT HUMIDITY 60.00
TIME 18.0000
NUMBER OF DATA POINTS 20
TRUE RANGE 0.00 TO 8.00
  
```

Combined "UP" and "DOWN" Data:

FIRST ORDER REGRESSION

$$T = 0.3011167 + 0.8788300M$$

TRUE VALUE	MEASURED VALUE	ERROR	CALCUTED VALUE	ERROR
0.80000	0.50000	-0.30000	0.74053	-0.05947
1.60000	1.38000	-0.22000	1.51390	-0.08610
2.40000	2.23000	-0.17000	2.26091	-0.13909
3.20000	3.20000	0.00000	3.11337	-0.08663
4.00000	4.08000	0.08000	3.88674	-0.11326
4.80000	5.02000	0.22000	4.71284	-0.08716
5.60000	5.90000	0.30000	5.48621	-0.11379
6.40000	6.80000	0.40000	6.27716	-0.12284
7.20000	7.70000	0.50000	7.06811	-0.13189
8.00000	8.60000	0.60000	7.85905	-0.14095
0.80000	0.57000	-0.23000	0.80205	0.00205
1.60000	1.60000	0.00000	1.70724	0.10724
2.40000	2.50000	0.10000	2.49819	0.09819
3.20000	3.50000	0.30000	3.37702	0.17702
4.00000	4.30000	0.30000	4.08009	0.08009
4.80000	5.30000	0.50000	4.95892	0.15892
5.60000	6.20000	0.60000	5.74986	0.14986
6.40000	7.20000	0.80000	6.62869	0.22869
7.20000	8.10000	0.90000	7.41964	0.21964
8.00000	8.60000	0.60000	7.85905	-0.14095
		MEAN= 0.26400		
		STANDARD DEVIATION= 0.35177		
		MAXIMUM ERROR= 0.90000		
		AT (7.20000, 8.10000)		
		MINIMUM ERROR= -0.30000		
		AT (0.80000, 0.50000)		
		SUM OF SQUARES= 3.74500		
		MEAN= -0.00000		
		STANDARD DEVIATION= 0.13583		
		MAXIMUM ERROR= 0.22869		
		AT (6.40000, 7.20000)		
		MINIMUM ERROR= -0.14095		
		AT (8.00000, 8.60000)		
		SUM OF SQUARES= 0.35054		

SECOND ORDER REGRESSION

$$T = 0.3660431 + 0.8375244M + 0.0044718M^2$$

TRUE VALUE	MEASURED VALUE	ERROR	CALCUTED VALUE	ERROR
0.80000	0.50000	-0.30000	0.78592	-0.01408
1.60000	1.38000	-0.22000	1.53034	-0.06966
2.40000	2.23000	-0.17000	2.25596	-0.14404
3.20000	3.20000	0.00000	3.09191	-0.10809
4.00000	4.08000	0.08000	3.85758	-0.14242
4.80000	5.02000	0.22000	4.68311	-0.11689
5.60000	5.90000	0.30000	5.46310	-0.13690
6.40000	6.80000	0.40000	6.26798	-0.13202
7.20000	7.70000	0.50000	7.08011	-0.11989
8.00000	8.60000	0.60000	7.89949	-0.10051
0.80000	0.57000	-0.23000	0.84488	0.04488
1.60000	1.60000	0.00000	1.71753	0.11753
2.40000	2.50000	0.10000	2.48780	0.08780
3.20000	3.50000	0.30000	3.35216	0.15216
4.00000	4.30000	0.30000	4.05008	0.05008
4.80000	5.30000	0.50000	4.93053	0.13053
5.60000	6.20000	0.60000	5.73059	0.13059
6.40000	7.20000	0.80000	6.62803	0.22803
7.20000	8.10000	0.90000	7.44338	0.24338
8.00000	8.60000	0.60000	7.89949	-0.10051
		MEAN= 0.26400		
		STANDARD DEVIATION= 0.35177		
		MAXIMUM ERROR= 0.90000		
		AT (7.20000, 8.10000)		
		MINIMUM ERROR= -0.30000		
		AT (0.80000, 0.50000)		
		SUM OF SQUARES= 3.74500		
		MEAN= -0.00000		
		STANDARD DEVIATION= 0.13312		
		MAXIMUM ERROR= 0.24338		
		AT (7.20000, 8.10000)		
		MINIMUM ERROR= -0.14404		
		AT (2.40000, 2.20000)		
		SUM OF SQUARES= 0.33670		

THIRD ORDER REGRESSION

$$T = 0.3039551 + 0.9138184M + -0.0160065M^{**2} + 0.0015001M^{**3}$$

TRUE VALUE	MEASURED VALUE	ERROR	CALCUTED VALUE	ERROR
0.80000	0.50000	-0.30000	0.75705	-0.04295
1.60000	1.38000	-0.22000	1.53848	-0.06152
2.40000	2.23000	-0.17000	2.27880	-0.12119
3.20000	3.20000	0.00000	3.11342	-0.08658
4.00000	4.08000	0.08000	3.86777	-0.13223
4.80000	5.02000	0.22000	4.67773	-0.12227
5.60000	5.90000	0.30000	5.44639	-0.15361
6.40000	6.80000	0.40000	6.24947	-0.15053
7.20000	7.70000	0.50000	7.07619	-0.12381
8.00000	8.60000	0.60000	7.93312	-0.06688
0.80000	0.57000	-0.23000	0.81991	0.01991
1.60000	1.60000	0.00000	1.73123	0.13123
2.40000	2.50000	0.10000	2.51190	0.11190
3.20000	3.50000	0.30000	3.37056	0.17056
4.00000	4.30000	0.30000	4.05668	0.05668
4.80000	5.30000	0.50000	4.92090	0.12090
5.60000	6.20000	0.60000	5.71186	0.11186
6.40000	7.20000	0.80000	6.61359	0.21359
7.20000	8.10000	0.90000	7.45293	0.25293
8.00000	8.60000	0.60000	7.93312	-0.06688
		MEAN= 0.28400		
		STANDARD DEVIATION= 0.35177		
		MAXIMUM ERROR= 0.90000		
		AT (7.20000, 8.10000)		
		MINIMUM ERROR= -0.30000		
		AT (0.80000, 0.50000)		
		SUM OF SQUARES= 3.74500		
		MEAN= 0.30306		
		STANDARD DEVIATION= 0.13174		
		MAXIMUM ERROR= 0.25293		
		AT (7.20000, 8.10000)		
		MINIMUM ERROR= -0.15361		
		AT (5.60000, 5.90000)		
		SUM OF SQUARES= 0.32995		

DATA IS BEST FITTED BY A THIRD ORDER REGRESSION CURVE

BIAS= 0.3039551

NONLINEARITY= 0.1138868

FIGURE OF MERIT= 1.6055220

"UP" Data:

FIRST ORDER REGRESSION

$$T = 0.3781891 + 0.8856659M$$

TRUE VALUE	MEASURED VALUE	ERROR	CALCUTED VALUE	ERROR
0.80000	0.50000	-0.30000	0.82102	0.02102
1.60000	1.38000	-0.22000	1.60341	0.00341
2.40000	2.23000	-0.17000	2.35322	-0.04678
