**THEORETICAL AND EMPIRICAL STUDY OF
SINGLE-SUBSTANCE, UPWARD TWO-PHASE
FLOW IN A CONSTANT-DIAMETER
ADIABATIC PIPE**

by

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ABSTRACT

A scheme is developed to describe the upward flow of a two-phase mixture of a single substance in a vertical adiabatic constant area pipe. The scheme is based on dividing the mixture into a homogeneous core surrounded by a liquid film. This core may be a mixture of bubbles in a contiguous liquid phase, or a mixture of droplets in a contiguous vapor phase. Emphasis is placed upon the latter case since the range of experimental measurements of pressure, temperature, and void fraction collected in this study fall in the "slug-churn"- "annular" flow regimes. The core is turbulent, whereas the liquid film may be laminar or turbulent. Turbulent stresses are modeled by using Prandtl's mixing-length theory.

The working fluid is Dichlorotetrafluoroethane $\text{CClF}_2\text{-CClF}_2$ known as refrigerant 114 (R-114); the two-phase mixture is generated from the single phase substance by the process of flashing. Previous studies showed that the Mach, Froude, and Reynolds numbers at the flashing front are the important parameters, particularly the Mach number which can be a measure of the compressibility of the working fluid. In this study, the effect of the Froude and Reynolds numbers on the liquid film characteristics is examined. The compressibility is accounted for through the acceleration pressure gradient of the core and not directly through the Mach number.

An expression for an interfacial friction coefficient between the turbulent core and the liquid film is developed; it is similar to Darcy's friction coefficient for a single phase flow in a rough pipe. The interfacial friction coefficient is a function of liquid film thickness and the well-known

universal constant in single phase flow. For a laminar film, the fully developed equation of motion is used, whereas for a turbulent film, an approach similar to the Karman-Pohlhausen approximation is used. Results indicate that for the range of Reynolds and Froude numbers considered, the liquid film is likely to be turbulent rather than laminar. The study also shows that two-dimensional effects are important, and the flow is never fully developed either in the film or the core. In addition, the new approach for the turbulent film is capable of predicting a local net flow rate that may be upward, downward, stationary, or stalled.

Since we can measure the local void fraction fairly accurately in our facility, we can examine currently used correlations and models for predicting void fraction. Thus, the drift flux model and other previously proposed void fraction correlations are examined. Results indicate that the drift flux model and the first developed void fraction correlation by Lockhart and Martinelli are the best among the examined correlations.

Finally, an actual steam-water geothermal well is simulated; it is based on actual field data from New Zealand. A similarity theory is used to predict the steam-water mixture pressure and temperature starting with laboratory measurements on the flow of R-114. Results indicate that the theory can be used to predict the pressure gradient in the two-phase region based on laboratory measurements.

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NOMENCLATURE

Latin letters

A	angle; proportionality constant
A	area (m^2)
A_c	cross-sectional area of the core (m^2)
A_d	cross-sectional area of the droplets (m^2)
A_g	cross-sectional area of the vapor (m^2)
A_f	cross-sectional area of the film (m^2)
a	coefficient; absolute
a^*	speed of sound at flashing (m/s)
b	coefficient
C	coefficient; constant
C_o	pseudo-slip (Zuber-Findlay distribution parameter)
c	coefficient
c_f	specific heat ($\text{kJ/kg } ^\circ\text{C}$)
c_p	specific heat ($\text{kJ/kg } ^\circ\text{C}$)
D	pipe inner diameter (m)
D_h	hydraulic diameter (m)
D_o	pipe outer diameter (m)
d	coefficient
Fr	Froude number
Fr^*	flash Froude number
f	friction coefficient; function
f_D	dissipation coefficient
f_i, f_j	functions

Gr	Grashof number
g	acceleration of gravity (m/s^2).
h	enthalpy (kJ/kg)
h_c	convective heat transfer coefficient ($\text{W/m}^2 \text{ } ^\circ\text{C}$)
h_{fg}	latent heat of evaporation (kJ/kg)
h_o	stagnation enthalpy (kJ/kg)
K	universal constant
K	proportionality constant in Henry's law
K_A	Armand coefficient
k	conductivity ($\text{W/m } ^\circ\text{C}$)
L	length (m)
l	mixing length (m)
l_c	core mixing length (m)
l_f	film mixing length (m)
l_ρ	mixing length associated with density (m)
l_w	mixing length associated with velocity (m)
Ma*	flash Mach number
m	parameter
$\dot{m}$	total mass flow rate (kg/s)
$\dot{m}_c$	core mass flow rate (kg/s)
$\dot{m}_d$	droplets mass flow rate (kg/s)
$\dot{m}_g$	vapor mass flow rate (kg/s)
$\dot{m}_l$	liquid film mass flow rate (kg/s)
n	parameter; number of moles
P	pressure (Pa)

P	point in space
P_a	normalized acceleration pressure gradient
P_h	normalized gravitational pressure gradient
P^o	normalized total pressure gradient
Pr	Prandtl number
$\dot{q}$	rate of heat transfer (W)
Re	Reynolds number
Re^*	flash Reynolds number
Re_f	liquid film Reynolds number
$\vec{r}$	position vector (m)
r	radial distance (m)
r	ratio
r_i	pipe centerline to interface distance (m)
r_o	pipe inner radius (m)
s	entropy (kJ/kg °C)
s_{fg}	entropy of evaporation (kJ/kg °C)
T	temperature (°C)
ΔT	time interval (s)
t	time (s)
V	volume (m ³)
v	specific volume (m ³ /kg)
v_f	liquid specific volume (m ³ /kg)
v_{fg}	specific volume of evaporation (m ³ /kg)
v_f^*	specific volume at flashing (m ³ /kg)
v_r	reduced volume (v/v_f)

$\vec{w}$	velocity vector (m/s)
w_c	local velocity in the core (m/s)
$\hat{w}_c$	average velocity in the core (m/s)
w_c^*	friction velocity (m/s)
$\hat{w}_d$	average velocity of droplets (m/s)
w_f^*	velocity at flashing (m/s)
$\hat{w}_f$	average velocity of liquid (m/s)
$\hat{w}_g$	average velocity of vapor (m/s)
w_i	interfacial velocity (m/s)
w_l	local velocity in liquid film (m/s)
$\hat{w}_l$	average velocity in liquid film (m/s)
$\hat{w}_r$	relative velocity (m/s)
w_v	mixture volumetric velocity (m/s)
w_{cm}	center of mass velocity (m/s)
w_{gd}	local drift velocity (m/s)
$\hat{w}_{fv}$	liquid velocity relative to the mixture volumetric velocity (m/s)
$\hat{w}_{gv}$	vapor velocity relative to the mixture volumetric velocity (m/s)
w_{sk}	superficial velocity of phase k (m/s)
w_{rms}	root-mean-square velocity (m/s)
x	quality
X	mole fraction
X_M	Lockhart-Martinelli variable
y	distance from the wall (m)
$\bar{y}$	normalized distance from the wall (y/r_o)
Z	parameter

z depth (m)

Greek letters

$\bar{\alpha}_c$ time-averaged void fraction in the core

α_f hold-up

α_g void fraction

β volume expansion coefficient; liquid fraction flowing in the film

δ film thickness

$\bar{\delta}$ normalized film thickness (δ/r_o)

δ' length

Δ difference; linear change

∇ gradient operator

γ density ratio

λ interfacial friction coefficient

μ dynamic viscosity (Pa·s)

ν kinematic viscosity (m²/s)

ψ total mass flux (kg/m²s)

ρ density (kg/m³)

ρ_m In-place density (kg/m³)

σ slip (velocity ratio)

θ constant

τ shear stress (Pa)

τ_i interfacial shear stress (Pa)

τ_w wall shear stress (Pa)

τ_i^o normalized interfacial shear stress

ξ pipe fraction occupied by liquid film

Subscripts

a apparent

c core

CO₂ carbon dioxide

k phase

f liquid

g vapor

H₂O water

i interface

max maximum

meas measured

o stagnation point

or orifice

r radial direction; reference state

res reservoir

s superficial; isentropic

sat saturation

wh well head

z axial direction

Symbols

$\langle \rangle$ space-average

$\langle \rangle_0$ space-average for a homogeneous flow

- time-average
- ~ local value; proportional
- ^ weighted value
- ' fluctuating value
- * flashing condition
- ∞ ambient condition; infinity

INTRODUCTION

The design of equipment in the process industry often involves two-phase flow. A literature survey in two-phase flow [1,2] shows that gas-liquid flow research was concerned primarily with two-phase pressure drop measurements in the period 1940-1950, and later on expanded greatly to include many other problems, especially in the period 1960-1970. The importance of two-phase pressure drop is reflected in the vast number of correlations that are available. The Lockhart-Martinelli correlation [3,4] is one of the best among the examined correlations [5], for all the flow regimes, although it gives no physical insight into the two-phase flow problem *per se*. The Lockhart-Martinelli correlation appears, however, to yield a higher error for one-component flows (i.e., steam-water) than for two-component flows (i.e., air-water).

A parameter equal in importance to the two-phase pressure drop is the void fraction; this has been the subject of a great deal of research. The knowledge of void fraction is important, for example, in the prediction of the slip ratio between the phases and in the calculation of pressure gradient. Most correlations developed for the prediction of void fraction were limited to specific flow regimes. An earlier investigation by Friedel [6] showed that the relationships for the mean void fraction by Hughmark and Rouhani are better than other correlations regardless of the type of fluid and regardless of the flow direction. Recently, Papathanassiou et al. [7] examined several void fraction correlations. They found that these correlations overestimated void fraction measured in the flow of a one-component fluid.

The very complex nature of two-phase flow, which requires extensive experimentation to model the physical problem as closely as possible, has led to

the development of flow maps [8,9]. These flow maps aim to identify flow regimes based on certain measured parameters. These maps show some deficiencies when used to predict flow patterns in other substances using similarity or scaling rules [10,11,12]. However, the Hewitt and Roberts flow map [8] is the most highly recommended to identify flow patterns for air-water and steam-water flows in vertical pipes.

In addition to the large number of correlations and flow maps, analytical work and modeling of multiphase problems contributed a great deal to identify important parameters as well as to understand the physics of two-phase flows. The wealth of contributions from all investigators has led to the publication of several books dealing exclusively with two-phase flows [e.g., 13,14]. While it is difficult to judge the trend that two-phase flow research will follow, it appears that the interest in two-phase flow is currently undergoing active development because of its importance, especially in the design of nuclear reactors. It appears also, that the emphasis in the period 1970-1986 is in the development and the treatment of the local instantaneous equations that govern two-phase flow problems [15-19].

A review article given by Stewart and Wendroff [20] discusses the different models and methods of solution in two-phase flow. Along with attempts at solving the mathematical problems, there have been many studies of measuring techniques used in two-phase flows, as sophisticated as is required by the different mathematical equations that describe a two-phase flow problem. The works of Hewitt et al. [21] and Jones et al. [22] reveal the difficulties due to the presence of the two-phases. Despite these difficulties, many techniques have been developed to measure some important parameters with some degree of accuracy, especially the measurement of void fraction. An excellent review of most instrumentation is given by Hewitt [23], but the

emphasis is on heat transfer because of the applications in the nuclear industry. The type of void fraction measurement required depends on the method of solving the local-instantaneous equations. If, for example, the instantaneous-local conservation laws are written in a differential form and solved numerically, then the solution will yield an instantaneous-local void fraction and may be compared with an experimental void fraction that is measured locally at a given instant of time. If the conservation equations are integrated over the cross-section of a pipe and solved in an integral form instead, then the solution will yield an average void fraction over the cross-sectional area, which may be compared with an experimental void fraction measured over the cross-section of a pipe. In order to tackle the problem, it seems that time and space averaging of the local-instantaneous equations are required chiefly for two reasons: the first reason is to reduce the number of dependent variables in order to obtain a tractable set of equations; the second one is to determine how the experimental measurements are acquired [16,24]. Hence, as far as the measurement of void fraction is concerned, the most widely used techniques are the local [25], line [26], area void fraction [27], and the quick-closing valve [28,29] for volume void fraction. The quick-closing valve is the least expensive and most easily constructed among all those techniques. In addition, experimentation shows that the reproducibility of measurements obtained with that technique is excellent, provided the valve closing time is short and the valves are synchronized properly [29].

Many models have been proposed to describe two-phase flow, and among these models the two-fluid formulation is naturally the most consistent one with the conservation laws. In this model [16,17,30-33] the equations are formulated in terms of different velocity fields for the phases. A study of this

model shows that the interfacial transfer terms are difficult to predict because of the complicated geometry at the interface. One way to eliminate this difficulty is to assume a homogeneous, one-dimensional, uniform-velocity two-phase flow [14]. By doing so, the governing equations can be combined to yield a set of two-phase mixture equations which resemble the usual equations of a single-phase flow. Although the homogeneous model is able to produce satisfactory results at high pressures and velocities, it has its limitations under different conditions. It appears that methods of analysis based on the homogeneous model are no longer relied upon [34] in two-phase flow. Therefore, accounting for the interfacial dynamics between the phases seems to be crucial if the predictions of the two-fluid model are to be of value. Bankoff [35] apparently was the first to account for the slip velocity between the phases by assuming a power law distribution for the velocity and the void fraction. Wallis [36] also correlated the slip at a later time. However, the elegant formulation of Zuber and Findlay [37], the so-called drift-flux model, appeared to be a turning point in two-phase flow research and the work of Ishii [38] in extending the drift-flux model to cover almost all flow regimes is very remarkable. The increased employment of the drift-flux model can be seen also in the work of many researchers [e.g., 39-41], especially in the prediction of nuclear reactor behavior by using large computer codes [42-44]. The most recent work is of Chexal et al. [45], in which drift-flux correlations have been developed without the need to know the flow regime *a priori*.

Most two-phase flows of engineering interest are turbulent. Therefore, a physically realistic model should account not only for interfacial dynamics, but also for the role of turbulent stresses. The previous analytical models which did not include turbulent stresses took this effect somewhat into account through developed semi-empirical correlations. Recently, the predictions of

velocity and void fraction profiles which incorporate the effect of turbulent stresses have shown an improvement in the calculation of the pressure drop in two-phase flows. Apparently, it is Levy [46,47] who attempted to change the approach to two-phase turbulent flow problems. He introduced a model based on Prandtl's mixing-length theory [48] to predict the slip between the two-phases and the pressure drop and by doing so, Levy made a major contribution to the theory of two-phase flow. The mixing-length theory seems to have some success, as reflected in the recent development of mathematical models to reduce the dependence on empirical correlations.

For example, Maeder et al. [49] studied flow regimes in which the liquid phase is the predominate one, i.e., "bubbly" or "slug" regimes. They proposed simple functions for the density distribution in the transverse direction to the flow, in order to model turbulent stresses using the mixing-length theory. They derived the velocity distribution in a differential form and solved it numerically. Although their model contains some adjustable parameters, they were able to show that most of the variation in density occurs very close to the wall.

Recently, Drew et al. [50] studied the effect of the radial distribution of void fraction in steady, fully-developed, adiabatic, air-water bubbly flow at low quality in a vertical pipe. Using the mixing length theory to model the turbulent stresses, they derived a void fraction profile in a differential form. Their predictions of void fraction profile seem to agree with the observed void fraction profiles of Serizawa et al. [51] at low quality ($x < 5 \times 10^{-4}$). However, there is a substantial controversy over the data of Serizawa et al. as indicated in the work of Theofanous et al. [52].

Levy et al. [53] attempted to model a fully developed and adiabatic two-phase annular flow in a horizontal pipe, using the mixing length theory.

In their model they divided the flow pattern into three regions: a film and a core separated by a transition layer. The results obtained in their work do not agree well with experiments. Maeder et al. [49] indicated that this was due to the particular choice of the closure equation for the shear stress which turned out to violate the second law of thermodynamics.

