Fermi National Accelerator Laboratory

FERMILAB-Pub-86/62-A
April 1986

DOUBLE INFLATION

Joseph Silk
Astronomy Department
University of California at Berkeley
Berkeley, CA 94720

Michael S. Turner
NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory
Box 500
Batavia, IL 60510

Departments of Physics and Astronomy and Astrophysics
Enrico Fermi Institute
The University of Chicago
Chicago, IL 60637

Abstract

The Zel'dovich spectrum of adiabatic density perturbations is a generic prediction of inflation. There is increasing evidence that when the spectrum is normalized by observational data on small scales, there is not enough power on large scales to account for the observed large-scale structure in the Universe. Decoupling the spectrum on large and small scales could solve this problem. As a means of decoupling the large and small scales we propose double inflation (i.e., two episodes of inflation). In this scenario the spectrum on large scales is determined by the first episode of inflation and those on small scales by a second episode of inflation. We present three models for such a scenario. By nearly saturating the large angular-scale cosmic microwave anisotropy bound, we can easily account for the observed large-scale structure. We take the perturbations on small scales to be very large, $\delta\rho/\rho \simeq 0.1-0.01$, which results in the production of primordial black holes (PBHs), early formation of structure, reionization of the Universe, and a rich array of as-

trophysical events. The Ω-problem is also addressed by our scenario. Allowing the density perturbations produced by the second episode of inflation to be large also lessens the fine-tuning required in the scalar potential and makes reheating much easier. We briefly speculate on the possibility that the second episode of inflation proceeds through the nucleation of bubbles, which today manifest themselves as empty bubbles whose surfaces are covered with galaxies.

I. INTRODUCTION

The inflationary scenario[1] provides highly specific initial data for the structure formation problem: the Zel'dovich spectrum of adiabatic density perturbations[2] (and in axion-dominated models isocurvature perturbations also[3]) and the prediction that $\Omega = 1$ (more precisely that the curvature signature $k = 0$). The overall normalization of the spectrum of perturbations is model-dependent and at present we must turn to cosmological data to normalize the spectrum. Since the shape of the spectrum is specified, the normalization to any one scale serves to set the amplitude of perturbations on all scales. Once the spectrum is normalized and the composition of the dark matter is specified (e.g., hot or cold), the initial data for the structure formation problem is completely determined (up to the present value of the Hubble constant $H_0 = 100h$ km sec^{-1} Mpc^{-1}) and the model can be tested by numerical simulations and analytic calculations.

Simulations and analytic calculations based upon these initial data suggested by the inflationary scenario result in a model of the Universe that appears to be deficient in large-scale structure. One manifestation of this deficiency is the cluster–cluster correlation function. It is enhanced over the galaxy–galaxy correlation function by a factor of ~ 20 on scales of $(20 - 100)h^{-1}$ Mpc (ref. 4), and it does not seem possible to explain this enhancement if all structure has formed from gaussian fluctuations that are described by a Zel'dovich spectrum. The observed frequency of the richest clusters of galaxies and of superclusters exceeds predictions of the cold dark matter scenario in which biasing is employed to allow $\Omega = 1$(ref. 5). Attempts to enhance the large-scale power are constrained by the rms fluctuations in galaxy counts on scales of a few Mpc, and cold dark matter cannot account for this large-scale structure. A hot dark, matter-dominated Universe, as expected if the one of the neutrino species has a mass of $\sim$ 30eV, has more success on large scales, but generally fails to make galaxies sufficiently early.

The discovery of widespread bubble structure on scales of $\sim (20 - 50)h^{-1}$ Mpc in the galaxy distribution[6] has renewed interest in theories of explosive galaxy formation[7]. These theories require large amplitude primordial fluctuations to produce the initial seeds. No such perturbations have hitherto been allowed in inflationary models, where the fluctuations are gaussian and scale invariant, and hence required by the extreme isotropy of the cosmic microwave background radiation to be of small amplitude ($\lesssim 10^{-4}$). Recently, there has been a possible detection[8] of large-scale peculiar velocities of $\gtrsim$ 500km s^{-1} on scales $\gtrsim 50h^{-1}$ Mpc; confirmation could provide direct evidence for considerably more large-scale gravitational potential energy-driven motions than available in any conventional (i.e. hot or cold dark matter-dominated) inflationary scenario. The reason for this is simple: there is only one natural length scale (λ_D), either the neutrino or baryon damping length or the horizon scale at matter-radiation equality, that enters into the residual matter fluctuation spectrum. With normalization specified by the small-scale galaxy distribution ($\sim 10h^{-1}$ Mpc), a scale-invariant primordial fluctuation spectrum one has no possibility of generating large-scale peculiar velocities of this magnitude. On scales above λ_D, the spectrum asymptotes to $|\delta_k|^2 \propto k$, and peculiar velocities cannot increase beyond the relatively low value reported on the Local Supercluster scale, on which $\Omega_{pec} \sim 200 - 300\text{km s}^{-1}$ (ref. 9).

One possible concern about increasing large-scale power is that it would create excessive microwave background anisotropy on large angular scales. The normalization of the perturbation spectrum is most tightly constrained by cosmological data on small scales, including the upper bound to the anisotropy of the microwave background[10] on 4.5′ and the galaxy–galaxy correlation function[11]. It is both interesting and important to note that when the spectrum is normalized to data on small scales, the predicted large angular scale microwave anisotropy is only $\simeq 3 \times 10^{-6}$ in a cold dark matter-dominated Universe. The present upper limit to the microwave background anisotropy over $10° - 90°$ is: $\delta T/T \lesssim 7 \times 10^{-5}$ (95 % confidence)[12] –about a factor of 20 larger than the predicted anisotropy for a cold dark matter-dominated Universe. Augmenting the rms amplitude of the large-scale fluctuations by a factor of 5 would raise $\delta T/T$ and v_{pec} by a similar factor, yet enhance the mass fluctuations on large scales, for example, as measured by the cluster–cluster correlation function, by ~ 25. Since in the standard cold dark matter scenario v_{pec} is low[12a] ($\sim 100\text{km s}^{-1}$ averaged on scales $\gtrsim 10\text{h}^{-1}$ Mpc), one has the possibility of generating large peculiar velocities, and accompanying large structures and voids. In a neutrino-dominated scenario, the large-scale power already suffices to give adequate (possibly excessive) v_{pec} and is capable of producing sufficiently large superclusters and voids, but small-scale structure (that is, galaxy formation) is not adequately explained.

We propose to decouple the power on small scales ($< 10\text{h}^{-1}$ Mpc) needed for galaxy formation, from that on large scales, needed to reproduce the observed large-scale structure. Our model will naturally lead to large amplitude primordial fluctuations on small scales in hot, cold, or explosive scenarios for galaxy formation, while providing sufficient power on large scales to account for the observations. Mass and light need only be effectively coupled on large scales, where if galaxy clusters are fair tracers of the underlying mass distribution, the correlation scale would be $\sim 20h^{-1}$ Mpc. The non-linear clustering of galaxies on scales below $\sim 10h^{-1}$ Mpc has been studied in N-body simulations[13], and shown, for a wide range of initial conditions to lead, at some specified epoch, to a correlation function of the requisite slope that is consistent with the galaxy correlation scale of $\sim 8h^{-1}$ Mpc.