3.20000	3.20000	0.00000	3.21232	0.01232
4.00000	4.08000	0.08000	3.99170	-0.00830
4.80000	5.02000	0.22000	4.82423	0.02423
5.60000	5.90000	0.30000	5.60362	0.00362
6.40000	6.80000	0.40000	6.40072	0.00072
7.20000	7.70000	0.50000	7.19782	-0.00218
8.00000	8.60000	0.60000	7.99492	-0.00508
		MEAN= 0.14100		
		STANDARD DEVIATION= 0.31328		
		MAXIMUM ERROR= 0.60000		
		AT (8.00000, 8.60000)		
		MINIMUM ERROR= -0.30000		
		AT (0.80000, 0.50000)		
		SUM OF SQUARES= 1.08210		
		MEAN= -0.00000		
		STANDARD DEVIATION= 0.01967		
		MAXIMUM ERROR= 0.02423		
		AT (4.80000, 5.02000)		
		MINIMUM ERROR= -0.04678		
		AT (2.40000, 2.23000)		
		SUM OF SQUARES= 0.00348		

SECOND ORDER REGRESSION

$$T = 0.3771057 + 0.8863678M + -0.0000772M^2$$

TRUE VALUE	MEASURED VALUE	ERROR	CALCUTED VALUE	ERROR
0.80000	0.50000	-0.30000	0.82027	0.02027
1.60000	1.38000	-0.22000	1.60015	0.00015
2.40000	2.23000	-0.17000	2.35332	-0.04668
3.20000	3.20000	0.00000	3.21269	0.01269
4.00000	4.09000	0.08000	3.99220	-0.00780
4.80000	5.02000	0.22000	4.82472	0.02472
5.60000	5.90000	0.30000	5.60399	0.00399
6.40000	6.80000	0.40000	6.40083	0.00083
7.20000	7.70000	0.50000	7.19756	-0.00244
8.00000	8.60000	0.60000	7.99415	-0.00585
		MEAN= 0.14100		
		STANDARD DEVIATION= 0.31328		
		MAXIMUM ERROR= 0.60000		
		AT (8.00000, 8.60000)		
		MINIMUM ERROR= -0.30000		
		AT (0.80000, 0.50000)		
		SUM OF SQUARES= 1.08210		
		MEAN= -0.00001		
		STANDARD DEVIATION= 0.01966		
		MAXIMUM ERROR= 0.02472		
		AT (4.80000, 5.02000)		
		MINIMUM ERROR= -0.04668		
		AT (2.40000, 2.23000)		
		SUM OF SQUARES= 0.00348		

THIRD ORDER REGRESSION

$$T = 0.3471680 + 0.9233398M + -0.0103912M^2 + 0.0007534M^3$$

TRUE VALUE	MEASURED VALUE	ERROR	CALCUTED VALUE	ERROR
0.80000	0.50000	-0.30000	0.80633	0.00633
1.60000	1.38000	-0.22000	1.60357	0.00357
2.40000	2.23000	-0.17000	2.36289	-0.03711
3.20000	3.20000	0.00000	3.22014	0.02014
4.00000	4.08000	0.08000	3.99259	-0.00741
4.80000	5.02000	0.22000	4.81578	0.01578
5.60000	5.90000	0.30000	5.58789	-0.01211
6.40000	6.80000	0.40000	6.38228	-0.01772
7.20000	7.70000	0.50000	7.18474	-0.01526
8.00000	8.60000	0.60000	7.99856	-0.00144
		MEAN= 0.14102		
		STANDARD DEVIATION= 0.31328		
		MAXIMUM ERROR= 0.60000		
		AT (8.00000, 8.60000)		
		MINIMUM ERROR= -0.30000		
		AT (0.80000, 0.50000)		
		SUM OF SQUARES= 1.08210		
		MEAN= -0.00452		
		STANDARD DEVIATION= 0.01709		
		MAXIMUM ERROR= 0.02014		
		AT (3.20000, 3.20000)		
		MINIMUM ERROR= -0.03711		
		AT (2.40000, 2.23000)		
		SUM OF SQUARES= 0.00283		

DATA IS BEST FITTED BY A THIRD ORDER REGRESSION CURVE

BIAS= 0.3471680

NONLINEARITY= 0.0281067

FIGURE OF MERIT= 0.2104568

"DOWN" Data:

FIRST ORDER REGRESSION

$$T = 0.2065277 + 0.8760109M$$

TRUE VALUE	MEASURED VALUE	ERROR	CALCUTED VALUE	ERROR
0.80000	0.57000	-0.23000	0.70585	-0.09415
1.60000	1.60000	0.00000	1.60814	0.00814
2.40000	2.50000	0.10000	2.39655	-0.00344
3.20000	3.50000	0.30000	3.27257	0.07257
4.00000	4.30000	0.30000	3.97337	-0.02663
4.80000	5.30000	0.50000	4.84939	0.04939
5.60000	6.20000	0.60000	5.63779	0.03779
6.40000	7.20000	0.80000	6.51381	0.11381
7.20000	8.10000	0.90000	7.30222	0.10222
8.00000	8.60000	0.60000	7.74022	-0.25978
		MEAN= 0.38700		
		STANDARD DEVIATION= 0.35982		
		MAXIMUM ERROR= 0.90000		
		AT (7.20000, 8.10000)		
		MINIMUM ERROR= -0.23000		
		AT (0.80000, 0.57000)		
		SUM OF SQUARES= 2.66290		
		MEAN= -0.00001		
		STANDARD DEVIATION= 0.11039		
		MAXIMUM ERROR= 0.11381		
		AT (6.40000, 7.20000)		
		MINIMUM ERROR= -0.25978		
		AT (8.00000, 8.60000)		
		SUM OF SQUARES= 0.10967		

SECOND ORDER REGRESSION

$$T = 0.3556061 + 0.7836761M + 0.0098362M^2$$

TRUE VALUE	MEASURED VALUE	ERROR	CALCUTED VALUE	ERROR
0.80000	0.57000	-0.23000	0.80550	0.00550
1.60000	1.60000	0.00000	1.63467	0.03467
2.40000	2.50000	0.10000	2.37627	-0.02373
3.20000	3.50000	0.30000	3.21897	0.01897
4.00000	4.30000	0.30000	3.90728	-0.09272
4.80000	5.30000	0.50000	4.78539	-0.01461
5.60000	6.20000	0.60000	5.59250	-0.00750
6.40000	7.20000	0.80000	6.50798	0.10798
7.20000	8.10000	0.90000	7.34873	0.14874
8.00000	8.60000	0.60000	7.82271	-0.17729
		MEAN= 0.38700		
		STANDARD DEVIATION= 0.35982		
		MAXIMUM ERROR= 0.90000		
		AT (7.20000, 8.10000)		
		MINIMUM ERROR= -0.23000		
		AT (0.80000, 0.57000)		
		SUM OF SQUARES= 2.66290		
		MEAN= -0.00300		
		STANDARD DEVIATION= 0.09204		
		MAXIMUM ERROR= 0.14874		
		AT (7.20000, 8.10000)		
		MINIMUM ERROR= -0.17729		
		AT (8.00000, 8.60000)		
		SUM OF SQUARES= 0.07624		

THIRD ORDER REGRESSION

