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M1 transitions between collective levels and F-spin purity

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ABSTRACT: M1 transitions between low lying collective levels in deformed nuclei are described within the IBM-2 framework. This is done by a special choice of the Hamiltonian which allows a simultaneous fit of energies, E2 and M1 transitions. Finally the results are interpreted using the F-spin concept.

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1. INTRODUCTION

The study of F-spin forbidden M1 transitions between low lying collective levels has gathered considerable interest lately. It was already known that these transitions are rather weak. But not until recently it has been realized that the observed weakness of these transitions might be due to the operation of a selection rule and an associated quantum number viz F-spin. This implies that upper limits on the purity of F-spin can be derived from the observed M1 strenghts of low lying collective transitions.

In this paper we will give a detailed report on calculations for ^{166}Er and compare them to results for ^{128}Xe and ^{168}Er . These calculations lead to F-spin impurities between 2 and 4% .

The investigation of this interesting subject requires new and very reliable data on E2/M1 mixing ratios of the gamma band transitions in some selected rare earth nuclei. This is done by measuring parts of conversion electron spectra following a neutron capture reaction. Because of the high requirements concerning energy resolution and statistics we use the beta spectrometer BILL at the ILL's high flux reactor in Grenoble.

With the Interacting Proton Neutron Boson Model (IBM-2) energies, E2 and M1 transitions can be described. But one cannot do extensive IBM-2 fits of energies, $B(E2)$ and $B(M1)$ values due to time limitations (note: One IBM-2 fit for ^{166}Er takes about 10h CPU time on a VAX 750 computer).

This paper shows an alternative way in getting good quality fits. The principal idea is the decoupling of the IBM-2 Hamiltonian and $T(E2)$ transition operator on one hand and the $T(M1)$ transition operator on the other hand. Starting with an IBM-1 calculation for the energies and $B(E2)$ values projection formulas are used to connect the IBM-1 parameters to the related IBM-2 ones. In such a way most of the IBM-2 parameters can be found. Only χ_V and ξ remain to be determined by the $B(M1)$ values and the 1^+ energy.

2. PROJECTION METHOD

The Interacting Boson Model (IBM) [1] has been successfully applied to the calculation of excitation energies and E2 transition probabilities. In its original form, the so called IBM-1, only one type of s and d bosons is considered. In its extended IBM-2 version, separate proton and neutron bosons were introduced [2]. The proton-neutron symmetry properties of the IBM-2 can be described using the F-spin concept [2] the boson analogue of fermion isospin. Proton and neutron bosons are assigned an F-spin $F=1/2$. The maximum value of F is $F_m = (N_\pi + N_\nu)/2$ where N_π (N_ν) is the number of π (ν)-bosons. States with $F=F_m$ are totally symmetric, have lowest energy values and are identical to the IBM-1 states, whereas in the general case, the eigenstates of the IBM-2 Hamiltonian are F-spin mixed.

First we started doing an IBM-1 calculation which is simpler and requires less computer time than the IBM-2 one. Having done this the IBM-1 parameters were reprojected on most of the IBM-2 parameters using the following trick: From ref. [3,12] it is known that IBM-2 operators can be projected on IBM-1 ones. It follows that IBM-1 parameters are functions of the IBM-2 parameters. Inverting these relations give IBM-2 parameters as functions of IBM-1 ones and we call this procedure reprojected. If the IBM-2 eigenstates are weakly F-spin admixed, the projected Hamiltonian can be used as a good approximation for calculating energies and B(E2)'s.

