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Kirkpatrick-Baez X-Ray- Microscope Optimization for Inertial-Confinement-Fusion Applications

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Abstract

Analytic approximations have been used to optimize single-channel and four-channel Kirkpatrick-Baez (K-B) x-ray microscopes in three ways: (I) for best effective solid angle at best resolution, (II) for best effective solid angle at a given resolution worse than the best, (III) for best effective solid angle at a specified breadth of field. All optimizations are also made consistent with clean channel response, to ensure that any high-energy second peak will be small compared with the primary channel response. This is achieved at a sacrifice of effective solid angle. We describe a "cross-talk" aberration not previously recognized in K-B x-ray microscopes. The analytic approximations presented in this paper, which describe the optical and constructional parameters of optimized K-B x-ray microscopes, are intended to allow scaling and localization in parameter space from which detailed numerical calculations can be used to fine-tune a microscope design.

1. Introduction

Perhaps the most significant application of x-ray imaging by grazing-incidence reflection from two orthogonal cylinders¹ has been the use of such Kirkpatrick-Baez x-ray microscopes in the laser fusion program. Such x-ray microscopes have been used for imaging laser fusion targets at Lawrence Livermore National Laboratory (LLNL) for several years (Fig. 1). During that time, several microscopes have been designed and fielded on the Janus, Argus, and Shiva laser systems. Kirkpatrick-Baez microscope systems to be used on the Helios laser facility at the Los Alamos National Laboratory have been constructed.

In successive designs, both laboratories have attempted to optimize microscope performance at wavelengths of interest. New microscope designs include better optical-surface quality and fixturing, and take account of better theoretical understanding of the optical design and reflecting materials required to optimize microscope resolution and collection efficiency. Here we report our calculations of the specifications of optimized Kirkpatrick-Baez x-ray microscopes.

Kirkpatrick-Baez x-ray microscopes (KBXRMS) are frequently mounted in the four-channel arrangement of Seward et al.² This arrangement (Fig. 2) provides four separate images, whose bandpasses are determined by the filter and reflector materials used. The microscope can be aligned by visible light because of the four-fold symmetry of the system; diffraction effects would make alignment of a single channel difficult. In order to produce an optic with four-fold symmetry to the required tolerances (in particular, tolerances on the radius of curvature and on surface roughness), the mirrors must be polished at the same time on the same lap. Because of the identical mirror radii and the close proximity of the two mirrors, the two orthogonal reflections undergone by a ray are at nearly the same angle of incidence.

In Sec. 2 we discuss practical considerations in optimizing the KBXRMS, such as metrology limitations, surface finish, commonality of focal region, cost, and ease of use.

Resolution and collection efficiency are both important in KB microscopes for laser fusion applications. We express collection efficiency in terms of the effective solid angle $\Omega_{\text{eff}} = \Omega_{\text{geom}} R^2$, where R is the mirror reflectivity. In Sec. 3 we formulate three optimization schemes: Type I, for best effective solid angle at best resolution; Type II, for best effective solid angle at a given, worse-than-best resolution; and Type III, for best effective solid angle at a given, fixed breadth of field.

Figure 1. Kirkpatrick-Baez x-ray microscope image of a 26-TW Shiva shot. The target was a glass microballoon mounted in a plate.

In Type I, best resolution is achieved when the geometrical aberration is approximately equal to the diffraction resolution. These results are similar to those of Prince.³ More precisely, the optimum resolution is achieved where the geometric resolution S_G is equal to $\eta_0 S_D$, where S_D is the diffraction resolution and η_0 is a constant of order unity. In Sec. 4 we calculate the resolution $S(\eta)$ (where $\eta = S_G/S_D$) from the formulations of Dyson⁴ and McGee et al.,⁵ and derive the optimum ratio η_0 .

In Sec. 3, we also show that (for each optimization scheme) the solid angle Ω of a microscope of optimum width is proportional to θ^n , where θ is the angle of incidence and n is an exponent that depends on the optimization scheme chosen. The effective solid angle is thus optimized by maximizing $\theta^n R^2(\theta)$ which is proportional to ΩR^2 . In Sec. 5, we consider this maximization by choice of angle and mirror material. Section 6 concludes the discussion of the optimization of a single-channel microscope. In Sec. 7 we consider the optimization of a symmetric, four-channel x-ray microscope with the broadest possible energy spread among the channels. We discuss the trade-offs that must be made because of differences in the magnitude of diffraction between the various energy channels.

Section 8 describes an aberration that arises from "cross-talk" between the two orthogonal mirrors as the small-angle approximation breaks down. This aberration, which goes as θ^3 , is severe for angles greater than 3 or 4 deg.

Section 9 is a summary and conclusion.

Figure 2. Ray diagram and image pattern for a four-channel KBXRMS arrangement.

2. Practical Considerations

First it is useful to consider some practical limitations in the design, fabrication, and use of a KBXRMS. In general, we will consider the four-channel x-ray microscope, but we will also discuss the single-channel system. Although each channel is a separate x-ray microscope, we require that the fields of view of the channels coincide almost exactly. The fields of view must be large enough to accommodate an entire laser fusion target (~ 1 mm) with reasonable resolution (5 to 10 μm). Over a somewhat smaller field of view, the resolution must be sufficient to distinguish the finest features expected (1 to 3 μm). The microscopes must work at an object distance large enough to avoid blast damage (to date, > 30 cm), and at a magnification high enough to produce images that are of useful size and compatible with the resolution element of the recording medium. The energy channels must give broad energy coverage without large gaps or significant overlap.

No method of coaligning the four channels is free of difficulty. Mounting the mirrors in a fixed cell may entail severe dimensional tolerances, some of which may require state-of-the-art metrology during fabrication. Mounting the mirrors in an adjustable cell increases the complexity and cost of the system and introduces a difficult alignment procedure. An adjustable system may go out of alignment during use. We believe that a properly designed fixed-mirror cell has the greatest promise of long-term stability and maintenance-free use, and appears to be most cost effective. A careful design will place the tightest tolerances on those parameters that are easiest to measure.

The mirrors for a laser fusion KBXRMS differ from flatness by a sagittal depth of only about $0.5 \mu\text{m}$. The best measurements of this depth are accurate to about $0.025 \mu\text{m}$, so that the radius of curvature and focal length of the mirror are known only to 5%. This variation is larger than the depth of field. By simultaneously polishing all mirrors in a set the same lap, as described earlier, one achieves identical focal lengths for all channels. The mirrors must be made from the same materials, because different materials would polish at different rates and develop different curvatures. A variety of reflector surfaces can be obtained by vacuum deposition of different materials on the polished surfaces.

The substrate material must be capable of taking a finish of order $10 \text{ \AA}$ rms. This severely restricts the choice of material. It is also useful to use a substrate that has a low atomic number (Z), because reflection might occur at the interface between a low- Z coating and a high- Z substrate. Low- Z substrate materials that take a good finish are silicon, quartz, and vitreous carbon. Beryllium is difficult to work with, because of its toxicity. Vitreous carbon has the lowest Z easily available, but is somewhat difficult to fabricate. Silicon is easy to polish, but is more expensive and has no reflectivity advantage over quartz.