$$T = 0.2375488 + 0.7257813M + -0.0284271M^2 + 0.0027733M^3$$

TRUE VALUE	MEASURED VALUE	ERROR	CALCUTED VALUE	ERROR
0.80000	0.57000	-0.23000	0.75652	-0.04348
1.60000	1.60000	0.00000	1.65738	0.05738
2.40000	2.50000	0.10000	2.41766	0.01766
3.20000	3.50000	0.30000	3.24846	0.04846
4.00000	4.30000	0.30000	3.91329	-0.08671
4.80000	5.30000	0.50000	4.75855	-0.04145
5.60000	6.20000	0.60000	5.54560	-0.05440
6.40000	7.20000	0.80000	6.46463	0.06463
7.20000	8.10000	0.90000	7.34511	0.14511
8.00000	8.60000	0.60000	7.86076	-0.13924
		MEAN= 0.38700		
		STANDARD DEVIATION= 0.35982		
		MAXIMUM ERROR= 0.90000		
		AT (7.20000, 8.10000)		
		MINIMUM ERROR= -0.23000		
		AT (0.80000, 0.57000)		
		SUM OF SQUARES= 2.66290		
		MEAN= -0.00320		
		STANDARD DEVIATION= 0.08469		
		MAXIMUM ERROR= 0.14511		
		AT (7.20000, 8.10000)		
		MINIMUM ERROR= -0.13924		
		AT (8.00000, 8.60000)		
		SUM OF SQUARES= 0.06466		

DATA IS BEST FITTED BY A THIRD ORDER REGRESSION CURVE
BIAS= 0.2375488
NONLINEARITY= 0.2127304
FIGURE OF MERIT= 1.0051560

Equations and Hysteresis:

THE COMBINED CURVE EQUATION IS $0.3039551 + 0.7138184M + -0.0160065M^2 + 0.0015001M^3$
THE UP CURVE EQUATION IS $0.3471680 + 0.7233398M + -0.0103912M^2 + 0.0007534M^3$
THE DOWN CURVE EQUATION IS $0.2375488 + 0.7257813M + -0.0284271M^2 + 0.0027733M^3$
THE MAXIMUM DISTANCE BETWEEN THE UP CURVE AND THE DOWN CURVE IS 0.30794

Appendix II

Description of On-Line Error Reduction Programs

1.0 On-Line Program for Simple Error Reduction:

This program consists only of a main routine. The data required is the coefficients from the regression equation which are defined by the use of a DATA statement. The data may be altered for a particular instrument by changing this statement. The measured value is entered from a terminal by the use of a read statement. It may be changed to be read from an A-to-D converter.

A true value is calculated inserting the measured value into a regression equation. A first, second, or third order regression equation may be used.

1.1 Description of Variables:

Refer to the program listing for the description of the following variables (Figure 2.2).

A, B, C, D - Regression coefficients from a third order
regression equation Ax^3+Bx^2+Cx+D

VMES - Measured value

TRUE - Best estimate of true

Input Format:

No input format is required for the regression coefficients since they are input by a DATA statement.

The measured value (VMES) is input using a F10.4 format specification but it may be altered as required.

Output:

The program prints out the input data (regression coefficients) with appropriate labels. The measured value (VMES) and corrected value (TRUE) are printed using a 2F10.4 format specification. The format definition may be changed as required. Refer to the example run for a typical printout (Figure 2.2).

2.0 On-Line Program for Error Reduction with Measurement Parameters

The input data for this program is a set of regression coefficients corresponding to different combinations of parameter conditions. For this example program, the two parameters considered are temperature and humidity.

To facilitate the association of parameters to coefficients, subscripted variables were utilized. Temperature and humidity were defined as single subscripted variables (eg. TEMP(I), HUM(I) and regression coefficients as double subscripted variable (eg. A(I,J), B(I,J) etc.) For example, if nine combinations of temperature and humidity are available with 3 values of temp and 3 values of humidity, then TEMP (1) through TEMP (3) will store the values of the three temperatures and HUM (1) through HUM(3) will store the values of the three humidities. Similarly, A(2,3) will store the first coefficient of the equation corresponding to the combination of parameters [TEMP(2), HUM(3)].

The measured value (VMES), temperature (TEMPR), and humidity (HUMID) are entered from a terminal using a read statement. It may be designed to be read from an A to D converter.

The calculation of the true value from the regression equation is done in subroutine COMP.

Linear interpolation is done in subroutine INTRP.

A block diagram that describes the functional parts of this program is in Figure II-1. Refer to the flow chart of figure to understand the overall procedure of the program.

Detailed description of the subroutines are presented in another section.

2.1 Description of variables of main program