The way we propose to decouple the large and small scales is to have two episodes of inflation, the first determining the spectrum on large scales and the second the spectrum on small scales. By so doing, the amplitude of perturbations on both scales can be increased. The amplitude on large scales need only be consistent with the cosmic microwave anisotropy upper limit on large angular scales and therefore can be up to a factor of 20 larger than the value predicted with a single episode of inflation. If $\delta\rho/\rho$ is sufficiently large on small scales, say 0.01–0.1, then small-scale structures will form very early on (around the time of decoupling) and the Universe will be reionized, thereby erasing any small-scale microwave anisotropy, and effectively sidestepping the small-scale anisotropy contraint. Throughout we will refer to the scale which separates large and small scales as M_{seed} and as we shall discuss, we will want to have M_{seed} between 10^2 and $10^{13} M_{\odot}$.

The astrophysical and cosmological implications of the double inflation scenario are very rich indeed. The existence of such large small-scale adiabatic density perturbations makes primordial black hole (PBH) formation very likely, and in fact it is possible that PBHs are the dark matter in this scenario. Early structure formation suggests that 'astrophysical fireworks' could play an important role in structure formation as envisaged in theories of explosive galaxy formation. Although PBHs could be the dark matter in this scenario, they need not be. For example, neutrinos could be the dark matter, in which case the damping scale which their free-streaming imprints upon the spectrum of large scale perturbations would naturally lead to structures on that scale, while the large amplitude fluctuations on small scales would lead to galaxy formation. The so-called Ω-problem (the fact that observations seem to indicate that $\Omega \simeq 0.1 - 0.3$, while inflation predicts that $\Omega = 1$) could be solved in the case that neutrinos are the dark matter, by the fact that neutrinos would likely be distributed smoothly on scales up to their damping scale ($\simeq 13h^{-2}$ Mpc), or in the case of cold dark matter, by the fact that only the enhanced correlations and peculiar velocity fields on large scales measure the true value of Ω. We briefly speculate on the possibility that, if the second episode of inflation proceeds through bubble nucleation, the bubble structures seen in the large-scale galaxy distribution may be a result of this process.

A key question that one must ask about double inflation is, how likely is it that there were two episodes of inflation, with the second lasting only 40 to 50 e-folds? In the past few years it has become clear that, inflation rather than occuring only in very special circumstances, seems to be generic to early Universe microphysics, being generally associated with the dynamics of a weakly-coupled scalar field. Almost any weakly-coupled scalar field which is displaced from the minimum of its potential will lead to inflation. Inflationary scenarios have been discussed in the context of spontaneous-symmetry breaking (SSB), compactification of extra spatial dimensions, induced gravity theories, supersymmetry/supergravity and superstring theories, and the dynamical evolution of any scalar field which finds itself displaced from the minimum of its potential. Thus it seems very likely that, if inflation ever occured, there were multiple stages of inflation. The only difficulty then is arranging for the last episode to last only 40 to 50 e-folds. [As we will discuss arranging for this to occur is very different, and we believe far more reasonable, than arranging for precisely enough inflation to result in the value of Ω being around 0.2 today.] We present three models where this occurs quite naturally. Since the reheating of the Universe is controlled by the last inflationary period, the fact that the perturbations produced by this episode of inflation are large greatly alleviates the reheating problem because the inflating field in this case need not be so weakly coupled.

[Another early Universe scenario which also leads to primordial density perturbations is that of cosmic strings (in this case isocurvature perturbations)[14]. The production of cosmic strings during a SSB transition with SSB scale of the order of $10^{16} - 10^{17} GeV$ results in very localized, large amplitude perturbations whose statistical properties are highly non-gaussian. Cosmic strings seem to be the principal alternative early Universe scenario with great promise for resolving both the large- and small-scale structure formation-problems. Cosmic strings do not, however, cast any light on the smoothness and flatness of the observed Universe, and an early epoch of inflation followed by string production does not seem viable. In order to work, the early epoch of inflation would have to produce density perturbations of amplitude no larger than about 10^{-4} and also reheat the Universe to a temperature of at least 10^{16} or $10^{17} GeV$. Such a high reheat temperature is difficult (if not impossible) to achieve along with small density perturbations. In addition, the production of long wavelength gravitational wave modes during inflation places an upper bound on the reheat temperature of about $3 \times 10^{16} GeV$.]

The outline of the paper is as follows. In the next section we discuss in some detail our models for double inflation. In the following section we describe the the rich multitude of astrophysical implications of double inflation. The final section is devoted to some concluding remarks.

II. Multiple Inflation

A. One-Shot Inflation

In the past five years we have learned that inflation seems to be generic to early Universe microphysics and that it invariably involves a weakly-coupled scalar field which for one reason or another finds itself displaced from the minimum of its potential.[1] Inflationary models involving SSB[15], compactification of extra dimensions[16], supersymmetry and/or supergravity[17–19] and superstrings, induced gravity theories[20], and scalar field dynamics in general[21] have been proposed. The fact that the inflating field must be weakly-coupled follows from requiring that the adiabatic density perturbations produced during inflation are of an acceptable magnitude. In a model which successfully implements inflation there is typically a dimensionless coupling in the scalar potential of the order of 10^{-15} or so[22]. In order that one-loop corrections not swamp this small coupling the inflating field must be very weakly-coupled to all other fields in the theory. Needless to say this means that it must be a gauge-singlet field. Because of its weak couplings, one would not expect the field to ever have been in thermal equilibrium (expect perhaps at energies much greater than m_{pl} where strong gravitational interactions might bring it into equilibrium). Thus as several authors have emphasized the initial value of the inflating field is not likely to be specified by dynamics (e.g., thermal considerations)[19,21] and its initial value must for now be taken as initial data.

Because of the variety of microphysical phenomena which lead to inflation, it seems very likely that, if inflation did occur during the early history of the Universe, it probably did so several times. Throughout we will adopt the point of view that initially ($t = t_{pl} \simeq 10^{-43} sec$?) there were a number of weakly-coupled scalar fields which were displaced from the minima of their potentials. Starting with this assumption, it then follows that a number of episodes of inflation would ensue, occuring in the order of the initial vacuum energies of the displaced scalar fields (the scalar field with the largest vacuum energy inflating first and so on).

