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ANALYTICAL EVALUATIONS OF THE ENDOCHRONIC
NONLINEAR CONCRETE MODEL

D. W. Goodpasture

E. G. Burdette

HTGR BASE TECHNOLOGY PROGRAM

Prestressed Concrete Nuclear Pressure
Vessel Development (189z 01331)

Milestone 1a

for

OAK RIDGE NATIONAL LABORATORY

Oak Ridge, Tennessee 37830

operated by

UNION CARBIDE CORPORATION

for the

DEPARTMENT OF ENERGY

Contract No. W-7405-eng-26

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THE UNIVERSITY OF TENNESSEE
Knoxville, Tennessee 37916

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FOREWORD

One task area of the Oak Ridge National Laboratory program of Prestressed Concrete Nuclear Pressure Vessel Development (189a No. 01331) is devoted to PCRV analysis methods development. The tasks described in this report constitute part of the overall effort to develop non-linear analysis methods. Thus far the work has been based primarily on the endochronic model developed by Z. P. Bazant of Northwestern University which was described in ORNL/Sub-4403/1 (Ref. 1). This second report of the analysis methods development activities presents comparisons of (1) the results of finite element analyses of concrete structures with test data, and (2) predicted and measured responses of homogeneous concrete test specimens subjected to uniaxial, biaxial and triaxial states of stress. In the first study, prestressed concrete beams tested under the ORNL study of grouted versus nongrouted prestressing tendons² were analyzed using the ADINA finite element code³ in which the endochronic model had been implemented by D. N. Fanning of ORNL. The second study was performed using a "constitutive model driver" program developed at ORNL by W. G. Dodge and D. N. Fanning. Comparisons were made between published multiaxial test data and calculated values obtained using the endochronic theory. This "model driver" provides the capability for examining the response of any hypothetical material as defined by candidate constitutive models to arbitrarily selected loading conditions.

Objective and Scope

Comparisons of the results of finite element (FEM) analyses were made with actual test data on prestressed concrete beam models. The ADINA computer program was used to perform the analyses with two material models for concrete. One material model was the concrete model furnished with ADINA and the second was the endochronic model as added by D. N. Fanning at Oak Ridge. Results of the prestressed concrete beam tests were compared to ADINA FEM results for both the regular and the endochronic material models.

Published results of concrete specimens tested in uniaxial, biaxial, and triaxial states of stresses were collected and reduced to computer data files. The data files were then used to compare with results predicted by the endochronic theory in another computer program. The results of these comparisons, the source of the reference material and a plot of the comparisons are included as Appendix A of this report. A total of thirty two files were created in this phase of the study.

Results

ADINA Analyses of Beam Tests

The test results chosen for comparison with the computer analyses were a series of grouted, post-tensioned, prestressed concrete beams, 6 inches (15.24 cm) by 12 inches (30.48 cm) by 10 feet (304.8 cm), each having one seven-wire 270^K strand ($\text{Area} = 0.1547\text{in}^2 = 1.0\text{cm}^2$). Two-dimensional plane stress analyses of the beams were conducted. The prestressing strand was represented by truss elements using an elastoplastic material

model which also incorporated strain hardening. These elements were connected to the 2-D concrete elements at all nodes along the strand to simulate the grouted case. Attempts were made to model the ungrouted tendons by connecting the truss elements to the concrete elements only at the beam ends and by providing fictitious vertical truss elements to transfer any vertical component of the prestressing force to the concrete. Problems were encountered and none of the runs were deemed successful. Therefore only the results for the grouted beams are presented herein.

Measured load-deflection curves from laboratory tests were obtained from ORNL. These curves represented the behavior of the beam up to and including failure. A satisfactory method of comparing the analytical results to the test results was sought. The best method of presentation is to superimpose the measured and analytical results on the same graph by plotting load vs deflection at the center line of the beam span. These graphs enable one to compare the point at which non-linearity occurs, the amount of ductility present (i.e. the amount of inelastic deflection prior to failure), and the magnitude of the ultimate load for each of the cases plotted.

Details of the prestressed concrete beam, G1, are given below:

Concrete: $f'_c = 5326$ PSI

Splitting tensile strength = 404 PSI

$E_c = 5.11 \times 10^6$ PSI

Steel: Area = 0.1547 in.²

$E_s = 28,410$ KSI

yield stress,

$F_y = 255$ KSI.

Strain hardening modulus - 625 KSI.

Effective prestress - 145 KSI.

The prestressing strand was located 2.5 inches above the bottom face of the beam. Loading was at the third points of the span. Figure 1 shows the load deflection curve of the test to failure of the beam G1.

Figure 2 illustrates the mesh used in the ADINA analyses. Only one half of the beam was modeled due to symmetry. The ADINA concrete model incorporated three features:

- (1) A non-linear stress-strain relationship with strain softening,
- (2) A failure envelope that defines cracking in tension and crushing in compression (Khan & Saugy),
- (3) A strategy to model the post-cracking and crushing behavior of the material.

The non-linear problem is solved incrementally. The non-linear problem is replaced by a piecewise linear problem, that is linear in each load increment. The accuracy of the results depends, therefore, not only on mesh size but also on load increment size.

Table I lists the variables and results for four analyses of beam G1 using ADNIA and the regular concrete model. The results for each of the four cases are shown in Figures 3 through 6, respectively. Although accuracy can be increased by reducing mesh size and load increment size, the purpose of these four cases was to provide a basis for comparison to the endochronic concrete model which follows.

TABLE I

CASE NUMBER	1	2	3	4
Shear reduction factor	0	0.2	0	0
Stiffness Reduction factor	0	0	0.2	0
Number of integration points	2	2	2	3
Ultimate load (kips)	5	8	23	8
Ultimate deflection (inches)	0.0277	18.11	3.258	0.993

The endochronic model, using the same mesh and load increment, shows an increase in accuracy as indicated in Figure 7. Figures 8 through 11 show the effects of varying different parameters. In each figure the two plots from Figure 7 are shown along with a third plot which reflects the effect of the variable. Figure 8 indicated that increased ductility is achieved by increasing the tensile strength of the concrete slightly. The same effect is achieved by using the total Lagrangian formulation as indicated in Figure 9. Changing the stiffness reduction factor from 0.2 to 0.4 has no effect as shown in Figure 10. A very slight increase in ductility and strength is achieved when three Gauss points are used instead of two as depicted in Figure 11.

Beams G2, G3, and G4 were also modeled with the endochronic concrete model. Figures 12, 13, and 14 reflect the good agreement between the analyses and the actual test results. Although the endochronic theory eliminates the need for a failure surface criterion in crushing, the model is essentially the same as the ADINA model in cracking.

Both models calculate principal tensile stresses at each step. Whenever this stress reaches a critical value, a crack is said to form. The constitutive matrix is replaced by an orthotropic matrix which is oriented in accord with the crack direction.

In both models, the orthotropic matrix can be modified by using a stiffness reduction factor between zero and one. When the factor is one, there is no reduction in stiffness across the crack. When the factor is zero, there is complete reduction in stiffness across the crack. Furthermore, both models incorporate a shear reduction factor. The endochronic model fixes this factor at one-half because of the

model's insensitivity to changes in its value.

Despite the similarity of the two models in cracking, the fact remains that the endochronic model gives superior results where cracking is the controlling mode of failure. This must be largely attributable to the different finite element implementation of the model. The ADINA finite element code prohibits equilibrium iteration for the ADINA model whereas iteration is performed with the user-defined endochronic model.

