A Sensitive Method for Detecting Stable Fractional Charges on Matter

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ABSTRACT

We describe in detail the design of an apparatus which will allow direct detection of stable fractional elementary charges if present on matter at the level of 10^{-24} per nucleon. This method depends upon production of a highly uniform and parallel stream of conductive spheres which are charge analyzed by passage through a static electric deflecting field.

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Introduction

It is well-known that electrically conductive liquid spheres may be formed at rates exceeding 10^5 sec^{-1} by acoustically modulating the pressure in a streaming jet of the liquid.¹ A carefully constructed nozzle to emit this jet together with accurately maintained parameters on the fluid allows production of a stream of spheres which are equal in mass $\delta m/m \leq 10^{-3}$, equal in speed, $\delta v/v \leq 10^{-3}$, and highly parallel, $\delta v_{\text{perp}}/v \leq 10^{-6}$.^{2,3} By properly influencing the electrical potential at the point of the jet where the spheres become separated, the electrical charges may be set as desired for each sphere separately.⁴

In the absence of applied electric fields, each sphere would continue to fly along very nearly the same straight trajectory regardless of its electrical charge. If a static electric field is maintained perpendicular to the unperturbed trajectory, the spheres with various distinct charges will follow separate paths in the plane defined by the unperturbed trajectory and the electric field vector. The net deflection of a sphere is approximately $qV_o L^2/mv^2 b$ where q is the charge on the sphere, V_o is the potential between the deflecting electrodes, L is the length of the deflecting electrodes, m is the mass of the sphere, v characterizes the velocity of the sphere, and b characterizes the separation between the deflecting electrodes. If the ratio $V_o L^2/mv^2 b$ is large enough, spheres having different numbers of elementary charges will have easily distinguishable trajectories.

Description of Proposed Apparatus

We propose to construct an apparatus which will allow us to measure the charges on $10\mu m$ diameter mercury spheres with a relative accuracy of 0.02 elementary charges. The apparatus would be built along the lines of Figure 1, consisting of a liquid drop emitting jet which has the following specifications: $v \approx 500 cm/sec$, $\delta v_{parallel} \leq 3 cm/sec$, $\delta v_{perp} \leq 10^{-3} cm/sec$, $\delta m \leq 3 \times 10^{-12} gm$. The jet axis would be aligned with the Earth's gravitational field, directed downward. The deflecting electrodes would be straight rigid metal bars, placed symmetrically about the unperturbed trajectory. The electrodes would be non-parallel with the throat end near the droplet source and the wider, mouth, end away from the source. The throat would have a $0.3 cm$ gap, the mouth would have a $3.0 cm$ gap. The electrodes would be $5.0 cm$ wide and $300.0 cm$ long. A groove $0.1 cm$ wide would penetrate the deflecting electrodes in the plane of the trajectories. This groove would facilitate harmless disposal of highly charged spheres.

For the above set of parameters, we have computed an analytic approximation to estimate the trajectories of the spheres. This analytic function is the exact solution to the problem:

$$m \frac{d^2 \vec{x}}{dt^2} = q \vec{E} + m \vec{g}$$

$$\vec{E} = \left(\frac{V_o}{b + 2z \tan(\Psi)}, 0, 0 \right)$$

$$\vec{g} = (0, 0, 983 cm/sec^2)$$

For purposes of discussion, let us define our coordinate system to have the $\hat{z}$ direction along the unperturbed trajectory and the $\hat{x}$ direction along the static electric field; the trajectories all lie within the $\hat{x} \times \hat{z}$ plane. Figure 2 shows the deflection of a sphere with unit charge as a function of z . The deflection is strictly linear in the charge on the sphere so that a sphere with two elementary charges will have exactly twice the x coordinate as the indicated function. Because the variously charged spheres become separated by multiples of this basic deflection, $787\mu m$, we can see that our stated goal of measuring charges to an accuracy of $0.02e$ corresponds to determining the deflections at the end of the deflecting electrodes accurate to $15\mu m$. We thus require apparatus which can determine the positions of the spheres to this accuracy. Granted that such an apparatus may be assembled, we will show that the remainder of the design is sufficient to preserve this desired resolution.

Let Ψ be the half-opening-angle between the deflecting electrodes. Figure 3 shows the Ψ dependence of the x deflection at the end of the electrodes when all other parameters are held constant. We may see from Figure 3 that it would be reasonable to open the mouth of the plates wider than $3.0 cm$ if it proves convenient for other reasons to do so.

