LA-UR -79-2368

TITLE: PHYSICAL CONDITIONS IN REVERSED FIELD PINCHES
AND THETA PINCHES AS INFERRED FROM SIMULATIONS

AUTHOR(S): Anthony Sgro, LASL, CTR-6

SUBMITTED TO: International School of Plasma Physics
Varennna, Italy - August 27-September 8,
1979.

MASTER

NOTICE
This report was prepared as an account of work
sponsored by the United States Government. Neither the
United States nor the United States Department of
Energy, nor any of their employees, nor any of their
contractors, subcontractors, or their employees, makes
any warranty, express or implied, or assumes any legal
liability or responsibility for the accuracy, completeness,
or usefulness of any information, apparatus, product, or
process disclosed, or represents that its use would not
infringe privately owned rights.

University of California

By acceptance of this article, the publisher recognizes that the
U.S. Government retains a nonexclusive, royalty-free license
to publish or reproduce the published form of this contribu-
tion, or to allow others to do so, for U.S. Government pur-
poses.

The Los Alamos Scientific Laboratory requests that the pub-
lisher identify this article as work performed under the aus-
pices of the U.S. Department of Energy.

LOS ALAMOS SCIENTIFIC LABORATORY

Post Office Box 1683 Los Alamos, New Mexico 87545

An Affirmative Action/Equal Opportunity Employer

PHYSICAL CONDITIONS IN REVERSED FIELD PINCHES
AND THETA PINCHES AS INFERRED FROM SIMULATIONS

Anthony Sgro

Los Alamos Scientific Laboratory
Los Alamos, New Mexico 87545 USA

ABSTRACT

The nature of the transport in certain pinches is investigated by comparing experimental data with computational predictions of the AURORA code. It is shown that anomalous resistivity and anomalous thermal conduction are present during the implosion phase of the ZT-S reversed field pinch, and that as equilibrium is approached the anomalous contribution to the transport coefficients decrease. Evidence is presented indicating that a low density wall emitted plasma occupies the region between the main plasma column and the wall. Simulations of a θ -pinch show that anomalous heating of the ions due to microturbulent fields makes a noticeable contribution to the ion energetics.

INTRODUCTION

Collisionless phenomena occur in many contexts ranging from astrophysical plasmas to plasmas of interest to controlled fusion research. In the latter context, the implosive formation of pinch equilibria is a topic of current interest. If the implosion time is less than or comparable to the plasma selfcollision time then the initial shock and subsequent approach to equilibrium is best described by a kinetic model. Since such a model must also account for magnetic field diffusion and is thus rather complicated, an accurate analytical treatment is difficult and the problem is usually treated numerically.

Anthony Sgro

Simulations which assume both kinetic ions and kinetic electrons are not feasible when studying macroscopic phenomena because the short length scales and time scales associate with the Debye length λ_D and the electron plasma frequency ω_{pe} make the calculation too costly. However, such "full particle" simulations have produced significant results concerning microturbulent processes in plasmas and the resulting anomalous transport coefficients. On the other hand, hybrid simulations, which assume kinetic ions and inertialess charge neutralizing fluid electrons, eliminate the short scales, and thus macroscopic simulations are feasible. In this case, the influence of the electron microdynamics on the macroscopic plasma evolution appears via "anomalous" bulk transport coefficients. By comparing the results of such simulations with experiments, information concerning these transport coefficients and other properties of the plasma may be inferred.

The one-dimensional hybrid code AURORA (Baker and coworkers, 1976; Baker and coworkers, 1978; Bengtson and coworkers, 1977) has been used to investigate conditions in several pinch plasmas, including the ZT-S reversed field pinch (Chodura, 1975; Liewer and Krall, 1973). This paper will describe the code and some interesting conclusions inferred from its use.

PHYSICAL MODEL

We assume a straight pinch having cylindrical symmetry, and assume that all variables are a function only of r , the distance from the z axis.