Refer to the program listing for description of the following variables (Figure II-2)

VMES - Measured value

TEMPR - Temperature value input

HUMID - Humidity value input

IHU, IHL - Subscript corresponding to upper and lower bound of humidity.

ITU, ITL - Subscript corresponding to upper and lower bound of
temperature

T1, T2, TRUE 1, TRUE 2 - Preliminary calculated true values.

TRUE - Final true value

TEMP - Temperature values at which coefficients are available.

HUM - Humidity values at which coefficients are available.

NTV - Number of temperature values.

NHV - Number of humidity values.

Figure II-1. STRUCTURE CHART OF PROGRAM FOR ON-LINE ERROR REDUCTION WITH MEASUREMENT PARAMETERS

2.2 Input

The regression coefficients are entered by a data statement and may be changed for a particular instrument.

The measured temperature, and humidity values are input from a terminal using a read statement. It may be modified to be input from a A to D converter or any other input device.

All input data is from the main program.

2.3 Output

The program prints out the set of regression coefficients with appropriate labels. The printout of the results include the measured value, corrected value, temperature, and humidity. Refer to the example run for a typical printout of the program.

2.4 Subroutines Required:

Subroutine FIND:

This subroutine receives a parameter value read on the main program and returns back the values of two subscripts corresponding to the upper and lower values of the parameter value read. For example, the subroutine receives a temperature avlue of 15°C and returns back ITV=2, ITL=3, measuring that the temperature value read is included in the range of TEMP(2) through TEMP(3).

Temperature and humidity values (TEMP,HUM) are defined as common variables in the main program and this subroutine.

Description of Variables of Subroutine FIND:

Refer to the program listing, section of subroutine FIND for the explanation of these parameters:

VAL - Parameter value transferred from main program.

NTYPE - Type of parameter transferred.

1 - temperature

2 - humidity

IV, IL - Subscript corresponding to upper and lower bound of parameter transferred from main program. These variables are returned to the main program.

Subroutine COMP

Computes a true value using a third order regression equation for a given set of environmental parameters. A first or second order regression equation may be used instead by changing the statement that defines the equation accordingly.

The subroutine receives from the main program the measured value (UMES), the subscript of the temperature value, and the subscript of the humidity value. It returns an estimate of the true value to the main program. For example, input values to the subroutine are:

VMES = X

I = subscript of temperature value.

J = subscript of humidity value.

The subroutine computes a true value using the third order regression equation:

$$\text{TRUE} = A(I,J)X^3 + B(I,J)X^2 + C(I,J)X + D(I,J)$$

and returns the true value to the main program.

The coefficients of the regression equation are defined as common variables in the main program and this subroutine.

Description of variables of subroutine COMP

Refer to the program listing, section of subroutine COMP for the explanation of these variables.

TRUE - 'true' value calculated from a regression equation. This value is transferred to the main program.

X - measured value. Transferred from main program.

I - subscript corresponding to temperature value.

J - subscript corresponding to humidity value.

Subroutine INTRP

Performs linear interpolation between two points having coordinates (X1, y1) and X2, y2). The subroutine receives the values of the coordinate at the two points (X1, y1), (X2, y2) with the value of the y-coordinate between X1 and X2 is returned to the main program.

The X values correspond to true values computed in subroutine COMP at temperature values (y) assuming the humidity is remained constant.

Description of Variables to Subroutine INTRP

Refer to program listing, section of subroutine INTRP for the explanation of these parameters:

X₁, X₂ - True values. Transferred from main program.

y₁, Y₂ - Temperature values transferred from main program.

y - Temperature value between y₁ and y₂ at which the true value is to be calculated by linear interpolation. Transferred from main program.

y - True value of temperature y. Transferred to main program.