Reflective coatings may degrade because of age and contamination. In this paper we only consider coatings that are stable in air and that can be cleaned if contaminated by organic material.

In some instances, the optimizations described below have been influenced by the practical considerations just stated.

3. Introduction to Optimizations Schemes

The two most important parameters of microscope performance for laser fusion applications are the resolution S at the center of the field of view and the collection efficiency, expressed as an effective solid angle Ω_{eff} . In Secs. 3-6, we approximately optimize S and Ω_{eff} for a single channel by judicious choice of the mirror width W , the grazing angle θ , and the reflecting material, of atomic number Z .

Figure 3 shows a two-dimensional cross section of the ray path associated with a single mirror in a KBXRMS. For small θ , the focusing effects of the two crossed mirrors in each channel are nearly independent. In the discussion below, we calculate the aberrations in one dimension of rays confined to a plane, and assume that the aberrated image produced by crossed mirrors is the product of the linear aberrations in the two dimensions. That is, we assume that the focal plane distribution $I(x, y)$ is given by the product

Figure 3. Two-dimensional cross section of the ray path associated with a single mirror in a KBXRMS. For low angles of incidence, the focusing effects of the two orthogonal mirrors in a channel are substantially independent. The microscope resolution can thus be analyzed in the two-dimensional geometry of the figure.

$I_1(x)I_2(y)$, where I_1 and I_2 are the one-dimensional focal-plane distributions introduced by the two mirrors. We test this assumption by ray traces in Sec. 8.

The two-dimensional "microscope" of Fig. 3 is subject to three principal aberrations. Because of the finite aperture of the optic, there is a diffraction aberration whose full base width (between the first nodes) is

$$S_D = \frac{2d_o \lambda}{W\theta}, \quad (1)$$

where d_o is the object-mirror distance, λ is the photon wavelength, and W is the mirror width. Here and below we make the small-angle approximation $\theta = \sin \theta = \tan \theta$. All aberration dimensions are referred to the object plane.

For a cylindrical mirror, "spherical" aberration causes rays reflecting from the front and back of the mirror to be focused at a different position than the central ray. This aberration can be eliminated by including a cubic term in the mirror figure (e.g., by using elliptical rather than circular curves),⁶ but introduction of this term greatly complicates mirror fabrication and does not eliminate diffraction or coma. In the geometric limit, the full width of the "spherical" aberration is

$$S_C = \frac{3}{16} \frac{W^2}{d_o} \theta \left(\frac{M^2 - 1}{M^2} \right), \quad (2)$$

where M is the microscope magnification.

Because the magnification is different at the front and back of the mirror, off-axis points will show a comatic aberration of full width

$$S_c = \frac{Wb}{d_o} \left(\frac{M+1}{M} \right), \quad (3)$$

where b is the off-axis distance in the object plane. (Equation 3 is only valid for $b > B_D \approx (3/8)W\theta(M-1)/M$). In many applications the best resolution is required only in a small region of the object plane, such as the center of a target, which can be selected before the exposure; the comatic aberration is then of secondary importance. If a third-order figure correction were made to eliminate spherical aberration, coma would become the major aberration to be balanced against diffraction.

The effective solid angle is given by

$$\Omega_{\text{eff}}(E) = \left[\frac{W\theta}{d_o} R(\theta, E) \right]^2, \quad (4)$$

where $R(\theta, E)$ is the mirror reflectivity at photon energy E . The mirror radius is generally so large that its curvature does not significantly affect the solid angle; the solid angle is that of a flat surface tilted at the mirror angle.

The parameters to be optimized (resolution S and effective solid angle Ω_{eff}) are mutually incompatible: a large effective solid angle requires a large mirror width W , which results in substantial spherical aberration and thus poor resolution. In the geometric limit, there is thus a trade-off between resolution and effective solid angle. We can improve resolution by decreasing W ($S \propto W^2$) while sacrificing Ω_{eff} . This trade-off occurs with θ held constant. The resolution cannot be improved indefinitely by decreasing W , because the diffraction resolution S_D becomes important at small W , and S increases as $1/W$ in the diffraction limit. Once W is decreased enough that S_C and S_D contribute about equally, further improvement in S can be obtained only by increasing θ . However, R^2 and thus Ω_{eff} decrease sharply as θ passes the critical angle θ_c .

Figure 4 shows the results of a numerical calculation to determine this trade-off locus. Each point on the locus represents the (W, θ) corresponding to the best possible S at a given Ω_{eff} . The calculation was made by searching along contours of constant Ω_{eff} in (W, θ) space for the smallest S , $S(\Omega_{\text{eff}})$. The vertical branch of the locus corresponds to the geometrical case; along the right-hand branch, one has the very

Figure 4. Optimization locus in the W - θ plane. Each point on the curve corresponds to the best possible resolution S at a given effective solid angle Ω_{eff} . To the left and right of each tick mark are the corresponding values of S and Ω_{eff} . The upper branch of the locus corresponds to the trade-off between S and Ω_{eff} in the geometric limit; the right branch shows the rapid loss of Ω_{eff} with little improvement in S , in the region where the geometric and diffraction resolutions are nearly equal. Along the right branch, the only improvement in S can come by increasing θ . The cross corresponds to the (W, θ) combination given by our Type I optimization. The locus is calculated for a nickel optic at $h\nu = 4$ keV, with a target-to-optic distance of 30 cm.

costly trade-off (in terms of effective solid angle) of increased grazing angle θ for resolution. The Ω_{eff} values given along the locus indicate the very rapid decrease in R^2 with angle as θ passes θ_c .

Figure 5 shows the path traced in the S - Ω_{eff} plane by the optimization locus. The bend in the locus corresponds to the bend in the plot of $S(\theta)$; the locus again shows that, below a certain resolution, further improvement comes at a great cost in collection efficiency.

Type I Optimization

Type I optimized microscopes will thus be found in the region of the bend, corresponding to the best resolution that can be obtained without great loss of efficiency. The optimization could be determined from numerical calculations such as those of Figs. 4 and 5; however, the purpose of this paper is to present analytical approximations to facilitate KBXRMS design. We find the optimized region analytically by minimizing the resolution at a given θ , to yield an optimum width W as a function of θ , and then choosing θ [and thus $W(\theta)$] to optimize Ω_{eff} . Because S is optimized over fixed θ rather than over fixed Ω_{eff} , this calculation does not guarantee a point on the optimization locus. However, numerical calculations show it is quite close (see the optimization point shown in Figs. 4 and 5).

Figure 5. Resolution S vs effective solid angle Ω_{eff} along the locus of Figure 4. Resolution can be effectively traded for solid angle along the geometric branch (steep part) of the curve to the right. To the left, resolution is gained very slowly with loss of effective solid angle. As in Fig. 4, the cross corresponds to the (W, θ) combination given by our Type I optimization.