Most recently, Abolfadl et al. [54] developed a model for vertical adiabatic annular two-phase flows, with liquid entrainment, using the mixing-length theory. They also proposed a correlation for the interfacial friction based on a large data bank. A comparison between their predictions and experimental data appears to be in good agreement. In addition and perhaps surprisingly, they used their model to predict the annular flow characteristics at zero wall shear stress, based on the experimental results of Hewitt et al. [55]. However, it is well known that the mixing-length theory is not valid when the wall shear stress is zero, as the so-called friction velocity becomes infinite in this case [56].

The object of this work is to study the characteristics of a single-component two-phase turbulent flow, using the mixing-length theory to shed some light on the physics of the problem and to take an approach that hopefully requires the least dependence on correlations, although admittedly these are unavoidable in two-phase flow problems. An additional objective is to examine different void fraction correlations, particularly the drift flux model because of its wide use, hoping to understand its advantages as well as its deficiencies, and to propose ways of improvement based on the experimental work that has been carried out using a single-component fluid in a vertical constant-diameter test section under adiabatic conditions [57].

It is not the purpose of this work to develop semi-empirical correlations of any kind, but to explore the dynamics of two-phase flow based

on a specific set of experimental measurements.

The report begins with introducing the experimental work in Chapter I; the reason is to provide the methodology of the test plan and explain the observations made in the test section. Very often, these observations motivate an experimentalist or a researcher to develop a scheme that might describe the physics of the problem as closely as possible. Chapter II introduces simply basic definitions that are pertinent to multiphase flow problems and shows how different parameters are calculated from some measured quantities. In Chapter III, two models are developed to describe the physics of two-phase flows. The predictions of the models are compared with experimental data in order to understand their advantages as well as their limitations. Chapter IV discusses some previous correlations that have been developed over the past few years by different researchers for their wide applications. And finally, we present in Chapter V simulation results of an actual geothermal well based on field data obtained in well NG-11 in New Zealand.

We should note that Chapters III, IV and V treat different subjects and are totally independent of each other.

I. THE EXPERIMENTAL FACILITY

The experimental work has been conducted at Brown University where a special facility was constructed for that purpose. A schematic of the facility where the test section is in a vertical position is shown in fig. 1.1. A complete description of the operation of the facility, the data acquisition system, and all instrumentations, such as venturi meters, temperature and pressure probes, are described in detail in [32,33,57]. In the current experimental work, the test section was slightly modified to accommodate quick-closing valves specially designed for the purpose of measuring void fraction. The system operates in an intermittent mode rather than a continuous one. Hence, a typical experiment last about 3 minutes. The working fluid is refrigerant 114 (R-114).

Measurements of temperature and absolute and differential pressure are made along the test section, and the void fraction measurements are made between two fixed locations along the test section. The mass flow rate is measured by a venturi meter located beneath the accumulator. The test section is located between two isolation butterfly valves (BFV-D and BFV-U) as shown in fig. 1.2. When the two butterfly valves are opened, the flow is initiated in the upward direction under the pressure difference between the accumulator and the condenser. The gate valve (GV), located downstream and just beneath the upper butterfly valve, allows the discharge area to vary from zero to that of the test section. At the inlet to the test section, there is a throttle valve (orifice plate) which is an aluminum disc with one hole or a number of holes drilled to provide a specific flow area, as shown in fig. 1.3. When the gate valve is slightly open, the fluid flows as a single substance up to and past the orifice plate and flashes somewhere in the test section to form a two-phase flow as soon as the pressure of the liquid drops to the saturation value

corresponding to the temperature of the fluid. As the gate valve is opened to increase the discharge area, the liquid may flash right at the orifice plate and continues upward as a two-phase flow. Hence, the gate valve can be adjusted to vary the degree of flashing in the test section by changing the back pressure on the throttle plate. Between the two extreme positions of the gate valve, different flows with different topology can be generated in the constant-length test section, that otherwise may occur in different sections along a much longer pipe. So if attention is focused at a fixed point along the test section, one will observe a "bubbly" flow near flashing, that changes to a "slug-churn" flow and ultimately to an "annular" flow as the gate valve is progressively opened from a closed position. When transition occurs to a "slug-churn" flow, the flashing front moves upstream and remains stationary at the orifice plate. In addition to flashing at the orifice plate, the flow is also choked there. This is tested by increasing the discharge area via the gate valve without causing any more flow to be induced through the nozzle located between the lower butterfly valve and the orifice plate.

In previous experiments [32,33], only scans of pressure and temperature were taken along the test section during the three-minute run. A scan is defined as a complete record of all outputs of the test section instruments at equal time intervals. The same method as in [33] is followed here for temperature and pressure measurements. However, with the new quick closing valves in the test section, the test plan was modified to permit the measurement of void fraction several times under the same stagnation conditions and constant mass flow rate.

1.1 Design, Construction and Use of Quick Closing Valves for Void Fraction Measurements

Two quick-closing valves (QCV) for void fraction measurements have been designed, constructed and assembled in our laboratory. The body of the QCVs consists of an outer casing, an outer cover plate, and two internal fittings that sandwich the inner plate which is driven bidirectionally by an air cylinder, as shown in fig. 1.4. The inner diameter of the hole of the inner plate matches the 0.0508 m glass test-section. The QCVs are leak proof. They are installed on the test section and are rigidly mounted against an I-beam as shown in fig. 1.5.

The closing time of the QCVs can be directly controlled by the air supply pressure. The closing time was measured at different air supply pressures prior to installation along the test section. This was accomplished by designing an electrical system which consists principally of two light sources set beneath the opening of the QCVs and two photovoltaic cells atop the same opening. The two cells are electrically connected to an oscilloscope. One cell is used to trigger and the other cell to stop two electrical input pulses. The closing time of the valve is determined by measuring the time interval between the two pulses. The bench test results revealed that the QCVs can be operated with a minimum closing time of 30 milliseconds. In addition, the two valves are checked for proper synchronization during the initial flight of the two inner plates. The maximum time difference between the two flights was four milliseconds.

Following the leak-tightness and speed-of-closing tests, the QCVs were tested on the test section to find the closing time which would minimize the waterhammer problems that might arise from rapid closing and the sudden halt of the flow. Such an over-pressure might cause damage to the instrumentation

or give rise to leaks in the test section. The valves are separated by 0.7 m and the test was conducted with a minimum of instrumentation: one absolute pressure probe was kept in place just below the upstream QCV. The tests indicated that the valves can be activated simultaneously at the minimum closing time without causing excessive over-pressure in the test section. This can be explained by the high compressibility of the working substance when it flows as a two-phase mixture. The vapor beneath the lower QCV acts as a cushion that absorbs the reflected pressure wave caused by the waterhammer effect.

Next, the QCVs were tested for the measurement of the total volume fraction occupied by liquid only between the inner plates. This volume fraction is defined as the hold-up. This is accomplished by initiating a two-phase flow as shown in fig. 1.6. Then, the valves are simultaneously closed; the condition of the flow just after shutting the QCVs is shown in fig. 1.7. And finally, the liquid level in the section between the QCVs is observed after the phases are separated as shown in fig. 1.8. No change in the liquid level was observed during a waiting period of 30 seconds. The hold-up was directly measured on a scale imprinted directly on the glass test section between the QCVs. The repeatability of hold-up measurements was established under conditions where the flow is extremely steady (i.e., when the gate valve is fully open and the flow is in the upper spectrum of an "annular" flow). This was accomplished by repeating the sequence of events shown in figs. 1.6, 1.7, and 1.8 about five times. The repeatability of the five hold-up measurements was excellent. It appears that whatever error is introduced in the hold-up when a mixture enters the lower QCV during the shutting period is cancelled by an opposite error introduced when a mixture leaves the upper QCV. After several preliminary test runs, the validity and the reliability of using the

QCVs for void fraction measurements was established. In order to measure the hold-up more than once under constant stagnation conditions and mass flow rate, a test plan was devised and an appropriate routine adopted as discussed below.

1.2 Test Plan

The test plan consists principally of the following steps:

- 1 - Pump R-114 to the accumulator and establish stagnation conditions.
- 2 - Record 8 scans of temperature and pressure probes with the test section filled with R-114 vapor only. The purpose of this step is for self-referencing and to remove the systematic error of a measurement. A complete description of the self-referencing procedure and accuracy test of the pressure transducers is given in [33].
- 3 - Open the gate valve (located downstream of test section) to a position predetermined to generate a flow with a certain topology and degree of flashing.
- 4 - Open both butterfly isolation valves, thereby initiating the flow.
- 5 - Wait 15 seconds.
- 6 - Record 8 scans of temperature and pressure along the test section and the temperature and pressure drop across the venturi.
- 7 - Close both QCVs and both butterfly valves, thereby halting the flow completely.

- 8 - Observe the liquid level in the section between the QCVs and read the hold-up off the scale.
- 9 - Open the QCVs.
- 10 - Repeat steps 4 to 9 five times.

The whole operation from the onset of preparing a run, execution of the test plan, and the method of data acquisition is completely automated and controlled by a Hewlett-Packard Model 85 desk-top computer. Hence, the computer serves as the controller and the monitor of the facility [32,33,57]. The only external human factor involved in this operation is at step 8 of the test plan.

Four series are conducted in our facility. Series 3-A and 4 are conducted in a vertical 0.0508 m glass test section and series 5 and 6 are conducted in a vertical 0.0254 m copper test section. Measurements of temperature, pressure, and hold-up were taken according to the scheme described above in series 3-A and 4, and measurements of temperature and pressure only were taken along the test section in series 5 and 6. The temperature and absolute pressure are measured at the inlet test section (stagnation point). These measurements indicate that the stagnation point is in a compressed state. Hence, the stagnation enthalpy is calculated at the measured stagnation temperature and pressure. Along the test section, temperature and differential pressures are measured at four stations, 1.248 m apart. The mass flow rate is calculated from a measurement of pressure drop across the venturi meter, using basic continuity and Bernoulli equations. In these equations the density of liquid R-114 is evaluated at the measured temperature in the venturi meter. The hold-up, α_f , is measured using two quick-closing valves at two fixed positions between stations 2 and 3 along the

test section. The basic parameters for series 3-A, 4, 5, and 6 are summarized in Table 1. The flash temperature, T^* , is simply the measured stagnation temperature. The definitions and calculation of the flash Mach number, Ma^* , flash Froude number, Fr^* , and flash Reynolds number, Re^* are given in Section II.

Experimental results showed that the interruption and resumption of the flow did not affect the mass flow rate and the stagnation conditions. In addition, the measurements of temperature and pressure along the test section showed excellent repeatability each time the flow is interrupted and then resumed during the last three sets of eight scan measurements. The first two sets of eight scans (a total of sixteen scans) were rejected since the temperature and pressure measurements represented a transient. Hence, out of five measured hold-ups, the last three readings are considered the true representative measurements.

The repeatability of hold-up measurements depended on the discharge area controlled by the gate valve. When the latter is fully open, the flow is well in the "annular" regime. In this case, the repeatability of hold-up measurements was very high. As the discharge area is reduced as the gate valve is progressively closed, the topology of the flow changed and the flow pattern changed from an "annular" flow to a "slug-churn" flow. However, the "slug-churn" flow regime did not include a series of identifiable vapor plugs. The generation of R-114 vapor rendered the structure of the "slugs" unstable and resulted in their destruction. Therefore, the "churning" flow was accompanied by a periodic flow of slugs of no particular shape. This flow appeared to be quasi-steady as opposed to the steady "annular" flow. So when the QCVs are closed, the hold-up measurement depends on whether or not a slug is caught over the 0.7 m isolated section at that instant of time. This

resulted in an unsatisfactory repeatability of hold-up measurements. For example, of the five hold-up measurements taken during a particular run, the following values were the last three taken: 25%, 29.5%, and 27.6%. The mean hold-up is 27.4% and the standard deviation is 2.3%. As a result, it was required to repeat each experiment at least three times for the same initial conditions and the same discharge flow area. Using this method, an excellent repeatability of hold-up averages was achieved. For example, three runs under the same conditions mentioned above resulted in the following averages: $27.4 \pm 2.3\%$, $27.1 \pm 4.0\%$, and $26.5 \pm 3.7\%$. The true average hold-up was considered to be the mean of the three means, and that is $27.0 \pm 0.8\%$.

Figure 1.9 shows the measured hold-up versus the quality x . The quality x is taken to be the average of two qualities evaluated at the measured temperatures at stations 2 and 3, since the hold-up α_f is measured between those two stations. The quality is calculated from an energy balance for an adiabatic control volume between the stagnation point and one of the four stations along the test section. The quality can be written from a simplified energy equation as

$$x = \frac{h_o - (h_f + gz)}{h_{fg}} \quad (1.1)$$

The application of eqn. (1.1) in this simplified form is examined and justified in [32,33]. The enthalpy of liquid h_f and the latent heat of evaporation h_{fg} are evaluated from properties of R-114 at the measured temperature by assuming thermodynamic equilibrium between the phases. The height z of a particular station from the stagnation point is also known as was reported earlier. The individual hold-up and quality measurements are reported in Table

2. The standard deviations of the qualities are 0.0007 and 0.001 for series 3-A and 4, respectively, unless otherwise noted with each individual quality measurement.

Figure 1.9 shows also how the hold-up decreases as the gate valve was progressively opened. Our experimental observations indicated that above a quality of 0.086, the flow appeared to be steady without any chaotic behavior ("annular" regime). But below a quality of 0.086 a transition occurred to a "churn-slug" flow regime as the gate valve was progressively closed. This was also accompanied by local circulation of the liquid adjacent to the wall and the liquid appeared to come to a halt; then the flow near the wall adjusted itself instantly to the bulk upward flow. This periodic phenomenon occurred during certain runs. In that sense, the flow appeared to be quasi-steady.

Series 3-A was conducted without insulating the test section, and that allowed a complete observation of the flow along the test section; whereas series 4 required insulation for accurate measurements. This decision was based on measured temperature differences between the adiabatic and diathermal cases. In series 3-A the temperature difference was at most 0.5°C , whereas differences as much as 2°C were measured in series 4. The measured temperature of the two-phase mixture in the adiabatic case was higher than that in the diathermal case. This result does not seem to be intuitive at all. A theoretical analysis is carried out to clarify this point and details are given in Appendix A.

One of the uses to which the Two-Phase Flow Facility can be put is the simulation of flow conditions in an actual geothermal well. Similarity analysis between R-114 and steam-water mixtures [12] reveals that it is not possible to achieve exact similarity, but that a reasonably good approximation should be possible. Hence, series 5 and 6 were conducted to confront the

similarity theory with two sets of measurements: one from an actual flowing geothermal well and one from the test facility, configured to match the well as closely as possible according to the theory.

A considerable modification of the test facility was required. The entire glass test section was replaced with a commercially available 0.0254 m copper tube. The test section was completely insulated and additional probes were added along the test section to permit an accurate determination of the flash horizon since this could no longer be ascertained visually. Similarity analysis [12] showed also that the mass flow rate had to be reduced drastically. This necessitated the fabrication of a new venturi flow meter. The new one was similar to the previous venturi meters used for different mass flow rates [32-33]. The mass flow rate measured by the new venturi flow meter was checked against the flow rate calculated from the level change of R-114 in the accumulator which is maintained under a constant pressure supplied by the booster (fig. 1.1). The discrepancy was less than 0.8% and no correction from the standard ASME flow formula was required.

II. AVERAGING AND BASIC DEFINITIONS

In two-phase flow, some averaging is required as in single-phase flows to replace a velocity distribution in one or two directions. This averaging will be done in the following way.