Equilibrium iteration is necessary when an incremental approach is used in solving non-linear problems. The numerical analysis journals indicate that there is extensive current research into non-incremental, non-linear solvers. The eventual use of such solvers in non-linear finite element codes will greatly reduce job costs and provide increased accuracy. A handicap of the endochronic parameter approach is its inability to account for the direction of the inelastic strain or damage. If a specimen is subjected to fatigue loading in one direction and then is loaded with a new orientation, the endochronic parameter cannot take this into consideration. The endochronic model should be viewed as a step toward a more extensive non-linear constitutive theory.

Concrete Data File

It is stated in the abstract of Reference 1 "by comparison with experimental data from literature, it is demonstrated that the proposed model predicts quite closely: stress-strain diagrams for concretes of different strength; uniaxial, biaxial and triaxial stress-strain diagrams and failure envelopes" The purpose of this phase of the project was to provide further documentation of the refined

endochronic model's capability for predicting the stress-strain relationship for concrete through its comparison with existing data on concrete subjected to multi-axial loading.

In the evaluation process the following procedures were decided upon:

- (1) For each reference containing stress-strain data for concrete under multi-axial loading, a brief abstract was written. A typical abstract contained source of data, test description, and specimen characteristics.
- (2) The data were then digitized and stored in a computer file.
- (3) The refined endochronic model was run for the conditions given with the data previously digitized. These results were also stored in a computer file.
- (4) The results obtained in Step 3 were compared with the information gathered in Step 2 and plotted together to create a documented hard copy.
- (5) The results from Step 4 were compared and a brief evaluation was written.
- (6) The final product is in the form of two files: an internal file which is the computer data bank, and an external file which includes abstracts, plots, comparisons, and evaluations. The external file is included as Appendix A.

Thirty-two files were established. The computer files and the hardcopy files were numbered correspondingly with File #1 being set

aside for an index to the files, computer program outlines and other pertinent notes.

The first stages of the project involved driving the model with data which were available in the Bazant report. This was done to assure that the program was copied correctly as well as to compare the results. It was found that test results and predicted curves were compatible except for an uncharacteristic dip in the prediction's stress-strain diagram which occurred in Files, 9, 10, 11, 12 involving Balmer triaxial test data (refer to Appendix A). The dip in the theoretical curve reappeared in File 15 which used Richart, Brandtzaeg and Brown triaxial test data. After reexamining the coding of the program, it was concluded that this incongruity was inherent in the model's equations.

In the latter stages of the project, data other than that used by Bazant were emphasized in the creation of new files. Five files based on ORNL data and six files based on data taken directly from the Richart, Brandtzaeg, Brown 1928 report (see Appendix A for complete reference) were created. The results of the model's predictions when compared to the uniaxial test data indicated a deviation between their modulus of elasticities. Furthermore, it appeared that the lower the compressive strength of the concrete the greater the deviation. For example, in the ORNL series, the best approximations were apparent in Files 23 and 20 with f'_c of 6430 PSI and 5540 PSI, respectively, while in the other files of that series with f'_c of 4240, 3540, 4140 PSI the model exhibited a smaller E value.

Likewise in files 29, 30, 31, 32, 33, and 34 using Richart, Brandtzaeg and Brown uniaxial test data the deviation in E values is

evident. The deviation may be attributed to the fact that the model is based only on the compressive strength parameter with no provision for the modulus of elasticity.

As stated previously, 32 files were established throughout this project. Obviously the greater the number of comparisons the better the evaluation of the model. Because all data to be digitized must be of graphical form, tabular data must be plotted by hand or disregarded. To alleviate this problem and increase the data availability, the ORNL experimental data digitizer should be modified to allow direct input of tabular data. This would increase the amount of useable data as well as eliminate errors induced by aligning the cursor with the actual data in the digitizing process.

Summary and Conclusions

Test results for a series of grouted, post-tensioned, prestressed concrete beams were compared to analyses performed with the ADINA computer code. Two different methods of modelling the concrete were utilized, namely the material model provided by the authors of ADINA and endochronic material model added to the ADINA code by ORNL. The endochronic model gives better results than the regular model as indicated in a series of graphs. However, it is disturbing to realize that the analytical procedure can produce a variety of results depending on the choice of certain input variables. Indeed, some of these variables do not have a rational basis for the selection. Therefore, care must be exercised and more than one analysis performed before one can be assured that the results are reasonable.

Problems were encountered in accessing the ORNL computers from UT. Therefore, it is recommended that future studies be performed with the computer directly from Oak Ridge rather than from Knoxville.

A total of thirty-two files of data from tests on concrete specimens was established. These files were both in computer format (tape) and hard copy (Appendix A) and included the source of the material and a brief evaluation on each file. The majority of the data were from references used by Bazant, et. al. in formulating the endochronic model, but eleven files were from other data. These results revealed good agreement for concretes of higher strength but agreement was not so good for lower concrete strengths. It is recommended that the computer program used to create these files be altered to allow input of tabular data in addition to the graphical format.

REFERENCES

1. Z. P. Bazant, P. D. Bhat and C. L. Sieh, Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures ORNL/SUB/4403-1, Northwestern University, Evanston, Illinois 60201, December 1976.
2. D. J. Naus and C. C. Hurtt, "Structural Model Testing for Prestressed Concrete Pressure Vessels: A Study of Grouted Versus Nongrouted Post-Tensioned Prestressing Tendon Systems, ORNL/TM-6478 Oak Ridge National Laboratory (to be published)
3. K. J. Bathe, ADINA, A Finite Element Program for Automatic Dynamic Incremental Nonlinear Analysis, Report No. 82448-1, MIT, Cambridge, Massachusetts (September 1975).

Figure 1. Test Results for beam G1

12 Truss Elements Connect with 2D

Elements at Nodes to Represent Steel

Figure 2. Mesh layout for ADINA FEM analysis

Figure 3. Beam G1 test results vs ADINA concrete model results, Case 1.

Figure 4. Beam G1, ADINA concrete model, Case 2.

Figure 5. Beam G1, ADINA concrete model, Case 3.

Figure 6. Beam G1, ADINA concrete model, Case 4.

Figure 7. Beam G1, endochronic concrete model, similar to Case 3 (see Figure 5)

Figure 8. Effects of changing ϵ_T from 0.00022 to 0.0003.

Figure 9. Effect of using total Lagrangian formulation.

Figure 10. Effect of changing stiffness reduction factor from .2 to .4

Figure 11. Effect of using 3 Gauss points vs 2 Gauss points

Figure 12. Beam G2 vs Endochronic Prediction.

Figure 13. Beam G3 vs Endochronic Prediction.

Figure 14. Beam G 4 vs Endochronic Prediction.

APPENDIX A

INDEX OF FILES

File No.