It will be shown in Sec. 4 that the spot size (resolution) S is a function of the quantity

$$\eta = \frac{S_G}{S_D} = \frac{3}{32} \frac{W^3 \theta^3}{d_o^2 \lambda} \left(\frac{M^2 - 1}{M^2} \right); \quad (5)$$

specifically,

$$S = F(\eta) \left[\eta^{-2/3} S_G(W, \theta) \right]. \quad (6)$$

It can be shown that $\eta^{-2/3} S_G(W, \theta)$ is proportional to $\theta^{-1/3}$, and that this quantity has no W dependence. The function F is selected so that $S \propto F(\eta)$ at constant θ ; the minimum of $F(\eta)$ corresponds to the preferred W . Since the spherical aberration is proportional to W^2 and the diffraction aberration goes as $1/W$, it is apparent that $F(\eta)$, and thus S , have minima at some value we will call η_0 . The condition for best resolution at fixed θ is $S_G = \eta_0 S_D$. Thus the mirror width yielding the best resolution at a given θ is

$$W_0(\theta) = \left[\eta_0 \frac{32}{3} \lambda \left(\frac{d_o}{\theta} \right)^2 \frac{M^2}{M^2 - 1} \right]^{1/3}. \quad (7)$$

The geometric solid angle is

$$\Omega = \left(\frac{W\theta}{d_o} \right)^2 = \left(\eta_0 \frac{32}{3} \lambda \frac{\theta}{d_o} \frac{M^2}{M^2 - 1} \right)^{2/3}. \quad (8)$$

We now have a function $W(\theta)$ that is optimized for the best resolution at that θ ; an example is plotted in Fig. 4.

Our optimization scheme (Sec. 5) is to pick the value of θ along the line in the W, θ plane that gives best resolution to optimize $\Omega_{\text{eff}}[\theta, W(\theta)]$. Since $\Omega_{\text{eff}} = \Omega R^2(\theta, E) \propto \theta^{2/3} R^2(\theta, E)$, we can optimize Ω_{eff} for $W = W(\theta)$ by maximizing $\theta^{2/3} R^2(\theta, E)$. (That is, we have $n = 2/3$ for Type I optimization.) The function $\theta^{2/3} R^2$ can have a broad maximum. In order that the (W, θ) combination selected be definitely on the flat, "high-resolution" branch of the $\Omega_{\text{eff}}(S)$ curve (Fig. 5), we select an θ_{opt} greater than the θ_{max} that maximizes $\theta^{2/3} R^2$. If we choose $\theta_{\text{opt}} > \theta_{\text{max}}$ such that $\theta_{\text{opt}}^{2/3} R^2(\theta_{\text{opt}}, E) = 0.8 \theta_{\text{max}}^{2/3} R^2(\theta_{\text{max}}, E)$, we ensure that the point selected is along the flat branch. This also ensures that there will be no large secondary peak in the x-ray transmission of the filtered x-ray microscope channel.

Type II Optimization

If it is necessary to collect more flux than is provided by Type I optimization, resolution may be traded away for solid angle (Type II optimization). In this case we wish to maximize Ω_{eff} (by increasing W and/or θ) at a specified resolution. For large Ω_{eff} , the resolution is determined by spherical aberration, diffraction can be ignored. In this case

$$W = \left(\frac{16}{3} S_C \frac{d_0}{\theta} \frac{M^2}{M^2 - 1} \right)^{1/2} \quad (9)$$

and

$$\Omega = \left(\frac{W\theta}{d_0} \right)^2 = \frac{\theta}{d_0} \frac{16}{3} S_C \frac{M^2}{M^2 - 1} \quad (10)$$

Thus a true Type II optimization would maximize Ω_{eff} by maximizing the figure of merit $\theta R^2(\theta, E)$ (note that this corresponds to $n = 1$). However, for a reflector with significant bulk absorption, and thus a slowly decreasing $R(\theta)$, the optimum angle would be significantly less than the critical angle (see Sec. 5). In order that the chosen K-edge filters properly define the high-energy cutoff of a channel, the mirror must have no significant reflectivity at approximately twice the desired K-edge energy, or else the filter would pass the signal at that energy. We choose to push the operating angle closer to the critical angle by operating at the "80% angle," θ_{opt} , as in the "best resolution" case.

Type III Optimization

If enough flux is available and best resolution is not required, it may be desirable to specify a field of view, in the sense that one wishes to optimize resolution at the edge of the field of view. To do this, one operates the microscope in the diffraction-limited regime on axis. The field is then defined to extend to the point at which the size of comatic aberration equals that of diffraction aberration:

$$S_C = S_D \quad (11)$$

This implies that

$$W = \left(\frac{2 d_0^2 \lambda}{b \theta} \frac{M}{M + 1} \right)^{1/2} \quad (12)$$

and

$$\Omega = \left(\frac{W\theta}{d_0} \right)^2 = \frac{2\lambda\theta}{b} \frac{M}{M + 1} \quad (13)$$

In this case, as for Type II optimization, $\theta R^2(\theta, E)$ must be maximized to maximize Ω (and, again, $n = 1$).

4. Evaluation of the Function $F(\eta)$

The resolution of a KBXRMS is easy to calculate in the limiting cases in which diffraction or spherical aberration dominate the point spread function. The region of greatest interest, however, is the region where diffraction and spherical aberration contribute about equally. In this region, a proper calculation using diffraction theory must be carried out. Dyson⁴ and McGee⁵ have made calculations of this type. In our notation, the object-plane equivalent intensity distribution in one dimension is given by

$$I(x) = \left(\frac{W\theta}{2 d_o} \frac{M+1}{M} \right)^2 \left\{ \int_0^1 \cos \left[\frac{2\pi}{3} \eta \left(u^3 - \frac{3ux}{S_G} \right) \right] du \right\}^2, \quad (14)$$

where $\eta = S_G/S_D$. In the limit $S_G \rightarrow 0$, this becomes

$$I(x) \propto \left[\frac{\sin \left(\frac{2\pi x}{S_D} \right)}{\frac{2\pi x}{S_D}} \right]^2, \quad (15)$$

which is the familiar result for diffraction by a slit. These results should be true for each of the orthogonal reflections in a KBXRMS, provided that the small-angle approximation made earlier remains valid.

The distribution $I(x)$ has been calculated numerically and is shown for a number of values of η in Fig. 6. Note that for small values of η , I becomes the standard slit-diffraction result; for large η , the envelope of the diffraction oscillations goes to the geometrical-optics limit. The distribution in two dimensions is just $I(x, y) = I(x)I(y)$, where we assume that the two mirrors are identical. The total resolution, S , is now defined in terms of the distribution $I(x, y)$ as the dimension of the smallest box $\Delta x = \Delta y = S$ that contains the fraction 0.815 of the focused intensity. This condition corresponds to the full width at base along each dimension in the strongly diffracted case. It is easily shown that S , so defined, is equivalent to S

Figure 6. Object-plane equivalent intensity distribution $I(x)$ for values of the geometric/diffraction resolution ratio η from 0.01 to 100. As η increases, the envelope of the distribution approaches the geometric-optics result, also shown.