The ions move in the four-dimensional phase space (w_r, v_θ, v_z, r) . The Vlasov equation is solved by the particle-in-cell technique (Nielson and Lindman, 1978). The equation of motion of each ion,

$$m_i \frac{dw}{dt} = eE_r + \left(\frac{w \times B}{c} \right) - \underline{P} + H_r \quad (1)$$

is solved by a second order Runge-Kutta scheme. Here, $\underline{P}$ is the (anomalous) momentum transferred from the ions to the electrons per ion per unit time due to

PHYSICAL CONDITIONS IN REVERSED FIELD PINCHES
AND THETA PINCHES AS INFERRED FROM SIMULATIONS

collisions, and is given by $\underline{P} = e\eta \cdot \underline{J}$, where $\eta (= \underline{\sigma}^{-1})$ is the (anomalous) resistivity tensor. H is the anomalous ion heating term. The ions are initially distributed in phase space according to a Maxwellian at the initial temperature and are subsequently accelerated by the imploding field. We thus know the ion distribution function $f(r, w, t)$, and by averaging, we find

$$n_i = \int f(r, w, t) d^3w, \quad v_{ir} = (1/n_i) \int w f(r, w, t) d^3w.$$

Charge neutrality then implies $n_e = n_i$, $v_{er} = v_{ir}$.

The effects of incomplete pre-ionization are included by randomly tagging the initial Vlasov particles as ions or neutrals depending on the pre-ionization level. Then, given n_e and T_e , the ionization rate and thus the number of neutrals ionized per cell per timestep may be calculated. This number of neutrals is selected at random and are henceforth tagged as ions.

The electrons are treated as a massless fluid. Since n_e and v_{er} are determined by the ion motion, the electron continuity equation is redundant. The electron momentum equation is

$$m_e \frac{dv_{er}}{dt} = 0 = -en_e \left(\underline{E} + \frac{\underline{v}_e \times \underline{B}}{c} \right) - \text{grad } p_e - n_e \underline{P}. \quad (2)$$

Let $\sigma_{\parallel}$, $\sigma_{\perp}$ be the electrical conductivity parallel to and perpendicular to B , respectively, and ϕ be the angle between the z axis and B . Further, let

$$\sigma_2 = \sigma_{\parallel} \sin^2 \phi + \sigma_{\perp} \cos^2 \phi,$$

$$\sigma_3 = \sigma_{\parallel} \cos^2 \phi + \sigma_{\perp} \sin^2 \phi,$$

$$\sigma_x = (\sigma_{\parallel} - \sigma_{\perp}) \sin \phi \cos \phi.$$

Anthony Sgro

From symmetry, $J_r = B_r = 0$. On introducing, the vector potential $\underline{A}$, the θ and z components of the electron momentum equation become

$$\begin{aligned} \frac{\partial^2 A_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial A_\theta}{\partial r} - \frac{1}{r^2} A_\theta &= \alpha_2 (\dot{A}_\theta + v_r \frac{\partial A_\theta}{\partial r} + \frac{v_r}{r} A_\theta) \\ &+ \alpha_x (\dot{A}_x + v_r \frac{\partial A_z}{\partial r}), \end{aligned} \quad (4a)$$

$$\frac{\partial^2 A_z}{\partial r^2} + \frac{1}{r} \frac{\partial A_z}{\partial r} = \alpha_x (\dot{A}_\theta + v_r \frac{\partial A_\theta}{\partial r} + \frac{v_r}{r} A_\theta) + \alpha_3 (\dot{A}_z + v_r \frac{\partial A_z}{\partial r}),$$

$$\alpha_2 = 4\pi\sigma_2/c^2, \alpha_3 = 4\pi\sigma_3 = 4\pi\sigma_2/c^2, \alpha_x = 4\pi\sigma_x/c^2. \quad (4a)$$

These equations are solved by a fully implicit space-centered finite-difference scheme.