2. Kupfer, Hilsdorf, and Rusch 1969; Uniaxial $f'_c = 4650$ PSI
3. Kupfer, Hilsdorf, and Rusch 1969; Biaxial $f'_c = 4650$
4. Hognestad, Hanson, McHenry 1955; Uniaxial $f'_c = 3000$
5. Hognestad, Hanson, McHenry 1955; Uniaxial $f'_c = 4650$
6. Hognestad, Hanson, McHenry 1955; Uniaxial $f'_c = 6200$
7. Balmer 1949; Triaxial $f'_c = 3570$ σ_2 & $\sigma_3 = -25,000$
8. Balmer 1949; Triaxial $f'_c = 3570$ σ_2 & $\sigma_3 = -20,000$
9. Balmer 1949; Triaxial $f'_c = 3570$ σ_2 & $\sigma_3 = -15,000$
10. Balmer 1949; Triaxial $f'_c = 3570$ σ_2 & $\sigma_3 = 10,000$
11. Balmer 1949; Triaxial $f'_c = 3570$ σ_2 & $\sigma_3 = -5,000$
12. Balmer 1949; Triaxial $f'_c = 3570$ σ_2 & $\sigma_3 = -2,500$
13. Balmer 1949; Triaxial $f'_c = 3570$ σ_2 & $\sigma_3 = -1,000$
14. Balmer 1949; Uniaxial $f'_c = 3570$
15. Richart, Brandtzaeg, Brown 1928; Triaxial 2840, σ_2 & $\sigma_3 = -2010$
16. Richart, Brandtzaeg, Brown 1928; Triaxial 2840, σ_2 & $\sigma_3 = -1090$
17. Richart, Brandtzaeg, Brown 1928; Triaxial 2840, σ_2 & $\sigma_3 = -550$
18. Richart, Brandtzaeg, Brown 1928; Uniaxial 2840
19. CRBR Data Uniaxial; $f'_c = 4240$
20. CRBR Data Uniaxial; $f'_c = 5540$
21. CRBR Data Uniaxial; $f'_c = 3540$
22. CRBR Data Uniaxial; $f'_c = 4140$
23. CRBR Data Uniaxial; $f'_c = 6430$
24. * Bad File
25. Green and Swanson; Triaxial σ_2 & $\sigma_3 = -2000$
26. Green and Swanson; Triaxial σ_2 & $\sigma_3 = -1000$
27. Green and Swanson; Uniaxial
28. * Bad File
29. Richart, Brandtzaeg, Brown; Uniaxial $f'_c = 3480$
30. Richart, Brandtzaeg, Brown; Uniaxial $f'_c = 2610$
31. Richart, Brandtzaeg, Brown; Uniaxial $f'_c = 3060$
32. Richart, Brandtzaeg, Brown; Uniaxial $f'_c = 2105$
33. Richart, Brandtzaeg, Brown; Uniaxial $f'_c = 2160$
34. Richart, Brandtzaeg, Brown; Uniaxial $f'_c = 1855$

Outline of Computer Programs

An example is given to aid in understanding the two programs used in digitizing the data and driving the model. The data of File # 15, a typical triaxial test, are provided below.

Experimental Data Digitizer

After loading the program tape in the console of the Tectronix 4051 and the tape on which the data is to be stored in the auxillary tape unit, the auto load button is pressed. The various programs available on that tape will appear. The experimental data digitizer is program number 18. Input values are underlined. A carriage return is required after each line of input.

- . Input: 18
Computer Response: What is the I/O channel number for the plotter?
- . Input: 2
Computer Response: What file is to be used for storing the experimental data?
- . Input: 15
Computer Response: Plot parameters are set by defining coordinates of 3 points. Enter coordinates of point 1. The data which is the form of a stress strain diagram is placed on the plotter. The computer is asking for 3 points to identify the axis of the data plot.
- . Input: 0,0
Computer Resopnse: Locate cursor and press call.
- . Input: (Locate cursor and press the call button on the plotter) (Repeat same steps for other two points)

Loading ratio: (cont.)

Stress (2,2) = 0

Stress (2,3) = 0

Stress (3,3) = 0

Loading increment = -1000

(Use user definable key to redefine plot variable)

Comp: Independant var:

Load? N

Time? N

Strain (1,1)? Input: Y

This axis horizontal? Input: Y

Dependant var:

Stress (1,1)? Input: Y

Stress (1,2)? Input: N

Stress (1,3)? Input: N

Stress (2,2)? Input: N

Stress (2,3)? Input: N

Stress (3,3)? Input: N

(Use user definable key to increment load, then use close file key to end file.)

Initially, all the normal and shear stresses are controlled.

The loading ratio on the normal stresses are set to 1 and the shear stress set to zero. Stress (2,2) and (3,3) are to be held at -2010 PSI so the load is to be incremented in 5 steps at -402 PSI to get the load condition to -2010 PSI. The plot definition is set with strain (1,1) as the independent variable and the normal stresses as dependent. The plot units are set. The load is then incremented. The controlled parameters are then redefined controlling all the normal and shear stresses. The loading ratio is set with stress (1,1) as 1 and the rest zero. The load is then incremented in -1000 PSI steps. The plot definition is redefined setting the strain (1,1) as the independent and stress (1,1) as the dependent variable.

File # 15 is a triaxial test, and in the plot definition we are plotting strain (1,1) as the independent variable and stress (1,1) as the dependent variable. The shear stresses and strains are set to zero. The normal strains (2,2) and (3,3) are not held constant and the normal stresses (2,2) and (3,3) are held at -2010 PSI.

Constitutive Model Driver

After loading the program tape in the console and the tape on which the predictions are to be stored into the auxillary tape input, press the auto load button. The Constitutive Model Driver is program number 2.

```
. Input: 2
. Comp: (The computer responds by displaying the user
        definable keys)
. Comp: Input file for storing results
. Input: 15
. Comp: Is this a restart?
. Input: N
. Comp: Input title
. Input: (Input the title you wish to appear on the plot)
. Comp: (The computer list various models)
. Input: 2 (for refined endochronic model)
. Comp: Input compressive strength
. Input: 2840
. Comp: Input I/O channel number of the plotter:
. Input: (32 for screen or 2 for plotter)
. Comp: Controlled parm:
        Stress (1,1)? Input: Y
        Stress (1,2)? Input: Y
        Stress (1,3)? Input: Y
        Stress (2,2)? Input: Y
        Stress (2,3)? Input: Y
        Stress (3,3)? Input: Y
        Loading ratio:
        Stress (1,1) = 1
        Stress (1,2) = 0
```

Loading ratio: (cont.)

Stress (1,3) = 0

Stress (2,2) = 1

Stress (2,3) = 0

Stress (3,3) = 1

Load Incr. = -402

Temp Incr. = 0

Plot definition:

Independent variable:

Load? N

Time? N

Volumetric strain? N

Strain (1,1)? Y

Is this axis horizontal? Y

Dependent var.

Stress (1,1)? Input: Y

Stress (1,2)? Input: N

Stress (1,3)? Input: N

Stress (2,2)? Input: Y

Stress (2,3)? Input: N

Stress (3,3)? Input: Y

Min/Max Plot Lim.

Strain = 0, -.015

Stress = 0, -11,000

Use user definable key to auto increment load - 5 steps. Then
redefine controlled parameters with the user definable key.

Comp: Controlled parm:

Stress (1,1)? Input: Y

Stress (1,2)? Input: Y

Stress (1,3)? Input: Y

Stress (2,2)? Input: Y

Stress (2,3)? Input: Y

Stress (3,3)? Input: Y

Loading ratio:

Stress (1,1) = 1

Stress (1,2) = 0

Stress (1,3) = 0

Comp: Plot definition

Independent var:

Load? Input: N (for no)

Time? Input: N

Strain (1,1)? Input: Y (for yes)

This axis horiz? Input: Y

Dependent var.

Stress (1,1)? Input: Y

Stress (1,2)? Input: N

Stress (1,3)? Input: N

Stress (2,2)? Input: N

Stress (2,3)? Input: N

Stress (3,3)? Input: N

Comp: Constant?