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defined as the smallest dimension, Δx , of a region in the one-dimensional distribution that contains 0.903 ($= 0.815^{1/2}$) of the energy, S can be written as

$$S = F(\eta) \eta^{-2/3} S_G, \quad (16)$$

where η is defined in Sec. 3; thus

$$\eta^{-2/3} S_G \propto \theta^{-1/3}. \quad (17)$$

At constant θ ,

$$S \propto F(\eta). \quad (18)$$

$F(\eta)$ is plotted in Fig. 7; F (and S , at constant θ) reach minima at

$$\eta = \eta_0 \approx 1.040. \quad (19)$$

Figure 8 shows $S(W, \theta)$ as a contour plot on the (W, θ) plane. Vertical cuts through this function give $S(W)$, which is proportional to $F(\eta)$, for various values of θ . A valley along the line $\eta = \eta_0 \propto W^{3/2} \theta^{-2}$ is apparent. The value of S decreases as one moves along the valley towards higher θ . This valley corresponds to the η_0 branch of the optimization locus (Figs. 4 and 5), along which W is optimized for resolution at a given θ ; the variations in Ω_{eff} and S correspond to the variation of θ .

Figure 7. Focal-plane distribution curves: $F(\eta)$ vs η .

Figure 8. Resolution S (in arbitrary units) as a function of W and θ . Vertical cuts through this function give $S(W)$, which is proportional to $F(\eta)$, for various values of θ .

5. Choice of Grazing Angle and Mirror Material

The optimization schemes of Sec. 3 determine the value of θ along the $W(\theta)$ curve that maximizes, or otherwise optimizes, $\theta^2 R^2(\theta, E)$. E is an average energy for a single-channel system. For a four-channel system, we design for the mean energy E_4 of the highest-energy channel, because the highest energy channel generally requires the largest solid angle.

The simple figure of a KBXRMS makes it possible to produce nearly perfect mirror surfaces. Theoretical calculations of surface reflectivity are thus appropriate. We use Eqs. (20)–(26) (the reflectivity formulas of Giacconi et al.⁷) to calculate $\theta^2 R^2(\theta, E)$ for various mirror materials:

$$R(\theta, E) = \frac{(\theta - a)^2 + b^2}{(\theta + a)^2 + b^2}, \quad (20)$$

where R is the reflected fraction, θ is the grazing angle in radians,

$$a = \frac{1}{2} \left[\sqrt{(\theta^2 - \theta_c^2)^2 + 4d^2} + \theta^2 + \theta_c^2 \right]^{1/2}, \quad (21)$$

and

$$b = \frac{1}{2} \left[\sqrt{(\theta^2 - \theta_c^2)^2 + 4d^2} - \theta^2 + \theta_c^2 \right]^{1/2}, \quad (21)$$

d is a coefficient related to the absorption, and θ_c is the critical angle for total external reflection:

$$d = \frac{\lambda \rho}{4\pi} \mu(E), \quad (23)$$

$$\theta_c = \sqrt{2\delta}. \quad (24)$$

where $\mu(E)$ is the mass absorption coefficient. The cross sections used are those of Veigele.⁸ The quantity δ is related to the phase velocity in the material, because the index of refraction n is given by

$$n = 1 - \delta - i\beta. \quad (25)$$

The quantity δ is calculated from the expression

$$\delta = \frac{2\pi r_0 c^2}{\omega^2} \left(N + \sum_H N_H \frac{\omega_{H1}^2}{\omega^2} \ln \left| \frac{\omega^2 - \omega_{H1}^2}{\omega_{H1}^2} \right| \right), \quad (26)$$

where r_0 is the classical electron radius, ω is the frequency of the radiation, N is the total electron density, N_H is the electron density associated with ω_{H1} , and ω_{H1} is the frequency associated with the absorption edge H , whose energy is E_H .

We have quoted these equations in detail to point out how our choices of material and angle differ from those based solely on electron density. From Eqs. (24) and (26), the critical angle is

$$\theta_c = 0.134 \lambda \left(\frac{Z^*}{A} \right)^{1/2} \rho^{1/2} \quad (\text{deg}), \quad (27)$$

where λ is in Å, ρ is the reflector density, A is the atomic number, and

$$Z^* = Z + \sum_H Z_H \frac{\omega_{H1}^2}{\omega^2} \ln \left| \frac{\omega^2 - \omega_{H1}^2}{\omega_{H1}^2} \right|, \quad (28)$$

where Z_H is the number of atomic electrons associated with edge H . For photon energies E , far from any absorption edge, the effective atomic number Z^* can be approximated as

$$Z^* \approx Z - \sum_{E_H > E} Z_H, \quad (29)$$

where the sum is over all electrons with binding energy E_H greater than the photon energy.

Unfortunately, the differences in reflection efficiency between different materials are themselves so small that this approximation is seldom accurate enough for our analysis. To make this point clear, Fig. 9 shows the relation between Z/A , Z^*/A and Z at $h\nu = 3$ keV. The quantity Z^*/A represents the efficiency per unit mass of the contribution of an element's electrons to x-ray reflection. As we increase Z , there is a step downward at each absorption edge, as another shell of electrons is lost from the reflection process. There is also a general downward trend based on the neutron excess of high- Z elements: from Fermi,⁹

$$\frac{Z}{A} = 1.873 + 0.0347 Z^{2/31}. \quad (30)$$

We particularly wish to point out the width of the depression of Z^*/A in the vicinity of an absorption edge; in fact, there is *no* region between edges where Z^*/A shows the gentle monotonic decrease of Z/A . The critical angle θ_c is proportional to both ρ and Z^*/A [see Eq. (27)]. Because of the downward trend of Z^*/A with Z , the benefit to θ_c from the high density of the noble metals is somewhat offset by their

Figure 9. Z/A and Z^*/A vs Z at $h\nu = 3$ keV. Z^* is the effective atomic number for an x-ray reflector; Z^*/A is the efficiency per unit mass of the contribution of an element's electrons that participate in the reflection process. (Electrons that are too tightly bound do not participate.) Note the downward step of Z^*/A at each absorption edge as Z increases.

high Z . Critical angle, however, is not the only important parameter for a reflector material. Figure 10 shows the figure of merit $\phi R^2(\phi)$ vs $\phi = \theta/\theta_c$ for a reflector material. (A similar curve can be drawn for the figure of merit $\theta^{2/3}R^2$.) The various curves correspond to different values of β/δ . For $\beta/\delta = 0$, there is no absorption, and the reflectivity is unity for all $\theta < \theta_c$. The quantity $\theta''R^2(\theta)$ is thus maximized at a value θ'' ; therefore $\theta_{opt} = \theta_c$. For $\beta/\delta > 0$, the θ_{opt} required to maximize $\theta''R^2(\theta)$ is less than θ_c . The peak figure of merit, $\theta''R^2|_{max}$, can be substantially lower than the zero-absorption value θ_c'' . For a high figure of merit in a K-B microscope system we require of a reflector material both a large critical angle and low absorption.

The considerations above constitute an introduction to Fig. 11, which gives the master curve for material selection. In Fig. 11 we plot $\theta''R^2|_{max}$ vs E for a number of elements and for $n = 2/3$ and 1; $\theta''R^2|_{max}$ is the maximum value of $\theta''R^2(\theta)$, at θ_{max} , for a given energy and material. At high energy, where absorption is not important ($\beta/\delta \rightarrow 0$), $\theta''R^2|_{max}$ approaches θ_c'' .