```

PROGRAM GFB
COMMON A3(3,4), B3(3,4), C3(3,4), D3(3,4), TEMP(3),
&HUM(4), NTV, NHV
DATA NTV, NHV, TEMP, HUM/3, 4, 10, 20, 30, 30, 40, 50, 60 /
DATA A3/. 002352, . 0018966, . 0021663, . 0010762, . 0009954, . 0010762,
&. 0012228, . 0007584, . 0010021, . 0009954, . 0002139, . 0018966/
DATA B3/-. 0393295, -. 0322456, -. 0360031, -. 022892, -. 022316, -. 022892,
&-. 0252228, -. 0198326, -. 0221367, -. 022316, -. 0111847, -. 0322456/
DATA C3/1. 131958, 1. 100418, 1. 115463, 1. 077987, 1. 081695, 1. 077987,
&1. 089202, 1. 077606, 1. 077819, 1. 081695, 1. 039657, 1. 100418/
DATA D3/-. 059845, -. 0162964, . 0445175, -. 0120773, . 0073853, . 0120773,
&-. 0278397, . 0098419, -. 0214539, -. 0073853, . 0289383, . 0162964/
WRITE(5,200)
WRITE(5,201)
WRITE(5,202)
N=3
WRITE(5,205)((TEMP(I), HUM(J), N, A3(I, J), B3(I, J), C3(I, J), D3(I, J),
&J=1, NHV), I=1, NTV)
WRITE(5,300)
WRITE(5,301)
10 WRITE(5,150)
READ(5,151)VMES
WRITE(5,152)
READ(5,151)TEMPR
WRITE(5,153)
READ(5,151)HUMID
CALL FIND(HUMID, 2, IHU, IHL)
CALL FIND(TEMPR, 1, ITU, ITL)
CALL COMP(T1, VMES, ITU, IHU)
CALL COMP(T2, VMES, ITL, IHU)
CALL INTRP(TRUE1, TEMPR, T1, TEMP(ITU), T2, TEMP(ITL))
CALL COMP(T1, VMES, ITU, IHL)
CALL COMP(T2, VMES, ITL, IHL)
CALL INTRP(TRUE2, TEMPR, T1, TEMP(ITU), T2, TEMP(ITL))
CALL INTRP(TRUE, HUMID, TRUE1, HUM(IHU), TRUE2, HUM(IHL))
WRITE(5,302)VMES, TRUE, TEMPR, HUMID
GO TO 10
150 FORMAT(1X, 'VMES= ')
151 FORMAT(F7.2)
152 FORMAT(1X, 'TEMPR= ')
153 FORMAT(1X, 'HUMID= ')
200 FORMAT(1H1, ///, 15X, 31H----- SET OF COEFFICIENTS -----, /)
201 FORMAT(1H0, 3X, 4HTEMP, 3X, 5HHUMID, 2X, 3HREG)
202 FORMAT(4X, 3H(C), 4X, 3H(%), 3X, 5HORDER, 8X, 1HA, 11X, 1HB, 11X, 1HC, 11X, 1HD
&)
205 FORMAT(3X, F5.1, 2X, F5.1, 4X, I1, 2X, 4F12.7)
300 FORMAT(2H1, ///, 25X, 19H----- RESULTS -----, /)
301 FORMAT(1H0, 7X, 14HMEASURED VALUE, 6X, 15HCORRECTED VALUE, 6X, 11HTEMPER
&ATURE, 7X, 8HHUMIDITY)
302 FORMAT(9X, F8.3, 12X, F8.3, 14X, F6.2, 10X, F6.2)
END

```

Figure II-2. LISTING OF ON-LINE MEASUREMENT ERROR REDUCTION PROGRAM

```

****
C**** LINEAR INTERPOLATION ****
C****
      SUBROUTINE INTRP(X, Y, X1, Y1, X2, Y2)
      IF(Y1-Y2)52, 51, 52
51  ALPH2=0.
      GO TO 53
52  ALPH2=(Y-Y1)/(Y2-Y1)
53  ALPH1=1-ALPH2
      X=ALPH1*X1+ALPH2*X2
      RETURN
      END