The electron energy equation may be written as

$$\left(\frac{\partial}{\partial t} + v_r \frac{\partial}{\partial r}\right)s = -\frac{\gamma R}{1} + \frac{(\gamma-1)\dot{Q}}{p_e} \quad (5)$$

where

$$s = \ln(p_e n^{-\gamma}),$$

$\gamma = 5/3$, R is the ionization rate, $\dot{Q}$ is the electron heating rate and p_e is the electron pressure.

$\dot{Q}$ may be expressed as

PHYSICAL CONDITIONS IN REVERSED FIELD PINCHES
AND THETA PINCHES AS INFERRED FROM SIMULATIONS

$$\dot{Q} = (\eta_H J_H^2 + \eta_L J_L^2 - n H_r) - R I_H + \frac{1}{r} \frac{\partial}{\partial r} (r \kappa_{\perp} \frac{\partial T_e}{\partial r}) + \sum_x n_e n_x \epsilon_x(T_e)$$

when, R is the ionization rate, $I_H = 13.6$ eV, $\kappa_{\perp}$ is the cross field thermal conductivity and $n_e n_x \epsilon_x(T_e)$ is the impurity radiation rate due to atom x .

Finally, the r component of the electron momentum equation yields, using $J_r = 0$,

$$E_r = \frac{1}{c} (V_{ez} B_{\theta} - V_{e\theta} B_z) - \frac{1}{ne} \frac{\partial p_e}{\partial r}. \quad (6)$$

This is the radial electric field required for quasi-neutrality.

The low density region between the pinched plasma and the wall requires special consideration. The simplest assumption is that the density of this region is so low that the fields are vacuum fields. This is easily done by setting $\vec{g} = \vec{n}^{-1} = 0$. However, it is not the only possible assumption.

During or after the implosion a low density plasma may be emitted from the wall. By making some simplifying assumptions, the principle effects of this emission can be approximated. It is assumed that a mass of plasma equal to 10% of the mass initially (at $t=0$) present in the wall is emitted during the course of the calculation. The plasma is assumed to leave the wall with the velocity $c \underline{E} \times \underline{B} / B^2$, which is in the radial direction. For times in which $\underline{E} \times \underline{B}$ pointed outward, this velocity is set to zero. This plasma is transported inward by assuming it also is described by Eqs. 1 to 7 and the Vlasov equation. This procedure determines v_r , which appears in the convection terms of the equations, i.e., $v_r B_z / c$, $v_r B_{\theta} / c$, and $v_r \partial s / \partial r$.

TRANSPORT COEFFICIENTS

Due to the steep gradients and high velocities which drive microturbulent fluctuations that are present during the implosion phase, the transport is anomalous. Liewer and Krall, using the known properties of various crossfield

Anthony Sgro

instabilities, have derived an anomalous resistivity algorithm which is applicable to θ pinches and is included in the code. When one considers a θ pinch only $\eta_{\perp}$ is required in the field diffusion calculation. However, in an RFP, $\eta_{\parallel}$ is also involved. Thus, while an ultimate goal would be an anomalous resistivity for an RFP similar to the one for the θ pinch, little theoretical information is currently available about anomalous parallel resistivity. Thus for an RFP we have taken a semi-empirical approach similar to that of Chodura (Sgro and Nielson, 1976). Chodura has found that the resistivity algorithm $\nu = c\omega_{pi}[1 - \exp(-|V_d|/fV_g)]$, $\eta_{\perp} = (m_e\nu/ne^2)$, where $c = 0.5$, $f = 5.0$, and $V_g = (\gamma kT_e/m_i)^{1/2}$ satisfactorily describes the behavior of the Garching θ pinch. We have generalized this algorithm as follows:

$$\nu_{\perp, \parallel} = c\omega_{pi}[1 - \exp(-|V_{d\perp, \parallel}|/f_3V_g)],$$

$$(\nu_{\perp, \parallel} = \max(\nu_{\perp, \parallel a}, \eta_{\perp, \parallel c}),$$

$$\eta = meV/ne^2, \quad (7)$$

where η_{cl} is the classical collision frequency and $c_{\parallel}$ and $c_{\perp}$ will be discussed below. For classical Coulomb collisions the collision frequency which defines the electrical resistivity also defines the thermal conductivity. In the following this is assumed to be a general property, so that the anomalous crossfield thermal conductivity is

$$\kappa_{\perp} = \frac{nk^2T_e}{m_eV_j} \frac{3.2}{1 + 0.8 \frac{\omega_{ci}^2}{V_j^2}}$$

Microturbulence also causes anomalous electron and ion heating. The algorithm of Liewer and Krall predicts these individual heating rates, while in the quasi-empirical model the fraction α of the resistivity heating going to the ions may be arbitrarily specified.