Figure 12 shows θ_{opt} (see Sec. 3) as a function of E and Z : in the figure $\theta_{opt} > \theta_{max}$ and $\theta_c^{2/3}R^2(\theta_{opt}, E) = 0.8\theta_c^{2/3}R^2(\theta, E)|_{max}$. The figure of merit $\theta^{2/3}R^2$ used in calculating Ω_{eff} must also be taken as $0.8\theta^{2/3}R^2(\theta, E)|_{max}$. Figure 11 is used to choose a mirror material for a given energy; Fig. 12 is then used to determine the angle for that material and energy.

It is apparent from Fig. 11 that no element is a good choice for a reflector near an absorption edge; for a given element, the variation in the figure of merit $\theta''R^2|_{max}$ at an edge is much greater than the variations from element to element. It is also apparent that the simple power-law behavior inferred in the absence of absorption and variation of Z^*/A ,

$$\theta''R^2|_{max} = \theta_c'' \sim \lambda'' \sim \left(\frac{1}{E}\right)^n, \quad (31)$$

holds only for the lowest Z (carbon) and at the highest energies ($E > 4$ keV). Otherwise a detailed calculation, such as that of Fig. 11, must be made.

Figure 11 shows that the choice of mirror material alternates between nickel and noble metals as edges come and go. Where the noble metals show absorption edges, nickel is the material of choice because of its relatively higher density ($\rho = 8.8$ g/cm³) and lower photoelectric absorption than high- Z materials.

Figure 10. The figure of merit $\phi R^2(\phi = \theta/\theta_c)$ vs ϕ , for various values of the absorption parameter β/δ . Only for zero absorption does the figure of merit peak at $\phi = 1$ ($\theta = \theta_c$). For gold at 4 keV, for example, $\beta/\delta = 0.33$, so that ϕR^2 peaks at $\phi = 0.55$.

Figure 11. The figure of merit $\theta^2 R^2|_{\max}$ vs $h\nu$ for several possible reflector materials and for $n = 2/3$ and 1.

Figure 12. Optimum reflection angle θ_{opt} for $n = 2/3$ and 1. The angle θ_{opt} is chosen so that $\theta_{\text{opt}} > \theta_{\text{max}}$ and $\theta_{\text{opt}}^{2/3} R^2(\theta, E) = 0.86^2 R^2(\theta, E)_{\text{max}}$.

The theoretical considerations above must be tempered by practical considerations: Figs. 11 and 12 include only materials that can take a high-quality polish. We conclude that to optimize collection efficiency for a given photon energy E , we should choose nickel or the noble metals, depending on E . For $E \geq 2$ keV, a region of likely interest, either nickel or chromium is the reflector of choice.

Power-law envelopes can be drawn to the curves of Figs. 11 and 12 to approximate the performance of the "best choice" mirror materials as a function of energy. These approximations will be used in the analytical evaluation of microscope performance in the following sections:

$$\theta^{2/3} R^2 \theta, E|_{\max} = 0.059 E^{-0.41}, \quad (32)$$

$$\theta R^2 \theta, E|_{\max} = 0.018 E^{-0.66}, \quad (33)$$

$$\theta_{\text{opt}} = 2.55 E^{-0.81} \quad (n = 2/3), \quad (34)$$

$$\theta_{\text{opt}} = 1.98 E^{-0.77} \quad (n = 1), \quad (35)$$

where the photon energy E is in keV and θ is in degrees. In finalizing the design, the true $\theta^2 R^2$ curves for the material chosen (Figs. 11 and 12) should be used.

The optimization schemes above are different from one designed to set up four energy channels with the greatest possible span of energy. Figure 13, showing four possible energy channels for a microscope

Figure 13. Effective solid angle (including filter transmission) for four channels of a KBXRMS. The optimized channel (4 keV) is a nickel reflector at 0.77-deg grazing angle; other channels are created by gold, chromium, and bare SiO₂ reflectors (all at 0.77-deg grazing angle) with appropriate filters.

optimized at 4 keV, illustrates this point. The calculations of Figs. 11 and 12 lead to the choice of nickel at 0.77-deg grazing angle to maximize $\theta^{2/3}R^2$ at 4 keV. The nickel channel is filtered with titanium to define its low-energy cutoff. Lower-energy channels are created by SiO₂ (fused silica), gold, and chromium reflectors, with other filters. To give the four mirrors identical figures, as is required in the four-channel scheme, they are polished simultaneously on the same lap; they are then coated with 500-Å layers of gold, nickel, and chromium.

The last channel, which uses gold as a reflector, has a higher energy response than the nickel channel for which the system was optimized; it has a mean energy of 5 keV. If we had chosen to optimize the system at 5 keV, however, rather than at 4 keV, we could have obtained a lower $\theta^{2/3}R^2$ with nickel at a smaller angle than with gold at 0.77 deg. The higher energy response of the gold channel thus does not contradict the optimization scheme: for maximum response at a given energy, we still choose nickel. If the spread between channels is more important than optimization of the highest-energy channel, we must use a different scheme. We did this in designing the system of Fig. 13, optimizing for the second-highest energy channel, then adding a gold channel in hopes of getting some signal, albeit at low intensity, in a higher-energy band.

6. Optimization of a Single-Channel KBXRMS

For each of the three optimization schemes, the information in the previous sections can be assembled into explicit analytical expressions for the constructional and operational parameters of a single-channel KBXRMS. These formulas are given in Tables 1, 2, and 3.

In general, the derivations of the formulas of Tables 1, 2, and 3 are obvious. The derivation of the formulas for depth and breadth of field deserves further comment, however.

In the geometrical limit, a δ -function (point) source object will produce an image whose size is given by Eq. (2). The reason for this is that the central ray (see Fig. 3) falls below the rays from the leading and trailing edge of the mirror at the image plane. The trailing-edge ray crosses the central ray a distance approximately $3W(M^2 - 1)/8$ in front of the image plane, while the leading-edge ray crosses the central ray approximately the same distance behind the image plane. In the region between these two points the rays cannot be approximated as a simple converging conical bundle. The resolution at the limits of this region is only twice that at its center, and thus does not change rapidly within the region. The image-plane depth of field, D_o , is therefore defined to be the average distance from the focal plane to the cross-over points:

$$D_{o, geom} = \frac{3}{8} W (M^2 - 1) = M^2 D_{o, geom} \quad (60)$$

The object-plane depth of field is related to the image-plane depth of field by the general relationship

$$D_o = \frac{D_i}{M^2} \quad (61)$$

In the case where diffraction dominates, a beam waist also exists. The length of the beam waist is defined to be the distance from best focus to the point where an (imaginary) undiffracted conical beam would have a width equal to the diffraction width. Thus the object-plane waist length, $D_{o, dif}$, is given by

$$D_{o, dif} = \frac{S_D d_o}{W \sin \theta} \quad (62)$$

Again, the resolution does not change rapidly in the region between best focus and $\pm D_o$. The ratio of the geometrical and diffraction depths of field is

$$\frac{D_{o, geom}}{D_{o, dif}} = 2\eta \quad (63)$$

Table 1. Type I optimization formulas. Type I optimization is for best effective solid angle (ΩR^2) at best resolution. Lengths are in millimetres and x-ray energies in keV.