Stress (1,2)? Input: Y

Value = ? Input: 0

Stress (1,3)? Input: Y

Value = ? Input: 0

Stress (2,2)? Input: Y

Value = ? Input: 0

Stress (2,3)? Input: Y

Value = ? Input: 0

Stress (3,3)? Input: Y

Value = ? Input: -2010

Strain (1,2)? Input: Y

Value = ? Input: 0

Strain (1,3)? Input: Y

Value = ? Input: 0

Strain (2,2)? Input: N

Strain (2,3)? Input: Y

Value = ? Input: 0

Strain (3,3)? Input: N

Comp: Locate cursor and press call for the following values:

Stress (1,1)

(Locate all data points in this manner)

(Press carriage return when finished)

Marking of Files

300,000 Bytes per tape (take $\approx$ 300 Byte/Point)

Input: Find 15 (File # which was last marked)

Input: Mark 5,15104 (5 indicates # of files to be marked)
(15,104 indicates length of file)

Kupfer, Hilsdorf, and Rüschi (1969)

TEST TYPE: Uniaxial

TEST DATE: 1969

SOURCE: Kupfer, H., Hilsdorf, H. K., and Rüschi, H., "Behavior of Concrete Under Biaxial Stresses, *Journal of the American Concrete Institute* 66, 656-66 (August 1969).

Bazant, Z. P., Bhat, P. D., and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: 20-cm X 20-cm X 5-cm prisms
15-mm maximum aggregate size
190 kg of cement per m³ of concrete
7-day moist cured
Stored at 65% R.H. and 65°F
28-day test age
 $f'_c = 4650$ psi

LOADING: Brush platens used
 ϵ_1 held constant

Kupfër, Hilsdorf, and Rusch (Uniaxial) 1969

Kypfer, Hilsdorf, Rush, 1969

Uniaxial, $f'_c = 4650$ psi

EVALUATION: As depicted by the accompanying stress-strain diagram, the model's prediction represents a good approximation of the actual behavior of the concrete. In the stress range from -2000 to -5000 psi, however, the model's predictions do deviate slightly. Within this range, the model predicts slightly higher strains than the actual data indicates, the maximum difference being approximately .0002 in/in.

Kupfer, Hilsdorf, and Rüsç (1969)

TEST TYPE: Biaxial

TEST DATE: 1969

SOURCE: Kupfer, H., Hilsdorf, H. K., and Rüsç, H., "Behavior of Concrete Under Biaxial Stresses," *Journal of the American Concrete Institute* 66, 656-66 (August 1969) [original].

Bazant, Z. P., Bhat, P. D., and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: $f'_c = 4650$ psi
20-cm X 20-cm X 5-cm prisms
15-mm maximum aggregate size
190 kg of cement per m^3 of concrete

Specimens were hand compacted and moist cured for 7 days after which they were stored at an R.H. of 65% at 65°F up to the test age of 28 days.

LOADING: During loading a constant strain rate was maintained that allowed maximum load to occur after approximately 20 min. Restraint free loading was accomplished through the use of brush bearing platens. These platens consist of individual steel filaments 3 mm X 5 mm with the length varying between 100 mm and 140 mm depending on the design strength of the concrete. Use of brush platens allows for no more than 0.5% deviation in the average principal stresses.

RESULTS: Failure mode consisted of numerous microcracks parallel to the specimens free surfaces with an additional major crack occurring at 18-27 degrees to the free surfaces. Results conclude the strength of concrete under biaxial compression is only approximately 16% greater than uniaxial compression.

KUPFER, HILSDORF, RUSCH 1969 (BIAXIAL-4650)

Kupfer, Hilsdorf, Rusch, 1969

Biaxial, $\sigma_1 = \sigma_2$, $f'_c = 4650$ psi

EVALUATION: In comparing the biaxial data to the model's predictions, the similarity to the results of the Kupfer et al uniaxial test found in File No. 2 is apparent. Again in the -2000 to -5000 stress range, the model predicts slightly larger strains.

Hognestad, Hanson, McHenry (1955)

TEST TYPE: Uniaxial

TEST DATE: 1955

SOURCE: Hognestad, E., Hanson, N. W., and McHenry, D., "Concrete Stress Distribution in Ultimate Strength," *Journal of the American Concrete Institute, Proceedings* 52(4), 455-79 (December 1955).

Bazant, Z. P., Bhat, P. D., and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: 6 in. X 12 in. cylinders
Type I cement
Elgin sand and gravel, 1 1/2 in. maximum aggregate
Moist cured then 5 days drying at 75°F
50%-80% R.H.
Age at test: 14 days
Loading rate not specified
 $f'_c = 3000$ psi

Hognestad, Hanson, McHenry (uniaxial) 1955

Hognestad, Hanson, McHenry, 1955
Uniaxial, $f'_c = 3000$ psi

EVALUATION: The curve of the test data and the curve of the endochronic theory have basically the same shape, but a smaller initial slope on the endochronic curve results in its predictions of larger strain values in the stress range from -1000 to -3000 psi.

Hognestad, Hanson, McHenry (1955)

TEST TYPE: Uniaxial

TEST DATE: 1955

SOURCE: Hognestad, E., Hanson, N. W., and McHenry, D., "Concrete Stress Distribution in Ultimate Strength," *Journal of the American Concrete Institute, Proceedings* 52(4), 455-79 (December 1955).

Bazant, Z. P., Bhat, P. D., and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: 6 in. X 12 in. cylinders
Type I cement
Elgin sand and gravel, 1 1/2 in. maximum aggregate
Moist cured then 5 days drying at 75°F
50%-80% R.H.
Age at test: 14 days
Loading rate not specified
 $f'_c = 4650$ psi

Hognestad, Hanson, McHenry (uniaxial -4650 psi)

Hognestad, Hanson, McHenry, 1955
Uniaxial, $f'_c = 4650$ psi

EVALUATION: In the previous comparisons, it was noted that the model predicted higher strains than were evidenced by the experimental data. In this case, the opposite holds true. In the stress range from -1000 to -5000 psi, the model predicts slightly lower strains, approximately .0001 in/in lower.

Hognestad, Hanson, McHenry. (1955)

TEST TYPE: Uniaxial

TEST DATE: 1955

SOURCE: Hognestad, E., Hanson, N. W., and McHenry, D., "Concrete Stress Distribution in Ultimate Strength," *Journal of the American Concrete Institute, Proceedings* 52(4), 455-79 (December 1955).

Bazant, Z. P., Bhat, P. D., and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: 6 in. X 12 in. cylinders
Type I cement
Elgin sand and gravel, 1 1/2 in. maximum aggregate
Moist cured then 5 days drying at 75°F
50%-80% R.H.
Age at test: 14 days
Loading rate not specified
 $f'_c = 6200$ psi

HOGNESTAD, HANSON, McHENRY 1955 (UNIAXIAL-6200)

Hognestad, Hanson, McHenry, 1955
Uniaxial, $f'_c = 6200$ psi

EVALUATION: As pointed out in File No. 5, the model predicted lower strains than the actual data exemplified. This discrepancy is much more apparent from the results of this file. In this case, the variance between the two strains reaches .0003 in/in.

Balmer (1949)

TEST TYPE: Triaxial

TEST DATE: 1949

SOURCE: Balmer, G. G., *Shearing Strength of Concrete Under High Triaxial Stress - Computation of Mohr's Envelope as a Curve*, Structural Research Laboratory Report No. SP-23, Denver, Colorado, Oct. 1949.

Bazant, Z. P., Bhat, P. D. and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: $f'_c = 3570$ psi
6 in x 12 in cylinders
Type II cement
water-cement-sand-gravel ratio, .058:1:2:86:4.47
Grand Coulee aggregate 1 1/2 in. maximum size.
fog cured 28 days at 70°F, then oven dried

LOADING: Hydrostatic pressure was applied first, then the axial load was superimposed. σ_2 and σ_3 were held constant at -25,000 psi.