/

PHYSICAL CONDITIONS IN REVERSED FIELD PINCHES
AND THETA PINCHES AS INFERRED FROM SIMULATIONS
ANOMALOUS TRANSPORT IN THE ZT-S REVERSED FIELD PINCH

If T_e is low enough, v_{ci} will be high enough to account for the observed diffusion. However, a low T_e is inconsistent with the fact that the plasma is Joule heated as the fields diffuse. Starting from the preionized plasma, the pinch evolves in the following manner. As the field at the wall rises a current sheath is formed near the outer boundary of the plasma and the plasma pinches. The temperature in the sheath region rises due to Joule heating and v_{ci} drops. The result is that the current sheath remains thin and most of the field is frozen out of the inner region of the pinched plasma, in contradiction to the experimental results. This is illustrated in Fig. 1. Note that the experimental field is fully diffused, with a sheath thickness of over 5 cm, while the computed field has a narrow sheath which is less than 2-cm thick. Note also that T_e is largest in the sheath region.

Previous work (Bengtson and coworkers, 1977; Sgro and Nielson, 1976; Sgro, 1978) has shown that the field diffusion during the implosion phase of many pinches is adequately described by Eq. (4) with $C = 1$. This is also true for the implosion phase of ZT-S. Figure 2a and b compare the calculated and measured fields during the implosion and early postimplosion phase. Except for the width of the calculated Z field profile being somewhat too narrow, the agreement is good. In the late postimplosion phase, however, the B_z field near the axis collapses to less than 1 kG (compared to the experimental value of over 2 kG), as illustrated in Fig. 2c. The most obvious interpretation is that the resistivity formula which describes the implosion yields values which are too large in the late postimplosion phase. A drop in anomalous resistivity at late times is expected, since the high velocities and steep gradients driving the turbulence during the implosion decrease afterward. In particular C has been set to 0.3 after the first peak compression (at 2 μ s) and to 0.1 after the third compression (at 5 μ s). The results of a calculation with this reduced resistivity is plotted with the dot-dashed lines in Fig. 2. In this case, the calculated B_z profile is more nearly in agreement with experiment.

The calculations described above include no electron energy losses. Consequently, the calculated T_e was too high. In Fig. 3a the T_e profiles of two

Fig. 1. Profiles of B_θ and T_e . Curves: simulations assuming classical transports; x and I, measurements.

Fig. 2. Profiles of B_θ and B_z at various times. Solid curves simulations with $c = 1$ at all times; dotted curves; c reduced at late times; x and o: data.

calculations at 12.5 μs are plotted, while the experimental temperatures at 0 cm, 1.5 cm, and 4.5 cm are plotted in Fig. 3a as vertical bars.