C****
C**** FIND UPPER & LOWER BOUND OF PARAMETERS ****
C****
      SUBROUTINE FIND(VAL, NTYPE, IU, IL)
      COMMON A3(3, 4), B3(3, 4), C3(3, 4), D3(3, 4), TEMP(3), HUM(4),
&NTV, NHV
      GO TO (1, 2), NTYPE
1  DO 30 I=1, NTV
      IF(VAL-TEMP(I))10, 12, 30
10  IU=I
      IL=I-1
      GO TO 35
12  IF(TEMP(I)-TEMP(1))17, 17, 10
17  IU=I
      IL=I
      GO TO 35
30  CONTINUE
      GO TO 35
2  DO 31 I=1, NHV
      IF(VAL-HUM(I))11, 14, 31
11  IU=I
      IL=I-1
      GO TO 35
14  IF(HUM(I)-HUM(1))16, 16, 10
16  IU=I
      IL=I
      GO TO 35
31  CONTINUE
35  RETURN
      END

C****
C**** COMPUTE TRUE VALUE ****
C****
      SUBROUTINE COMP(TRUE, X, I, J)
      COMMON A3(3, 4), B3(3, 4), C3(3, 4), D3(3, 4), TEMP(3), HUM(4),
&NTV, NHV
      TRUE=A3(I, J)*X**3+B3(I, J)*X*X+C3(I, J)*X+D3(I, J)
      RETURN
      END

```

Figure II-2 (cont.). LISTING OF ON-LINE MEASUREMENT ERROR REDUCTION PROGRAM

3.0 On-line Program for Reduction of Hysteresis Errors:

The function of this program is very similar to the one for error reduction with measurement parameters. The main difference is that the true values for given parameters must be calculated using three different equations instead of a single one. These three true values are obtained from the equations for the UP, COMBINED, and DOWN curves and are used in subroutine HYSTE for hysteresis error correction.

The input data for this program is a set of regression coefficients corresponding to the UP, combined, and down curves for different combinations of temperature and humidity.

Changes in the program, as compared to the program for correction with measurement parameters, include the addition of a do-loop to repeat the process of calculating true values three times, with proper statements to define these true values accordingly.

Changes in the subroutine COMP, FIND, and INTRP are discussed in a separate section together with the description of subroutine HYSTE for hysteresis error. A simplified flow chart is shown in Figure 4.8 and a structure chart that describes the functional parts of this program is in Figure II-3. Refer to the program listing of both programs to denote changes (Figure II-2 and II-4).

3.1 Description of variables of main program

VMES - measured value

TEMPR - temperature value input

HUMID - humidity value input

IHU, IHL - Subscript corresponding to upper and lower bound of
humidity

Figure II-3. STRUCTURE CHART OF PROGRAM FOR ON-LINE REDUCTION OF HYSTERESIS ERROR

ITU, ITL - Subscript corresponding to upper and lower bound of
temperature

T1, T2, TRUE1, TRUE2 - Preliminary calculated true values

TRUE - True value before hysteresis correction

TEMP - Temperature value at which coefficients are available

HUM - Humidity value at which coefficients are available.

NTV - Number of temp. values

NHV - Number of humidity values

VPI - True value at up curve

CMI - True value at combined curve

DNI - True value at down curve

INIT - Reset indicator for subroutine HYTE

BI - Final true value with hysteresis error corrected.

3.2 Subroutines Required:

a. Subroutine FIND

No changes are done in this subroutine.

b. Subroutine COMP

A variable (IX) was included in the list of variables transferred from the main program to determine from which equation is the true value calculated.

If IX equals:

1. use UP curve equation
2. use combined curve equation
3. use down curve equation

Other variables are as defined in previous program.

c. Subroutine INTRP

No changes are done in this subroutine.

Subroutine HYTE

This subroutine corrects for hysteresis error based on the algorithm described earlier in the text. It receives the calculated true values from the UP, combined, and down curves for a given measured value and returns to the main program an unfound estimate of the true value.

Description of Variables for Subroutine HYTE

Refer to the program listing for a description of the following variables:

INIT - variable to reset algorithm

VPI - True value at UP curve

CMI - True value at combined curve

DNI - True value at down curve

VMI - Most recent measured value

BI - Most recent Best estimate at true value

FI - Preliminary best true value.

VMIMS - Previous true value at UP curve

DNIMI - Previous true value at down curve

Slope - Indicates direction of movement at present measured value

sloped - Indicates previous direction of movement C = value of K