Let us consider a point P in space characterized by a phase function $\tilde{\alpha}$ defined as follows:

$$\tilde{\alpha}_k(\vec{r}, t) \equiv \begin{cases} 0 & \text{if P is not in phase k} \\ 1 & \text{if P is in phase k.} \end{cases} \quad (2.1)$$

The time-averaged fraction occupied by phase k at point P is defined as

$$\bar{\alpha}_k(\vec{r}) = \frac{1}{T} \int_T \tilde{\alpha}_k dt. \quad (2.2)$$

In a turbulent flow the time interval T should be larger than the turbulence time scale and bounded by an upper limit time such that the regime under consideration is stable.

If we consider any parameter Z associated with a phase k (vapor or liquid), its time-average has to be modified from what has been done in turbulent, single-phase flows. In the latter case the time-average of some function f is given by

$$\bar{f} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T \tilde{f} dt. \quad (2.3)$$

In single-phase flow the point at which $\tilde{f}$ is measured is always occupied by the same phase, whereas in a two-phase flow that point in space may be occupied by either vapor or liquid. Therefore, the time interval over which

a parameter Z is being averaged should be the sub-interval over which phase k occupied point P , i.e.,

$$\bar{Z}_k = \frac{\frac{1}{T} \int_T \bar{\alpha}_k \tilde{Z}_k dt}{\frac{1}{T} \int_T \bar{\alpha}_k dt}, \quad (2.4)$$

where $\tilde{Z}_k = \tilde{Z}_k(\vec{r}, t)$ is the instantaneous value of the parameter Z at point P .

From the Reynolds decomposition used in turbulent single-phase flows, it is known that if a function $\tilde{f}_i$ is decomposed into a mean value $\bar{f}_i$ and a fluctuating term f_i' , we get

$$\overline{f_i'} = \overline{\tilde{f}_i - \bar{f}_i} = 0, \quad (2.5)$$

and

$$\overline{\tilde{f}_i \tilde{f}_j} = \bar{f}_i \bar{f}_j + \overline{f_i' f_j'}. \quad (2.6)$$

Now eqn. (2.4) can be written as

$$\overline{\tilde{\alpha}_k \tilde{Z}_k} = \bar{\alpha}_k \bar{Z}_k. \quad (2.7)$$

From a comparison of eqn. (2.7) with eqn. (2.6), it appears that the term $\overline{\tilde{\alpha}_k' Z_k'} = 0$. This can be true if one assumes ideally that the phase function $\tilde{\alpha}_k$ is purely a step function without fluctuations. Therefore, in two-phase flow $\bar{Z}_k$ is not necessarily equal to zero as in single-phase flow and that will create problems in the solution of the set of equations with turbulent terms. The next step is to space-average the previously time-averaged quantities; this will depend on the measuring technique.

In the case of the quick-closing valve the space-average fraction occupied by phase k should be defined as

$$\alpha_k \equiv \langle \bar{\alpha}_k \rangle = \frac{1}{V} \int_V \bar{\alpha}_k dV \quad (2.8)$$

where V is the total volume of the domain between the two quick-closing valves. If the quick-closing valves are not too far apart and if that the void fraction profile does not change between them, then α_k can be approximated by

$$\alpha_k \equiv \langle \bar{\alpha}_k \rangle = \frac{1}{A} \int_A \bar{\alpha}_k dA \quad (2.9)$$

where A is the area of the domain (i.e., the cross-sectional area). The average value of $\bar{Z}_k(\vec{r})$ over a cross-section is defined as

$$\hat{Z}_k = \frac{\frac{1}{A} \int_A \bar{\alpha}_k \bar{Z}_k dA}{\frac{1}{A} \int_A \bar{\alpha}_k dA} = \frac{\langle \bar{\alpha}_k \bar{Z}_k \rangle}{\langle \bar{\alpha}_k \rangle} \quad (2.10)$$

It should be noted that in eqn. (2.10), $\langle \bar{\alpha}_k \rangle$ and $\hat{Z}_k$ are assumed to be uncorrelated in space.

Using the previous definitions, we can write the continuity equation for phase k as

$$\langle \psi x_k \rangle = \langle \alpha_k \rho_k w_k \rangle \quad (2.11)$$

The overscore has been dropped for convenience. If we assume a constant mass flux ψ and a constant ρ_k at a cross-section, the continuity equation can be written as

$$\frac{\psi}{\rho_k} \langle x_k \rangle = \psi v_k \langle x_k \rangle = \langle \alpha_k w_k \rangle \quad (2.12)$$

If we sum over both phases, we get

$$\psi \sum_{k=1}^2 v_k \langle x_k \rangle = \sum_{k=1}^2 \langle \alpha_k w_k \rangle \quad (2.13a)$$

or

$$\psi v = \langle w_v \rangle, \quad (2.13b)$$

where

$$v = \sum_{k=1}^2 v_k \langle x_k \rangle, \quad (2.14)$$

and

$$\langle w_v \rangle = \sum_{k=1}^2 \langle \alpha_k w_k \rangle \quad (2.15)$$

At the cross-section where flashing occurs, the void fraction approaches zero and the mixture volumetric velocity approaches the liquid velocity. Therefore, eqn. (2.13b) becomes

$$\psi v_f^* = \langle w_f^* \rangle \quad (2.16)$$

At the flashpoint, denoted by the symbol "*", the mixture is assumed to be a saturated liquid. From eqns. (2.13) and (2.16), we can form a reduced volume as the ratio

$$v_r = \frac{v}{v_f} = \frac{v_f^*}{v_f} \frac{\langle w_v \rangle}{\langle w_f^* \rangle} \quad (2.17)$$

Later on, the reduced volume v_r appears as the independent variable in some graphs. Some authors use $\langle w_v \rangle$ instead. The two ways are equivalent as seen from eqn. (2.17). Traditionally, $\langle w_v \rangle$ is chosen as an independent parameter because it can be measured accurately from a knowledge of the mass flow rates of the individual phases; and in most practical cases v_f^*/v_f is very close to unity.

Using eqn. (2.10) we can write

$$\langle \alpha_k w_k \rangle = \langle \alpha_k \rangle \hat{w}_k \quad (2.18)$$

Therefore, eqn. (2.12) becomes

$$\psi \langle x_k \rangle = \rho_k \hat{w}_k \langle \alpha_k \rangle \quad (2.18a)$$

For the vapor phase this becomes,

$$\psi \langle x \rangle = \rho_g \hat{w}_g \langle \alpha_g \rangle, \quad (2.19)$$

and for the liquid phase this becomes,

$$\psi \langle 1-x \rangle = \rho_f \hat{w}_f \langle \alpha_f \rangle \quad (2.20)$$

Hence, from eqns. (2.19) and (2.20) we can form the ratio

$$\sigma = \gamma \frac{\langle x \rangle}{1-\langle x \rangle} \frac{\langle \alpha_f \rangle}{\langle \alpha_g \rangle} \quad (2.21)$$

where,

$$\langle \alpha_g \rangle = 1 - \langle \alpha_f \rangle \quad (2.22)$$

is the void fraction.

$$\gamma = \rho_f / \rho_g \quad (2.23)$$

is the liquid-to-vapor density ratio, and

$$\sigma = \hat{w}_g / \hat{w}_f \quad (2.24)$$

is the velocity or slip ratio of the phases.

At flashing, there are three important parameters [33]. These are the flash Froude number

$$Fr^* = \frac{\langle w_f^* \rangle}{(gD)^{1/2}}, \quad (2.25)$$

the flash Reynolds number

$$Re^* = \frac{\rho_f^* \langle w_f^* \rangle D}{\mu_f^*} = \frac{\psi D}{\mu_f^*}, \quad (2.26)$$

and the flash Mach number

$$Ma^* = \frac{\langle w_f^* \rangle}{a^*}, \quad (2.27)$$

Next let us clarify how other parameters can be calculated from the actual measurements mentioned in Section I.

From the measurement of hold-up α_f , the void fraction α_g is simply determined from eqn. (2.22). From a knowledge of hold-up and void fraction and the average quality x , the vapor velocity $\hat{w}_g$ and liquid velocity $\hat{w}_f$ are calculated from eqns. (2.19) and (2.20), respectively. Hence the slip ratio can be calculated easily from eqn. (2.24). In eqns. (2.19) and (2.20) the vapor density ρ_g and liquid density ρ_f are evaluated at an average temperature measured at the same section where the hold-up is known. The mass flux ψ is calculated from the measured mass flow rate using the venturi meter and the cross-sectional area of the pipe.

Knowing the above parameters, we can easily calculate the mixture volumetric velocity w_v from eqn. (2.13b) where the mixture specific volume v is calculated from eqn. (2.14). From the calculated mixture specific volume v and the liquid density ρ_f , where $\rho_f = 1/v_f$, the reduced volume can then be calculated as defined in eqn. (2.17).

At the flashpoint, the flash temperature is considered to be the same as the stagnation temperature in the adiabatic test section. Hence, the specific volume of liquid at the flashpoint v_f^* can be determined from thermodynamic properties of R-114. The velocity of liquid at the flashpoint w_f^* can be calculated from eqn. (2.16) and the flash Froude number Fr^* becomes known from eqn. (2.25). The flash Reynolds number Re^* becomes also known from eqn. (2.26) in which the dynamic viscosity of R-114 is evaluated at the flash temperature.

The sonic velocity at the flashpoint a^* is evaluated under the assumption that the mixture is homogeneous (i.e., the slip ratio σ is unity). The sonic velocity is defined as

$$a_f^* \equiv \left[\frac{\partial P}{\partial \rho} \right]_{s, \text{sat}}^{1/2} \quad (2.28)$$

Maeder et al. [58] give an expression for the sonic velocity for the case of liquid-vapor, two-phase equilibrium systems; and in the case of saturated liquid ($x \rightarrow 0$), the sonic velocity is given by

$$a_f^* = h_{fg}^{1/2} \frac{v_f}{v_{fg}} \left[\frac{s_{fg}}{c_{p,f}} \right]^{1/2} \quad (2.29)$$

where the thermodynamic properties on the right-hand side of eqn. (2.29) are evaluated at the flash temperature. Hence, the flash Mach number Ma^* can be evaluated from its definition in eqn. (2.27).

The calculated slip ratio σ is presented vs. the reduced volume v_r for series 3-A and 4 in figs. 2.1 and 2.2, respectively. Usually, the flow develops from a "bubbly" flow near flashing, where the reduced volume v_r has a value of 1, to a "slug-churn" flow and ultimately to an "annular" flow. In series 3-A the "bubbly" flow near flashing could not be achieved under the experimental conditions available. In fig. 2.1 our experimental observations indicated the appearance of periodic slugs of no particular shape below $v_r = 16$. It is interesting to note in fig. 2.1. that within the "slug-churn" and "annular" flow regimes, the slip ratio tends to reach a maximum as v_r increases. As the mass flow rate is reduced from 2.10 kg/s in series 3-A to 1.05 kg/s in series 4, the slip ratio appears to steadily increase on the average with v_r . The slip ratio exhibits also a higher scatter in fig. 2.2 than in fig. 2.1. In fig. 2.2 the

flow is "bubbly" at approximately $v_r < 5$ and above that, a transition occurs to a "churn" flow.

It is interesting that the behavior of slip ratio in figs. 2.1 and 2.2 was also observed by Agostini et al. [29], in their density measurements of steam-water mixtures flowing in a tubular channel under adiabatic conditions.

And finally, we should indicate that Hewitt and Roberts' flow map [8] can be used to identify flow regimes. This map is mainly developed to be used for air-water and steam-water mixtures; and the coordinate variables that appear in this map must be evaluated using air-water or steam-water properties. Since the fluid used in our facility is R-114, then a similarity between R-114 and steam-water must be established to find the steam-water map coordinates that are equivalent to a liquid-vapor mixture of R-114, e.g., [12]. If this similarity analysis is carried out, then the Hewitt and Roberts' flow map [8] predicts that series 3-A falls into the "wispy-annular" regime, whereas series 4 falls into the "annular" regime except for points at $v_r < 5$. The "wispy-annular" regime is characterized by clouds of liquid droplets moving within the central vapor core as shown in fig. 2.3. In either series 3-A or 4, the Hewitt and Roberts' flow map does not predict any "slug-churn" flow which was clearly observed experimentally in the glass test-section in our facility.

III. MODELING OF UPWARD TWO-PHASE FLOW

3.1 General Physical Description

The motivation behind the study of two-phase upward flow in a constant area pipe stems from the interest of pressure drop predictions in two-phase flows of steam-water encountered in geothermal wells. The single-phase fluid, initially in a compressed state (stagnation condition), flashes as soon as the pressure drops to the saturation value corresponding to the saturation temperature of the fluid. Usually, nucleation sites such as deposits, which effectively reduce the flow area and thus cause a drop in pressure, enhances flashing.

Upon flashing a "bubbly" flow is formed, where bubbles travel at velocities higher than the adjacent fluid, but these velocities have usually comparable magnitudes. The "bubbly" flow is characterized by dispersed bubbles in a contiguous liquid medium, which may move with or without coalescence. Without coalescence, the "bubbly" flow is usually stable. But as soon as the bubbles begin to coalesce, larger bubbles in the form of plugs begin to form, and a transition is accompanied from the "bubbly" regime to a new one, called "slug" or "plug" flow. The criterion for the occurrence of such a transition has been studied by many researchers [i.e., 59]. The "plug" flow is characterized by elongated, bullet-like bubbles called Taylor bubbles. The diameter of such bubbles approaches that of the pipe. Liquid film appears to be flowing downward on the sides of the bubble, although the net flow of liquid and vapor may be upward. In the wake of the vapor bubble, smaller bubbles are formed. As mentioned in [59], the "plug" flow may become unstable due to either the coalescence of the plug bubbles, or due to the instability of the liquid film [60,61].

When the plugs are disrupted, a transition occurs to an "annular" flow. The transition is called "churn" flow, and is characterized by an oscillatory motion. In the "annular" flow, the liquid flows as a film on the wall, if the pipe is adiabatic, and the vapor moves in a central core carrying with it droplets at practically the same velocity. The droplets in the core and the liquid film form the discontinuous phase, while the vapor phase is contiguous. Usually, the velocity of the fluid in the core is much greater than that of the surrounding liquid film. In addition, the interface between the film and the core is characterized by a wavy surface of different amplitudes. The wavy interface is sheared off by the vapor and that leads to the entrainment of droplets in the vapor core [14,62,63].

The mechanism of entrainment can occur in many ways. The waves may be sheared-off at the top if they travel in the same direction as the vapor, or may be undercut if the waves travel in an opposite direction to the vapor. Entrainment can also take place due to bubbles bursting at the surface of the liquid film. Along with entrainment, droplets may impinge from the vapor core onto the film. These mechanisms also take place in a two-component two-phase flow, such as an air-water mixture. However, the phenomenon of entrainment is more complicated in a single-component two-phase flow such as steam-water or freon, because vapor condensation can result in liquid in the form of droplets or a film; and liquid evaporation from the film and droplets can result in more vapor in the core.

3.2 Physical Modeling: The Misty Core/Liquid Film Model

In the present case, the development of a model was enhanced by the direct observations of the two-phase flow through a transparent glass test section. The ability of the model in describing the physics of two-phase flow

will be tested on the grounds of direct measurements of pressure, temperature and hold-up. The two-phase flow of a single component is upward in a constant-diameter, adiabatic pipe. The model describes a flow configuration which consists of a liquid film surrounding a core of liquid and vapor; hence, the inner wall of the pipe is assumed to be always wet. The core can be visualized as consisting of dispersed bubbles in a contiguous liquid phase, or dispersed droplets in a contiguous vapor phase. In the case of dispersed bubbles in the core, the liquid film is part of the contiguous liquid phase; moreover, the local density decreases slowly from the liquid density at the interface to the local mixture density at the pipe centerline, as shown in fig. 3.1. However, in the case of dispersed droplets in a vapor core, the film is part of the discontinuous liquid phase, and the density decreases rather sharply from the average liquid density at the interface to a characteristic average core density, as shown in fig. 3.2.