To analyze the evolution of the scalar fields which inflate we will use the semi-classical equations of motion and assume that the field is smooth in some sufficiently large region of space so that spatial inhomogeneities in the scalar field do not interfere with inflation. The equation of motion for the field ϕ is

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + V'(\phi) = 0 \tag{2.1}$$

where prime denotes a derivative with respect to ϕ, overdot a derivative with respect to time, $H \equiv \dot{R}/R$ is the expansion rate of the Universe, Γ is the decay width of the ϕ particle, and in our units $\hbar = c = k_B = 1$. During an episode of inflation the energy density of the Universe, ρ, is dominated by potential energy and

$$H^2 = \frac{8\pi}{3m_{pl}^2} V(\phi) \tag{2.2}$$

During the "slow rollover" period (when $V'' \leq 9H^2$) the $\ddot{\phi}$ term can be neglected and

$$\dot{\phi} \simeq -V'/(3H) \tag{2.3a}$$

$$dN \simeq -3H^2 d\phi/V' \tag{2.3b}$$

where $N \equiv \int H dt$ so that $R \propto \exp(N)$. The amplitude of the adiabatic density perturbations produced during inflation on the comoving scale λ, when that scale re-enters the horizon during the post-inflation period is

$$(\delta\rho/\rho)_H \equiv k^{3/2}|\delta_k|/(2\pi)^{3/2} = \begin{cases} \mathrm{H}^2/(\pi^{3/2}\dot{\phi}) & \mathrm{RD} \\ \mathrm{H}^2/(10\pi^{3/2}\dot{\phi}) & \mathrm{MD} \end{cases} \tag{2.4}$$

where $H^2/\dot{\phi}$ is evaluated when the scale λ crossed outside the horizon during inflation and δ_k is the Fourier component on the comoving scale k ($\equiv 2\pi/\lambda$). Perturbations on scales which re-enter the horizon when the Universe is radiation-dominated (RD) do so as pressure waves of the amplitude specified in Eqn(2.4); by the time the Universe becomes matter-dominated the amplitude of the perturbation in the non-baryonic, dark matter component has grown by a factor of a few, and

$$(\delta\rho/\rho)_{EQ} \simeq H^2/\dot{\phi} \tag{2.5}$$

Perturbations on scales which re-enter the Universe when it is matter-dominated (MD) do as growing mode perturbations with the amplitude specified in Eqn(2.4). A perturbation on the comoving scale λ (where R is normalized to be unity today) crosses outside the horizon $N(\lambda)$ e-folds before the end of inflation where

$$N \simeq 46 + \ln(\lambda/\mathrm{Mpc}) + \frac{1}{3}\ln[(V^{1/2}/10^{-8}m_{pl}^2)(T_{RH}/10^{-9}m_{pl})] \tag{2.6}$$

For reference the scale 1 Mpc corresponds to $1.5 \times 10^{11}h^2 M_\odot$ and the scale which just enters the horizon at the epoch of matter-radiation equivalence is $13h^{-2}$ Mpc. In a one shot inflation scenario with cold dark matter, the cosmological data on small scales require that the spectrum be normalized so that $H^2/\dot{\phi} \simeq 10^{-4}$. This means that on large scales $(\delta\rho/\rho)_H \simeq 3 \times 10^{-6}$. The quadrupole anisotropy $(\delta T/T)_Q \simeq (\delta\rho/\rho)_H$ on very large scales, so that with the above normalization $(\delta T/T)_Q = 3 \times 10^{-6}$. The exponential expansion period of inflation ends when the scalar field begins to evolve rapidly (on a timescale short compared to H^{-1}) and eventually oscillates about the minimum of its potential. This oscillatory phase begins when $\phi \equiv \phi_{end}$, where

$$V''(\phi_{end}) \simeq 9H^2(\phi_{end})$$

Eventually these coherent oscillations decay (in a time Γ^{-1}) into radiation (from the particle point of view, ϕ particles are decaying), reheating the Universe to a temperature

$$T_{RH} = min[V(\phi_{end})^{1/4}, (\Gamma m_{pl})^{1/2}] \tag{2.7}$$

For further discussion about 'one-shot' inflation we refer the reader to Turner in ref. 1.

B. Double Inflation

We have just outlined how a single epoch of inflation proceeds, and now we will consider double inflation. Of course double inflation can easily be generalized to multiple inflation. Consider two scalar fields ϕ_1 and ϕ_2 whose scalar potentials are $V_1(\phi_1)$ and $V_2(\phi_2)$, and whose initial values are ϕ_{1i} and ϕ_{2i}. We shall assume that both scalar fields are sufficiently smooth in a given region of the Universe for inflation to proceed and that the two fields are either not coupled to each other, or are sufficiently weakly-coupled to each other, so that they can be considered to be independent of one another. We take $V(\phi_{1i}) > V(\phi_{2i})$ so that ϕ_1 inflates first. Define N_1 and N_2 to be the number of e-folds of inflation during the first and second episodes of inflation respectively. In order for double inflation to solve the smoothness and flatness problems $N_1 + N_2$ must be greater than about 55 or so, **cf.** Eqn (2.6) with $\lambda = 3000$ Mpc. However, unless N_2 is less than about 55 all scales in the observable Universe today ($\lambda \lesssim 3000 Mpc$) will have crossed outside the horizon during the second episode of inflation, making the first episode of no concern to us at all. Recall that $N(\lambda)$ is the number of e-folds from the time the scale λ crosses outside the horizon, until the end of inflation. Perturbations on scales corresponding to

$$N(\lambda) > N_2$$

will have their amplitudes set by the first episode of inflation and those with

$$N(\lambda) < N_2$$

will have their amplitudes set by the second episode of inflation. We will call the mass scale which separates the two parts of the spectrum of density perturbations M_{seed}, and as we will discuss in Sec. III, we would like M_{seed} to be between about 10^2 and $10^{13} M_{\odot}$. This corresponds to $\lambda_{seed} \simeq (3 \times 10^{-4} - 3)$ Mpc and $N_2 \simeq 38 - 47$. [Here and throughout we will ignore the 'ln term' in Eqn(2.6).] In addition we wish to have $(\delta\rho/\rho)_H \simeq (2-7) \times 10^{-5}$ on large scales (the larger value saturating the upper limit to the quadrupole anisotropy) to have as much power as possible on large scales. On small scales we want $(\delta\rho/\rho)_H \simeq 0.01 - 0.1$. The spectrum we desire from double inflation is shown schematically (at the epoch of matter-radiation equivalence) in Fig. 1. The most difficult aspect of double inflation is to achieve between 38 and 47 e-folds of inflation during the second episode of inflation. Before we outline our three models for doing so let us point out that achieving this differs greatly from having just the right amount of inflation to have $\Omega \simeq 0.1 - 0.3$ today. In order to have $\Omega \simeq 0.1 - 0.3$ the number of e-folds of inflation must be set to a precision of ± 0.3. In addition, the number of e-folds of inflation required to solve the homogeneity and isotropy problems is, upto a log-term, equal to that required to solve the flatness problem[23]. That means that if the number of e-folds of inflation is tuned to give $\Omega \simeq 0.1 - 0.3$ today, then the horizon problem has just barely been solved. Since the isotropy of the microwave background is sensitive to perturbations with wavelengths larger than the present horizon[24]: $(\delta T/T)_Q \simeq (\delta\rho/\rho)_H (H^{-1}/\lambda)^2$, for $\lambda \gtrsim H^{-1}$, having just enough inflation to solve the horizon problem seems likely to be inconsistent with the isotropy of the microwave background. We will now discuss our three models for double inflation.