Balmer, 1949

Triaxial, $\sigma_2 = \sigma_3 = -25,000$ psi, $f'_c = 3570$ psi

EVALUATION: In examining the stress strain diagrams of Balmer's data and the model's prediction, one sees that in the tests using higher confining pressures, σ_2 and $\sigma_3 = -25,000$, the model predicts consistently higher strains than are actually observed. The variance in strain values ranges from .005 to 0 in/in.

Balmer (1949)

TEST TYPE: Triaxial

TEST DATE: 1949

SOURCE: Balmer, G. G., *Shearing Strength of Concrete Under High Triaxial Stress - Computation of Mohr's Envelope as a Curve*, Structural Research Laboratory Report No. SP-23, Denver, Colorado, Oct. 1949.

Bazant, Z. P., Bhat, P. D. and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: f'_c - 3570 psi
6 in x 12 in cylinders
Type II cement
water-cement-sand-gravel ratio, .058:1:2.86:4.47
Grand Coulee aggregate 1 1/2 in. maximum size.
fog cured 28 days at 70°F, then oven dried

LOADING: Hydrostatic pressure was applied first, then the axial load was superimposed. σ_2 and σ_3 were held constant at -20,000 psi.

BALMER 1949, TRIAXIAL, (SIG2=SIG3=20000PSI)3570PSI

Balmer, 1949

Triaxial, $\sigma_2 = \sigma_3 = 20,000$ psi, $f'_c = 3570$ psi

EVALUATION: As shown in the previous files, the model's predictions thus far represent a reasonable approximation of the test data. This is again illustrated by the results of this file in which the curves of Balmer's test data and the model's predictions are practically congruent.

An inconsistency does exist, however, in the model's inability to predict the strength of the concrete out in the ultimate range. The curve of the model tapers off more quickly than that of the actual data.

Balmer (1949)

TEST TYPE: Triaxial

TEST DATE: 1949

SOURCE: Balmer, G. G., *Shearing Strength of Concrete Under High Triaxial Stress - Computation of Mohr's Envelope as a Curve*, Structural Research Laboratory Report No. SP-23, Denver, Colorado, Oct. 1949.

Bazant, Z. P., Bhat, P. D. and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: $f'_c = 3570$ psi
6 in. x 12 in. cylinders
Type II cement
water-cement-sand-gravel ratio, 0.58:1:2.86:4.47
Grand Coulee aggregate 1 1/2 in. maximum size.
fog cured 28 days at 70°F, then oven dried

LOADING: Hydrostatic pressure was applied first, then the axial load was superimposed. σ_2 and σ_3 were held constant at -15,000 psi.

Balmer, 1949

Triaxial, $\sigma_2 = \sigma_3 = 15,000$ psi, $f'_c = 3570$ psi

EVALUATION: Once again, the model's predictions coincide nicely with the actual test data. This comparison does, however, show a peculiarity in the theoretical curve which has not been observed until now. An uncharacteristic dip occurs in the curve at approximately a strain of .060 in/in.

Balmer (1949)

TEST TYPE: Triaxial

TEST DATE: 1949

SOURCE: Balmer, G. G., *Shearing Strength of Concrete Under High Triaxial Stress - Computation of Mohr's Envelope as a Curve*, Structural Research Laboratory Report No. SP-23, Denver, Colorado, Oct. 1949.

Bazant, A. P., Bhat, P. D. and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: $f'_c = 3570$ psi
6 in x 12 in cylinders
Type II cement
water-cement-sand-gravel ratio, .058:1:2.86:4.47
Grand Coulee aggregate 1 1/2 in. maximum size.
fog cured 28 days at 70°F, then oven dried

LOADING: Hydrostatic pressure was applied first, then the axial load was superimposed. σ_2 and σ_3 were held constant at -10,000 psi.

BALMER, 1949, TRIAXIAL, (SIG2=SIG3=10000)3570PSI

Balmer, 1949

Triaxial, $\sigma_2 = \sigma_3 = 10,000$ psi, $f'_c = 3570$ psi

EVALUATION: File 10 exhibits the same type of dip in the model's curve as was previously noted in File No. 9. This time the dip occurs at a strain of approximately .040 in/in. The model also tapers off slightly out toward the ultimate strength range. Otherwise, the model is consistent with the test data.

Balmer (1949)

TEST TYPE: Triaxial

TEST DATE: 1949

SOURCE: Balmer, G. G., *Shearing Strength of Concrete Under High Triaxial Stress - Computation of Mohr's Envelope as a Curve*, Structural Research Laboratory Report No. SP-23, Denver, Colorado, Oct. 1949.

Bazant, Z. P., Bhat, P. D. and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: $f'_c = 3570$ psi
6 in x 12 in cylinders
Type II cement
water-cement-sand-gravel ratio, Q58:1:2.86:4.47
Grand Coulee aggregate 1 1/2 in. maximum size.
fog cured 28 days at 70°F, then oven dried

LOADING: Hydrostatic pressure was applied first, then the axial load was superimposed. σ_2 and σ_3 were held constant at -5,000 psi.

BALMER, 1949, TRIAXIAL, (SIG2=SIG3=5000PSI) 3570PSI

Balmer, 1949

Triaxial, $\sigma_2 = \sigma_3 = 5000$ psi, $f'_c = 3570$ psi

EVALUATION: Like File No. 9 and 10, the curve of the model's predictions again tapers off exhibiting a stress variance of approximately 5000 psi from the actual data. The uncharacteristic dip in the model's curve is also present at a strain of approximately .020 in/in.

Balmer (1949)

TEST TYPE: Triaxial

TEST DATE: 1949

SOURCE: Balmer, G. G., *Shearing Strength of Concrete Under High Triaxial Stress - Computation of Mohr's Envelope as a Curve*, Structural Research Laboratory Report No. SP-23, Denver, Colorado, Oct. 1949.

Bazant, Z. P., Bhat, P. D. and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: $f'_c = 3570$ psi
6 in x 12 in cylinders
Type II cement
water-cement-sand-gravel ratio, Q58:1:2.86:4.47
Grand Coulee aggregate 1 1/2 in. maximum size.
fog cured 28 days at 70°F, then oven dried

LOADING: Hydrostatic pressure was applied first, then the axial load was superimposed. σ_2 and σ_3 were held constant at -2,500 psi.

BALMER, 1949, TRIAXIAL, (SIG2=SIG3=2500PSI)3570PSI

Balmer, 1949

Triaxial, $\sigma_2 = \sigma_3 = 2500$ psi, $f'_c = 3570$ psi

EVALUATION: The results of this file again show similar characteristics to that of all the other files involving the Balmer series of multiaxial tests. The dip is present, and a tapering off towards the ultimate is evident.

Balmer (1949)

TEST TYPE: Triaxial

TEST DATE: 1949

SOURCE: Balmer, G. G., *Shearing Strength of Concrete Under High Triaxial Stress - Computation of Mohr's Envelope as a Curve*, Structural Research Laboratory Report No. SP-23, Denver, Colorado, Oct. 1949.

Bazant, Z. P., Bhat, P. D. and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: $f'_c = 3570$ psi
6 in x 12 in cylinders
Type II cement
water-cement-sand-gravel ratio, Q58:1:2.86:4.47
Grand Coulee aggregate 1 1/2 in. maximum size.
fog cured 28 days at 70°F, then oven dried

LOADING: Hydrostatic pressure was applied first, then the axial load was superimposed. σ_2 and σ_3 were held constant at -1,000 psi.