Starting from an IBM-2 operator Q , we obtain the projected operator $Q_{F_m F_0}$ through

$$Q_{F_m F_0} = P_\pi^+ : F_+^{(F_m - F_0)} Q F_-^{(F_m - F_0)} : P_\pi / N_{F_m F_0}^2$$

The colons indicate that the operator inside should be in normally ordered form. F_+ (F_-) are the F-spin step operators [2,3]. F_0 is the F-spin projection quantum number. The operator

$F_+^{(F_m - F_0)}$ acting on the state $|F_m F_0\rangle$ produce the maximum weight state $|F_m F_m\rangle$, i.e. a state built only out of π -bosons; $N_{F_m F_0}^2$ is the norm of $F_+^{(F_m - F_0)} |F_m F_0\rangle$. P_π is the projection operator which removes from Q any term containing ν -boson annihilation operators. The subscript $F_m F_0$ of Q specifies the projection referring to a given nucleus with $F_0 = (N_\pi - N_\nu)/2$. $Q_{F_m F_0}$ contains only π -boson operators, which are trivially mapped on the boson operators of IBM-1. The IBM-1 operators are given to the left and the reprojected IBM-2 ones are shown to the right.

IBM-1

reprojected IBM-2

Hamiltonian

$$H = (\kappa'/2)L^2 + (\kappa/2) Q^2(\chi_S) \quad H = \varepsilon_2 (n_{d_\pi} + n_{d_\nu}) + \omega L^2 + \kappa_{\pi\nu} Q_\pi(\chi_\pi) \cdot Q_\nu(\chi_\nu) + \kappa_{\pi\nu} \chi_V^2 T_\pi^{(2)} \cdot T_\nu^{(2)} + \xi M$$

Quadrupole operator

$$T(E2) = q Q(\chi_S) \quad T(E2) = q_\pi Q_\pi(\chi_\pi) + q_\nu Q_\nu(\chi_\nu)$$

where $L = L_\pi + L_\nu$ is the total angular momentum,

$$q_S = (q_\pi + q_\nu)/2, \quad q_V = (q_\pi - q_\nu)/2, \quad \chi_S = (\chi_\pi + \chi_\nu)/2, \quad \chi_V = (\chi_\pi - \chi_\nu)/2$$

The equality of the parameter χ_S in the IBM-1 Hamiltonian and the E2 transition operator implies: $q_\pi N_\pi = q_\nu N_\nu$. Most of the IBM-2 parameters ($\varepsilon_2, \omega, \kappa_{\pi\nu}, q_\nu, q_\pi, \chi_S$) are functions of the IBM-1 parameters ($\kappa', \kappa, \chi_S, q$) and the following equations hold:

$$\begin{aligned} \varepsilon_2 &= -\kappa(4 - \chi_S^2); \quad \omega = \kappa'/2; \quad \kappa_{\pi\nu} = N(N-1)\kappa/(2N_\pi N_\nu); \\ \chi_\pi &= \chi_S + \chi_V; \quad \chi_\nu = \chi_S - \chi_V; \quad q_\pi = qN/(2N_\pi); \quad q_\nu = qN/(2N_\nu) \end{aligned} \quad (1)$$

3. DESCRIPTION OF M1 TRANSITIONS

Besides energies and $B(E2)$'s, the IBM-2 can describe properties which are beyond the scope of IBM-1, such as mixed symmetry states [2,3] or M1 transitions between low lying

collective states. As shown before we have constructed an IBM-2 Hamiltonian and E2 transition operator with $\chi_V=0$ (Note: In this case no M1 transitions take place):

$$H = \epsilon_2 (n_{d_\pi} + n_{d_V}) + \omega L^2 + \kappa_{\pi V} Q_\pi(\chi_\pi) \cdot Q_V(\chi_V) + \kappa_{\pi V} \chi_V^2 T_\pi^{(2)} \cdot T_V^{(2)} + \xi M \quad (2)$$

$$T(E2) = g_\pi Q_\pi(\chi_\pi) + g_V Q_V(\chi_V) \quad (3)$$

In order to describe M1 transitions we take χ_V values different from zero and use the following magnetic dipole operator:

$$T(M1) = \sqrt{3/(4\pi)} (g_\pi L_\pi + g_V L_V) \mu_N \quad (4)$$

The important feature of this parametrization is the small dependence of energies and B(E2) values on χ_V .

This can easily be seen by the projected operators which are free of χ_V .