(1) Modified Shafi-Vilenkin-Pi Inflation. Consider two gauge singlet scalar fields ϕ_1 and ϕ_2 with scalar potentials

$$V_1(\phi_1) = \lambda_1 \phi_1^4 \tag{2.8a}$$

$$V_2(\phi_2) = \lambda_2 \sigma^4/2 + \lambda_2 \phi_2^4 [\ln(\phi_2^2/\sigma^2) - 1/2], \tag{2.8b}$$

where $\lambda_1 \simeq 10^{-10}, \lambda_2 \simeq 10^{-9}$, and $\sigma \simeq 10^{16} - 10^{17} GeV$. For their initial values, we shall take $\phi_{1i} \simeq few\ m_{pl}$ and $\phi_{2i} \simeq 0$. In this model the initial vacuum energy density associated with ϕ_1 is much greater than that associated with ϕ_2 and so the field ϕ_1 inflates first, followed later by ϕ_2. The scalar field ϕ_1 is one of Linde's 'chaotic inflaters'[21] and in our toy model serves no other purpose. The scalar field ϕ_2 serves to induce GUT symmetry breaking, just as in the models of Shafi and Vilenkin[15], and Pi[15]. Its vacuum expectation value, σ, induces a negative mass squared for the scalar field which breaks the GUT down to $SU(3) \otimes SU(2) \otimes U(1)$ (or some intermediate group); e.g., for an $SU(5)$ model, ϕ_2 induces a negative mass squared for the 24-dim Higgs representation. In this toy model, ϕ_2 must be weakly-coupled to other fields in the theory (quartic couplings to other scalar fields $\lesssim 10^{-4}$ and Yukawa couplings to fermion fields $\lesssim 10^{-2}$) in order that one-loop corrections not spoil the flatness of V_2. We will now briefly outline inflation in this model. From Eqns(2.3,2.4) it follows that

$$N_1 \simeq \pi^2 \phi_{1i}^2 / m_{pl}^2 \tag{2.9a}$$

$$H^2/\dot{\phi} \simeq 4\sqrt{2}\lambda_1^{1/2} N^{3/2} \tag{2.9b}$$

$$V_1(\phi_{1end}) \simeq 2 \times 10^{-12} m_{pl}^4 \tag{2.9c}$$

where $\phi_{1end}^2 = m_{pl}^2/2\pi$. Clearly for $\phi_{1i} \gtrsim few\ m_{pl}$ there will be sufficient inflation during the first epoch so that the total amount of inflation will be enough to solve the horizon problem. Assuming that $N_2 \simeq 38 - 47$, the largest scales in the observable Universe today cross outside the horizon when $N_1 \simeq 0(10)$. Therefore the value $\lambda_1 \simeq 10^{-10}$ results in perturbations of amplitude about 3×10^{-5} on large scales, thereby nearly saturating the present upper limit to the quadrupole anisotropy. When $\phi_1 \simeq \phi_{1end}$, the field ϕ_1 begins to oscillate about $\phi_1 = 0$. A scalar field oscillating about a quartic potential behaves like matter with an equation of state, $p = \rho/3$, so that the energy density in these oscillations decreases as R^{-4} (ref. 25). Since another period of inflation follows, it is irrelevant whether or not the energy density in ϕ_1 oscillations decays into radiation and reheats the Universe.

The second epoch of inflation begins when the energy density in ϕ_1 oscillations falls to a value which is comparable to the vacuum energy locked up in the potential energy density in the scalar field ϕ_2. When this occurs ϕ_2 starts to evolve and it follows from the equations of motion for ϕ_2 that

$$N_2(\phi_2) \simeq \frac{\pi}{30} \frac{\sigma^4}{{\phi_2}^2 {m_{pl}}^2} \tag{2.10a}$$

$$H^2/\dot{\phi} \simeq 16\lambda^{1/2} N^{3/2} \tag{2.10b}$$

where $N_2(\phi_2)$ is the number of e-folds of inflation that the Universe undergoes while ϕ_2 evolves from ϕ_2 to σ. Recall that the initial value of the scalar field ϕ_2 was zero, which formally would lead to $N_2 \to \infty$. However, during the first inflationary period, deSitter space produced quantum fluctuations in ϕ_2 will lead to fluctuations of order[26]

$$\Delta\phi_2 \simeq {N_1}^{1/2} H_1/2\pi$$

where $H_1 \simeq 3 \times 10^{-5} m_{pl}$ is the value of the Hubble constant during the first period of inflation. Because of these quantum fluctuations, ϕ_2 is in essence pushed out to a value of order $\Delta\phi_2$ and therefore the initial value of ϕ_2 used in the semi-classical equations of motion should be of order

$$\phi_{2i} \simeq 0(few H_1/2\pi)$$

For the parameters choosen this results in the second episode of inflation automatically lasting of order 10-100 e-folds, about the desired amount.

When $9H^2 \simeq V_2''$, which occurs for $\phi_2 \simeq 0.5\sigma^2/m_{pl}$, the scalar field ϕ_2 begins to evolve rapidly (compared to the expansion timescale) and then oscillates about the minimum of its potential. The energy density in these coherent oscillations is

$$\rho_2 \simeq V_2(\phi_{2end}) \simeq \lambda_2 \sigma^4/2 \tag{2.11}$$

The decay of these oscillations (due to the $\Gamma\dot{\phi}$ term in Eqn(2.1)) results in the reheating of the Universe. Because ϕ_2 is not nearly as weakly-coupled as in the usual one-shot inflation scenario of Shafi and Vilenkin, and Pi, reheating to a high enough temperature for baryogenesis to take place ($T_{RH} \gtrsim 10^9 GeV$ or so) should be much easier to achieve. Of course in supersymmetric models this could exacerbate the problem with the overproduction of gravitinos, since gravitino production is proportional to the reheat temperature[27].

We note that the energy density tied up in ϕ_1 oscillations is not a matter of concern, as at the onset of the second episode of inflation it is only comparable to V_2, and by the end of inflation it has been reduced relative to V_2 by a factor of $exp(4N_2)$. Since the energy density in ϕ_1 oscillations always decreases as R^{-4} it can never become a significant component of the energy density in the Universe.

[We note that in our first model, the potential V_1 could just as well have been the potential[28]

$$V_1(\phi_1) \simeq m^2 {\phi_1}^2$$

with $m^2 \simeq 10^{-9}$. Likewise, the second potential need not have been of the Coleman-Weinberg form; a potential of the form:[22] $V_2 = V_0 + \alpha{\phi_2}^2 - \beta{\phi_2}^3 + \lambda_2{\phi_2}^4$, where $\sigma = 0.75\beta/\lambda_2$ and $V_0 = \lambda_2\sigma^4/3$, with $\lambda_2 \simeq 10^{-10}$, $\beta \simeq 3 \times 10^{-11} m_{pl}$, and $\alpha \leq 10^{-14} m_{pl}^2$ would serve just as well.]