BALMER, 1949, TRIAXIAL, (SIG2=SIG3=1000PSI)3570PSI

Balmer, 1949

Triaxial, $\sigma_2 = \sigma_3 = 1000$ psi, $f'_c = 3570$ psi

EVALUATION: In this plot, the model predicts a stress approximately -2000 psi less than that of the actual test data. Also, the dip which characterized the previous files is not present in this plot. This could be due to the scale of the graph or perhaps the load increment used in driving the model.

Balmer (1949)

TEST TYPE: Uniaxial

TEST DATE: 1949

SOURCE: Balmer, G. G., *Shearing Strength of Concrete Under High Triaxial Stress - Computation of Mohr's Envelope as a Curve*, Structural Research Laboratory Report No. SP-23, Denver, Colorado, Oct. 1949.

Bazant, Z. P., Bhat, P. D. and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: $f'_c = 3570$ psi
6 in x 12 in cylinders
Type II cement
water-cement-sand-gravel ratio, 0.58:1:2.86:4.47
Grand Coulee aggregate 1 1/2 in. maximum size.
fog cured 28 days at 70°F, then oven dried

LOADING: Hydrostatic pressure was applied first, then the axial load was superimposed. σ_2 and σ_3 were held constant at 0 psi.

BALMER, 1949, UNIAXIAL, 3570PSI

Balmer, 1949

Uniaxial, $f'_c = 3570$ psi

EVALUATION: As evidenced by the stress-strain diagram, the model predicts a stress of approximately -2300 psi at a strain of .004 in/in whereas the actual data shows a stress of approximately -3600 psi. This stress continues out to a strain of .005 in/in.

Richart, Brandtzaeg, Brown, (1928)

TEST TYPE: Triaxial

TEST DATE: 1928

SOURCE: Richart, F. E., Brandtzaeg, A., Brown, R. L., *A Study of the Failure of Concrete Under Combined Compressive Stresses*, University of Illinois Engineering Experiment Station, Bulletin, No. 185, Nov. 1928, pp. 1-23.

Bazant, Z. P., Bhat, P. D., and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: $f'_c = 2840$ psi
4 in x 8 in cylinders
Type I cement
cement-sand-gravel ratio, 1:2.1:2.5
Wabash River sand, 3/8 in. maximum aggregate size
Forms were removed 24 hours after pouring, and specimens were placed in a moist closet for curing. After being allowed to dry out in the laboratory air for one day, the specimen was tested at the age of 28 days.

LOADING: The method of loading consisted of applying a small initial end load to the specimen then applying a liquid pressure (oil) to the sides of the cylinder to a predetermined value (σ_2 and $\sigma_3 = -2010$ psi). The test then proceeded by increasing the axial load.

RICHART, BRANDTZAEG, BROWN, 1928, (TRIAXIAL, SIG2=SIG3=-2010PSI) $f'_c=2840$ PSI

Richart, Brandtzaeg, Brown, 1928

Triaxial, $\sigma_2 = \sigma_3 = 2010$ psi, $f'_c = 2840$ psi

EVALUATION: The dip in the theoretical curve which was present in the Balmer series reappears again in this plot. After finding no mistakes in the coding of the program, it appears that this incongruity is inherent in the model's equations. The dip in this plot occurs at a strain of approximately $-.006$ in/in.

Furthermore, instead of the model's curve tapering off and predicting lower stresses as in Files 8, 10, 11, 12, 13, and 14, the model's curve projects above the actual data thus predicting higher stresses.

Richart, Brandtzaeg, Brown, (1928)

TEST TYPE: Triaxial

TEST DATE: 1928

SOURCE: Richart, F. E., Brandtzaeg, A., Brown, R. L., *A Study of the Failure of Concrete Under Combined Compressive Stresses*, University of Illinois Engineering Experiment Station, Bulletin, No. 185, Nov. 1928, pp. 1-23.

Bazant, Z. P., Bhat, P. D., and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: $f'_c = 2840$ psi
4 in x 8 in cylinders
Type I cement
cement-sand-gravel ratio, 1:2.1:2.5
Wabash River sand, 2/8 in. maximum aggregate size
Forms were removed 24 hours after pouring, and specimens were placed in a moist closet for curing. After being allowed to dry out in the laboratory air for one day, the specimen was tested at the age of 28 days.

LOADING: The method of loading consisted of applying a small initial end load to the specimen then applying a liquid pressure (oil) to the sides of the cylinder to a predetermined value (σ_2 and $\sigma_3 = -1090$ psi). The test then proceeded by increasing the axial load.

RICHART, BRANDTZAEG, BROWN, 1928, (TRIAxIAL, SIG2=SIG3=-1090PSI) $f'_c=2840$ PSI

RICHART, BRANDTZAEG, BROWN, 1928, (TRIAxIAL, SIG2=SIG3=-1090PSI) $f'_c=2840$ PSI

Richart, Brandtzaeg, Brown, 1928

Triaxial, $\sigma_2 = \sigma_3 = 1090$ psi, $f'_c = 2840$ psi

EVALUATION: Like in File No. 15, the model's overestimation of the stresses is apparent here also. The actual and theoretical curves fit nicely up to a strain of approximately $-.004$ and then a stress difference of about 500 psi maximum occurs.

Richart, Brandtzaeg, Brown, (1928)

TEST TYPE: Triaxial

TEST DATE: 1928

SOURCE: Richart, F. E., Brandtzaeg, A., Brown, R. L., *A Study of the Failure of Concrete Under Combined Compressive Stresses*, University of Illinois Engineering Experiment Station, Bulletin, No. 185, Nov. 1928, pp. 1-23.

Bazant, Z. P., Bhat, P. D., and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: $f'_c = 2840$ psi
4 in x 8 in cylinders
Type I cement
cement-sand-gravel ratio, 1:2.1:2.5
Wabash River sand, 3/8 in. maximum aggregate size
Forms were removed 24 hours after pouring, and specimens were placed in a moist closet for curing. After being allowed to dry out in the laboratory air for one day, the specimen was tested at the age of 28 days.

LOADING: The method of loading consisted of applying a small initial end load to the specimen then applying a liquid pressure (oil) to the sides of the cylinder to a predetermined value (σ_2 and $\sigma_3 = -550$ psi). The test then proceeded by increasing the axial load.

RICHART, BRANDTZAEG, BROWN, 1928, (TRIAXIAL, SIG2=SIG3=-550PSI) $f'_c=2840$ PSI

Richart, Brandtzaeg, Brown, 1928

Triaxial, $\sigma_2 = \sigma_3 = 550$ psi, $f'_c = 2840$ psi

EVALUATION: In this plot, the actual and theoretical curves coincide nicely. There is perhaps a 100 psi maximum stress variance of the actual data over the theoretical curve.

Richart, Brandtzaeg, Brown, (1928)

TEST TYPE: Uniaxial

TEST DATE: 1928

SOURCE: Richart, F. E., Brandtzaeg, A., Brown, R. L., *A Study of the Failure of Concrete Under Combined Compressive Stresses*, University of Illinois Engineering Experiment Station, Bulletin, No. 185, Nov. 1928, pp. 1-23.

Bazant, Z. P., Bhat, P. D., and Shieh, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976) [secondary source].

SPECIMEN: f'_c - 2840 psi
4 in x 8 in cylinders
Type I cement
cement-sand-gravel ratio, 1:2.1:2.5
Wabash River sand, 3/8 in. maximum aggregate size
Forms were removed 24 hours after pouring, and specimens were placed in a moist closet for curing. After being allowed to dry out in the laboratory air for one day, the specimen was tested at the age of 28 days.