```

PROGRAM ALE
COMMON AU3(5,4), BU3(5,4), CU3(5,4), DU3(5,4), TEMP(5),
&HUM(4), NTV, NHV, AC3(5,4), BC3(5,4), CC3(5,4), DC3(5,4),
1AD3(5,4), BD3(5,4), CD3(5,4), DD3(5,4)
DATA NTV, NHV, TEMP, HUM/5, 4, 10., 20., 30., 40., 50., 30., 40., 50., 60. /
DATA AC3/. 0014362., 0034752., 0063326., 0064702., 0054765., 0017457,
!. 0035357., 0067375., 006181., 0051358., 0015342., 0037262., 0058968,
". 0063088., 0025086., 0015383., 0030534., 0049005., 0058653., 0013011/
DATA BC3/-. 0017643, -. 0385437, -. 0746078, -. 0765686, -. 0639038,
!-. 0213223, -. 0392227, -. 0794601, -. 0725098, -. 0588303, -. 0175686,
"- . 0411453, -. 0704155, -. 074398, -. 0280113, -. 016758, -. 034729, -. 0583
#115, -. 0704765, -. 0133591/
DATA CC3/. 9797058, 1. 0359726, 1. 1373596, 1. 1286621, 1. 08078, . 990387,
!1. 0365448, 1. 1485138, 1. 1136475, 1. 0581741, . 964119, 1. 039978, 1. 1275
"94, 1. 1223602, . 9661255, . 9589005, 1. 02565, 1. 096405, 1. 1161041,
#. 9042969/
DATA DC3/. 3280792., 2143097., 1164474., 102478., 1186676., 3248291,
!. 2171097., 1077347., 1128769., 1476059., 3589478., 2163696., 1195908,
". 1026917., 2194977., 3655701., 2162094., 1020584., 1033859., 3066635/
DATA AU3/. 0012918., 0028262., 0047464., 0057218., 0049341., 0020134,
!. 0030096., 0048039., 0050561., 0050828., 0014548., 0031083., 0034707,
". 0050707., 0025949., 0010387., 0022092., 0012908., 0042851., 0003475/
DATA BU3/-. 0188274, -. 0314732, -. 0559578, -. 0662537, -. 0563698,
!-. 0260792, -. 0349884, -. 0554008, -. 0579987, -. 0566654, -. 0206985,
"- . 0369244, -. 0426559, -. 0589676, -. 0294209, -. 0149498, -. 0285988,
#-. 0172615, -. 0505486, -. 0046597/
DATA CU3/. 9930954, 1. 011931, 1. 0847855, 1. 0878067, 1. 0500412, 1. 00563
!05, 1. 036499, 1. 0746918, 1. 0631332, 1. 0360413, . 9878693, 1. 0481567,
"1. 0591202, 1. 0729523, . 9653549, . 9730301, 1. 0348206, . 9993973, 1. 05121
#61, . 9038391/
DATA DU3/. 3913116., 2707443., 1945267., 1785431., 1868286., 3882675,
!, 2624359., 2032852., 1949921., 2381439., 4413567., 2544937., 2042084,
". 1803513., 3018875., 4611092., 2537003., 1722641., 2023849., 3540955/
DATA AD3/. 0019355., 0044346., 0081394., 0075028., 0062616., 0018823,
!. 0043004., 0090652., 0075939., 005758., 0022848., 0045137., 0084927,
". 0077848., 0030112., 0031476., 0040829., 0085666., 0078549., 0027814/
DATA BD3/-. 0217247, -. 0497169, -. 0961647, -. 0910797, -. 0749931,
!-. 022541, -. 0465775, -. 1085472, -. 0912018, -. 0694923, -. 0244904, -. 0474
"472, -. 099884, -. 0932312, -. 0355301, -. 0342522, -. 0429535, -. 0992737,
#-. 0962486, -. 0285244/
DATA CD3/. 989624, 1. 0683289, 1. 2015228, 1. 1874847, 1. 1268158, 1. 00097
!66, 1. 0487213, 1. 2408905, 1. 1819916, 1. 1173553, . 9851456, 1. 039856,
"1. 2015228, 1. 186203, 1. 0064697, 1. 0102921, 1. 0241699, 1. 1908569, 1. 205
#6122, . 9268646/
DATA DD3/. 2371597., 141243., 0242538., 0060425., 0329895., 23246,
!. 1577072, -. 0078201., 0099792., 0147095., 2231598., 1671829., 0255966, -
". 0078506., 0909576., 1957092., 1672668., 0309906, -. 0244904., 2369156/
WRITE(5,200)
WRITE(5,204)
WRITE(5,201)
WRITE(5,202)

```

Figure II-4. LISTING OF ON-LINE PARTIAL HYSTERESIS ERROR REDUCTION PROGRAM

```

N=3
WRITE(5,205)<<(TEMP(I), HUM(J), N, AU3(I, J), BU3(I, J), CU3(I, J
1), DU3(I, J), J=1, NHV), I=1, NTV)
WRITE(5,206)
WRITE(5,201)
WRITE(5,202)
WRITE(5,205)<<(TEMP(I), HUM(J), N, AC3(I, J), BC3(I, J), CC3(I, J
1), DC3(I, J), J=1, NHV), I=1, NTV)
WRITE(5,207)
WRITE(5,201)
WRITE(5,202)
WRITE(5,205)<<(TEMP(I), HUM(J), N, AD3(I, J), BD3(I, J), CD3(I, J
1), DD3(I, J), J=1, NHV), I=1, NTV)
WRITE(5,300)
WRITE(5,301)
INIT=0
10 WRITE(5,152)
READ(5,151)VMES
WRITE(5,153)
READ(5,151)TEMPR
WRITE(5,154)
READ(5,151)HUMID
CALL FIND(HUMID, 2, IHU, IHL)
CALL FIND(TEMPR, 1, ITU, ITL)
DO 40 IX=1, 3
CALL COMP(T1, VMES, ITU, IHU, IX)
CALL COMP(T2, VMES, ITL, IHU, IX)
CALL INTRP(TRUE1, TEMPR, T1, TEMP<ITU>, T2, TEMP<ITL>))
CALL COMP(T1, VMES, ITU, IHL, IX)
CALL COMP(T2, VMES, ITL, IHL, IX)
CALL INTRP(TRUE2, TEMPR, T1, TEMP<ITU>, T2, TEMP<ITL>))
CALL INTRP(TRUE, HUMID, TRUE1, HUM<IHU>, TRUE2, HUM<IHL>))
GO TO (1, 2, 3), IX
1 UPI=TRUE
GO TO 40
2 CMI=TRUE
GO TO 40
3 DNI=TRUE
40 CONTINUE
CALL HYTE<INIT, VMES, UPI, DNI, CMI, BI>
WRITE(5,302)VMES, BI, TEMPR, HUMID
GO TO 10
151 FORMAT(F7.2)
152 FORMAT(1X, 'MEASURED VALUE= ')
153 FORMAT(1X, 'TEMPERATURE= ')
154 FORMAT(1X, 'HUMIDITY= ')
200 FORMAT(1H1, '///, 15X, 31H----- SET OF COEFFICIENTS -----, /)
201 FORMAT(1H0, 3X, 4HTEMP, 3X, 5HHUMID, 2X, 3HREG)
202 FORMAT(4X, 3H<C>, 5X, 3H<?>, 2X, 5HORDER, 6X, 1HA, 11X, 1HB, 11X, 1HC, 11X, 1HD
&)

```

Figure II-4 (page 2). LISTING OF ON-LINE PARTIAL HYSTERESIS ERROR REDUCTION PROGRAM