4. FITTING PROCEDURE

The IBM-1 calculations are done in a consistent Q formalism (CQF) [7,10] i.e. the χ of the Hamiltonian is equal to the χ of the transition operator. We want to add that in IBM-2 we no longer have consistent Q formalism due to the introduction of the operator $\kappa_{\pi V} \chi_V^2 T_\pi^{(2)} \cdot T_V^{(2)}$.

The IBM-1 parameters can be reprojected on IBM-2 parameters by the formulas (1) where $\chi_V=0$ and $\xi=\infty$.

Subsequently we fix the values of the boson g-factors (4) to $g_\pi=1$ and $g_V=0$ (schematic model) (Note: For M1 transitions only $g_V=(g_\pi-g_V)/2$ is important). In a second step, M1 transitions and the description of the lowest collective 1^+ level energy are introduced by adjusting χ_V and ξ . The excitation energy of the lowest collective 1^+ state determines the Majorana parameter ξ . If the Majorana parameter ξ is small it induces low lying mixed

symmetry states by lowering the originally ($\xi=\infty$) high lying F_m-n ($n > 0$) states which then mix with the already low lying F_m states. (Note: The 1^+ state is a F_m-1 state).

The last parameter χ_V is determined by the B(M1) values. For practical reasons we use branching ratios and (E2/M1) mixing ratios instead. As we adjusted g_V to 0.5 (schematic model), the parameter χ_V is not determined model independent only the product $g_V\chi_V$ is specified to be -0.3. As stated before the fit can be performed in such a way due to the small dependence of energies and B(E2) values on χ_V and ξ .

5. EXAMPLE

^{166}Er provides a good example for the fit procedure given above because numerous experimental data are available.

The IBM-1 parameters for ^{166}Er are:

$N = 15$, $\chi_S = -0.49$, $\kappa = -0.0362$ MeV, $\kappa' = 0.01348$ MeV, $q = 1$

The parameter χ_S is taken from [7]. The parameters κ and κ' are fitted to the 2_1^+ and 2_2^+ excitation energy.

From the IBM-1 parameters, our relations (1) and an adopted 1^+ energy of 3.16 MeV, we find for the following IBM-2 parameters:

$N_\pi=7$; $N_V=8$; $\epsilon_2 = 0.06805$ MeV ; $\omega = 0.00674$ MeV ;
 $\kappa_{\pi V} = -0.06788$ MeV ; $\xi = 0.295$ MeV ; $q_V = 0.9375$;
 $q_\pi = 1.0714$; $g_\pi = 1$; $g_V = 0$; $\chi_S = -0.49$.

The adopted 1^+ energy of 3.16 MeV determines the Majorana parameter $\xi = 0.295$ MeV.

The parameters $\kappa_{\pi V}\chi_V^2$, χ_π and χ_V depend on χ_V and three typical cases of χ_V are listed here:

	$\chi_V=0$	$\chi_V=-0.6$	$\chi_V=-1.0$
$\kappa_{\pi V}\chi_V^2$	0	-0.02444	-0.06788
χ_π	-0.49	-1.09	-1.49
χ_V	-0.49	0.11	0.51

table 1

Comparison between exp. and calculated energies for ^{166}Er

E : experimental energy ; E^a : IBM-2 energy ($\chi_V = 0$)

E^b : IBM-2 energy ($\chi_V = -0.6$) ; E^c : IBM-2 energy ($\chi_V = -1.0$)

I	K	E [MeV]	$10^3(E^a/E-1)$	$10^3(E^b/E-1)$	$10^3(E^c/E-1)$
0	0	0	0	0	0
2	0	0.08057	5	5	5
4	0	0.26498	34	38	38
6	0	0.54544	44	47	47
2	2	0.78589	-2	-1	8
3	2	0.8594	13	13	20
4	2	0.9562	34	35	42
5	2	1.0753	55	55	58
6	2	1.2159	68	68	72

In all IBM-2 calculations the 1^+ energy was set to 3.16 MeV.