3.3 Assumptions, Mathematical Tools and Remarks

In the following, the governing equations needed to describe the foregoing model will be developed from first principles on the basis of the following assumptions:

1. The film thickness will be assumed of constant thickness δ in this model, as shown in fig. 3.3, in the sense that it is a time- and space-averaged thickness, i.e.,

$$\delta = \frac{\int \int \delta(\vec{r}, t) \, d\vec{r} \, dt}{\int \int d\vec{r} \, dt} \quad (3.1)$$

In an actual two-phase flow, the liquid film thickness varies in space and time. Sometimes large-amplitude, rolling waves at the interface may well reach to regions near the centerline of the pipe. So without a constant film thickness δ as defined in eqn. (3.1) the solution of the problem becomes extremely difficult.

2. The flow is axisymmetric and one-dimensional; i.e., the azimuthal velocity w_ϕ and the radial velocity w_r are zero.

The assumption of an axisymmetric, one-dimensional flow in a pipe is of practical importance in many engineering problems. In single-phase pipe flow, the analytical expressions based on that assumption have been used successfully in many design problems and agree very well with measurements. The assumption of zero radial velocity is justified in the next assumption.

3. The flow is piece-wise frozen; i.e., there is no mass transfer between the phases over a small length along the test section. This assumption is approximately valid if over that small length of the test section the gradient of the quality dx/dz is small.

Usually when there is mass transfer between the phases, there is a velocity normal to the interface between the phases, either between the film and the core or between the droplets and the vapor phase within the core. In addition, there could be another radial velocity w_r associated with the droplets that may be sheared-off at the film-core interface, or may be deposited from the core onto the film. Hence, these velocities give rise to a momentum exchange at the interface. This momentum exchange has to be accounted for in modeling the turbulent stresses using the mixing-length theory, a difficult problem in this case. This is mainly due to the fact that there is more than one characteristic velocity in the liquid film. These include the average velocity of liquid film in the flow direction, the mass transfer radial velocity

at the interface and the shearing-off-deposition radial velocity of droplets. In such a case where there is more than one velocity scale, the mixing-length theory may become inconsistent as discussed by Tennekes et al. [64] for the case of mass transfer near a rigid wall. Grover [65] uses the mixing-length theory and allows for mass transfer without considering additional velocity scales, but his model is mostly empirical and has no theoretical justification. Therefore, a correct description of the physics of an axisymmetric, one-dimensional, two-phase flow using the mixing-length theory can only be accomplished within the limitations of the above assumptions, and by neglecting any momentum exchange associated with radial velocity mass transfer or shearing-off-deposition at the interface.

4. If in addition to neglecting the radial velocity we neglect the relative velocity of droplets with respect to the vapor in the flow direction, then the flow in the core may be treated as homogeneous, i.e., there is no local slip between the phases in the core. The modeling based on homogeneous core allows us to treat it like single-phase flow.

5. The liquid phase is incompressible, whereas the core is compressible and turbulent.

If we examine the flash Reynolds number Re^* for series 3-A and 4 in Table 1, we find that the liquid at the flashpoint is already turbulent. As soon as the liquid flashes in the pipe, the velocity of the generated vapor increases and the vapor phase is expected to be turbulent even in the presence of liquid droplets in the core. As the vapor expands or accelerates in the flow direction, it is expected that the vapor phase to be compressible.

Nikitopoulos et al. [33] showed that compressibility can result from phase change or can be attributed to the presence of the individual phases. In most practical cases, the former case is the dominant one. But, since we

assumed a piece-wise frozen flow, the core is compressible because of expansion due to the pressure drop. Hence, the flow may not be fully developed because it is constantly accelerating.

As far as the liquid flowing in the film is concerned, it may be laminar or turbulent. Both cases will be studied later on.

6. The temperature at any cross-section is constant; otherwise, the physics of the problem can be needlessly obscured and very complicated if the temperature is allowed to vary radially at a cross-section of the pipe.

The conservation laws are written next using the cylindrical coordinates as shown in fig. 3.4. The continuity equation is given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z} (\rho w_z) + \frac{1}{r} \frac{\partial}{\partial r} (\rho r w_r) = 0, \quad (3.2)$$

where the terms involving radial velocity are retained for the time being. The equation of motion is given by

$$\rho \left(\frac{\partial w_z}{\partial t} + w_r \frac{\partial w_z}{\partial r} + w_z \frac{\partial w_z}{\partial z} \right) = \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rz}) + \frac{\partial}{\partial z} \tau_{zz} - \frac{\partial P}{\partial z} - \rho g, \quad (3.3a)$$

in the z-direction, and by

$$\rho \left(\frac{\partial w_r}{\partial t} + w_r \frac{\partial w_r}{\partial r} + w_z \frac{\partial w_r}{\partial z} \right) = \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rr}) + \frac{\partial}{\partial z} \tau_{rz} - \frac{\partial P}{\partial r}, \quad (3.3b)$$

in the r-direction, where

$$\tau_{rz} = \mu \left[\frac{\partial w_z}{\partial r} + \frac{\partial w_r}{\partial z} \right] , \quad (3.3c)$$

$$\tau_{zz} = \mu \left[2 \frac{\partial w_z}{\partial z} - \frac{2}{3} \nabla \cdot \vec{w} \right] , \quad (3.3d)$$

and

$$\tau_{rr} = \mu \left[2 \frac{\partial w_r}{\partial r} - \frac{2}{3} \nabla \cdot \vec{w} \right] , \quad (3.3e)$$

where

$$\nabla \cdot \vec{w} = \frac{1}{r} \frac{\partial}{\partial r} (r w_r) + \frac{\partial w_z}{\partial z} . \quad (3.3f)$$

Let us consider, in general, a compressible turbulent flow and assume that the temperature does not fluctuate, which means that the physical properties of the fluid, including the viscosity, do not fluctuate. Let

$$w_z = \bar{w}_z + w'_z . \quad (3.4a)$$

$$w_r = w'_r . \quad (3.4b)$$

$$\rho = \bar{\rho} + \rho' . \quad (3.4c)$$

$$P = \bar{P} + P' . \quad (3.4d)$$

Note that

$$\bar{w}_r = 0 . \quad (3.4e)$$

If we combine eqns. (3.2), (3.3), and (3.4) and neglect triple correlations, we get after some algebraic manipulations

$$\frac{\partial}{\partial z} (\bar{\rho} \bar{w}_z) + \frac{1}{r} \frac{\partial}{\partial r} \left[r \overline{\rho' w'_r} \right] + \frac{\partial}{\partial z} (\overline{\rho' w'_z}) = 0 , \quad (3.5a)$$

for the continuity equation;

$$\frac{\partial}{\partial t} (\overline{\rho' w_z'}) + \frac{\partial}{\partial z} (\bar{\rho} \bar{w}_z'^2) = \frac{1}{r} \frac{\partial}{\partial r} [r(\bar{\tau}_r + \tau_r')] + \frac{\partial}{\partial z} (\bar{\tau}_z + \tau_z') - \frac{\partial \bar{P}}{\partial z} - \bar{\rho} g \quad (3.5b)$$

for the equation of motion in the z-direction, and

$$\frac{\partial}{\partial t} (\overline{\rho' w_r'}) + \frac{1}{r} \frac{\partial}{\partial r} [r \bar{\rho} \bar{w}_r'^2] = -\frac{2}{3} \frac{1}{r} \frac{\partial}{\partial r} (r \bar{\tau}_z) + \frac{\partial}{\partial z} (\bar{\tau}_r + \tau_r') - \frac{\partial \bar{P}}{\partial r}, \quad (3.5c)$$

for the equation of motion in the r-direction; where

$$\bar{\tau}_r = \mu \frac{\partial \bar{w}_z}{\partial r}, \quad (3.5d)$$

$$\bar{\tau}_z = \mu \frac{\partial \bar{w}_r}{\partial z}, \quad (3.5e)$$

$$\tau_r' = -[\bar{\rho} \overline{w_r' w_z'} + \bar{w}_z \overline{\rho' w_r'}], \quad (3.5f)$$

$$\text{and} \quad \tau_z' = -[2 \bar{w}_z \overline{\rho' w_z'} + \bar{\rho} \overline{w_z'^2}]. \quad (3.5g)$$

The laminar viscous stresses are $\bar{\tau}_r$ and $\bar{\tau}_z$; τ_r' and τ_z' are the turbulent stresses which arise from the combined effect of velocity fluctuations $w_r' w_z'$, and $w_z'^2$ products, and the velocity and density fluctuation products, $\rho' w_r'$ and $\rho' w_z'$.

An order of magnitude analysis shows that eqns. (3.5a)-(3.5c) reduce to

$$\frac{\partial}{\partial z} (\bar{\rho} \bar{w}_z) + \frac{1}{r} \frac{\partial}{\partial r} [r \bar{\rho}' w_r'] = 0, \quad (3.6a)$$

$$\frac{\partial}{\partial z} (\bar{\rho} \bar{w}_z^2) = \frac{1}{r} \frac{\partial}{\partial r} [r(\bar{\tau}_r + \tau_r')] - \frac{\partial \bar{P}}{\partial z} - \bar{\rho} g, \quad (3.6b)$$

$$\text{and} \quad \frac{1}{r} \frac{\partial}{\partial r} [r \bar{\rho} \bar{w}_r'^2] = - \frac{\partial \bar{P}}{\partial r}. \quad (3.6b)$$

But

$$\left| \frac{\partial \bar{P}}{\partial r} \right| < < \left| \frac{\partial \bar{P}}{\partial z} \right|,$$

in most pipe flows; therefore, we can neglect pressure variation at a cross-section in the pipe. This, along with Assumption 6 of a constant temperature at a cross-section, establishes thermodynamic equilibrium between the phases if the pressure corresponds to the saturation temperature of the fluid, and implies also that the surface tension is negligible. These assumptions have to be relaxed near flashing, where nucleation is the dominant factor, and the bubbles are much smaller than those encountered in a normal "bubbly" flow.

In turbulent flows the modeling of turbulent stresses remains the most difficult to achieve. However, in many engineering applications, particularly in pipe flows, the mixing-length theory proves to be very successful and predicts two-phase pressure drops satisfactorily [66]. Using Prandtl's mixing-length theory [67], and following the work of Maeder et al. [49], the components of the turbulent stress τ_r' are written as:

$$\overline{\rho w_r' w_z'} = - \bar{\rho} l_w^2 \left| \frac{\partial \bar{w}_z}{\partial r} \right| \frac{\partial \bar{w}_z}{\partial r}, \quad (3.7a)$$

and

$$\overline{w_z \rho' w_r'} = - \bar{w}_z l_w l_\rho \left| \frac{\partial \bar{\rho}}{\partial r} \right| \frac{\partial \bar{w}_z}{\partial r}. \quad (3.7b)$$

If we assume, as a first hypothesis, that the mixing lengths l_w and l_ρ are equal, then τ_r' becomes

$$\tau_r' = l^2 \left[\bar{\rho} \left| \frac{\partial \bar{w}_z}{\partial r} \right| + \bar{w}_z \left| \frac{\partial \bar{\rho}}{\partial r} \right| \right] \frac{\partial \bar{w}_z}{\partial r}. \quad (3.8)$$

Now, we will consider the case that corresponds to an annular flow, fig. 3.2. In this model, the velocity is continuous, but the density is discontinuous at the interface between the liquid film and the core. In addition, as shown in fig. 3.2,

$$\frac{\partial \bar{\rho}}{\partial r} \approx 0; \quad r \neq r_i. \quad (3.9)$$

That is, the density is constant in either the film or the core. Hence, eqn. (3.8) becomes in this case

$$\tau_r' = \bar{\rho} l^2 \left| \frac{\partial \bar{w}_z}{\partial r} \right| \frac{\partial \bar{w}_z}{\partial r}. \quad (3.10)$$

Equation (3.10) simply reduces to eqn. (3.7a), but without making the assumption of an incompressible core. At the interface $r = r_i$, $\tau_r' \rightarrow \infty$ as

$\partial \bar{\rho} / \partial r \rightarrow \infty$. Hence, the only way to remove this singularity at the interface in this model is by setting $\bar{\rho} = 0$ in eqn. (3.7b) which means that fluctuations in densities are dropped, and the turbulent stresses in a turbulent film and the core arise simply from velocity fluctuations only.

3.4 The Interfacial Friction Coefficient

The presence of the liquid film has an effect on the turbulent core boundary layer at the interface, similar to the roughness effect of a wall on a turbulent boundary layer in single-phase flow. Through a dimensional analysis it is well known in turbulent single-phase flow that the pressure loss or the wall shear stress is proportional to the dynamic pressure. The proportionality factor is the well known Darcy's friction factor which is usually dependent on the Reynolds number and the roughness of the pipe. In a similar way, a proportionality factor, the interfacial friction coefficient between the interfacial shear stress and some characteristic dynamic pressure may be established. This may serve as a closure for the interfacial shear stress and may be incorporated in the solution of equations of motion, in which the interfacial shear stress appears by applying the boundary conditions at the interface as will be seen later on.

Let us consider the equation of motion

$$\frac{\partial}{\partial z} (\bar{\rho} \bar{w}_z^2) + \frac{\partial \bar{P}}{\partial z} + \bar{\rho} g = \frac{1}{r} \frac{\partial}{\partial r} [r(\bar{\tau}_r + \tau_r')]. \quad (3.6b)$$

The continuity equation reduces to

$$\frac{\partial}{\partial z} (\bar{\rho} \bar{w}_z) = 0, \quad (3.11)$$

because in eqn. (3.6a) the term

$$\overline{\rho' w_r'} = 0 .$$

based on eqn. (3.9). The equation of motion can be written as

$$\bar{\rho} \bar{w}_z \frac{\partial \bar{w}_z}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} (r \tau) - \frac{d\bar{P}}{dz} - \bar{\rho} g , \quad (3.12)$$

where

$$\tau = \bar{\tau}_r + \tau_r' . \quad (3.12a)$$

If we assume that in the core

$$\bar{\rho} \bar{w}_z \frac{\partial \bar{w}_z}{\partial z} \approx \bar{\rho}_c \hat{w}_c \frac{\Delta \hat{w}_c}{\Delta z} , \quad (3.13)$$

then upon integrating eqn. (3.12) within the core, an expression for the interfacial shear stress can be written as

$$\tau_i = \frac{r_i}{2} \left[\frac{d\bar{P}}{dz} + \bar{\rho}_c g + \bar{\rho}_c \hat{w}_c \frac{\Delta \hat{w}_c}{\Delta z} \right] . \quad (3.14)$$

Usually, the dynamic and the hydrostatic pressure gradients in the core are smaller than the magnitude of the static pressure gradient. Hence τ_i is negative in eqn. (3.14). So, let us define

$$\tau_i = -\frac{1}{2} \lambda \bar{\rho}_c |\hat{w}_c - \bar{w}_i| [\hat{w}_c - \bar{w}_i] , \quad (3.15a)$$

and if, $\hat{w}_c > \bar{w}_i$, then

$$|\tau_i| = \frac{1}{2} \lambda \bar{\rho}_c [\hat{w}_c - \bar{w}_i]^2, \quad (3.15b)$$

where

$$\hat{w}_c = \frac{\int_0^{r_i} \bar{w}_c dr}{\int_0^{r_i} dr}. \quad (3.15c)$$

We should note that eqn. (3.15c) is compatible with eqn. (2.10) if the time-averaged void fraction in the core $\bar{\alpha}_c$ is constant (the void fraction in the core is simply the area of vapor per unit area of the core at a cross-section). However, the core void fraction $\bar{\alpha}_c$ is constant because the core density which is defined for a homogeneous core as

$$\bar{\rho}_c = \bar{\alpha}_c \bar{\rho}_g + [1 - \bar{\alpha}_c] \bar{\rho}_f \quad (3.16)$$

is assumed to be uniform at a cross-sectional area in our model as shown in fig. 3.2, and since the vapor density, $\bar{\rho}_g$, and liquid density, $\bar{\rho}_f$, are a function of temperature only which is also assumed to be uniform at a cross-sectional area.