Grazing angle (degrees):

$$\theta = 2.55 E^{-0.81} \quad (n = 2/3), \quad (36)$$

Mirror width:

$$W = \left[\frac{32}{3} \eta_0 \lambda \left(\frac{d_0}{\theta} \right)^2 \frac{M^2}{M^2 - 1} \right]^{1/3} = 0.188 \eta_0^{1/3} d_0^{2/3} E^{0.267} \left(\frac{M^2}{M^2 - 1} \right)^{1/3} \quad (37)$$

Mirror radius of curvature:

$$r = \frac{2M d_0}{M + 1 \theta} = 44.9 E^{0.81} d_0 \frac{M}{M + 1} \quad (38)$$

Mirror sagittal depth:

$$\sigma = \frac{r \theta^2}{8} = 9.86 \times 10^{-5} E^{-0.397} \eta_0^{2/3} d_0^{1/3} \left[\frac{M(M+1)}{M-1^2} \right]^{1/3} \quad (39)$$

Resolution:

$$S_0 = F(\eta_0) \eta_0^{-2/3} S_G = 2.96 \times 10^{-4} F(\eta_0) d_0^{1/3} E^{-0.397} \left(\frac{M^2 - 1}{M^2} \right)^{1/3} \quad (40)$$

Effective solid angle (sr):

$$\Omega_{\text{eff}} = \left(\frac{W \theta}{d_0} R \right)^2 = 2.64 \times 10^{-5} E^{-1.08} \left(\frac{\eta_0}{d_0} \frac{M^2}{M^2 - 1} \right)^{2/3} \quad (41)$$

Object-plane depth of field:

$$D_{\text{object}} = \pm \frac{3}{8} W \frac{M^2 - 1}{M^2} = 0.0706 \eta_0^{1/3} d_0^{2/3} E^{0.267} \left(\frac{M^2 - 1}{M^2} \right)^{2/3} \quad (42)$$

Object-plane breadth of field:

$$B_0 = \pm \frac{3}{8} W \theta \left(\frac{M - 1}{M} \right) = 3.14 \times 10^{-3} \eta_0^{1/3} d_0^{2/3} E^{-0.603} \left[\frac{M - 1^2}{M(M + 1)} \right]^{1/3} \quad (43)$$

Thus for Type I optimization, for which $\eta = \eta_0 \approx 1$, the geometrical depth of field dominates.

The breadth of field is limited by the relation between focal length and the angle of incidence of the rays upon the mirror, which is

$$f = \frac{r \sin \theta}{2} \quad (64)$$

Thus the focal length shifts as the image point moves transversely away from the center of the object plane. The breadth of field is defined as the distance from the center of the object plane at which the focal length has changed by an amount equal to the object-plane depth of field. Using this definition, and the geometrical or diffraction-limited depth of field as appropriate, one readily obtains Eqs. (43), (51) and (59).

Table 2. Type II optimization formulas. Type II optimization is for best effective solid angle at a given, less-than-best resolution. Lengths are in millimetres and x-ray energies in keV.

Grazing angle (degrees):

$$\theta = 1.98 E^{-0.77} \quad (44)$$

Mirror width:

$$W = \left(\frac{16}{3} S_G \frac{d_o}{\theta} \frac{M^2}{M^2 - 1} \right)^{1/2} = 12.4 (S_G d_o)^{1/2} E^{0.385} \left(\frac{M^2}{M^2 - 1} \right)^{1/2} \quad (45)$$

Mirror radius of curvature:

$$r = \frac{2M d_o}{M + 1} = 57.9 d_o E^{0.77} \left(\frac{M}{M + 1} \right) \quad (46)$$

Mirror sagittal depth:

$$\sigma = \frac{1}{3} S_G \frac{M}{M - 1} \quad (47)$$

Resolution (given, for Type II optimization):

$$S \approx S_G \geq 2S_D \quad (48)$$

Effective solid angle (sr):

$$\Omega_{eff} = \left(\frac{W\theta}{d_o} R \right)^2 = 0.0768 \frac{S_G}{d_o} E^{-0.66} \frac{M^2}{M^2 - 1} \quad (49)$$

Object-plane depth of field:

$$D_{o,perm} = \pm \frac{3}{8} W \frac{M^2 - 1}{M^2} = \pm 4.66 \left(S_G d_o \frac{M^2 - 1}{M^2} \right)^{1/2} E^{0.385} \quad (50)$$

Object-plane breadth of field:

$$B_o = \pm \frac{3}{8} W\theta \left(\frac{M - 1}{M} \right) = \pm 0.161 \left(S_G d_o \frac{M - 1}{M + 1} \right)^{1/2} E^{-0.385} \quad (51)$$

Table 3. Type III optimization formulas. Type III optimization is for best effective solid angle at a given, fixed breadth of field. Lengths are in millimetres and x-ray energies in keV.

Grazing angle (degrees):

$$\theta = 1.98 E^{-0.77} \quad (52)$$

Mirror width:

$$W = d_0 \left(\frac{2\lambda}{b\theta} \frac{M}{M+1} \right)^{1/2} = 8.47 \times 10^{-3} d_0 b^{-1/2} E^{-0.115} \left(\frac{M}{M+1} \right)^{1/2} \quad (53)$$

Mirror radius of curvature:

$$r = \frac{2Md_0}{M+1\theta} = 57.9 d_0 E^{0.77} \left(\frac{M}{M+1} \right) \quad (54)$$

Mirror sagittal depth:

$$\sigma = \frac{W^2}{8r} = 1.55 \times 10^{-7} \frac{d_0}{bE} \quad (55)$$

Resolution:

$$S \approx S_D = \frac{2d_0\lambda}{W\theta} = 8.47 \times 10^{-3} E^{-0.115} \left(b \frac{M+1}{M} \right)^{1/2} \quad (56)$$

Effective solid angle (sr):

$$\Omega_{eff} = \left(\frac{W\theta}{d_0} R \right)^2 = 3.57 \times 10^{-8} b^{-1} E^{-1.66} \quad (57)$$

Object-plane depth of field:

$$D_{obj} = \pm S_D \frac{d_0}{W\theta} = \pm 28.9 b E^{0.77} \left(\frac{M+1}{M} \right) \quad (58)$$

Object-plane breadth of field (given, for Type III optimization):

$$b \approx b_0 > \frac{3}{8} W\theta \left(\frac{M-1}{M} \right) \approx 1.10 \times 10^{-4} d_0^{2/3} E^{-0.590} \left[\frac{M-1}{M+1} \right]^{1/3} \quad (59)$$

7. Optimization of a Four-Channel KBXRMS

Some of the constraints used to optimize four-channel K-B x-ray microscopes result from the considerations of Sec. 2. To obtain coincident fields of view, we require identical focal lengths $f = (r \sin \theta)/2$. As described earlier, all radii are made identical by making the mirrors from the same material and polishing them simultaneously with the same lap. To reduce difficulties in polishing, it is helpful to make all mirrors of the same width.