Now there are several effects which may alter the actual trajectories of the charged spheres from the analytic solution shown on Figure 2. These effects may be subdivided into five main classifications: 1.) Random influences on the formation and flight of the individual spheres. 2.) Aerodynamic and electrical interactions between spheres as they pass through the apparatus. 3.) Electrical interactions of the spheres with their image charges induced on the deflecting electrodes. 4.) Inaccuracy of the analytically solvable electric field configuration. 5.) Actual changes of charge on the spheres in flight. Each of these perturbing effects will be discussed separately:

Random Influences on the Formation and Flight of the Individual Spheres

Granted that the drop forming jet performs to specifications, we may easily compute the *R.M.S.* fluctuation in the deflection due to random influences on the formation of the spheres, $\delta x \leq \sqrt{(11.0)^2 + (4.2)^2 + (6.3)^2} \mu m = 13.4 \mu m$

If we were to consider the aerodynamic effects on the flight of the spheres in the standard atmospheric pressure, we would find that the spheres would stop before they had travelled the 300cm length of the deflecting electrodes. Clearly the apparatus must be run in a vacuum to keep aerodynamic perturbations down to a minimum. If the vacuum is sparse enough so that the mean free path of the residual gas molecules is much greater than the diameter of the spheres, we may neglect all cooperative effects in the atmospheric fluid flow problem: the problem becomes a simple matter of Newtonian fluid mechanics.

Recall that in Newtonian fluid mechanics, to compute the spectrum of impulses to the sphere requires merely to compute the impulses delivered to the sphere by the individual gas atoms and then integrate over the range of impact parameters and incident velocities which describe the sphere and the gas, respectively. Thus we need a microscopic model of the collisions of the gas molecules with the spheres. We consider here five separate models for this interaction: 1.) Gas molecules adhere inelastically to the spheres on impact. 2.) Gas molecules scatter specularly. 3.) Gas molecules scatter elastically but with randomized directions. 4.) Gas molecules are promptly re-emitted but are thermalized with respect to the sphere. 5.) Gas molecules adhere inelastically, but for each absorbed molecule, another molecule is evaporated, that is, it is randomly emitted from the surface of the sphere in thermal equilibrium.

Each of the five preceding microscopic models were implemented into a numerical computation of the resulting spectrum of impulses. The computation was, in each case, exact up to numerical quadrature errors. No approximations were made regarding the speed of the sphere relative to the speed of sound in the gas, *etc.* The results of these computations are shown in Figures 4-8. The important conclusion that we may draw from Figures 4-8 is that the distribution of impulses felt by the sphere is sharply cut off so that the Central Limit Theorem allows

us to compute the distribution function for summed contributions due to many gas molecule collisions with the sphere. The formula which governs the size of the random fluctuations in deflection due to aerodynamic effects may be stated:

$$\delta x = N^{1/2} \frac{L}{\text{collisions}} \frac{\delta p_{\text{perp}}}{\rho_{\text{sphere}}}$$

We estimate $\delta v_{\text{perp}}/\rho_{\text{sphere}}$ from the worst case of the five Newtonian fluid mechanics impulse spectra and we obtain the result $\delta x \leq 0.7 \mu m$ for effects due to aerodynamics.

To perform the preceding computation, we incidentally computed the net drag due to aerodynamics and the random variability in this drag. These results are shown in Figures 9-13. On the average, about 9×10^{10} gas molecules collide with each sphere as it travels through the deflecting electrodes. The total slowing of the spheres will obey $\delta v_{\text{parallel}} \leq 4.0 \text{ cm/sec}$ and, of course, the variability in this drag is about 5 orders of magnitude smaller. The net change in deflection due to aerodynamic drag is less than $1 \mu m$.

Aerodynamic and Electrical Interactions between Spheres as they Pass through the Apparatus

Aerodynamic interactions between the spheres in flight are caused by neighboring spheres obstructing the right-of-way for gas molecules which otherwise would have collided with the sphere of interest. Such aerodynamic effects cannot be of magnitudes greater than the total deflections due to aerodynamic effects on solitary flight of the spheres. We may, therefore, neglect these potentially troublesome aerodynamic interactions between spheres.