(2) Tumbling Inflation. Our second model is not really a model, but a germ of an idea which is a generalization of the trick used in model (1) to achieve $N_2 \simeq 40 - 50$. Recall the basic idea of the trick in (1) was to use the first epoch of inflation to push the scalar field which inflates second out far enough on its potential so that it only inflates 40 or 50 e-folds. This idea is very easy to generalize to many fields. One could imagine that all the fields which inflate start close to the origin of their respective potentials (here we are assuming that the minimum of the potential is not at the origin). As each stage of inflation proceeds, deSitter space produced quantum fluctuations push all the other fields waiting to inflate further out on their respective potentials, so that when they do ultimately inflate they do not have as far to evolve and thereby will inflate less than the previous field. It is easy to imagine a pattern of successive epochs of inflation where the first field inflates 10^6 e-folds, the next 10^4, and so on until a field only inflates 40 e-folds or so, with all the subsequent fields so far out on their potentials that they do not inflate at all. The number

of e-folds of inflation in each successive episode of inflation tumbles downward with the last 'hitting' at value of order 10–100.

(3) Late Chaos. For our purposes, Linde's chaotic inflation[21] has the attractive feature that the number of e-folds of inflation $N \simeq \pi(\phi_i/m_{pl})^2$. That means for $\phi_i \simeq 0(few\ m_{pl})$ inflation lasts 30 or so e-folds–just the number we want! In this model we make the scalar field ϕ_2 the chaotic inflater.

Again we have two weakly-coupled scalar fields ϕ_1 and ϕ_2 with scalar potentials

$$V_1(\phi_1) \simeq \lambda_1\sigma^4/2 + \lambda_1{\phi_1}^4[\ln({\phi_1}^2/\sigma^2) - 1/2] \tag{2.12}$$

$$V_2(\phi_2) \simeq \lambda_2{\phi_2}^4 \tag{2.13}$$

where $\lambda_1 \simeq 10^{-12}$, $\sigma \simeq 300 m_{pl}$, and $\lambda_2 \simeq 10^{-8}$. The initial values of the scalar fields ϕ_1 and ϕ_2 are taken to be: $\phi_{1i} \simeq \phi_{2i} \simeq few\ m_{pl}$.

In this model scalar field ϕ_1 inflates first and produces density perturbations of order 10^{-4} on large scales. Then field ϕ_2 inflates, producing density perturbations of order $0.01 - 0.1$ on scales smaller than M_{seed}. The quantum fluctuations in ϕ_2 produced during the first episode of inflation are smaller than the initial value we have chosen for ϕ_2 and so thereby do not affect the initial value of ϕ_2. The second episode of inflation lasts the requisite number of e-folds because the initial value of ϕ_2 is of the order of a $few\ m_{pl}$. This model is really a toy model and we will not attempt to tie it to any particle physics phenomenology. We present it as another qualitatively different way to achieve 40-50 e-folds of inflation during the second episode of inflation.

(4) Bubbles. Finally, we mention a very speculative possibility. Heretofore, we have considered potentials without barriers between the initial value of ϕ and the value of ϕ at the minimum of its potential. Suppose the potential for ϕ_2 had such a barrier. Then the

second episode of inflation would have to commence by ϕ_2 tunneling through this barrier via the nucleation of bubbles. Within these bubbles the usual slow-rollover transition takes place. On scales much larger than the scale of these bubbles (which today should be of the order λ_{seed}) the Universe is very smooth by virtue of the first episode of inflation. On the intermediate scales, i.e., those comparable to λ_{seed}, the bubbles should leave a significant imprint on the Universe, likely, density perturbations of order unity. Speculating a bit further, it is not impossible that these could lead to the large bubble structures (of size $\sim (10-30)h^{-1}$ Mpc) recently discovered in the distribution of galaxies[6]. At present we are studying this possibility in more detail.

Let us end this section by briefly summarizing. We have argued that multiple stages of inflation are very likely (if the Universe ever inflated). If the last of these stages lasted less than about 55 e-folds, then the spectrum of density perturbations on presently observable scales is determined by at least two of the episodes of inflation, implying that the amplitude of perturbations on large and small scales need not be the same. We have shown two plausible mechanisms whereby the most recent episode of inflation might only last 40 or 50 e-folds, and using these mechanisms we have constructed models where the perturbations on large scales would have amplitude of order 10^{-4} and those on small scales would have amplitude of order $10^{-2}-10^{-1}$. While we have focussed on using two distinct and decoupled scalar fields to implement double inflation, it is not completely implausible that a single field, with a sufficiently complicated scalar potential, could also do the trick. Now we will explore the plethora of astrophysical and cosmological implications of such a spectrum of density perturbations.

III. Astrophysical Implications

A. Galaxy Formation by Accretion

With the rms density fluctuations $\delta\rho/\rho \sim 0.1$, galaxy formation can begin as early as $z \sim 1000$, with rare objects becoming non-linear much earlier. Objects that condense at $z \sim 1000$ have a mean internal density, neglecting any dissipation, of $\sim 10^3 h^2 M_\odot pc^{-3}$, and can be identified with bulges or nuclei of galaxies. Such objects would be common if the Universe is dominated by cold dark matter, and we should therefore ensure that the enhanced fluctuation amplitudes as large as $\delta_{rms} \sim 0.1$ do not extend beyond $\sim 10^9 - 10^{10} M_\odot$. This presumes that such early forming objects survive and are not tidally disrupted and incorporated into more diffusive, more massive structures as clustering continues. The rare objects, however, certainly should survive. Typical dark halos, of density $\sim 0.01 M_\odot pc^{-3}$, collapsed at $z \lesssim 20h^{-2/3}$, and would form by accretion around the early forming cores. Secondary infall of cold dark matter onto cores should be approximately spherically symmetric, and yields a density profile $\rho(r) \propto r^{-2}$ (ref. 29).

In a Universe dominated by hot dark matter, the formation of bound structures by linear fluctuations is inhibited until the neutrino Jeans mass $\sim 10^8(1+z)^{3/2} M_\odot$ drops below the seed fluctuation scale[30]. However, non-linear lumps that form from rare peaks in the fluctuation distribution will accrete more loosely bound halos of non-relativistic matter. We shall argue below that to avoid excessive black hole formation at earlier epochs, non-linear adiabatic fluctuations are probably too rare ($\sim 10\sigma$ fluctuations) to help galaxy formation. To form galaxies in a hot dark matter Universe we need fluctuations that become non-linear by $z \sim 2$, and hence the corresponding mass-scale is $\gtrsim 10^{10}(0.1/\delta_{rms})^{3/2} M_\odot$. However, freestreaming erased all linear adiabatic fluctuations in the hot dark matter below a mass $\sim 10^{15}h^{-2}M_\odot$. Baryonic fluctuations would survive on scales above the baryon damping length, equivalent to a mass scale $\sim 10^{12}\Omega_b^{-1/2}h^{-5/2}M_\odot$, but with amplitude reduced by a factor $\sim \Omega_b/\Omega_\nu \simeq 0.1$. We conclude that in a neutrino-dominated Universe, the galaxy forming fluctuations must have large amplitude ($\gtrsim 0.01$) on scales $10^{10} - 10^{13}M_\odot$, whereas in a cold dark matter dominated Universe, where accretion occurs much earlier, the relevant scale is $\lesssim 10^9 M_\odot$ if $\delta_{rms} \sim 0.1$.