The grazing angle θ is fixed at the time of construction. However, as we shall see, θ is not necessarily exactly the same for all mirrors.

Choosing a Compromise Grazing Angle and Mirror Width

The result of these constraints is that we must modify the single-channel optimizations. The four-channel KBXRMS is to be optimized with compromise values of the mirror width, W' , and grazing angle, θ' . The object distance d_o and magnification M are also the same for each channel. The goal of the Type I four-channel optimization is to find W' and θ' for which the resolution and effective solid angle are degraded as little as possible for each of the channels. The greatest disparity occurs between the channels of highest (E_h) and lowest (E_l) energies. It can be readily shown that minimizing the differences for these two channels will also approximately minimize the differences for the two intermediate channels. We require that the two channels are degraded by an equal fractional amount, so that we put

$$\frac{S'_l(E_l)}{S_l(E_l)} = \frac{S'_h(E_h)}{S_h(E_h)}. \quad (65)$$

In the primed quantities, which represent four-channel optimization, W' , θ' , d_o , and M are the same for both subscripts, h and l ; in the unprimed quantities, which represent single-channel optimization, W and θ are different. Equation (5) shows that if W' , θ' , d_o , and M are held constant, $\eta' \propto E$.

Equation (65) can be readily evaluated using Eqs. (16) and (40). $S_l(E_l)$ and $S_h(E_h)$ are obtained by inserting E_l and E_h for E in Eq. (40). Equation (65) becomes

$$F\{\eta'(E_h)\} = \left(\frac{E_h}{E_l}\right)^{0.270} F\{\eta'(E_l)\}. \quad (66)$$

A constant η_h can then be defined:

$$\eta_h = \eta'(E_h), \quad (67)$$

such that

$$\eta'(E) = \frac{E}{E_h} \eta_h. \quad (68)$$

Equation 66 then becomes an equation in one unknown:

$$F(\eta_h) = \left(\frac{E_h}{E_l}\right)^{0.270} F\left[\left(\frac{E_l}{E_h}\right) \eta_h\right]. \quad (69)$$

This equation can be solved graphically or numerically from the data in Fig. 7. A crude solution can be obtained by using the asymptotic approximations to $F(\eta)$:

$$F(\eta) = \eta^{-1/3}, \quad \eta \lesssim \eta_0 \approx 1 \quad (70)$$

$$F(\eta) \approx 0.82 \eta^{2/3}, \quad \eta \geq \eta_0. \quad (71)$$

These approximations lead to

$$\eta_h \approx 1.22 \left(\frac{E_h}{E_l}\right)^{0.603}. \quad (72)$$

A typical value of E_h/E_l is 4, which yields $\eta_h \approx 2.8$; consideration of Fig. 7 shows this to be an overestimate. Given that a value of η_h can be obtained by one of several methods, the optimization can proceed using η_h as a known constant. Starting with W' , it is straightforward to proceed through Table 1. It is assumed that Ω_{eff} is optimized for the highest energy channel (E_h), since typically the least flux is available

at the highest energies. All equations corresponding to those in Table 1 [except Eq. (40), for the resolution] are generated by simply substituting η_h for η_0 and E_h for E . The equation for the resolution of any channel is generated from Eqs. (16) and (68):

$$S(E) = \frac{E}{E_h} F\left[\left(\frac{E}{E_h}\right) \eta_h\right] \left[\frac{3}{4} \frac{d_0 \lambda_h^2}{\theta_h} \left(\frac{M^2 - 1}{M^2} \right) \right]^{1/3} \\ = 2.96 \times 10^{-3} \frac{E}{E_h} F\left[\left(\frac{E}{E_h}\right) \eta_h\right] d_0^{1/3} E_h^{-0.392} \left(\frac{M^2 - 1}{M^2} \right)^{1/3}. \quad (73)$$

The fractional degradation of the resolution is

$$\tilde{S} = \frac{F(\eta_h)}{F(\eta_0)}. \quad (74)$$

Type II optimization of a four-channel KBXRMS is simpler than Type I optimization. In Type II optimization, the microscope is operated in the geometrically limited mode to gain solid angle at the expense of resolution. The resolution limit, S_G , is specified. In this case, the effective solid angle of the higher-energy channels should be made as large as possible. This is done by evaluating θ_{opt} and Ω_{eff} at $E = E_h$. All Type II optimization equations follow by making $E = E_h$ in Table 1.

In Type III optimization, it is assumed that flux is plentiful, and that resolution requirements are relaxed. An x-ray optical system with a specified large, flat field of view is to be produced. Such a microscope has relaxed alignment tolerances and is very easy to use. Breadth of field is greater at lower energies and larger angles. Therefore, in Type III optimization one sets $E = E_i$ in all equations in Table III except Eq. (56), for the resolution. The on-axis resolution for any channel is obtained by scaling Eq. (56) with energy:

$$S(E) = \frac{E_i}{E} S_D(E_i) = 8.47 \times 10^{-3} \frac{E_i^{0.885}}{E} \left(b \frac{M + 1}{M} \right)^{1/2}. \quad (75)$$

Asymmetries in Mirror Cell Geometry

One set of mirrors must be mounted slightly in front of the other in a KBXRMS mirror cell. Their centerlines must, of course, be at least one mirror width (W) apart. However, the depth of field is $\pm 3W/8$, implying that if the radii, grazing angles, and focal distance are the same, the focal points of the two sets of mirrors will not coincide. Since the radii are equal, the grazing angle must be changed slightly so that the foci of both sets of mirrors will fall in the same plane. In the four-mirror configuration of Fig. 2, this is most easily done by making the spacers for the front and back sets of mirrors of slightly different thickness. A slightly narrower spacing than nominal for the front mirror pair makes their focal length slightly shorter; a slightly wider than nominal spacing for the back mirror pair makes their focal length longer.

A brief calculation shows that the appropriate spacer thicknesses are

$$T_{\pm} = 2d_0 \left\{ d_0 \left[1 - \left(\frac{M-2}{M} \right) \left(\frac{e}{2d_0} \right)^2 \right] \pm \frac{e}{2} \left[\left(\frac{2M-1}{M} \right) - \frac{1}{M} \left(\frac{e}{2d_0} \right)^2 \right] \right\}, \quad (76)$$

where e ($e > W$) is the center-to-center mirror spacing; the positive sign holds for the back spacer, and the negative sign for the front spacer. The magnification of the KBXRMS is, of course, slightly different (1 or 2%) in the two orthogonal directions, but this is intrinsic to the KBXRMS geometry. The problem will also arise in a single-channel KBXRMS, but it can be adjusted out during alignment; this cannot be done for a four-channel microscope.

8. Three-Dimensional Ray Traces: The Cross-Talk Aberration

The optimization results above are based on a two-dimensional analysis of aberrations, which assumes that all rays lie in a plane; we implicitly assume that the focusing actions of the orthogonal reflectors are independent. Intuitively, one would expect the independence of the two reflections to break down as the grazing angles become large. We have written a ray-tracing program to follow rays through a crossed-reflector system in three dimensions to look for such effects. The programs use geometric optics only, and do not include diffraction effects.