We may not neglect the effects due to electrical interactions between the spheres in flight. These electrical interactions may be subdivided into four classifications: 1.) Interaction between the net charge on the neighboring sphere with the net charge on the sphere of interest. 2.) Interaction of the electrical dipole moment on the neighboring sphere with the net charge on the sphere of interest. 3.) Charge/dipole moment interaction. 4.) Dipole/dipole interaction. For the design parameters of interest here, the dipole/dipole interactions result in the greatest perturbations.

We have computed the perturbing electric potential, its gradient, dyadic and third rank differential tensor due to the presence of a neighboring sphere with arbitrary charge and with electric dipole moment as induced by the applied electric field. These quantities are computed, correctly taking into account boundary conditions due to the presence of the equipotential surfaces (the deflecting electrodes modelled as infinite, flat planes inclined at the correct angles to one another.) Since we expect to be dealing with spheres with less than 100 elementary charges at all times, we have taken the conservative expedient of setting the charge on the neighboring

sphere to be $100e$. The results of this computation will be presented in terms of the minimum allowable distance between consecutive spheres. Our criterion of acceptability is that the electrical force remain smaller than 10^{-2} of the force on an elementary charge at the location of the sphere of interest. This severe restriction must be maintained for all positions of the sphere of interest on its flight through the apparatus for acceptability to be certified. The results of this computation are shown in Figure 14. This Figure is the worst case computed for inter-sphere separation of $d=0.065cm$. In conclusion, there will be no significant interaction between the spheres in the apparatus so long as the pressure is maintained at $0.185\mu m\ Hg$, the vapor pressure of mercury at $0^{\circ}C$ and that the inter-sphere separation exceeds $0.065cm$.

The question of how to set the inter-sphere separation as a matter of practice now raises itself. We intend to make use of the ability to set the electrical charges separately for each sphere. If the stream of spheres emerging from the nozzle is separated, say, by $25\mu m$, then we must dispose of 25 spheres for each one sphere which is actually used. The spheres to be disposed of must, of course, be highly charged so that they pass quickly out of the deflecting electric field. Provided that the spheres to be disposed of are each given very nearly identical charges, their net deflection of any sphere of interest will remain a *non-random* function of the charge on the sphere of interest. Thus, it would appear simple enough to cope with this potentially troublesome obstacle.

Electrical Interactions of the Spheres with their Image Charges Induced on the Deflecting Electrodes

This effect is not as significant as those treated in the two preceding sections of this report because it produces results which are not random. The net effect of self-interaction is slightly to alter the otherwise strictly linear relation between deflection and charge. The magnitude of the self-interaction is approximately similar to the magnitude of the electrical interactions between neighboring spheres and it is, therefore, quite negligible. The self-interaction is included in the final computation of the trajectories which will be described below.

Inaccuracy of the Analytically Solvable Electric Field Configuration

This effect also is relatively unimportant because it also is not random. This effect is even smaller than the previously described effect and it, too, is included in the final computation to be described below.

Actual Changes of Charge on the Spheres in Flight

Charge changing reactions may be subdivided into three classifications: 1.) Interactions with residual gas molecules. 2.) Interactions with field emission electrons and secondary charged particles from the deflecting electrode surfaces. 3.) Cosmic-ray reactions. Provided the distribution is similar to thermal in the residual gas, there will be no need to consider any significant population of ionized atoms. The ionization potential of mercury atoms is 10.47 eV. Thus 1.) may be considered to apply only to collisions in which a neutral mercury atom in the gas somehow becomes ionized in collision with a sphere. Again, we must expect the large barrier of the ionization potential of mercury greatly to inhibit this process. Classification 2.) is a potential difficulty to successful operation of the envisioned apparatus. We hope to avert charge changes due to emissions from the surfaces by keeping the electric fields low enough. The field at the neck of the deflecting electrodes will be about 10 kV/cm. By comparison, field emissions begin at about 300 kV/cm for most listed materials. Field emission rates depend very steeply upon the magnitude of the applied field intensities so that we have good reason to expect that our design is comfortably conservative with respect to 2.). We may place strict upper bounds upon the importance of 3.) in changing the charges. The fraction of spheres which encounter charges liberated in the residual gas by cosmic rays is quite low, computed by standard means it is seen to be less than 10^{-12} . The fraction of spheres from which charges are liberated by direct impact of cosmic rays may be written:

$$P_{direct} = \frac{2\pi^2 a^3 L \Phi N_e \alpha^2 (\hbar c)^2}{m_e c^2 E_0 v}$$