The effect of the wavy film is primarily at the interface, which acts to destabilize the viscous sublayer in the core, and results in an effectively thinner sublayer. Moreover, if the liquid film flow is laminar, then the core boundary layer will consist of a transition layer and a completely rough regime. In this case, the interfacial friction coefficient λ will depend on the Reynolds number of the core, and on the film thickness. However, if the

liquid film flow is turbulent, then the core boundary layer will be in the completely rough regime, and λ will depend mainly on the film thickness. At this time we will neglect the transition layer, whether the film is laminar or turbulent. As a result, the viscous stress in the core can be neglected, and eqns. (3.12) and (3.14) reduce to

$$\frac{1}{r} \frac{\partial}{\partial r} (r\tau') = 2 \frac{\tau_i}{r_i} \quad (3.17)$$

Using the wall coordinate defined as

$$y = r_o - r, \quad (3.18)$$

we find that eqn. (3.17) becomes

$$\frac{1}{r_o - y} \frac{\partial}{\partial y} [(r_o - y) \tau_r'] = - \frac{2\tau_i}{(r_o - \delta)}; \delta \leq y \leq r_o \quad (3.19a)$$

and

$$\tau_r' = - \bar{\rho}_c l_c^2 \left| \frac{d\bar{w}_c}{dy} \right| \frac{d\bar{w}_c}{dy} \quad (3.19b)$$

An additional effect of the wavy film near the interface is that the velocity distribution in the core is less steep than near a smooth interface. In single-phase flow, Prandtl's assumption that the mixing length is proportional to the distance from the wall, i.e.,

$$l_c = Ky, \quad (3.20a)$$

was considered for turbulent flows along a smooth flat plate. If we shift the reference frame at the interface toward the wall by an amount δ' , then we can treat the interface as being hydraulically smooth as suggested by Rotta

[68] for the turbulent mean velocity distribution near a rough wall as shown in fig. 3.5. Therefore, eqn. (3.20) is modified to a near-wall mixing length equation of the form

$$l_c = K(y + \delta') , \quad (3.20b)$$

where δ' is considered to be proportional to the film thickness in a two-phase flow. In single-phase turbulent flow, δ' is proportional to the pipe roughness. The universal constant, K , has the empirical value of 0.4 in single-phase flows. In two-phase turbulent flows, K appears to vary. Gill et al. [69] studied the effect of flow rates on K , and found it to vary approximately between 0.1 and 0.4. When the interface is smooth, usually at low liquid flow rates, the value of K is close to 0.4. On the other hand, at higher liquid flow rates, and hence greater droplet entrainment, the value of K decreases. It appears that droplets subdue turbulence in the core as will be explained later on.

Now, using eqn. (3.20b), we can integrate eqns. (3.19) and find:

$$\bar{\rho}_c K^2 (\delta' + y)^2 \left| \frac{d\bar{w}_c}{dy} \right| \frac{d\bar{w}_c}{dy} = - \frac{\tau_i(r_o - y)}{(r_o - \delta)} . \quad (3.21)$$

If we define a friction velocity as

$$w_c^{*2} = \frac{|\tau_i|}{\bar{\rho}_c} , \quad (3.22a)$$

and let

$$\bar{w}_c^+ = \frac{\bar{w}_c}{w_c^*} , \quad (3.22b)$$

and

$$y^+ = (y + \delta') \frac{w_c^*}{v_c}, \quad (3.22c)$$

then, for an axisymmetric flow, eqn. (3.21) becomes

$$K^2 y^{+2} \left(\frac{d\bar{w}_c^+}{dy^+} \right)^2 = \frac{r_o^+ - y^+}{r_o^+ - \delta^+}, \quad (3.23a)$$

or

$$\frac{d\bar{w}_c^+}{dy^+} = \frac{1}{Ky^+} \frac{(r_o^+ - y^+)^{1/2}}{(r_o^+ - \delta^+)^{1/2}}. \quad (3.23b)$$

Since $y^+ \ll r_o^+$, $(r_o^+ - y^+)^{1/2} \approx r_o^{+1/2} \left[1 - \frac{1}{2} \frac{y^+}{r_o^+} \right]$.

Integrating eqn. (3.23b) will give

$$\bar{w}_c^+ - \bar{w}_i^+ = K^{-1} \left[1 - \frac{\delta^+}{r_o^+} \right]^{-1/2} \left[\ln \frac{y^+}{\delta^+} - (y^+ - \delta^+) \frac{1}{2r_o^+} \right], \quad (3.24)$$

where,

$$\bar{w}_i^+ = \frac{\bar{w}_i}{w_c^*}, \quad (3.24a)$$

$$r_o^+ = (r_o + \delta') \frac{w_c^*}{v_c}, \quad (3.24b)$$

and

$$\delta^+ = (\delta + \delta') \frac{w_c^*}{v_c} \quad (3.24c)$$

If the velocity profile is very steep, we can approximate the average velocity $\hat{w}_c^+$ by integrating eqn. (3.24) from the interface to the pipe centerline; that is

$$\hat{w}_c^+ = \frac{\int_{\delta^+}^{r_o^+} \bar{w}_c^+ dy^+}{\int_{\delta^+}^{r_o^+} dy^+} \quad (3.25)$$

Hence, upon integration, we get

$$\hat{w}_c^+ - \bar{w}_i^+ = K^{-1} \left[1 - \frac{\delta^+}{r_o^+} \right]^{-1/2} \left[1 + \frac{1}{4} \left[1 - \frac{\delta^+}{r_o^+} \right] + \left[1 + \frac{\delta^+}{r_o^+} \right] \ln \left[\frac{\delta^+}{r_o^+} \right] \right] \quad (3.26)$$

It should be noted that $\delta^+ \neq 0$; that is, the wall is always wet. Using eqns. (3.22) to eliminate $\hat{w}_c^+ - \hat{w}_i^+$ between eqns. (3.15b) and (3.26), an interfacial friction coefficient expression can be written as

$$\lambda = \frac{2 \left[1 - \frac{\delta^+}{r_o^+} \right] K^2}{\left[1 + \frac{1}{4} \left[1 - \frac{\delta^+}{r_o^+} \right] + \left[1 + \frac{\delta^+}{r_o^+} \right] \ln \left[\frac{\delta^+}{r_o^+} \right] \right]^2} \quad (3.27)$$

For a given constant K , the interfacial shear stress depends on the film thickness and the relative velocity $\hat{w}_c - \bar{w}_i$. Conversely, if the interfacial shear stress is known or determined experimentally, then an estimate of K in two-phase flows can be made. In the model developed by Macder et al.

[49], K is one of their adjustable parameters.

Having derived an expression for the interfacial friction coefficient λ , we now proceed to study the dynamics of the liquid film whether it is a laminar or a turbulent flow.

3.5 Physics of the Liquid Film

Let us consider the case of a fully developed, incompressible liquid film. Eqns. (3.6a) and (3.6b) give

$$\frac{1}{r} \frac{\partial}{\partial r} (r\tau) = \frac{d\bar{P}}{dz} + \rho_f g, \quad (3.28a)$$

where

$$\tau = \mu_\epsilon \frac{d\bar{w}_f}{dr}, \quad (3.28b)$$

and

$$\mu_\epsilon = \mu_f + \rho_f l_f^2 \left| \frac{d\bar{w}_f}{dr} \right|. \quad (3.28c)$$

In terms of the wall coordinate, these equations can be written also as

$$\frac{d}{dy} [\tau(r_o - y)] = - (r_o - y) \left[\frac{d\bar{P}}{dz} + \rho_f g \right], \quad (3.29a)$$

where

$$\tau = - \mu_\epsilon \frac{d\bar{w}_f}{dy}. \quad (3.29b)$$

If eqn. (3.28a) is integrated from the interface to any point in the film, it becomes

$$r\tau - r_i\tau_i = \frac{1}{2} [r^2 - r_i^2] \left[\frac{d\bar{P}}{dz} + \rho_f g \right] \quad (3.30)$$

At $y = 0$, $\ell_f = 0$, and $\mu_e = \mu_f$. Therefore, eqns. (3.29) become

$$-r_o \mu_f \left. \frac{d^2 \bar{w}_\ell}{dy^2} \right|_{y=0} - \tau_w = -r_o \left[\frac{d\bar{P}}{dz} + \rho_f g \right], \quad (3.31a)$$

and eqn. (3.30) becomes

$$r_o \tau_w - r_i \tau_i = \frac{1}{2} [r_o^2 - r_i^2] \left[\frac{d\bar{P}}{dz} + \rho_f g \right]. \quad (3.31b)$$

If eqn. (3.14) is substituted into eqn. (3.31b) for the interfacial shear stress τ_i and if the wall shear stress is eliminated between eqns. (3.31), the curvature of the velocity profile at the wall can be written as

$$\mu_f \left. \frac{d^2 \bar{w}_\ell}{dy^2} \right|_{y=0} = \frac{1}{2} \left[\frac{d\bar{P}}{dz} + \rho_f g \right] + \frac{1}{2} \frac{r_i^2}{r_o^2} \left[\bar{\rho}_f g - \rho_c g - \bar{\rho}_c \hat{w}_c \frac{\Delta \hat{w}_c}{\Delta z} \right]. \quad (3.32)$$

For a thin liquid film, the interfacial radius r_i is of the same order of magnitude as the pipe radius r_o ; and if the hydrostatic and acceleration pressure gradients in the core are much smaller than the hydrostatic pressure gradient in the liquid film, then it can be shown from eqn. (3.32) that the sign of $(d^2 \bar{w}_\ell)/(dy^2)$ is given approximately by the term

$$\frac{1}{2} \frac{d\bar{P}}{dz} + \rho_f g .$$

$$\text{If } \frac{1}{2} \left| \frac{d\bar{P}}{dz} \right| < \rho_f g , \text{ then } \frac{d^2 \bar{w}_l}{dy^2} > 0 .$$

Equivalently, this corresponds to an adverse pressure gradient. If, on the other hand

$$\frac{1}{2} \left| \frac{d\bar{P}}{dz} \right| > \rho_f g , \text{ then } \frac{d^2 \bar{w}_l}{dy^2} < 0 ;$$

and this corresponds to a favorable pressure gradient. Moreover, the liquid film may be flowing totally or partially upward, or downward, depending on the thickness of the film and the flow rates of the phases.

These cases are summarized in fig. 3.6 for the case where

$$\frac{1}{2} \left| \frac{d\bar{P}}{dz} \right| < \rho_f g .$$

Near the interface the velocity profile is usually concave upward for a laminar film, and concave downward for a turbulent film. In case a, the vapor velocity is small and has no effect on the film which is flowing totally downward. The interfacial shear stress exerted by the vapor on the film is small and can be neglected. This case resembles Couette flow. In case b, the core flows at the minimum velocity to exert on the film a shear such that the interfacial velocity is zero. This case resembles Poiseuille flow. In case c, the interfacial velocity becomes upward as the vapor velocity increases beyond

the value of case b. In this case some of the liquid moves upward, but the net liquid flow is downward. In case d, the vapor velocity increases to a point where the net liquid flow is zero. Hence, the liquid appears to be stationary and is accompanied by local rotation. In case e, the net liquid flow is upward, and in case f, the vapor velocity reaches a point where the wall shear stress is zero. Normally, this point is called the separation point which occurs whether the boundary layer is laminar or turbulent [70]. In case g, the film flow is completely upward and the wall shear stress is negative (downward).

In most upward two-phase annular flows, the liquid film flow varies between cases d to g. When the wall shear stress fluctuates about zero asymmetrically, it is possible for a backflow to take place over large areas along the wall (e.g., case e). Our observation indicated, however, that this backflow was intermittent and occurred over small regions along the pipe wall and then the film flow re-adjusted itself back to the main upward flow. So, the "churn-annular" flow was characterized by a periodic intermittent backflow over small areas of the pipe wall. In this case the streamline was assumed to be attached to the wall at all times and the boundary layer approximation still held.

In the following, the cases of laminar and turbulent film will be examined.

3.6 The Case of a Laminar Liquid Film

If the flow in the film is assumed to be laminar, then a relationship between the film mass flow rate and its thickness can be easily established. This is done first by finding the velocity distribution in the film. To see that, let us consider the equation of motion given by eqn. (3.29a) and integrate

between the interface and any point in the film to find the shear stress distribution:

$$\tau = \frac{(r_o - \delta)}{(r_o - y)} \tau_i - \frac{1}{2} \left[\frac{(r_o - \delta)^2}{(r_o - y)} - (r_o - y) \right] \left[\frac{d\bar{P}}{dz} + \rho_f g \right]. \quad (3.33)$$

If the film thickness is thin in comparison to the radius of the pipe, i.e.,

$$\bar{\delta} \equiv \frac{\delta}{r_o} \ll 1,$$

and if we neglect terms of order $\bar{\delta}^2$ and higher, then eqn. (3.33) becomes

$$\tau = \tau_i [1 + \bar{y} - \bar{\delta}] + r_o \left[\frac{d\bar{P}}{dz} + \rho_f g \right] [\bar{\delta} - \bar{y}], \quad (3.34)$$

where

$$\bar{y} \equiv \frac{y}{r_o}.$$

For a laminar film,

$$\tau = -\mu_f \frac{d\bar{w}_f}{dy}. \quad (3.35)$$

This along with eqn. (3.34) yields the velocity distribution in the film; that is

$$\bar{w}_f = -\frac{r_o \bar{y}}{\mu_f} \left[\tau_i [1 - \bar{\delta}] + r_o \left[\frac{d\bar{P}}{dz} + \rho_f g \right] \left[\bar{\delta} - \frac{1}{2} \bar{y} \right] \right]. \quad (3.36)$$

In particular, the interfacial velocity is given by

$$w_i = -\frac{r_o \bar{\delta}}{\mu_f} \left[\tau_i (1 - \bar{\delta}) + r_o \frac{\bar{\delta}}{2} \left(\frac{d\bar{P}}{dz} + \rho_f g \right) \right] . \quad (3.36a)$$

Using the velocity distribution $\bar{w}_l$, the mass flow rate in the film can be found from

$$\dot{m}_l = \int_0^{\bar{\delta}} 2\pi r_o^2 \rho_f \bar{w}_l (1 - \bar{y}) d\bar{y} . \quad (3.37a)$$

Hence,

$$\dot{m}_l = -\frac{2\pi r_o^3}{\nu_f} \bar{\delta}^2 \left[\frac{\tau_i}{2} \left(1 - \frac{4}{3} \bar{\delta} \right) - r_o \frac{\bar{\delta}}{3} \left(1 - \frac{5}{8} \bar{\delta} \right) \left(\frac{d\bar{P}}{dz} + \rho_f g \right) \right] . \quad (3.37b)$$

Let β be an index that indicates the fraction as well as the direction of the total liquid flowing in the film in an upward or downward direction. Hence,

$$\dot{m}_l = \beta (1 - x) \dot{m} . \quad (3.38a)$$

If β is positive, the net liquid film flow is upward; if it is negative, the net liquid film flow is downward; and if β is equal to zero, the net flow is zero and the flow is stationary. Therefore, β can be negative, zero or positive, but always less than one. Hence, a negative $\dot{m}_l$ simply indicates that the net flow of the film is downward (i.e., fig. 3.6c). Most of us are used to treating

the mass flow rate as a positive quantity. Of the total liquid flowing in the pipe, the fraction entrained in the core is simply $1 - \beta$ and the mass flow rate of the droplets is given by

$$\dot{m}_d = (1 - \beta)(1 - x) \dot{m}. \quad (3.38b)$$

Hence, if β is negative, then the entrainment of droplets in the core will increase since the total mass flow rate of liquid is conserved for a frozen flow.