Ray traces were performed for systems using cylindrical and spherical mirrors. At grazing angles, the focal length of a spherical mirror perpendicular to the plane of incidence is greater than that in the plane of incidence by the factor θ^{-1} . The transverse focusing effect of spherical mirrors is thus a small perturbation for angles of order 1 deg. Of course, the two-dimensional calculations of the previous sections apply equally whether the arc is the cross section of a three-dimensional cylinder or a sphere.

The ray-tracing program was written specifically for Kirkpatrick-Baez microscope systems. A packet of 441 rays, in a 21×21 square grid, is started on the surface of the first mirror. The incidence angles of these rays are determined by tracing them back to the source; they are then followed in reflection from the two mirrors and onto the focal plane. The focal pattern is optimized by minimizing its full width in both directions; this is done by varying the mirror radii and the separation of the second pair of mirrors. The focal-plane plot often preserves the grid structure of the ray packet. This can be useful in understanding the nature of the aberrations.

In a three-dimensional geometry, it is necessary to specify the relative orientation of the orthogonal mirrors (three choices are shown in Fig. 14). We define the microscope axis as the line passing through the midpoints between each pair of mirrors. The normal to the mirrors is perpendicular to the microscope axis in the standard four-channel geometry (Fig. 14a). The mirrors can thus be conveniently registered against a block with plane-parallel faces. Other four-channel geometries are possible (Figs. 14b, 14c); in these, the mirrors are tipped toward or away from the source. The spacing of the mirrors thus changes, and the appropriate registration block does not have parallel faces. Another operational inconvenience is the larger mirror-cell assembly (14b) or larger image-plane pattern (14c). We have no present plans at LLNL or Los Alamos to field a microscope with a non-parallel geometry.

Our calculations show a "cross-talk" aberration, so called because the reflections from the two pairs of mirrors are not independent. That is, the aberration results from the coupling of the reflections from the two pairs of mirrors. The aberration becomes significant for grazing angles as small as 3 or 4 deg, and becomes dominant at large angles.

Figure 15 shows the focal-plane distribution associated with the cross-talk aberration. If the aberration is not present (it can be eliminated by tipping the mirrors as in Fig. 14b), the focal pattern is dominated by spherical aberration (Fig. 15a). The grid pattern is folded on its center in the x and y axes; rays from the center of the aperture are focused to the lower left corner of the pattern. The focal-plane deviation of a ray from the central ray is proportional to the square of the distance, on the mirror surface, between the ray and the central ray; this is as expected for spherical aberration. For example, rays from the front and back of the first mirror that hit the center of the second mirror deviate toward increasing x , and coincide at the lower right corner of the focal pattern.

Figure 15b shows an image for the same microscope design as in Fig. 15a, but with the mirrors in the standard parallel geometry of Fig. 14a. The cross-talk aberration now dominates. Rays from the front and back of the first mirror that would have gone to the same y location in the image plane, in the absence of cross-talk, now do not do so; instead, the lines of the grid pattern are opened into parabolas. We do not completely understand the cause of the cross-talk aberration.

For any mirror configuration, the full width of the spherical aberration can be calculated by means of Eq. (2):

$$S_C = \frac{3}{16} \frac{W^2}{d_c} \theta \left(\frac{M^2 - 1}{M^2} \right). \quad (2)$$

The calculated S_C agrees exactly with that of the ray trace for cases such as Fig. 15a. We have investigated the scaling of the cross-talk aberration with mirror length and grazing angle by tracing a number of

Figure 14. Possible relative orientations of the mirrors in a four-channel KBXRMS. One plane of symmetry is in the plane of the paper; the other is perpendicular to the plane of the paper. (a) is the standard configuration. The configuration in (b) requires a large mirror cell; that in (c) gives a larger image-plane pattern.

Figure 15. Focal-plane point spread functions calculated by the ray-tracing programs. In (a), the mirrors are tipped into the configuration of Fig. 14b, eliminating the cross-talk aberration; the pattern is dominated by spherical aberration. In (b), the mirrors are in the configuration of Fig. 14a, and the pattern is dominated by the cross-talk aberration. In both cases, the microscopes are 40 cm from the object, with a reflection angle of 3.7 deg and a magnification of 3. The mirrors are 0.95 cm wide.

designs. The full width S_{ct} of the cross-talk aberration is obtained by subtracting the calculated spherical aberration S_c from the full width S_{tot} of the ray-trace focal pattern. Figure 16 shows the effect of such scaling on S_{tot} , S_c , and S_{ct} . The baseline configuration from which mirror length and grazing angle are varied is that of Fig. 15b: magnification 3, object-to-first-mirror distance 40 cm, grazing angle 3.7 deg, mirror length 0.95 cm, and no separation between the first and second pairs of mirrors. The aberrations are measured in the x direction; the y -direction aberration scales the same way.

Figure 16 shows that simple scaling laws exist for S_{ct} . The cross-talk aberration is proportional to $W\theta^3$, the product of mirror width W and the cube of the grazing angle θ . Further ray-tracing calculations indicate that S_{ct} can be obtained from the equations

$$S_{ct} = 0.0535 \left(1 + \frac{1}{M} \right) \theta^3 W \quad \mu\text{m (cylinders)} , \quad (77)$$

$$S_{ct} = 0.0961 \left(1 + \frac{2}{M} \right) \theta^3 W \quad \mu\text{m (spheres)} . \quad (78)$$

where θ is in degrees and W is in cm. Note that spherical aberration scales as $W^2\theta$, so that it dominates at small angles for wide mirrors. The comatic aberration is not changed by the degradation of the focused image; the full width of the image still increases linearly with the off-axis spacing.

The cross-talk aberration is not significant for the Kirkpatrick-Baez microscopes that have been fielded in the laser fusion program. These devices all have grazing angles of order 1 deg and mirror widths of order 1 cm, so that the cross-talk aberration is insignificant for both spherical and cylindrical optics. At larger angles, cylindrical mirrors may be preferred because of the factor-of-2 less cross-talk. In cases where cross-talk is significant, ray traces show that it may be eliminated (for cylinders only) by resort to the mirror configurations of Figs. 14b or 14c, either of which would require a non-parallel registration block. Spherical optics show no improvement when so configured.

Figure 16. Effect of parameter scaling on the geometrical aberrations S_{tot} (total), S_G (spherical), and S_{ct} (cross-talk) for spherical and cylindrical mirrors. The baseline case is that of Fig. 15b. (a,b) Effect of scaling the angle θ ; (c,d) effect of scaling mirror width W .

Figure 16. Con't.

Summary

We have approximately optimized single-channel and four-channel Kirkpatrick-Baez (K-B) x-ray microscopes in three ways: (I) for best effective solid angle at best resolution; (II) for best effective solid angle at a given resolution worse than the best; and (III) for best effective solid angle at a specified breadth of field. We have provided analytical expressions giving the constructional and optical parameters of K-B x-ray microscopes optimized in these ways. We have described a "cross-talk" aberration not previously recognized in these microscopes.

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