PHYSICAL CONDITIONS IN REVERSED FIELD PINCHES
AND THETA PINCHES AS INFERRED FROM SIMULATIONS

If no energy losses are included in the calculation, T_e increases secularly with time due to Joule heating. The dot dashed curve in Fig. 3a represents such a calculation with the reduced resistivity and vacuum fields near the wall. In the calculation $T_e > 100$ eV everywhere, while the data imply $T_e \lesssim 25$ eV everywhere. The dashed curve in Fig. 3a represents a calculation that includes impurity radiation, (0.7% oxygen) anomalous cross field thermal conduction and electron ion equilibration, and assumes a wall emitted plasma in the outer region. In this case, there is good agreement between the calculations and the inner three experimental points, and poorer agreement with the outer point. From Fig. 3b it may be seen that the density at the radius of the outer temperature measurement is quite low. Thus, it may be that the thermal balance assumed in the model is inaccurate in the low density region. Alternately, since the density is so low the measurement has more uncertainty at this radius and the actual error bar might be longer than indicated in the figure. The average power loss over the 15 μ s duration of the calculation is 5.2×10^{11} erg $\text{cm}^{-1} \text{s}^{-1}$. As compared with the measured average power loss over the 30 μ s stable discharge time of 5 to 8×10^{11} erg $\text{cm}^{-1} \text{s}^{-1}$.

No single loss mechanism is able to reduce the computed temperature to the experimental value at all experimental points. Radiation and equilibration are most effective in the highest density region, while the effect of conduction is most noticeable in the low density region near the outer boundary, when the temperature gradients tend to be steepest.

In Fig. 4 field profiles from a calculation in which radiation, equilibration, and anomalous thermal conduction are included are compared with experimental data. The solid and dashed curves refer the assumption of vacuum or wall emitted plasma near the wall. Either assumption about this region yields profiles which sensibly describe the diffusion of the θ field to the axis and the overall behavior of the Z field. However, if vacuum fields are assumed near the wall, the resulting $1/r$ dependence of B_θ in this region causes B_θ to be noticeably higher than the measurements at certain radii and times. Furthermore, the width of the Z profile agrees more favorably with experiment at later times when wall emitted plasma is assumed to occupy the outer region. Thus, the wall emitted plasma assumption yields marginally better results for this particular ZT-S shot. However, in some other shots, the measured B_θ fields do vary approximately a $1/r$

Anthony Sgro

Fig. 3. a) Profiles of T_e vs. r . dashed dot: simulations with no electron energy losses; dashed: includes energy losses; I: data. b) a typical simulation profile of n vs. r .

Fig. 4. Profiles of B_θ and B_z at various times. Solid curves: vacuum near the wall; dotted curves: low density plasma near the wall; x, 0; data.

near the wall. Thus, the question of whether, in ZT-S, this region contains vacuum fields or a wall emitted plasma with non-negligible current is variable on a shot-to-shot basis. This issue must be treated as a "boundary condition," in the sense that the situation is inferred from experiment and the appropriate conditions are included in the calculation. In the end, the issues are wall

PHYSICAL CONDITIONS IN REVERSED FIELD PINCHES
AND THETA PINCHES AS INFERRED FROM SIMULATIONS

physics and wall contamination, which are not addressed by the computational model.

The reduction of ϵ and the reduction of gradients in the fields as time progresses implies that the resistivity used in the calculations approaches the classical value. This is confirmed in Fig. 5, in which $\eta_{\perp}/\eta_{cl}$ and $\eta_{\parallel}/\eta_{cl}$ from the calculation in Fig. 4 with the wall emitted plasma are plotted at an early time and a late time. At $5 \mu s$ the resistivity is as much as five times classical in the plasma column ($r < 3.8 \text{ cm}$) and 100 times classical in the wall emitted halo, while at $12.5 \mu s$ the resistivity is essentially classical in the plasma column and 10 times classical in the halo. In this figure η_{cl} is the classical resistivity along the field, so that the classical value for $\eta_{\perp}/\eta_{cl}$ is ~ 2 .

ANOMALOUS ION HEATING IN THE SCYLLA IV-P θ PINCH

The Scylla IV-P θ pinch is a high-voltage experiment in which T_i reaches the keV range. One of the interesting diagnostics on this experiment is a measurement of the neutron emission as a function of time. Since AUROPA calculates the exact

Fig. 5. Ratio of perpendicular and parallel resistivity to the classical value at two times.

Anthony Sgro

ion distribution function, a comparison with the experiment can be made without assuming the ions are instantaneously Maxwellized and thus have a high energy tail. We point out that the ion selfcollision time of 5 μ s is greater than the entire simulation time of 4.5 μ s considered below. The anomalous transport algorithm of Liewer and Krall, appropriate to θ pinches, (Park, 1978) is used in the calculation. This algorithm predicts the anomalous ion and electron heating rates.