Where a is the radius of the sphere, Φ is the cosmic ray flux, N_e is the number density of electrons in the sphere, m_e is the mass of the electron, E_0 is an energy which characterizes knock-on electrons which escape the sphere, and v is the speed of the sphere. Substituting the design parameters of the apparatus, we find the direct cosmic ray background of charge changes to be less than 10^{-10} . We conclude that charge changing interactions will occur below the rate of 10^{-10} of the spheres measured. Even granted this very low background rate, we would wish to install into the apparatus more than the single position sensing detector which would be needed in the absence of background. Three position sensitive detectors should assure us of a considerable reduction in charge changing events when we require electronically that each acceptable event should have signatures in all three detectors consistent with there having been no change of charge in flight.

Results from the Final Trajectory Computation: Conclusions

We have created a program to compute the summed effects of all of the above perturbations to the analytically estimated trajectories. Each of these perturbations was computed by assuming that the sphere of interest follows close enough to its unperturbed trajectory so that negligible error is induced by computing the perturbing forces at positions along the unperturbed trajectory. The largest deflection from the analytic solution which we have computed is less than $8 \mu m$. The small magnitude of this perturbation verifies the computational simplification and also allows us to claim that the charge resolution of the apparatus will, indeed, meet the design criterion of $0.02e$.

Mass Flow Rate: Ultimate Sensitivity

The rate at which mass is examined by this apparatus is, according to design, $> 5 \times 10^{-5} gm/sec = 3 \times 10^{19} Nucleons/sec$. This corresponds to one sphere every $1.3 \times 10^{-4} sec$. In order to be able to place a limit of better than 10^{-24} fractional charges per nucleon, we will need to operate the apparatus for $10^5 sec$ for a confidence level of 95%.

We close by considering the relation of the presently described experiment upon the fractional charge claims by the Stanford Low Temperature Laboratory.^{5,6} They claim 6 separate observed fractional charges after examining 9 niobium spheres. Their spheres weigh, in total, $7 \times 10^{-4} gm$. There is room for grave doubt as to the validity of their claim, however, due to their questionable statistical analysis together with the presence of various unexplained spurious effects and non-repeatabilities in their experiment. If their claim is actually correct, then we should be able to detect 2400 fractionally charged spheres during our $10^5 sec$ of running. This is to be compared with the expected background of about 0.1. The statistical significance of our proposed experiment should be, therefore, amply sufficient to refute the Stanford claim if it is, indeed, incorrect; on the other hand, we should be able to provide unquestionable evidence of the effect if their claim is correct.

Figure Captions

1. **Perspective View of Proposed Apparatus.** A.) Nozzle emitting mercury spheres. B.) Deflecting electrodes. C.) Imaginary plane bisecting the nozzle and the deflecting electrodes. All trajectories lie within this plane. D.) Trajectory of highly charged spheres to be disposed of so as to increase the inter-sphere separation. E.) Position sensing devices. F.) Slots in deflecting electrodes. G.) Direction of gravitational acceleration.
2. **Analytic Trajectory of Sphere with Unit Charge.** The largest value of z coordinate corresponds to the location of the mouth end of the deflecting electrodes. The deflection of a unit charged sphere is, thus, $787\mu m$.
3. **Dependence of Net Deflection upon Opening Half-angle between Deflecting Electrodes.**
4. **Spectrum of Impulses Delivered Perpendicular to the Trajectory of a Sphere by the Residual Gas Atoms.** The horizontal axis is labelled in arbitrary momentum units with a shifted zero. The numerical entries at bottom labelled "mean" and "sigma" are in proper dimensionless units, $\sqrt{2m_{Hg}kT}$. The vertical axis records the number of pseudo-events which produce momentum transfers lying within the appropriate bin; bins marked with half of an event actually have zero occupants. This figure applies for microscopic model 1.) as defined in the text.
5. Same as Fig.4 except for model 2.)
6. Same as Fig.4 except for model 3.)
7. Same as Fig.4 except for model 4.)
8. Same as Fig.4 except for model 5.)
9. **Spectrum of Impulses Delivered Parallel to the Trajectory of a Sphere by the Residual Gas Atoms.** Same units as Fig.4. This figure applies to model 1.)
10. Same as Fig.9 except for model 2.)
11. Same as Fig.9 except for model 3.)
12. Same as Fig.9 except for model 4.)
13. Same as Fig.9 except for model 5.)
14. **Worst Case of Perturbation Due to Electrical Forces from Neighboring Sphere.**