The experimental energies were taken from ref. [5] and the maximum deviation between experimental and theoretical values is 7% . Note the weak dependence of the theoretical energies on χ_V even though ξ is small.

table 2

Mixing ratios $\delta^2 = I_\gamma(E2)/I_\gamma(M1)$ in intra-band transition in the γ -band of $\text{Er}166$

a: δ^2 determined by the Alaga rule; b: IBM-2 fit ($\chi_V=0$)
c: IBM-2 fit ($\chi_V=-0.6$); d: IBM-2 fit ($\chi_V=-1.0$)

	δ^2 (exp.)	δ^2 (a)	δ^2 (b)	δ^2 (c)	δ^2 (d)
$5^+ \rightarrow 4^+$	2.8(+2.8;-0.9)	2.60	∞	2.04	0.84
$6^+ \rightarrow 5^+$	2.5(+2.5;-1.1)	3.28	∞	1.97	0.81

Table 2 lists experimental and theoretical mixing ratios of ^{166}Er [6]. The theoretical ones were derived by the Alaga rule [11] and various IBM-2 fits. Only the IBM-2 fit with $\chi_V=-0.6$ can be accepted to reproduce the experimental values which fixes the product $g_V\chi_V$ to -0.3.

table 3

Branching ratios (br. ratios) of the γ -band in ^{166}Er B : exp. branching ratio ; B^a : IBM-2 br. ratio $\chi_V = 0$ B^b : IBM-2 br. ratio $\chi_V = -0.6$; B^c : IBM-2 br. ratio $\chi_V = -1.0$

I_i	I_f	K_f	B	B^a	B^b	B^c	M1
2	0	0	100	100	100	100	
2	2	0	108.5	109.4	109.7	108.8	+
2	4	0	2.55	1.91	1.89	1.81	
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3	2	0	100	100	100	100	+
3	4	0	13.6	18.2	18.4	18.2	+
3	2	2	0.068	0.026	0.038	0.054	+
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4	2	0	100	100	100	100	
4	4	0	180	187	191	190	+
4	6	0	3.71	2.22	2.18	2.00	
4	2	2	2.00	1.60	1.58	1.53	
4	3	2	0.57	0.20	0.30	0.42	+
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5	4	0	100	100	100	100	+
5	6	0	15.7	16.2	16.9	17.1	+
5	4	2	0.29	0.17	0.25	0.34	+
5	3	2	4.10	3.59	3.47	3.24	
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6	4	0	100	100	100	100	
6	6	0	187	223	234	232	+
6	4	2	35.5	35.9	34.6	30.8	
6	5	2	1.29	0.94	1.37	1.84	+

Experimental branching ratios of the γ -band in ^{166}Er [5] and theoretical values are shown in table 3. The transitions which can have M1 admixtures are labelled by a plus sign (+). In these cases increasing χ_V leads to an increase of the branching ratios whereas for pure E2 they remain nearly constant. This confirms the good quality of the fit procedure even for a small Majorana parameter. This independence is due to the small F-spin impurity which is less than 3% .

table 4

		F-spin amplitudes of the first 2^+ of the γ -band		
	$g_V \chi_V$	F_m	F_{m-1}	F_{m-2}
^{128}Xe	-0.11	0.9806	0.1547	0.1207
^{168}Er	-0.3	0.9900	0.1307	0.0520
^{166}Er	-0.3	0.9922	0.1115	0.0545

The high purity of F-spin is also confirmed in table 4 where the product $g_V \chi_V$ and the F-spin amplitudes are given for the nuclei ^{128}Xe , ^{168}Er and ^{166}Er [8,9]. Some care should be taken in interpreting the $g_V \chi_V$ value of ^{128}Xe where the 1^+ level energy was set to 2.3 MeV.

6. SUMMARY

M1 transitions between low lying collective levels in deformed nuclei are described within the IBM-2 framework. This is done by a special choice of the Hamiltonian which allows a simultaneous fit of energies, E2 and M1 transitions. We have obtained a good description for ^{166}Er and have compared F-spin impurities and values of the parameter $g_V \chi_V$ which determines the B(M1) strengths for ^{128}Xe , ^{166}Er and ^{168}Er . F-spin impurities between 2 and 4 % have been found.

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