LOADING: The method of loading consisted of applying a small initial end load to the specimen then applying a liquid pressure (oil) to the sides of the cylinder to a predetermined value (σ_2 and $\sigma_3 = -0$ psi). The test then proceeded by increasing the axial load.

RICHART, BRANDTZAEG, BROWN, 1928, (UNIAXIAL) $f'_c = 2840 \text{ PSI}$

Richart, Brandtzaeg, Brown, 1928
Uniaxial, $f'_c = 2840$ psi

EVALUATION: File 18 exhibits practically the same relationship between the actual data and the theoretical data as File 17. The model under estimates the stress values by approximately 250 psi maximum.

CRBR Concrete Test Specimens

TEST TYPE: Uniaxial

TEST DATE: 1978

SOURCE: Data was obtained from Charles Barry Oland at ORNL.

SPECIMEN: $f'_c = 4240$ PSI
6 x 12 in cylinder
Type I Portland Cement

NOTE: Although the title of the plots indicate high temperature compressive tests, the data is actually from normal room temperature compressive test. These test were used to formulate a design mix to be subjected to high temperature test at a later date.

CRBR High Temp Concrete Test
Uniaxial, $f'_c = 4240$ PSI

EVALUATION: Aside from a small difference in the modulus of elasticity of the two curves, the prediction represents a reasonable approximation of the data. The two curves have the same general shape as well as same f'_c .

CRBR Concrete Test Specimens

TEST TYPE: Uniaxial

TEST DATE: 1978

SOURCE: Data was obtained from Charles Barry Oland at ORNL.

SPECIMEN: $f'_c = 5540$ PSI
6 x 12 in cylinder
Type I Portland Cement

NOTE: Although the title of the plots indicate high temperature compressive tests, the data is actually from normal room temperature compressive test. These test were used to formulate a design mix to be subjected to high temperature test at a later date.

CRBR High Temp Concrete Test
Uniaxial, $f'_c = 5540$ PSI

EVALUATION: Good results. Close approximation. Same
curve shape and f'_c .

CRBR Concrete Test Specimens

TEST TYPE: Uniaxial

TEST DATE: 1978

SOURCE: Data was obtained from Charles Barry Oland at ORNL.

SPECIMEN: $f'_c = 3540$ PSI
6 x 12 in cylinder
Type I Portland Cement

NOTE: Although the title of the plots indicate high temperature compressive tests, the data is actually from normal room temperature compressive test. These test were used to formulate a design mix to be subjected to high temperature test at a later date.

(CRBR High Temp Concrete Test
Uniaxial, $f'_c = 3540$ PSI

EVALUATION: Similarly to File #19, there is a deviation of
the modulus of elasticity of the two curves.

CRBR Concrete Test Specimens

TEST TYPE: Uniaxial

TEST DATE: 1978

SOURCE: Data was obtained from Charles Barry Oland at ORNL.

SPECIMEN: $f'_c = 4140$ PSI
6 x 12 in cylinder
Type I Portland Cement

NOTE: Although the title of the plots indicate high temperature compressive tests, the data is actually from normal room temperature compressive test. These test were used to formulate a design mix to be subjected to high temperature test at a later date.

CRBR High Temp Concrte Test
Uniaxial $f'_c = 4140$ PSI

EVALUATION: Once again a difference in the initial slopes of the curves is apparent.

CRBR Concrete Test Specimens

TEST TYPE: Uniaxial

TEST DATE: 1978

SOURCE: Data was obtained from Charles Barry Oland at ORNL.

SPECIMEN: $f'_c = 6430$ PSI
 6 x 12 in cylinder
 Type I Portland Cement

NOTE: Although the title of the plots indicate high temperature compressive tests, the data is actually from normal room temperature compressive test. These test were used to formulate a design mix to be subjected to high temperature test at a later date.

CRBR High Temp Concrete Test
Uniaxial, $f'_c = 6430$ PSI

EVALUATION: In this comparison, the model represents a good approximation of the data. In the CRBR concrete test comparisons the files which displayed the closest approximation to the actual data were the files with the higher f'_c . For example, the best approximations were apparent in this File and File # 20 with f'_c of 6430 PSI and 5540 PSI. In the other files with compressive strength of 4240, 3540, 4140 PSI the model exhibited a smaller modulus of elasticity than that of the actual data.

Green, Swanson, 1973

TEST TYPE: Triaxial

TEST DATE: 1973

SOURCE: Green, S.J., and Swanson, S.R., *Static Constitutive Relations for Concrete*, Technical Report No. AFWL-TR-72-2, Terra-Tek, Inc., Salt Lake City, Utah, April, 1973.

Bazant, Z.P., Bhat, P.D. and Sheih, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976).
(Secondary Source).

SPECIMAN: $f'_c = 7020$ PSI
2.7 in x 6 in cylinder
Type I Portland cement
3/8" max. size, smooth aggregate
Age 28 days
Agg./cement ratio = 6.3/1 (by weight)
W/C Ratio = .6/1 (by weight)

LOADING: Hydrostatic pressure applied first, then axial; strain rate 10^{-4} /second
 σ_2 & $\sigma_3 = -2000$ PSI

FIT OF GREEN & SWANSON TRIAXIAL DATA (SIG2=SIG3=-2 KSI)

Green and Swanson, 1973
Triaxial, σ_2 & σ_3 = -2000 PSI

EVALUATION: As shown the plot of the data and the prediction coincide nicely. The general shape of the curves as well as the strengths are the same.

Green, Swanson, 1973

TEST TYPE: Triaxial

TEST DATE: 1973

SOURCE: Green, S.J., and Swanson, S.R., *Static Constitutive Relations for Concrete*, Technical Report No. AFWL-TR-72-2, Terra-Tek, Inc., Salt Lake City, Utah, April, 1973.

Bazant, Z.P., Bhat, P.D. and Sheih, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976).
(Secondary Source).

SPECIMAN: $f'_c = 7020$ PSI
2.7 in x 6 in cylinder
Type I Portland cement
3/8" max. size, smooth aggregate
Age 28 days
Agg./cement ratio = 6.3/1 (by weight)
W/C Ratio = .6/1 (by weight)

LOADING: Hydrostatic pressure applied first, then axial; strain rate 10^{-4} /second
 σ_2 & $\sigma_3 = 1000$ PSI

Green and Swanson, 1973

Triaxial, σ_2 & σ_3 = -1000 PSI

EVALUATION: The curves reflect the same shape, however, as evidenced by the plot, the model over estimates the strength by approximately 2000 PSI.

(Green, Swanson, 1973

TEST TYPE: Uniaxial

TEST DATE: 1973

SOURCE: Green, S.J., and Swanson, S.R., *Static Constitutive Relations for Concrete*, Technical Report No. AFWL-TR-72-2, Terra-Tek, Inc., Salt Lake City, Utah, April, 1973.

Bazant, Z.P., Bhat, P.D. and Sheih, C., *Endochronic Theory for Inelasticity and Failure Analysis of Concrete Structures*, Structural Engineering Report No. 1976-12/259, Department of Civil Engineering, Northwestern University, Evanston, Illinois (1976).
(Secondary Source).