The inhomogeneous fluctuation spectrum cannot extend much beyond $10^{12} - 10^{13}M_\odot$, or we would begin to violate constraints that derive from the clustering of galaxies. Hierarchical clustering occurs continuously, with the typical scale becoming non-linear today over $\sim 8h^{-1}$ Mpc or $\sim 3\times 10^{14}h^{-1}M_\odot$ (ref. 11). Hence, fluctuations containing $\sim 10^{12}M_\odot$ would have gone non-linear at $z \sim 300$ and could arise from primordial fluctuations with $\delta_{rms} \sim 0.01$. To avoid excessive clustering inhomogeneity in a cold dark matter-dominated Universe, the large amplitude mass fluctuations should not extend beyond $\sim 10^{10}\delta_{rms}^{-1}M_\odot$. This is a weaker, but more rigorous, constraint than the smaller mass scale inferred from the argument given previously which utilized the internal density, since the number of surviving objects is certain in a continuous clustering, non-dissipative cold dark matter scenario. Non-linear gravitational clustering ensues that the galaxy correlation function will have the correct slope for an appropriate choice of δ_{rms} and seed mass, although there is a legitimate worry as to whether in the transition region where the correlation function is of order unity, there may be excessive correlations induced by the large-scale fluctuation component. This potential difficulty is avoided in a hot dark matter model where the large coherence length effectively suppresses contributions from the large-scale fluctuations to the small-scale correlations.

B. Reionization and Population III

The plausible possibility exists in our model of substantial heat and ionization sources in the form of non-linear objects collapsing at $z \sim 10^3$ and developing large accretion luminosities. Recombination of the Universe may never occur at $z \sim 1000$. Even if the Universe has always remained ionized, with $\Omega_b \sim 0.1$ there are negligible associated distortions to the blackbody spectrum of the cosmic microwave background, either due to free-free emission distorting the Rayleigh-Jeans region or Comptonization distorting the Wien region.[31] Primordial anisotropy in the cosmic microwave background is smoothed out to an angular scale of $\sim 10\Omega_b^{-1/3}$ degrees. On smaller angular scales secondary fluctuations are induced by the small-scale peculiar velocity field on the last scattering surface at z_s, of order

$$\delta T/T = (\delta\rho/\rho)_\lambda(\lambda/ct_s) \simeq 10^{-4}(\delta_{rms}/0.1)(z_s/100)\theta \qquad (3.1)$$

for angular scales $\theta \lesssim 0.5'(M/10^{12}M_\odot)^{1/3}$, and decreases as $\theta^{-5/2}$ on larger angular scales. Above $\sim 10°$ the usual Sachs-Wolfe anisotropy induced by gravitational potential fluctuations dominates.

Chemical evolution commences at z $\sim$ 1000, once the first clumps collapse and form massive stars. If seeds of mass $10^9 M_\odot$, identified with the forming bulges of a galaxies, were to form stars with a standard initial mass function, each would produce upto $\sim 10^7 M_\odot$ of metals by a redshift $z \sim 10 - 100$. Most of this enriched matter would be ejected from the central bulge of the protogalaxy, and diluted by $\sim 10^{11} M_\odot$ of gas that is in the process of accreting to form a typical luminous galaxy. Consequently the enrichment expected for the halo generally amounts to about $Z \lesssim 10^{-4} Z_\odot$. One would not expect to find any stars as gas surviving today with $Z = 0$. Observations are not inconsistent with this: the most metal-poor halo star known has $Z \sim 10^{-4} Z_\odot$ (ref. 32), and the most primitive known gas, namely, the Lyman alpha clouds seen toward quasars, has $Z < 10^{-3} Z_\odot$ (ref. 33).

C. Dark Matter

The large amplitude, small-scale fluctuations yield a natural source of dark matter. Rare density peaks with $\delta\rho/\rho > 1$ are able to collapse just after horizon crossing, and will form black holes.[34] To avoid overdominating the present Universe with black holes, the fluctuation amplitude must be low enough that this seldom occurs today, since PBHs which formed very early on will dominate the mass contribution of PBHs. The total mass in a horizon volume ($\simeq \rho t^3$) when the Universe is radiation-dominated is about $3 \times 10^{13} g (T/10^{10} GeV)^{-2}$, and so a PBH of mass m_{bh} formed when $T \simeq 2 \times 10^9 GeV (m_{bh}/10^{15} g)^{-1/2}$. The fraction of critical density contributed by PBHs which formed when the temperature of the Universe was T is: $\Omega_{bh} \simeq 2f \times 10^{17} (T/10^9 GeV) \simeq 4f \times 10^{17} (m_{bh}/10^{15} g)^{-1/2}$, where f is the fraction of the mass of the Universe converted into PBHs. Since the inflation-produced density fluctuations are gaussian, that fraction is $f \sim \delta_{rms} exp(-\nu^2/2)$, where $\nu\delta_{rms} \simeq 1$ is the amplitude at horizon crossing required for PBH formation. PBHs less massive than about $10^{15} g$ will have evaporated by today, and so the greatest contribution to Ω_{bh} will be from PBHs formed when $T \simeq 3 \times 10^9 GeV$. Requiring that $\Omega_{bh} \leq 1$ implies that PBHs must have formed from 10σ fluctuations, or that $\delta_{rms} \lesssim 0.1$.

The density fluctuations that form primordial black holes are assumed to have amplitude unity at horizon crossing. This 'threshold amplitude' is probably uncertain by a factor of at least 2. Moreover, the galaxy forming fluctuations underwent sub-horizon growth during the radiation era enhancing their amplitude at the epoch of matter-radiation equality by about a factor of 3–5. Thus requiring $\Omega_{bh} = 1$ and $\delta_{rms} = 0.1$, probably means that $\delta_{rms}^{EQ} \sim 0.3$ on galaxy forming scales.

There is one severe difficulty with $\Omega_{bh} \simeq 1$, however. Evaporating black holes of mass $\sim 10^{15}$g have temperatures $\sim$ 10MeV and are expected to produce a shower of readily observable gamma rays.[34] This does of course assume that black holes do evaporate[34a] and that the black hole environment is sufficiently transparent to permit the gamma rays produced near the event horizon to escape freely. If the gamma rays do escape, the evaporating PBHs will produce a diffuse gamma ray flux that should be detectable. In fact, utilizing the observed gamma ray background as an upper limit on possible black hole evaporation implies that $\Omega_{bh} \lesssim 10^{-6}$. This objection only applies to black holes that evaporate at $z < 1000$. Since $\tau_{bh} \simeq 10^{-28}$ sec $(m_{bh}/g)^3$, this corresponds to black holes of mass in the narrow range $10^{14} - 10^{15}$g, produced at $T \sim 2 \times 10^9 - 10^{10}$GeV.

This leaves us with two options: reheating to a temperature less than about 10^9GeV which would mean that no dangerously observable black holes form as a by-product of double inflation while $\Omega_{bh} = 1$ (provided that the rms fluctuation amplitude is carefully adjusted).[35] Alternately, we can simply lower δ_{rms}, to below 0.1, and form far fewer black holes. Some black hole formation seems inevitable, however, and a plausible implication of double inflation might be that dark matter associated with galaxies (namely halos) would be PBHs (or population III black holes), while the dark matter distributed over large scales that contributes $\Omega \simeq 0.9$ or so could be in the form of weakly-interacting, relic particles.