Let us define a liquid film Reynolds number as

$$Re_f = \frac{\hat{w}_f D_h}{\nu_f}, \quad (3.39)$$

where the hydraulic diameter D_h is given by

$$D_h = \frac{4\pi(r_o^2 - r_i^2)}{2\pi r_o} = 4\delta. \quad (3.40)$$

Since,

$$\dot{m}_f = \pi(r_o^2 - r_i^2) \bar{\rho}_f \hat{w}_f, \quad (3.41)$$

the Reynolds number Re_f can be written also as

$$Re_f = \beta(1 - x) \frac{\psi D}{\mu_f} \quad (3.42)$$

where

$$\psi = \frac{\dot{m}}{A} \quad (3.43)$$

Most of us are familiar with Re_f as a positive quantity. Since β can be negative, then a negative Re_f simply means that the net flow in the liquid film is locally downward. Therefore, Re_f can be written also as

$$Re_f = \beta(1 - x) \frac{\mu_f^*}{\mu_f} Re^* \quad (3.44)$$

Conversely, the fraction of liquid flowing in the film can be expressed as

$$\beta = \frac{1}{1 - x} \frac{\mu_f}{\mu_f^*} \frac{Re_f}{Re^*} \quad (3.45)$$

In most practical applications, the flashing Reynolds number Re^* is very large. The viscosity ratio is of order one. Hence, for small qualities, the amount of entrained liquid in the core depends on the state of the flowing film and on the flashing state. For example, if $x = 0.1$ and if the flow in the film is laminar, we find that 99% of the liquid is entrained in the core if $Re_f = 1000$ and $Re^* = 100,000$. The maximum Re_f corresponds to no entrainment in the core; that is ($\beta = 1$)

$$Re_{f,max} = (1 - x) \frac{\mu_f^*}{\mu_f} Re^* \quad (3.46)$$

Next, if eqn. (3.38a) and eqns. (3.42) - (3.45) are substituted into eqn. (3.37b), then a normalized interfacial shear stress can be written as

$$\tau_i^o = - \frac{2}{\left[1 - \frac{4}{3} \bar{\delta}\right]} \left[\frac{2}{\bar{\delta}^2} \frac{\rho_f^*}{\rho_f} \frac{Re_f}{Re^{*2}} Fr^{*2} \left[\frac{\mu_f}{\mu_f^*} \right]^2 + \frac{\bar{\delta}}{3} \left(1 - \frac{5}{8} \bar{\delta}\right) \left[\frac{\rho_f}{\rho_f^*} + P^o \right] \right], \quad (3.47a)$$

where

$$\tau_i^o = \frac{\tau_i}{\rho_f^* g r_o}, \quad (3.47b)$$

$$P^o = \frac{1}{\rho_f^* g} \frac{d\bar{P}}{dz}. \quad (3.47c)$$

On the other hand, the interfacial shear stress is also given by eqn. (3.14), in which the core average velocity and density must be specified.

To find the core velocity, let us write the mass flow rate of the droplets in the core as

$$\dot{m}_d = \rho_f \hat{w}_d A_d = (1 - \beta)(1 - x) \dot{m}. \quad (3.48)$$

Now,

$$\frac{A_d}{A} = 1 - \frac{A_g}{A} - \frac{A_f}{A} = (1 - \bar{\delta})^2 - \alpha_g. \quad (3.49)$$

From the continuity of the vapor, the void fraction α_g is given by

$$\alpha_g = \frac{x\psi}{\rho_g \hat{w}_g}. \quad (3.50)$$

Since the core is homogeneous, then $\hat{w}_c = \hat{w}_g = \hat{w}_d$. Therefore, upon combining eqns. (3.48)-(3.50), $\hat{w}_c$ can be expressed as

$$\hat{w}_c = \frac{\psi}{\rho_f} \frac{x\gamma + (1 - \beta)(1 - x)}{[1 - \bar{\delta}]^2} ; \quad (3.51a)$$

or

$$\frac{\hat{w}_c}{w_f^*} = \frac{\rho_f^*}{\rho_f} \frac{x\gamma + (1 - \beta)(1 - x)}{[1 - \bar{\delta}]^2} . \quad (3.51b)$$

To find the core density, let us start with the definition of the core specific volume given by

$$v_c = x_c v_g + (1 - x_c) v_f , \quad (3.52a)$$

where

$$x_c = \frac{\dot{m}_g}{\dot{m}_g + \dot{m}_d} = \frac{x}{x + (1 - \beta)(1 - x)} . \quad (3.52b)$$

Hence

$$\bar{\rho}_c = \frac{1}{v_c} = \rho_f \frac{1 - \beta(1 - x)}{x\gamma + (1 - \beta)(1 - x)} . \quad (3.53)$$

Therefore, the normalized interfacial shear stress given by eqn. (3.14) becomes

$$\tau_i^o = \frac{1}{2} [1 - \bar{\delta}] [P^o + P_h + P_a] \quad (3.54a)$$

where

$$P_h = \frac{\rho_f}{\rho_f^*} \frac{1 - \frac{\mu_f}{\mu_f^*} \frac{Re_l}{Re^*}}{x(\gamma - 1) + 1 - \frac{\mu_f}{\mu_f^*} \frac{Re_l}{Re^*}} \quad (3.54b)$$

is the normalized hydrostatic pressure gradient of the core, and

$$P_a = Fr^{*2} \frac{\rho_f^*}{\rho_f} \frac{D}{[1 - \bar{\delta}]^4} \left[1 - \frac{\mu_f}{\mu_f^*} \frac{Re_l}{Re^*} \right] \frac{\Delta[x(\gamma - 1)]}{\Delta z} \quad (3.54c)$$

is the normalized acceleration pressure gradient of the core.

For given initial flashing conditions, fluid properties, and Re_l , eqns. (3.47) and (3.54) can be solved simultaneously to predict the interfacial shear stress required to support a film of thickness δ . Apparently, eqns. (3.47) and (3.54) yield two solutions. This can be seen from the graph of the normalized shear stress τ_i^0 vs. the normalized film thickness $\bar{\delta}$ in fig. 3.7. The straight line corresponds to eqns. (3.54) and the other curve corresponds to eqns. (3.47). On the one hand, one solution yields a very thin film and an interfacial velocity much smaller than the average velocity of the core; on the other hand, the second solution yields a thicker film and an interfacial velocity that is close to the average core velocity. In the latter case the coupling between the interfacial rolling waves and the core is strong.

The continuity of the liquid film requires that

$$\rho_f \hat{w}_l \xi = \beta(1 - x) \psi, \quad (3.55)$$

where ξ is the fraction of the pipe occupied by the liquid film. For the droplets, continuity requires

$$\rho_f \hat{w}_d (1 - \alpha_g - \xi) = (1 - \beta)(1 - x) \psi. \quad (3.56)$$

Using eqns. (3.50), (3.55) and (3.56) and setting $\hat{w}_d = \hat{w}_g$, it can be shown that

$$\frac{\alpha_f}{\alpha_g} = \frac{(1-x)}{\gamma x} \left[1 - \beta + \beta \frac{\hat{w}_g}{\hat{w}_l} \right] . \quad (3.57)$$

Continuity of the liquid in the film and the core requires that

$$\rho_f \alpha_f \hat{w}_f = (1-x) \psi . \quad (3.58)$$

Hence, from eqns. (3.50) and (3.58), the slip ratio can be written as

$$\sigma \equiv \frac{\hat{w}_g}{\hat{w}_f} = \frac{\gamma \alpha_f}{\alpha_g} \frac{x}{1-x} . \quad (3.59)$$

If eqn. (3.57) is substituted in eqn. (3.59), then the slip ratio can be written in terms of the average velocity of the film as

$$\sigma = 1 - \beta + \beta \frac{\hat{w}_g}{\hat{w}_l} . \quad (3.60)$$

So, if there is total entrainment, $\beta = 0$, then $\sigma = 1$ and the flow is homogeneous.

Going back to fig. 3.7, the slip ratio σ is found to be very close to unity for the thin film solution which corresponds to a homogeneous flow in the pipe. In this case, the average velocity of the film is small and the film appears to cling to the wall. However, for the thicker film, the slip ratio σ is in the range of 3 to 5.

Next, we will examine the effect of the quality and pipe diameter on entrainment. We have illustrated earlier that at small qualities and large flashing Reynolds number, almost all the liquid is entrained in the core. If the quality is not constrained, would it be possible to have an "annular" flow with a laminar flow in the film and practically no entrainment in the core?

In order to understand the physics of this particular configuration, let us consider eqns. (3.54) without the acceleration pressure gradient term. In this case τ_i^o becomes simply

$$\tau_i^o = \frac{1}{2} [1 - \bar{\delta}] [P^o + P_h] . \quad (3.61)$$

For no entrainment in the core ($\beta = 1$), τ_i^o reduces to

$$\tau_i^o = \frac{1}{2} [1 - \bar{\delta}] \left[P^o + \frac{\rho_g}{\rho_f^*} \right] ; \quad (3.62)$$

or, solving for P^o in terms of τ_i^o , eqn. (3.62) becomes

$$P^o = 2\tau_i^o [1 + \bar{\delta}] - \frac{\rho_g}{\rho_f^*} . \quad (3.62a)$$

If P^o is substituted into eqn. (3.36a) and (3.47a), the interfacial velocity and shear stress become

$$w_i = - \frac{D^2 g \bar{\delta} \rho_f^*}{4\mu_f} \left[\tau_i^o + \frac{\bar{\delta}}{2} \left(\frac{\rho_f - \rho_g}{\rho_f^*} \right) \right] , \quad (3.63)$$

and

$$\tau_i^o = -2 \left[\frac{2(1-x) v_f \psi}{\bar{\delta}^2 D^2 g \rho_f^*} + \frac{\bar{\delta}}{3} \left(1 - \frac{5}{8} \bar{\delta} \right) \left(\frac{\rho_f - \rho_g}{\rho_f^*} \right) \right] . \quad (3.64)$$

If τ_i^o is substituted into eqn. (3.63), w_i becomes

$$w_i = - \frac{2(1-x)}{\bar{\delta}} w_f^* \left(\frac{\rho_f^*}{\rho_f} \right) + \frac{\bar{\delta}^2 D^2 g (\rho_f - \rho_g)}{24 \mu_f} \left(1 - \frac{5\bar{\delta}}{2} \right) . \quad (3.65)$$

For $\beta = 1$, the average core velocity given by eqn. (3.51b) reduces to

$$\hat{w}_c = \frac{\psi x}{\rho_g [1 - \bar{\delta}]^2} = \frac{4mx}{\pi D^2 \rho_g [1 - \bar{\delta}]^2} \quad (3.66)$$

For upward flows the condition that

$$\hat{w}_c > w_i \quad (3.67)$$

requires the quality to satisfy a minimum value for no liquid entrainment in the core. This relationship is derived from eqns. (3.65)-(3.67) and given by

$$x_{\min} \geq \frac{1 + \frac{\pi g D^4 \bar{\delta}^3}{96 \dot{m} v_f} [\rho_f - \rho_g] \left(1 - \frac{5\bar{\delta}}{2}\right)}{1 + \gamma \bar{\delta} [1 + 2\bar{\delta}]} \quad (3.68)$$

The effect of the fluid temperature on $x_{\min}$ is shown in fig. 3.8 for the conditions indicated in the figure. It can be seen that at a given quality, there are generally two solutions for the normalized film thickness $\bar{\delta}$; this can be noticeable on the curve that corresponds to the high temperature. Again, the two solutions for a thin and thick film correspond to a homogeneous flow and a flow of slip ratio in the range of 3 to 5, respectively. In most geothermal applications the quality of steam in the "annular" regime corresponds to about 10% quality of R-114 in the laboratory conditions. Hence, for these conditions it is impossible to avoid entrainment in the core.

For a constant mass flow rate and a given fluid temperature, the effect of the pipe diameter on the minimum quality is illustrated in fig. 3.9.

At a very small normalized film thickness $\bar{\delta}$, $x_{\min}$ is independent of the diameter. But at thicker films, the effect is more pronounced. At larger diameter, there are generally two films at a given quality. As we have demonstrated in fig. 3.7, the thinner film corresponds to a homogeneous flow. As the pipe diameter increases, the film thickness decreases which is expected for a constant mass flow rate. Since the vapor velocity increases in inverse proportion to the square of the pipe diameter, the film becomes very thin at the limit and flows at relatively very low velocities, like rain drops on a glass. The flow in this case becomes simply a stratified flow over a vertical flat plate.

So, for a small diameter pipe, small qualities, and high flashing Reynolds number, it is practically impossible to avoid entrainment in the core for a laminar flow in the film. As a matter of fact, it turns out that almost all liquid is entrained in the core. As we have concluded earlier from fig. 3.7, the slip ratio is not necessarily close to unity, but might be of order 5. This depends on the ratio of the fraction of liquid flowing in the film and the film average velocity, $\beta/\hat{w}_f$. If β and $\hat{w}_f$ are small, then their ratio may be much larger than 1, which means that σ is larger than 1 as can be seen from eqn. (3.60).

The choice of one of the two solutions in the case of a laminar film is governed by the value of the measured slip. In other words, eqns. (3.47) and (3.54) should be solved simultaneously for a unique solution using eqn. (3.60) as a constraint.

Is it possible for this mechanism to take place in a turbulent film? Apparently, there are no similarities as we will see later on. But let it suffice to say that the maximum liquid film Reynolds number attainable is given by eqn. (3.46). This means that it is possible to have a very small number of

droplets entrained in the core. So now, let us turn our attention to the case of a turbulent liquid film.

3.7 The Case of a Turbulent Liquid Film

At higher liquid film Reynolds numbers, the flow becomes turbulent. Similar to single-phase flows in a pipe, we distinguish an inner region or a viscous sublayer near the wall, where the momentum exchange is accounted for by viscous shear stresses, and an outer layer or a fully turbulent region where the momentum exchange is much higher than that accounted for by viscous stresses.

In modeling the turbulent stresses in pipe flows, the mixing-length theory has proved to be very successful. Extensive work has been done to model the mixing length l_f correctly. For the region closest to the wall, Reichardt [71] showed that l_f is proportional to the square of the distance from the wall. In the neighborhood of the wall, von Karman [71] showed that l_f is proportional to the distance from the wall, and far from the wall, Prandtl [71] showed that the mixing length l_f is a constant.

From all the proposed empirical expressions for the mixing length, it can be concluded that l_f depends strongly on the distance from the wall. Most of the correlations proved very successful mainly for a boundary layer with no pressure gradient or with mild adverse pressure gradient. However, when the hydrostatic pressure gradient of the phase is three to five times larger than the magnitude of the static pressure gradient, the flow field is effectively subjected to a strong adverse pressure gradient. In this case, the flow is characterized by intermittent stalls and some flow reversal may occur near the wall as mentioned earlier. Physically, one would expect a small shear stress at the wall. It can be in a positive or negative direction and most likely

fluctuates about a value of zero. Hence, the turbulent stress τ'_z given by eqn. (3.5g) may become important [72,73]. If the flow is to adjust itself in this effective adverse pressure gradient without stall, then this has to be done at the expense of a very large wall shear stress.

Previous researchers [46,47,53] applied the mixing-length theory and the universal velocity distributions to the film in an "annular" flow, in which the hydrostatic pressure of the liquid is negligible (i.e., horizontal flows). In this case, eqn. (3.32) predicts a negative curvature for the velocity profile if the term $\rho_f g$ is not present (effective favorable pressure gradient) as shown in fig. 3.10. Hence, it is expected that the universal velocity laws developed for a single-phase flow in a pipe should be applicable in this case for a turbulent liquid film. In a vertical flow, the use of single-phase mixing length expressions is not justified where the hydrostatic pressure gradient in the film is much larger than the static pressure drop. However, if the interfacial shear stress is very large, then the curvature of the velocity profile may be negative, and the use of single-phase universal velocity distributions might be justified.