DISCUSSION AND CONCLUSIONS

One conclusion inferred from this study concerns the nature of the resistivity required to match the experimental field behavior of ZT-S. Calculations including only classical transport yield field profiles which do not agree with the experimental data of ZT-S. Thus, an anomalous resistivity is present. In order to match the later time field behavior, this anomalous resistivity must be reduced as time progresses, presumably because the gradients and drift velocities driving the turbulent fluctuations decrease. This presents the interesting possibility that in future machines, such as ZT-40, which will hopefully confine the plasma for much longer times, the transport may approach its classical value after a long enough time.

The two dimensional toroidal calculations of Park and Chu (Liewer and Krall, 1973; Sgro, 1978), also demonstrated that the anomalous contribution to the transport coefficients decreased with time and that these coefficients approached their classical values as equilibrium is approached. Another aspect of anomalous transport, namely anomalous ion heating, is illustrated in the simulations of Scylla IV-P. In these simulations, this heating mechanism accounts for a shift of the peak in the neutron emission rate curve relative to the maximum of the applied field.

An interesting illustration of the presence of anomalous ion heating is the following. In the experiment, the peak of the neutron emission rate occurred 0.7 μ s after the maximum in the applied field. This behavior also occurred in the calculations and was traced to the fact that the peak of the anomalous ion heating rate was delayed by the same time. In this experiment, the field

PHYSICAL CONDITIONS IN REVERSED FIELD PINCHES
AND THETA PINCHES AS INFERRED FROM SIMULATIONS

gradients were still relatively strong at these times, so that the reduction in the anomalous transport had not yet occurred.

REFERENCES

- D. A. Baker, L. C. Burkhardt, J. N. DiMarco, A. Haberstich, R. L. Hagenson, R. B. Howell, H. J. Karr, and A. E. Schofield (1976). Current Temperature and Confinement Time Scaling in Toroidal Reversed Field Pinch Experiments ZT-1 and ZT-S, Plasma Physics and Controlled Nuclear Fusion Research.
- D. A. Baker, C. J. Luchenauer, L. C. Burkhardt, C. K. Chu, J. N. DiMarco, J. N. Downing, A. Haberstich, R. B. Howell, A. R. Jacobson, H. J. Karr, P. C. Liewer, E. M. Little, M. D. Machalek, W. Park, A. E. Schofield, A. G. Sgro, K. S. Thomas, and P. C. T. Van der Laan (1978). LASL Toroidal Reversed Field Pinch Program, Plasma Physics and Controlled Nuclear Fusion Research, Innsbruck (International Atomic Energy Agency, Vienna) to be published.
- R. H. Bengtson, S. A. Eckstrand, A. G. Sgro, and C. W. Nielson (1977). Plasma Rotation During Implosion in a ζ Pinch, Phys. Rev. **39**, 889.
- R. Chodura (1975). A Hybrid Fluid-Particle Model of Ion Heating in High Mach Number Shocks, Nucl. Fusion **15**, 35.
- P. C. Liewer and N. A. Krall (1973). Self Consistent Approach to Anomalous Resistivity Applied to Theta Pinch Experiments, Phys. Fluids **16**, 953.
- C. W. Nielson and E. L. Lindman (1973). Proceedings of the Sixth Conference on Numerical Simulation of Plasmas, Lawrence Berkeley Laboratory Conference Report 73-0710 (Lawrence Berkeley Laboratory, Berkeley California).
- W. Park (1978). Dissertation (Columbia University, New York.)
- A. G. Sgro and C. W. Nielson (1976). Hybrid Model Studies of Ion Dynamics and Magnetic Field Diffusion During Pinch Implosions, Phys. Fluids **19**, 126.
- A. G. Sgro (1978). Calculations of Incomplete Preionization in High Voltage Theta Pinches, Phys. Fluids **21**, 1510.