D. Large-scale Structure

Saturating the large-scale microwave background anisotropy limit allows us to enhance δ_{rms} on large scales by a factor of upto $\sim$ 20 relative to the standard cold dark matter spectrum normalized to $\delta M/M = 1$ at $8h^{-1}$ Mpc. This means that if galaxy clusters are fair tracers of large-scale power, our double inflation model naturally yields the observed enhancement of the cluster-cluster correlation function ($\propto \delta_{rms}{}^2$) on scales of $(20–100)h^{-1}$ Mpc if we raise δ_{rms} by a factor of 5. Of course, this means that we are predicting that $\delta T/T \sim 2 \times 10^{-5}$ (95% confidence) over angular scales larger than 10°. The prediction of $\delta T/T$ is similar for either hot or cold dark matter. There is no need for any large-scale power enhancement relative to a conventional neutrino-dominated model, however, because these models are normalized differently, and already go a long way towards explaining the large-scale structure. Conventional neutrino-dominated models fail to account for galaxy formation; our addition of non-linear small-scale power provides local seeds for forming galaxies earlier than in the conventional hot dark matter scenario. If one accepts the linear theory prediction of excessive large-scale peculiar velocities in a neutrino-dominated model,[35a] which requires our local supercluster to be in an improbably quiescient region of the Universe, we might actually wish to reduce the amplitude of the large-scale fluctuations by about a factor of 2. However, this still allows very considerable pancaking to have occurred by the present epoch, and should be capable of yielding the desired large-scale structure of the galaxy distribution.

The neutrino model, in essence, becomes a combined hot and cold dark matter scenario. Double inflation with massive neutrinos both solves the Ω problem and forms galaxies, in addition to helping to resolve the large-scale structure problem. The neutrino damping scale is $\sim$ 40 Mpc (for h = 1/2) and naturally leads to large voids and superclusters of galaxies. At the same time, the reluctance of neutrinos to undergo small scale clustering provides a natural resolution of the Ω problem: the neutrino density is uniform on scales

$\ll \lambda_D$. In this scenario dark halos most likely would consist of PBHs or population III relics.

E. Explosive Galaxy Formation

Double inflation provides a natural origin for the rare seeds on which explosive amplification models of galaxy formation[7] are based. The idea here is that injection of energy by the formation of many supernovae that form in a collapsing seed cloud sweeps up a shell of baryonic matter. A mass amplification ($M_{fragment}/M_{seed}$) of about 10^4 occurs when the swept-up shell becomes gravitationally unstable to fragmentation: the fragments collapse and inject more energy as stars form. If seeds form at a late epoch ($z < 10$), the cooling efficiency limits the explosive amplification to two generations[7], and $10^6 M_\odot$ seeds are required to build structure upto $10^{14} M_\odot$. The resulting voids that are produced are too small to account for the observed bubble structure of the large scale galaxy distribution, and this problem must be supplemented by gravitational instability of hot or cold dark matter as outlined above. However our model favors seed formation at epochs z $\sim$ 100, when Compton cooling allows multiple generations of explosive amplification to occur. The initial seeds need be only $10 - 100 M_\odot$ to form supernovae, and the early hierarchical explosion models form large holes[36,37]. In this case, the amplitude of the large-scale fluctuations could be much less than envisaged here: one need not appeal to gravitational processes at all.

IV. CONCLUDING REMARKS

The inflationary Universe scenario is by far one of the most attractive scenarios for the very history of the Universe. However, conventional inflation (or 'one shot' inflation as we refer to it) is in danger of being falsified by observational data. As we have discussed, the one shot scenario does not seem to be able to account for the abundance of large-scale structure observed–voids, superclusters, bubbles, the correlation of clusters, and large peculiar velocities (if real), and also predicts that $\Omega = 1$, whereas observations indicate that the amount of matter distributed on scales $\lesssim 10$ Mpc only accounts for $\Omega \simeq 0.1 - 0.3$. Many of these difficulties can be traced to the fact that the spectrum of primordial density perturbations on large and small scales is directly coupled.

We have proposed to remedy this apparent conflict by having two episodes of inflation, thereby decoupling the primordial spectrum of density perturbations on large and small scales. Because of the diversity of microphysical situations in which inflation has been shown to occur, we believe it is likely that, if inflation ever occured, it probably occured several times. The only real obstacle to double inflation is arranging that the second episode only last 40-50 e-folds, and we have presented 3 plausible mechanisms to accomplish this.

The spectrum of density perturbations that we advocate is characterized by an amplitude of order $10^{-5} - 10^{-4}$ on the large scales, which is about a factor of 3-10 larger than in the conventional one shot scenarios with cold dark matter, but similar to what is needed for hot dark matter, comes close to saturating the large angular scale microwave anisotropy bounds, and should easily produce sufficient large scale structure. On small scales, $M \lesssim M_{seed} \simeq 10^2 - 10^{13} M_\odot$, the amplitude of perturbations is $\simeq 0.01 - 0.1$, which results in the formation of PBHs, early structure formation, the possibility of population III objects, explosive galaxy formation, and in the case of neutrinos as the dark matter, a combined cold/hot scenario. If the second episode of inflation proceeded through the nucleation of bubbles, there is even the possibility that the observed bubble structure in the distribution of galaxies is a direct consequence of these inflation produced bubbles. While some of the simplicity and definiteness of the 'one shot' scenario are sacrificed in double inflation, the returns are very great indeed. In addition, physics at two extremes–the large scale structure in the Universe and the diversity of microphysical phenomena which give rise to inflation, may be telling us that we have to consider multiple inflation.

ACKNOWLEDGMENTS

This work was supported in part by the DOE (at Chicago, Berkeley, and Fermilab), the NASA (at Berkeley and Fermilab), and by the Alfred P. Sloan Foundation.

REFERENCES

1. A. Guth, *Phys.Rev.***D23**, 347(1981); A. Linde, *Phys.Lett.***108B**, 389(1982); A. Albrecht and P.J. Steinhardt, *Phys.Rev.Lett.***48**, 1220(1982). For a recent review and pedagogical discussion of inflation, see M.S. Turner, 'The Inflationary Paradigm', in*Proc. of the Cargese School on Fundamental Physics and Cosmology*, eds. J. Audouze and J. Tran Thanh Van (Editions Frontieres, Gif-Sur-Yvette, 1985), or in *The Architecture of the Fundamental Interactions at Short Distances*, eds. P. Ramond and R. Stora (North-Holland, Amsterdam, 1986).
2. S.W. Hawking, *Phys.Lett.***115B**, 295 (1982); A.A. Starobinskii, *Phys.Lett.***117B**, 175 (1982); A. Guth and S.-Y. Pi, *Phys.Rev.Lett.***49**, 1110(1982); J.M. Bardeen, P.J. Steinhardt, and M.S. Turner, *Phys.Rev.***D28**, 679(1983).
3. P.J. Steinhardt and M.S. Turner, *Phys.Lett.***129B**, 51(1983); D. Seckel and M.S. Turner, *Phys.Rev.***D32**, 3178 (1985); A. Linde, *Phys.Lett.***158B**, 375(1985).
4. N. Bahcall and R. Soneira, *Astrophys.J.***270** 20,(1983).
5. J.M. Bardeen, J.R. Bond, N. Kaiser, and A. Szalay, *Astrophys.J.*, in press(1986).