In the following a scheme based on the Karman-Pohlhausen approach [74] is devised to describe the velocity profile in the liquid film where the interfacial shear stress is very small. This is based on describing the velocity profile using a polynomial of a certain degree, as long as there are enough boundary conditions to determine the coefficients of the polynomial. In the case of the liquid film, the following boundary conditions are used. At the wall, the no-slip condition requires

$$\bar{w}_z \big|_{y=0} = 0 \quad (3.69)$$

At the wall, we are going to neglect the effect of the wall curvature and write

$$\mu_f \left. \frac{d^2 \bar{w}_l}{dy^2} \right|_{y=0} = \frac{d\bar{P}}{dz} + \rho_f g . \quad (3.70)$$

In the outer layer of the liquid and very close to the liquid-vapor interface, the film is treated as a free shear layer and the mixing length l_f is approximately constant and proportional to the width of the liquid film, that is,

$$l_f = \theta \delta , \quad (3.71)$$

where, θ ranges from 0.075 to 0.09 [75]. At the interface, the viscous shear stress is neglected and the boundary conditions are given by

$$\bar{w}_l \Big|_{y=\delta} = \bar{w}_i , \quad (3.72)$$

for the velocity, and

$$\tau = \tau_i = - \rho_f (\theta \delta)^2 \left[\frac{\partial \bar{w}_l}{\partial y} \right]^2 \Big|_{y=\delta} , \quad (3.73)$$

for the interfacial shear stress. Following the ideas of Dickinson et al. [32], we hypothesize that

$$\bar{w}_l \Big|_{y=\delta} = \bar{w}_i = \frac{1}{2} (\hat{w}_c + \hat{w}_l) . \quad (3.74)$$

Therefore, based on the foregoing four boundary conditions, we assume a velocity profile in the form of

$$\frac{\bar{w}_f}{\bar{w}_i} = \bar{a} + b\bar{y} + c\bar{y}^2 + d\bar{y}^3 \quad (3.75)$$

The no-slip condition requires that $\bar{a} = 0$. The first and second derivatives of $\bar{w}_f$ with respect to $\bar{y}$ are thus given by

$$\frac{d\bar{w}_f}{d\bar{y}} = w_i [b + 2c\bar{y} + 3d\bar{y}^2] \quad (3.76)$$

and
$$\frac{d^2\bar{w}_f}{d\bar{y}^2} = w_i [2c + 6d\bar{y}] \quad (3.77)$$

At $y = 0$, eqns. (3.70) and (3.77) can be combined to find coefficient c ; that is

$$c = \frac{r_o^2}{2 \mu_f \bar{w}_i} \left[\frac{d\bar{P}}{dz} + \rho_f g \right] \quad (3.78)$$

If we apply the boundary conditions at $y = \delta$, then eqns. (3.72) and (3.75) can be combined to give

$$b\bar{\delta} + c\bar{\delta}^2 + d\bar{\delta}^3 = 1 \quad (3.79)$$

and eqns. (3.73) and (3.76) give

$$-\rho_f [\theta \bar{\delta}]^2 [b + 2c\bar{\delta} + 3d\bar{\delta}^2]^2 \bar{w}_i^2 = \tau_i \quad (3.80)$$

Once the velocity profile is determined, the mass flow rate in the film can be found from

$$\dot{m}_l = \beta (1 - x) \dot{m} = 2\pi r_o^2 \rho_f \bar{w}_i \int_0^{\bar{\delta}} [b\bar{y} + c\bar{y}^2 + d\bar{y}^3](1 - \bar{y}) d\bar{y} \quad (3.81)$$

which upon integration becomes

$$\begin{aligned} \dot{m}_l = \beta (1 - x) \dot{m} = 2\pi r_o^2 \rho_f \bar{w}_i \bar{\delta}^2 & \left[b \left[\frac{1}{2} - \frac{1}{3} \bar{\delta} \right] + c\bar{\delta} \left[\frac{1}{3} - \frac{1}{4} \bar{\delta} \right] \right. \\ & \left. + d\bar{\delta}^2 \left[\frac{1}{4} - \frac{1}{5} \bar{\delta} \right] \right] \quad (3.82) \end{aligned}$$

Next we will normalize the equations as we did in the case of a laminar film. Using eqns. (3.43), (3.50), (3.51b), (3.55), and (3.56), the interfacial velocity can be written as

$$\begin{aligned} \frac{\bar{w}_i}{w_f^*} = \frac{1}{2} \frac{\hat{w}_c + \hat{w}_l}{w_f^*} = \frac{1}{2} \frac{\rho_f^*}{\rho_f} \frac{1}{[1 - 2\bar{\delta}]} & \left[x(\gamma - 1) + 1 \right. \\ & \left. + \frac{1}{2\bar{\delta}} \beta (1 - x) [1 - 4\bar{\delta}] \right] \quad (3.83) \end{aligned}$$

If we substitute the term $\beta(1-x)$ from eqn. (3.82) into eqn. (3.83), the interfacial velocity becomes

$$\frac{\bar{w}_i}{w_f^*} = \frac{\frac{1}{2} \frac{\rho_f^*}{\rho_f} \frac{x(\gamma - 1) + 1}{[1 - 2\bar{s}]}}{1 - \frac{\bar{s}}{2} \frac{[1 - 4\bar{s}]}{[1 - 2\bar{s}]} \left[b \left[\frac{1}{2} - \frac{1}{3} \bar{s} \right] + c\bar{s} \left[\frac{1}{3} - \frac{1}{4} \bar{s} \right] + d\bar{s}^2 \left[\frac{1}{4} - \frac{1}{5} \bar{s} \right] \right]} \quad (3.84)$$

In a normalized form, c becomes

$$c = \frac{r_o^2 \rho_f^* g}{2 \mu_f \bar{w}_i} \left[P^o + \frac{\rho_f}{\rho_f^*} \right], \quad (3.85)$$

where P^o , P_h , and P_a are defined in eqns. (3.47c), (3.54b) and (3.54c), respectively. And, finally, eqn. (3.80) is written as

$$\frac{\rho_f}{\rho_f^*} [\theta \bar{s}]^2 \frac{\bar{w}_i^2}{r_o g} [b + 2c \bar{s} + 3d \bar{s}^2]^2 = -\frac{1}{2} [1 - \bar{s}] [P^o + P_h + P_a]. \quad (3.86)$$

If we examine eqns. (3.79), (3.82), (3.84), (3.85), and (3.86), we see that the coefficients b , c , and d cannot be determined explicitly; but we can say in principle that the interfacial velocity can be eliminated in favor of the

normalized film thickness $\bar{\delta}$ and the coefficients b , c , and d . Hence, these coefficients are mainly functions of the film thickness δ , the pipe geometry, the static pressure gradient, the initial conditions of the fluid at flashing, the local properties of the fluid at a given cross-sectional area, and the Reynolds number of the liquid film. So, if the initial conditions and the local properties of the fluid are specified, the coefficients b , c , and d become simply functions of the static pressure gradient, the film thickness and the liquid film Reynolds number, that is,

$$b = b \left[\delta, \frac{d\bar{P}}{dz}, Re_l \right]; \quad (3.87a)$$

$$c = c \left[\delta, \frac{d\bar{P}}{dz}, Re_l \right]; \quad (3.87b)$$

$$\text{and} \quad d = d \left[\delta, \frac{d\bar{P}}{dz}, Re_l \right]. \quad (3.87c)$$

In the Karman-Pohlhausen approximation, these coefficients are functions of the local thickness of the boundary layer and the pressure gradient $d\bar{P}/dz$. Hence, if we specify in addition Re_l , then the coefficients b , c , and d are functions of δ and $d\bar{P}/dz$ only.

3.8 Method of Solution and Results

The success of the MC/LF models based on laminar and turbulent films will be tested in the following section based on experimental data of series 3-A and 4. Equations (3.47) and (3.54) will be examined in the case of

a laminar film and eqns. (3.79), (3.82), and (3.84-86) will be examined simultaneously in the case of a turbulent film.

The solution of these equations is based on the measurements of temperature, pressure drop, and hold-up at a fixed section of the pipe where the QCVs are mounted. The temperature used to evaluate the physical properties of the mixture that appear in these equations is taken to be the average of two temperature readings at two stations just upstream (station 2) and downstream (station 3) of the QCVs. The total pressure drop is taken to be the measured pressure drop between the stations mentioned above. The hold-up measured between the two quick-closing valves is assumed to be the average hold-up at the mid-section between the QCVs. As far as the quality is concerned, we have indicated how it is calculated in eqn. (1.1).

So, the data used in the above-mentioned equations represent local average values - local in a sense they are the averages of parameters at a fixed cross-section of the test section. But since we generate different fluid conditions at any fixed cross-section by adjustment of the gate valve (which changes the actual discharge area), we note that these data represent average points of piece-wise sections of a much longer test section with a constant discharge area.

3.8.1 Method of Solution for the Case of a Laminar Film

The purpose of the solution is to predict how much liquid is entrained in the core, and to evaluate the interfacial shear stress required to support a film of thickness δ . Once the film thickness and the amount of entrainment are predicted, the void fraction can be easily found from eqns. (3.50) and (3.51). Hence, the success of assuming a laminar film can be judged by a comparison of the predicted void fraction or the hold-up with the

measured hold-up shown in fig. 1.9.

Unfortunately, the case of a laminar film turns out to be unsuccessful in the prediction of hold-up or the slip ratio for series 3-A and 4. So let us examine the simultaneous solution of eqns. (3.47) and (3.54) to explain the failure as a result of assuming a laminar film.

If we examine eqns. (3.47) and (3.54) for fixed stagnation conditions at the inlet of the test section, fixed temperature, and total pressure drop at the given cross-section between the QCVs, then we find that there are three unknowns in two equations, namely,

$$\tau_i^\circ, Re_f \text{ and } \bar{\delta}.$$

Hence, one of these three variables must be treated as a parameter in order to have a closed set of equations. The liquid film Reynolds number Re_f is chosen as the parameter, and eqns. (3.47) and (3.54) are solve simultaneously and iteratively.

The solution of these equations can be physically realistic if the interfacial velocity, w_i , does not exceed the core average velocity, $\hat{w}_c$, given that other constraints are met; for example, a liquid fraction β that is always less than one, and a normalized film thickness $\bar{\delta}$ that is positive and much smaller than 1.

The upper limit of Re_f is taken to be 2000. The first numerical observation that was made from varying Re_f is that the scheme predicts an interfacial or wall shear stress which appeared to be insensitive to changes in the Re_f for fixed temperature and pressure drop in the test section that is examined. This is shown in fig. 3.11 where the normalized interfacial and wall shear stresses are not only independent of Re_f but are practically equal which suggests a linear velocity profile in the liquid film (plane Couette flow). The

interfacial stress comes out of the simultaneous solution of eqns. (3.47) and (3.54); and the wall shear stress is calculated from (3.31b). We should note that in Couette flow it is well known that the velocity of the moving plane (the interface in this case) is twice the average velocity of the flowing liquid film.

As the Reynolds number Re_f is varied, it is found also that the core average velocity, $\hat{w}_c$, is insensitive to changes in Re_f ; but the interfacial velocity w_i is found to increase with Re_f . The average core velocity is calculated from eqn. (3.51a) once β and $\bar{\delta}$ are found from eqns. (3.47) and (3.54); and the interfacial velocity is calculated from eqn. (3.36a). Figure 3.12 shows the interfacial velocity w_i and the core average velocity $\hat{w}_c$ vs. Re_f . It can be seen that beyond a certain Re_f the interfacial velocity exceeds the core average velocity, which is not physically realistic. Hence, the upper cutoff for Re_f was taken to be 1000 for series 3-A. Similar results showed that the upper cutoff for Re_f had to be 100 for series 4. Results indicated also that the slip ratio and hold-up were insensitive to changes in Re_f . Their magnitudes will be compared with experimental values. Besides the constraint imposed on Re_f , the scheme shows its deficiency by its inability to be extended to low values of quality x , because it begins to predict interfacial velocities greater than the average core velocity. The lower cutoff for the quality was 0.06 for series 3-A and 0.086 for series 4 (see fig. 1.9).

Next, we present the predictions of the following variables at fixed $Re_f = 1000$ for series 3-A and $Re_f = 100$ for series 4: β , τ_i° , τ_w° , w_i , α_f , σ , and λ . The points shown in figs. 3.13-3.16 do not represent experimental values for the above parameters, but are the results of solving eqns. (3.47) and (3.54) simultaneously based on measured temperature and pressure drop only.

The liquid fraction flowing in the film vs. the reduced volume is shown in fig. 3.13. It can be seen that β increases by a maximum of 16% for series 3-A and is practically constant for series 4 as v_r is increased. It can be observed also that at least 99% of the liquid is entrained in the core. Hence at very high flash Reynolds numbers, it is expected that the liquid is completely entrained in the core since

$$Re_f \ll Re^*$$

and β is very small in this case as can be seen from eqn. (3.45).

The scheme predicts a very thin film. As a result, the shear stress distribution across the film is practically constant as shown in figs 3.14 and 3.15 for series 3-A and 4, respectively, where the normalized interfacial shear stress is plotted vs. the normalized wall shear stress. Another consequence is that the interfacial velocity is predicted to be approximately 1.95 times the average velocity of the liquid film as shown in fig. 3.16. This agrees very closely with the result from the "continuity wave" theory that predicts $w_i = 2\hat{w}_f$ for a constant shear stress in a very thin film. It agrees also with the result of a plane Couette flow.

The laminar film scheme however shows its major deficiency with respect to the prediction of hold-up. The predicted hold-up versus the experimental hold-up is shown in fig. 3.17. As can be clearly seen, this scheme underpredicts the hold-up. Conversely, it predicts larger void fractions.

The predicted slip ratio is found from eqn. (3.60) and comparison with the experimental slip ratio is shown in fig. 3.18. This scheme predicts practically a homogeneous flow, i.e., $\sigma \approx 1$, whereas our measurements show σ varies from about 2.2 to 4.8. We should note that in the case of a laminar film, there are two solutions for the film thickness δ that satisfy eqns. (3.47)

and (3.54) simultaneously, as shown in fig. 3.7. We also mentioned that the slip ratio σ is found to be very close to unity for the thin film solution, and is of order 3 to 5 for the thick film solution. But, the scheme predicted larger interfacial velocities than the core average velocity in some cases. For the majority of points, the solution satisfies a homogeneous flow.

The interfacial friction coefficient, λ , is calculated using eqns. (3.15b) and (3.53), and results are shown in fig. 3.19. In addition, two correlations are shown for comparison: that of Wallis [14]

$$\lambda = 0.005 \left[1 + 150 \bar{\delta} \right], \quad (3.89)$$

and that of Bharathan [76]

$$\lambda = 0.005 + 406 \left[\frac{\bar{\delta}}{2} \right]^{2.04}. \quad (3.90)$$

Wallis' correlation is one of the very earliest correlations developed and very popular. Bharathan's correlation is used because it predicts much larger λ than Wallis' correlation at very large $\bar{\delta}$ as will be shown in the case of a turbulent film. It can be seen in fig. 3.19 that for the range of film thickness predicted by this scheme, Bharathan's correlation predicts virtually a constant λ , whereas Wallis' correlation predicts λ that increases by over 50% as $\bar{\delta}$ varies from 0 to 3.5×10^{-3} . It can be seen also that very few points fall near these two correlations. Moreover, the trends are completely different. Whereas Wallis' and Bharathan's correlation predict an increasing λ with $\bar{\delta}$, the laminar film scheme predicts otherwise, as seen in fig. 3.19. Perhaps, this could be another indication of the deficiency of a laminar film assumption for series 3-A and 4.