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Fig. 1

JNPERTURBED TRAJECTORY

09.46.59 19 JUN 79

NO. 1

Fig. 2

REFLECTION MAXIMUM FOR 1000 MOLECULES, TYPE 5

11.30.44 10 JUN 79

NO. 10

Fig. 8

[illegible]

THIS HAS BEEN
NOTHER T.D.C.
SPF'DY PLOT
ELAPSED TIME=
8.059 SEC

601

07.98

60.00

45.00

25.22

•

STATIONARY STATE

$$V = \frac{1}{2} \rho \int_{-L}^L \dot{y}^2 dx = \frac{1}{2} \rho \int_{-L}^L \left(\frac{\partial y}{\partial t} \right)^2 dx$$

N
U
M
B
E
R

O
F

I
N
V
E
N
T
S

D
E
P

B
I
T
S

1000.

398.1

158.5

63.10

25.12

10.00

3.981

1.585

.6310

.2512

.1000E+00

0.00

 20.00 40.00 60.00 80.00 100.00
MOMENTUM IN ARBITRARY SHIFTED UNITS

X-AXIS MARGINAL

MEAN= 5.05E+01

SIGMA= 2.89E+01

TOTAL= 5.05E+03

YMIN= 1.00E+00

YMAX= 1.00E+02

Y-AXIS MARGINAL

MEAN= 1.00E+02

SIGMA= 2.40E+02

TOTAL= 1.00E+04

YMIN= 5.00E 01

YMAX= 9.93E+02

AS SCATTER PLOT

XBAR= 5.07E+03

YBAR= 3.03E-04

AS DISTRIBUTION

Y(X)

XBAR= 5.05E+01

YBAR= 2.97E+03

SIGMA= 4.55E+00

NORM= 1.00E+04

X(Y)

YBAR= 1.02E+02

YBAR= 6.76E+04

SIGMA= 2.43E+02

NORM= 5.05E+03

THIS HAS BEEN

ANOTHER T.O.P.

SPEEDY PLOT

ELAPSED TIME=

7.070 SEC

Fig. 10

$x = 117$ $2.16'' = 0.2$ $\Sigma TGM =$ $4.12'' = 0.1$

THIS HAS BEEN
ANOTHER T.D.C.
SPEEDY PLOT
ELAPSED TIME=
7.924 SEC


```

*****
.1000E-01 *****
M +
A +
X +
I +
M .6310E-02 +
U +
N +
F +
O .3981E-02 +
R +
C +
E +
I .2512E-02 +
N +
U +
N +
I .1585E-02 +
T +
S +
O +
F .1060E-02 +
E +
L +
E .6310E-03 +
N +
T +
A +
R .3981E-03 +
Y +
F +
O +
R .2512E-03 +
C +
E +
.1585E-03 +
.1000E-03 +
.560E-02 .2700E-02 .3000E-03 .5000E-03 .2700E-02 .4500E-02
*****
MAXIMUM CHARGE= 4.50E-16 R POSITION= 3.20E+03 SEPARATION= 6.50E-02 DEFLECTING POTENTIAL= 1.70E+02
NUMBER OF SECTORS= 700 POSITION VECTOR= 3.00E+00 J. 0.
***** SEPARATION BETWEEN PLATES IS 2.69E-12

```

X-AXIS MARGINAL

MEAN= 7.40E-18

SIGMA= 2.53E-03

TOTAL= 2.22E-16

XMIN= 4.21E-03

XMAX= 4.20E-03

Y-AXIS MARGINAL

MEAN= 4.76E-03

SIGMA= 2.11E-03

TOTAL= 1.43E-01

YMIN= 7.32E-04

YMAX= 7.23E-03

AS SCATTER PLOT

XVBAR= 1.09E-11

XVCOR= 2.86E-06

AS DISTRIBUTION

V(X)

XBAR= 2.29E-08

X2BAR= 3.80E-06

SIGMA= 1.95E-03

NORM= 1.43E-01

X(Y)

YBAR= 3.97E-03

Y2BAR= 1.46E-05

SIGMA= 1.98E-03

NORM= 6.51E-02

THIS HAS BEEN

ANOTHER T.P.C.

SPEEDY PLOT

ELAPSED TIME=

6.181 SEC