6. V. deLapparent, M. Geller, and J. Huchra, *Astrophys.J.* **302**, L1(1986).

7. S. Ikeuchi, *P.A.S.J.* **33**, 211(1981); J.P. Ostriker and L.L. Cowie, *Astrophys.J.* **243**, L127(1981).

8. N. Bahcall, in *Galaxy Distances and Deviations from Universal Expansion*, eds. B. Madore and R. Tully, in press(1986); D. Burstein, R. Davies, S. Faber, D. Lynden-Bell, R. Terlevich, and M. Wagner, ibid.

9. M. Aaronson, G. Bothum, J. Mould, J. Huchra, R.A. Schomer, and M.G. Connell, *Astrophys.J.*, in press(1986).

10. J. Uson and D. Wilkinson, *Nature* **312**, 427(1984).

11. M. Davis and P.J.E. Peebles, *Astrophys.J.* **267**, 465(1983).

12. D. Fixsen, D. Chang, and D. Wilkinson, *Phys.Rev.Lett.* **50**, 620(1983); D. Lubin, T. Villela, G. Epstein, and G. Smoot, *Astrophys.J.* **298**, L1(1985).

12a. N. Vittorio and J. Silk, *Astrophys.J.* **293**, L1(1985)

13. M. Davis, G. Efstathiou, C. Frenk, and S.D.M. White, *Astrophys.J.* **292**, 371(1985); C. Frenk, S.D.M. White, and M. Davis, *Astrophys.J.* **271**, 417(1983).

14. For a discussion of the role cosmic strings may play in structure formation, see, e.g., A. Vilenkin, *Phys.Rep.* **121**, 263 (1985); Ya.B. Zel'dovich, *Mon.Not.r.Astron.Soc.* **192**, 663 (1980); A. Albrecht and N. Turok, *Phys.Rev.Lett.* **54**, 1868 (1985); A. Szalay and D. Schramm, *Nature* **314**, 718 (1985); N. Turok, *Phys.Rev.Lett.* **55**, 1801 (1985).

15. Q. Shafi and A. Vilenkin, *Phys.Rev.Lett.* **52**, 691(1984); S.-Y. Pi, *Phys.Rev.Lett.* **52**, 1725(1984).

16. See, e.g., Q. Shafi and C. Wetterich, *Phys.Lett.* **129B**, 387 (1983); **152B**, 51 (1985).

17. For a recent review of the CERN SUSY/SUGR inflation models, see D.V. Nanopoulos,

Comments on Astrophysics **X**, 219 (1985).

18. B. Ovrut and P.J. Steinhardt, *Phys.Rev.Lett.* **53**, 732 (1984); *Phys.Lett.* **147B**, 263 (1984).

19. R. Holman, P. Ramond, and G.G. Ross, *Phys.Lett.* **137B**, 343 (1984).

20. F. Accetta, D. Zoller, and M.S. Turner, *Phys.Rev.* **D31**, 3046 (1985); B.L. Spokoiny, *Phys.Lett.* **147B**, 39 (1984).

21. A. Linde, *Phys.Lett.* **129B**, 177 (1983).

22. P.J. Steinhardt and M.S. Turner, *Phys.Rev.* **D29**, 2162 (1984), or M.S. Turner in ref. 1.

23. For a detailed discussion of the amount of inflation required to solve the horizon and flatness problems, see M.S. Turner, ref. 1.

24. L.P. Grischuk and Ya.B. Zel'dovich, *Astron.Zh.* **44**, 209(1978) [*Sov.Astron.* **22**, 125(1978)].

25. M.S. Turner, *Phys.Rev.* **D28**, 1243 (1983).

26. A. Linde, *Phys. Lett.* **116B**, 335 (1982); A. Vilenkin and L. Ford, *Phys.Rev.* **D26**, 1231 (1983); A. Guth and S.-Y. Pi, *Phys.Rev.*, (1985).

27. J. Ellis, J. Kim, and D.V. Nanopoulos, *Phys.Lett.* **145B**, 181 (1984); L.L. Krauss, *Nucl.Phys.* **B227**, 556 (1983); M.Yu. Khlopov and A.D. Linde, *Phys.Lett.* **138B**, 265 (1984).

28. Inflationary models which employ a potential of the form, $V = m^2\phi^2$, have been discussed by L. Jensen, unpublished(1985) and I.G. Moss, unpublished(1985), and are analyzed by M.S. Turner, ref. 1.

29. J.R. Gott, *Astrophys.J.* **201**, 296 (1975).

30. J.R. Bond, G. Efstathiou, and J. Silk, *Phys.Rev.Lett.* **45**, 1980 (1980).

31. A. Stebbins and J. Silk, *Astrophys.J.* **300**, 1 (1986).

32. M. Bessel and J.S. Norris, *Astrophys.J.* **285**, 622 (1984).

33. F.H. Chaffee, D.B. Folt, J. Bechtold, and R.J. Weymann, *Astrophys.J.* **301**, 116(1986).

34. B.J. Carr, *Astrophys.J.* **201**, 1(1975); B.J. Carr, *Astrophys.J.* **206**, 8 (1976).

34a. T.D. Lee has recently raised the question of whether or not black holes evaporate at all; see, T.D. Lee, "Do Black Holes Radiate as Blackbodies?", Columbia Univ. preprint (1985).

35. As discussed in Sec. II, the end of inflation is followed by a period of time when the energy density of the Universe is dominated by coherent scalar field oscillations, and then finally by reheating when $t \simeq \Gamma^{-1}$. [If Γ is sufficiently large, $\Gamma \gtrsim H$, then the Universe immediately reheats at the end of inflation.] During the coherent oscillation- dominated phase, the energy density of the Universe behaves like a non-relativistic fluid, decreasing as R^{-3}. In discussing PBH formation we have tacitly ignored the coherent oscillation-dominated phase. Whether or not PBHs can form during this phase is an interesting question. If one were very conservative and assumed that they could, then the condition that no PBHs less massive than $10^{15}g$ formed is: $V(\phi_{end})^{1/4} \lesssim 10^{9}GeV$.

35a. N. Kaiser, *Astrophys.J.* **273**, L17(1984).

36. B.J. Carr and S. Ikeuchi, *Mon.Not.r.Astron.Soc.* **213**, 497(1985).

37. E. Bertshinger, *Astrophys.J.* **295**, 1 (1985).

FIGURE CAPTIONS

Figure 1–A schematic representation of the spectrum of primordial density perturbations at the epoch of matter–radiation equivalence, in the double inflation scenario (solid curve), and in the one shot inflation scenario (broken curve). The broken-dotted curve shows the effect of the neutrino damping scale on the large-scale portion of the spectrum. M_{EQ} is the mass scale which enters the horizon at the matter–radiation epoch.