```

205 FORMAT(3X, F5. 1, 2X, F5. 1, 4X, I1, 4F12. 7)
204 FORMAT(1H0, 15X, 31H      -----UP CURVE-----      )
206 FORMAT(1H0, 15X, 31H      -----COMBINED CURVE-----   )
207 FORMAT(1H0, 15X, 31H      -----DOWN CURVE-----      )
300 FORMAT(2H1, /, /, /, 25X, 19H----- RESULTS -----, /)
301 FORMAT(1H0, 7X, 14HMEASURED VALUE, 6X, 15HCORRECTED VALUE, 6X, 11HTEMPER
&ATURE, 7X, 8HHUMIDITY)
302 FORMAT(9X, F8. 3, 12X, F8. 3, 14X, F6. 1, 10X, F6. 1)
      END
C****
C****  LINEAR INTERPOLATION  ****
C****
      SUBROUTINE INTRP(X, Y, X1, Y1, X2, Y2)
      IF(Y1-Y2)52, 51, 52
51  ALPH2=0.
      GO TO 53
52  ALPH2=(Y-Y1)/(Y2-Y1)
53  ALPH1=1-ALPH2
      X=ALPH1*X1+ALPH2*X2
      RETURN
      END
C****
C****  FIND UPPER & LOWER BOUNDS OF PARAMETERS  ***
C****
      SUBROUTINE FIND(VAL, NTYPE, IU, IL)
      COMMON AU3(5, 4), BU3(5, 4), CU3(5, 4), DU3(5, 4), TEMP(5),
&HUM(4), NTV, NHV, AC3(5, 4), BC3(5, 4), CC3(5, 4), DC3(5, 4),
1AD3(5, 4), BD3(5, 4), CD3(5, 4), DD3(5, 4)
      GO TO (1, 2), NTYPE
1  DO 30 I=1, NTV
      IF(VAL-TEMP(I))10, 12, 30
10  IU=I
      IL=I-1
      GO TO 35
12  IF(TEMP(I)-TEMP(1))17, 17, 10
17  IU=I
      IL=I
      GO TO 35
30  CONTINUE
      GO TO 35
2  DO 31 I=1, NHV
      IF(VAL-HUM(I))11, 14, 31
11  IU=I
      IL=I-1
      GO TO 35
14  IF(HUM(I)-HUM(1))16, 16, 10
16  IU=I
      IL=I
      GO TO 35
31  CONTINUE
35  RETURN
      END

```

Figure II-4 (page 3). LISTING OF ON-LINE PARTIAL HYSTERESIS ERROR
REDUCTION PROGRAM

```

***
*** COMPUTE TRUE VALUE ***
C***
SUBROUTINE COMP(TRUE,X,I,J,N)
COMMON AU3(5,4),BU3(5,4),CU3(5,4),DU3(5,4),TEMP(5),
&HUM(4),NTV,NHV,AC3(5,4),BC3(5,4),CC3(5,4),DC3(5,4),
1AD3(5,4),BD3(5,4),CD3(5,4),DD3(5,4)
GO TO (1,2,3),N
1 TRUE=AU3(I,J)*X**3+BU3(I,J)*X*X+CU3(I,J)*X+DU3(I,J)
GO TO 10
2 TRUE=AC3(I,J)*X**3+BC3(I,J)*X*X+CC3(I,J)*X+DC3(I,J)
GO TO 10
3 TRUE=AD3(I,J)*X**3+BD3(I,J)*X*X+CD3(I,J)*X+DD3(I,J)
10 RETURN
END

```

```

C***
C*** CORRECT FOR HYSTERESIS ERROR ***
C***
SUBROUTINE HYTE(INIT,VMI,UPI,DNI,CMI,BI)
C=8.0
IF(INIT.EQ.1) GO TO 100
BI=CMI
SLOPE=1
GO TO 200
100 SLOPE=VMI-VMIM1
IF((SLOPE1*SLOPE).GE.0.0) GO TO 110
BI=CMI
GO TO 200
110 FI=DNI+(UPI-DNI)*(BIM1-DNIM1)/(UPIM1-DNIM1)
IF(SLOPE) 120,130,130
120 BI=FI-(FI-DNI)*(VMIM1-VMI)/C
GO TO 200
130 BI=FI+(UPI-FI)*(VMI-VMIM1)/C
GO TO 200
200 BIM1=BI
VMIM1=VMI
UPIM1=UPI
DNIM1=DNI
SLOPE1=SLOPE
RETURN
END

```

Figure II-4. (page 4). LISTING OF ON-LINE PARTIAL HYSTERESIS ERROR REDUCTION PROGRAM

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