SPECIMAN: $f'_c = 7020$ PSI
2.7 in x 6 in cylinder
Type I Portland cement
3/8" max. size, smooth aggregate
Age 28 days
Agg./cement ratio = 6.3/1 (by weight)
W/C Ratio = .6/1 (by weight)

LOADING: Hydrostatic pressure applied first, then axial; strain rate 10^{-4} /second
 σ_2 & $\sigma_3 = 0$ PSI

FIT OF GREEN & SWANSON UNIAXIAL DATA

Green and Swanson, 1973
Uniaxial, $f'_c = 7020$

EVALUATION: In this comparison the compressive strengths coincide, however, as depicted by the plot there is variance in the shape of the two curves. At a strain of $-.002$ in/in the model predicts a stress of approximately 7000 PSI. The actual data shows a stress of approximately 500 PSI less.

Richart, Brandtzaeg, Brown, (1928)

TEST TYPE: Uniaxial

TEST DATE: 1928

SOURCE: Richart, F.E., Brandtzaeg, A., Brown, R.L., *A Study of Failure of Concrete Under Combined Compressive Stressess*, University of Illinois Engineering Experiment Station, Bulletin, No. 185, Nov. 1928.

SPECIMAN: $f'_c = 3480$ PSI
4 in x 8 in cylinders
Type I Portland cement
Wabash River sand, 2/8 in. max. agg.
Cement-Sand-Gravel Ratio, 1:1:2
Forms removed 24 hours after
pouring and moist cured. Usually
allowed to air dry 1 day before
testing at age 28 days.

LOADING: Same as the commonly employed with
the standard 6 x 12 in. cylinder
compression test.

Richart, Brandtzaeg, Brown, 1928
Uniaxial, $f'_c = 3480$ PSI

EVALUATION: As the plot of the curves show, the model is a good approximation of the data. It should be noted that the data for this series of Richart, Brandtzaeg, Brown test was taken from small plots and that small error might have been induced from the data digitizing.

Richart, Brandtzaeg, Brown, (1928)

TEST TYPE: Uniaxial

TEST DATE: 1928

SOURCE: Richart, F.E., Brandtzaeg, A., Brown, R.L., *A Study of Failure of Concrete Under Combined Compressive Stressess*, University of Illinois Engineering Experiment Station, Bulletin, No. 185, Nov. 1928.

SPECIMAN: ^E
 $f'_c = 2610$ PSI
4 in x 8 in cylinders
Type I Portland cement
Wabash River sand, 2/8 in. max. agg.
Cement-Sand-Gravel Ratio, 1:1:2
Forms removed 24 hours after
pouring and moist cured. Usually
allowed to air dry 1 day before
testing at age 28 days.

LOADING: Same as the commonly employed with
the standard 6 x 12 in. cylinder
compression test.

RICHART, BRANDTZAEG, BROWN (UNIAXIAL, $f'_c = 2610 \text{ PSI}$)

(Richart, Brantzaeg, Bwoen 1928
Uniaxial, $f'_c = 2610$

EVALUATION: Similar to previous files involving CRBR data, the model displays a smaller modulus of elasticity than is presented by the actual data.

Richart, Brandtzaeg, Brown, (1928)

TEST TYPE: Uniaxial

TEST DATE: 1928

SOURCE: Richart, F.E., Brandtzaeg, A., Brown, R.L., *A Study of Failure of Concrete Under Combined Compressive Stressess*, University of Illinois Engineering Experiment Station, Bulletin, No. 185, Nov. 1928.

SPECIMAN: $f'_c = 3060$ PSI
4 in x 8 in cylinders
Type I Portland cement
Wabash River sand, 2/8 in. max. agg.
Cement-Sand-Gravel Ratio, 1:1:2
Forms removed 24 hours after
pouring and moist cured. Usually
allowed to air dry 1 day before
testing at age 28 days.

LOADING: Same as the commonly employed with
the standard 6 x 12 in. cylinder
compression test.

RICHART, BRANDTZAEG, BROWN (UNIAXIAL, $f'_c = 3060 \text{ PSI}$)

Richart, Brandtzaeg, Brown, 1928
Uniaxial, $f'_c = 3060$ PSI

EVALUATION: Once again a deviation between the modulus of elasticity of the model's prediction and that of the actual data occurs.

Richart, Brandtzaeg, Brown, (1928)

TEST TYPE: Uniaxial

TEST DATE: 1928

SOURCE: Richart, F.E., Brandtzaeg, A., Brown, R.L., *A Study of Failure of Concrete Under Combined Compressive Stressess*, University of Illinois Engineering Experiment Station, Bulletin, No. 185, Nov. 1928.

SPECIMAN: $f'_c = 2105$ PSI
4 in x 8 in cylinders
Type I Portland cement
Wabash River sand, 2/8 in. max. agg.
Cement-Sand-Gravel Ratio, 1:2.1:2.5
Forms removed 24 hours after
pouring and moist cured. Usually
allowed to air dry 1 day before
testing at age 28 days.

LOADING: Same as the commonly employed with
the standard 6 x 12 in. cylinder
compression test.

RICHART, BRANDTZAEG, BROWN (UNIAXIAL, $f'_c=2105\text{PSI}$)

Richart, Brandtzaeg, Brown, 1928
Uniaxial, $f'_c = 2105$

EVALUATION: The discrepancy between the E values of the two curves is apparent again. It appears that the smaller the f'_c the greater the deviation of of the two E values.

Richart, Brandtzaeg, Brown, (1928)

TEST TYPE: Uniaxial

TEST DATE: 1928

SOURCE: Richart, F.E., Brandtzaeg, A., Brown, R.L., *A Study of Failure of Concrete Under Combined Compressive Stressess*, University of Illinois Engineering Experiment Station, Bulletin, No. 185, Nov. 1928.

SPECIMAN: $f'_c = 2160$ PSI
4 in x 8 in cylinders
Type I Portland cement
Wabash River sand, 2/8 in. max. agg.
Cement-Sand-Gravel Ratio, 1:2.1:2.5
Forms removed 24 hours after
pouring and moist cured. Usually
allowed to air dry 1 day before
testing at age 28 days.

LOADING: Same as the commonly employed with
the standard 6 x 12 in. cylinder
compression test.

RICHART, BRANDTZAEG, BROWN (UNIAXIAL, $f'_c = 2160 \text{ PSI}$)

(Richart, Brandtzaeg, Brown, 1928
Uniaxial, $f'_c = 2160$ PSI

EVALUATION: Again, the model predicts a smaller E value
than actual occurs.

Richart, Brandtzaeg, Brown, (1928)

TEST TYPE: Uniaxial

TEST DATE: 1928

SOURCE: Richart, F.E., Brandtzaeg, A., Brown, R.L., *A Study of Failure of Concrete Under Combined Compressive Stressess*, University of Illinois Engineering Experiment Station, Bulletin, No. 185, Nov. 1928.

SPECIMAN: $f'_c = 1855$ PSI
4 in x 8 in cylinders
Type I Portland cement
Wabash River sand, 2/8 in. max. agg.
Cement-Sand-Gravel Ratio, 1:2.1:2.5
Forms removed 24 hours after
pouring and moist cured. Usually
allowed to air dry 1 day before
testing at age 28 days.

LOADING: Same as the commonly employed with
the standard 6 x 12 in. cylinder
compression test.

RICHART, BRANDTZAEG, BROWN (UNIAXIAL, $f'_c = 1855 \text{ PSI}$)

Richart, Brandtzaeg, Brown, 1928
Uniaxial, $f'_c = 1855$ PSI

EVALUATION: Since the f'_c of this concrete is 1855 PSI, we would expect a large deviation in the E values of the model and the actual data. This is indeed apparent from the plot. Therefore, in this series of Richart, Brandtzaeg, Brown uniaxial tests the greatest deviation between model and data occurs in the lower strength concretes.