In concluding this section, we should state that instead of choosing the liquid film Reynolds number, Re_f , as a parameter and then solving eqns. (3.47) and (3.54) simultaneously for the interfacial shear stress, one could use instead an additional constraint such as eqn. (3.60). This equation relates the slip ratio to the liquid fraction in the film and to the average velocities of the core and liquid film. However, we found from the previous analysis that the solution tends to satisfy a homogeneous flow ($\sigma \approx 1$), whereas the experimental slip varies approximately between 2.2 and 4.8. Hence, with the set of eqns. (3.47), (3.54), and (3.60), no solution was generally found that met the condition of an average velocity in the core larger than the interfacial velocity.

3.8.2 Method of Solution for the Case of a Turbulent Film

The purpose of the solution is the same as in the case of a laminar film; that is, to predict the interfacial shear stress required to support a film of thickness δ and to predict the amount of liquid entrained in the core.

First of all, the success of the turbulent scheme is revealed by its ability to cover the whole range of experimental data shown in fig. 1.9 for both series. Second, this model is capable of predicting a flow in the film such that the net flow may be upward, downward, or stationary. It also predicts whether or not the flow is stalled; i.e., the wall shear stress is zero.

If we examine eqns. (3.79), (3.82) and (3.84-86) for fixed stagnation conditions at the inlet of the test section, fixed temperature, and total pressure drop at the given cross-section between the QCVs, then we find that there are six unknowns in five equations. These unknowns are:

the coefficients b , c , and d ;

and $\bar{w}_l$, $\bar{\epsilon}$, and Re_l .

Hence, additional information must be known, such as the hold-up measured between the QCVs, if none of the above unknowns are to be treated as a parameter. Instead of hold-up, we can choose the slip ratio given by either eqn. (3.59) or eqn. (3.60) as additional information since it is related to the measured hold-up by eqn. (3.59). So, the additional sixth equation is

$$\sigma = 1 - \beta + \beta \frac{\hat{w}_g}{\hat{w}_l}, \quad (3.60)$$

which is true whether the film is laminar or turbulent. In eqn. (3.60) the experimental slip ratios are used; hence, we end up with six unknowns in six equations, and the set of equations is closed. We should note at the onset that the hold-up predicted by this scheme should be the same as the experimental hold-up since these are related by eqn. (3.59) to the slip ratio which is used in this scheme as an additional input parameter to the equations.

At the onset we should state again that the points appearing in figs. 3.20-3.32 are not experimental points, but are the solution of the above equations using the experimental temperature, total pressure drop and slip ratio as input parameters.

The liquid fraction in the film, β , is plotted vs. the reduced volume v_r in fig. 3.20. The minus sign is indicated on the axis to identify those cases where the net flow appeared to be locally downward. It can be seen that β varies over a wide range. For low reduced volume v_r , the net flow in the liquid is locally downward (fig. 3.6c), and since we assume no mass transfer between the phases, then more liquid has to be formed in the core in the form of droplets to conserve the total mass of the liquid. For the cases where

β is close to zero, the average velocity in the liquid film is very small and the flow in the liquid film will appear to be stationary (fig. 3.6d). In this case, the total liquid mass flow rate is the same as the mass flow rate of the droplets. Usually, this halt has a very short duration and the flow in the film adjusts very quickly to the outer flow in the core and the total mixture flows in the upward direction. All these interesting phenomena were observed in the glass test section. At high reduced volume (upper spectrum of an "annular" flow regime), the liquid becomes almost nonentrained as β approaches 1. In each series, we notice that the liquid fraction in the film tends on the average to increase linearly with the reduced volume. The few points at the highest reduced volume in series 3-A are simply a scatter and one should not interpret β to be reaching a maximum. One would tend to correlate the liquid fraction to the reduced volume for different mass flow rate, but this should be avoided presently because a correlation can be only meaningful for a wide range of mass flow rates.

The interfacial velocity versus the average velocity in the liquid film is shown in figs. 3.21 and 3.22 for series 3-A and 4, respectively. For $\hat{w}_l$ well above a halt point, $\bar{w}_i$ is approximately between 2 and 3.5 times $\hat{w}_l$. Again, these figures suggest a linear relationship between the interfacial velocity and the average velocity in the liquid film.

Figure 3.23 shows the interfacial shear stress versus the wall shear stress for series 3-A and 4, respectively. The value of the interfacial shear stress comes out of the simultaneous solution of the equations we mentioned earlier. The wall shear stress, τ_w , is evaluated by multiplying eqn. (3.76) by the dynamic viscosity of the liquid, μ_f , and setting $\bar{y}$ equal to zero. If we compare the magnitude of the stresses, it can be seen that τ_i is much larger than τ_w in the case of a turbulent film as opposed to equal stresses in the

laminar film. Moreover, the wall shear stress reverses direction in series 3-A. This means that the state of the fluid switched from case c to case g via case f in fig. 3.6. As the wall shear stress changes from a negative value to a positive one, the state of fluid changes correspondingly from case g to cases e, d, or c if the liquid fraction in the film is positive, zero, or negative, respectively. In series 4, the wall shear stress remains positive, which indicates that the state of the fluid switched between cases c, d, or e for a negative, zero, or positive liquid fraction in the film, respectively. If we inspect fig. 3.23, we observe that in series 3-A, there are four states that correspond to case g (negative wall shear stress) and approximately two states near stall as in case f (close to zero wall shear stress). In series 4, the wall shear stress remains positive; hence, the state of the points changes from case c to case e as the liquid fraction in the film increases in the flow direction. So, we can see that most states correspond to a positive wall shear stress and to cases c-e in fig. 3.6.

Most of the previous investigations, in which the film is turbulent, show a very large shear stress at the wall and in one direction which corresponds to a very steep velocity gradient at the wall as shown in fig. 3.10. In this case, it is expected that the slip ratio will be close to unity. Unfortunately, none of the previous investigations reported slip ratios to my knowledge. Hence, whenever the wall shear stress reverses direction, the fully developed, one-dimensional equation of motion given by eqns. (3.28) fails to predict the velocity distribution in the liquid film; this also implies the interfacial and wall shear stresses cannot be related anymore by eqn. (3.31b).

So, the liquid film may be constantly accelerating and it is expected that the liquid film velocity will change in the flow direction. This is clearly shown in figs. 3.24 and 3.25 for the average velocity and the Reynolds number

in the film versus the reduced volume for series 3-A. The same conclusion is true for series 4.

As far as the interfacial friction coefficient, λ , is concerned, it is calculated using eqns. (3.15b) and (3.53), and results are shown in fig. 3.26 along with the Wallis and Bharathan correlations. Very few points fall near Bharathan's correlation and about 67% of the points fall near Wallis' correlation. Although the trends of these points appear to be different, it can be shown using a linear least square fit technique that the interfacial friction coefficient exhibits a positive slope with respect to the normalized film thickness. Even if we except the highest three values of λ in series 4 to improve the fit, a positive slope will result.

Most researchers usually define the interfacial shear stress closure to be proportional to the density and the square of the velocity of vapor, instead of the definition given by eqn. (3.15b). This is based on negligible droplet entrainment in the core and on time-average interfacial velocity much smaller than the average velocity of vapor. If we calculate the interfacial friction factor, λ , based on these approximations, we find the points to follow a different trend from what we showed in fig. 3.26. The new trend is consistent with [14] and [76] as shown in fig. 3.27, and about 82% of the points fall near Wallis' correlation.

However, this scheme predicts a time-average interfacial velocity which is of the same order of magnitude as the core average velocity as shown in fig. 3.28. From a physical point, it is expected the interfacial shear stress to have a smaller magnitude when the interface moves than when it is stationary, and the density of the core in the presence of droplets is of the same order of magnitude as the vapor density. Therefore, eqn. (3.15b) should be the appropriate closure for the interfacial shear stress when the interfacial

velocity is not negligible.

It appears then that the interfacial friction coefficient, λ , is sensitive to how the interfacial shear stress closure is defined and does not appear to be simply related to just the film thickness as in [14] and [76]. This point requires further analysis and experimentation particularly in single-substance, two-phase flow such as R-114.

The most interesting result is about the universal mixing-length constant K . K is calculated from the expression given by eqn. (3.27) once the interfacial friction coefficient λ is calculated from eqn. (3.15b). We should note that when we form the ratio δ^+/r_o^+ in eqn. (3.27), the quantity w_c^*/v_c cancels out and its calculation is not required. Figures 3.29 and 3.30 show the effect of the total liquid flow rate on K for $\delta' = 0.035 \delta$ and $\delta' = 0.35 \delta$, respectively, for series 3-A. $\delta' = 0.035 \delta$ is the experimental result found from the effect of pipe roughness on a turbulent boundary layer in single-phase flow [68]. In two-phase flow, the film thickness is expected to be 5 or in some cases 10 times greater than the pipe roughness. Hence, it is natural as a first approximation to take δ' to be 5 or 10 times larger than 0.035δ . We observe that at low liquid mass flow rate, the model predicts some points above $K = 0.6$, but for $\delta' = 0.35 \delta$, all points are below 0.6. Miya [77] suggested a value of $K = 0.6$. However, Gill et al. [69] predict values of K between approximately 0.1 and 0.4. At higher liquid flow rates, this scheme predicts also values of K between 0.1 and 0.4. The same conclusion is also true for series 4 as shown in figs. 3.31 and 3.32.

We also notice that the universal constant K exhibits a particular trend with the liquid quality. One could easily correlate the two parameters K and $1-x$, but more experiments at different mass flow rates are recommended in order to establish a sound correlation.

The variation of the universal constant K from the established value of 0.4 in single-phase flow has a meaningful effect on the velocity distribution in the core in two-phase flow. To illustrate this, it would be plausible to assume Prandtl's velocity distribution law in the homogeneous core; that is,

$$\bar{w}_c^+ \equiv \frac{\bar{w}_c}{w_c^*} = \frac{1}{K} \ln y + C; \quad \delta \leq y \leq r_o. \quad (3.91)$$

The constant of integration C can be eliminated in favor of the maximum velocity at the pipe centerline, and eqn. (3.91) can be modified to

$$\frac{\bar{w}_{c,\max} - \bar{w}_c}{w_c^*} = \frac{1}{K} \ln \frac{r_o}{y}. \quad (3.92)$$

At a fixed position y in the pipe and for a constant friction velocity we can see from eqn. (3.92) that the difference between the maximum velocity and the local velocity in the core is inversely proportional to the universal constant K . So, at large values of K (possibly $K \geq 0.4$), this velocity difference can be considered small (characteristic of turbulent velocity profile) relative to the difference in a laminar profile, for example. In this case, replacing the local density and velocity by their averages as in eqn. (3.13) is a good approximation. However, as the liquid quality $(1-x)$ increases, K decreases as was shown in figs. 3.29-32. Hence, the velocity difference in eqn. (3.92) increases and the velocity profile becomes less shallow; and upon integrating eqn. (3.12) a momentum correction factor is required if the assumption made in eqn. (3.13) is to hold.

If single-phase flow correction factors are used which are close to unity, then it is expected that the values of K to remain practically the same as those predicted by this scheme. On the other hand, two-phase flow may require correction factors different from unity. In such a case, the value of K becomes highly dependent on the velocity profile in the core and vice versa.

In concluding this section, we state that at high flash Reynolds number, Re^* , the liquid film is most likely to be turbulent. In the case of a compressible core where acceleration pressure drop is important, it appears that this latter term becomes also important in the liquid film. Moreover, when the wall shear stress is small, the velocity in the transverse direction becomes important. Hence, a more rigorous approach for a turbulent liquid film would be to consider two-dimensional boundary-layer equation and march the solution numerically starting at flashing. This might create some obstacles since the prediction of a flow reversal might prohibit marching the solution in one direction because of singularities very near the separating point [78].

Based on this analysis, it seems reasonable, in the design of geothermal wells, to assume no entrainment in the core at large reduced volumes. With regard to the interfacial friction coefficient, λ , it appears that the expression given by eqn. (3.27) can be used only for the case where the liquid film is turbulent.

In general, the MC/LF model appears to reveal the physics of the liquid film based on an assumed velocity profile in the film. The assumption of a piece-wise frozen flow and a piece-wise constant film thickness seems to predict reasonable results as we have shown earlier in the graphs. However, the only way to measure the success of the model is to compare at least one of the predicted parameters with measured values. Among the interfacial shear stress, the wall shear stress, the fraction of liquid flowing in the film and the

film thickness, it appears that the last parameter can be reasonably measured.

Improvement of the MC/LF can be tested if local density and velocity in the core are considered instead of averaged density and averaged velocity. In addition, momentum exchange associated with the transverse exchange of droplets at the film-core interface and with mass transfer should be considered. This, however, may necessitate a completely different approach other than the mixing-length theory as we explained in assumption 3.

IV. VOID FRACTION CORRELATIONS

4.1 Formulation of the Problem

In this section the drift flux model as well as various correlations will be examined to predict the void fraction and the slip ratio between the phases, and compare the predicted values with experimental one.

The mechanism of slip in the two-phase flow problem can be modeled with the fewest complications. If there is no slip, the problem can be formulated by assuming a one-dimensional, uniform and homogeneous two-phase flow as shown in fig. 4.1. It is homogeneous in a sense that the velocities of both phases are equal. This simple model eliminates the need to specify interfacial transfer terms as they do not appear in the conservation laws [14]. In addition, if thermodynamic equilibrium is assumed between the phases, the mixture density becomes simply the thermodynamic density. This means that a direct measurement of the void fraction is not necessary; it can be deduced simply from a knowledge of the mass flow rates of the individual phases. The homogeneous model is not adequate at low pressure and mass fluxes, but is still used as a base model to indicate the trend among parameters or correlations for some specific applications.

The next level of sophistication in formulating a two-phase flow problem with slip is to assume a one-dimensional, non-homogeneous flow, as shown in fig. 4.2 [36]. In this model the phases have uniform but different velocities. The formulation can also be done by assuming a homogeneous, two-dimensional flow as shown in fig. 4.3 [35]. In this model both phases have the same velocity locally. A higher level of sophistication is to formulate the problem by assuming, a non-homogeneous, two-dimensional flow, as shown in fig. 4.4 [37]. This model is apparently a combination of the Wallis and Bankoff models, and is the so-called drift flux model. The word

"drift" results from the analogy with molecular diffusion of chemical species in multicomponent systems [79]. In these systems the formulation is based on the relative velocity of one species with respect to the mixture volumetric flow and is called the drift velocity of the species.

4.1.1 Drift Flux Model

In the drift flux model, Zuber et al. [37] developed a closure for the drift velocity of vapor or gas relative to the mixture using the continuity equation only. In addition, this model corrects for the momentum and kinetic energy as is the case in single-phase flows [80] when we deal with averages. A complete formulation of the drift flux model, which incorporates momentum and energy conservation equations is given by Ishii [38].

The derivation of the drift flux model due to Zuber and Findlay [37], but which includes here time averages explicitly, proceeds as follows. Let us consider eqn. (2.15) and write it explicitly as

$$\overline{\langle w_v \rangle} = \overline{\langle \alpha_f w_f \rangle} + \overline{\langle \alpha_g w_g \rangle} = \overline{\langle \alpha_f w_f + \alpha_g w_g \rangle} , \quad (4.1)$$

then we can define the local volumetric velocity as

$$\overline{w_v} = \overline{\alpha_g w_g} + \overline{\alpha_f w_f} . \quad (4.2)$$

The local drift velocity of phase k is defined as the relative velocity of that phase with respect to an observer moving with the volumetric velocity, namely,

$$\overline{w_{kd}} = \overline{w_k} - \overline{w_v} . \quad (4.3)$$

So, the local drift velocity of the vapor phase may be written as

$$\bar{w}_{gd} = \bar{w}_g - \bar{w}_v . \quad (4.4)$$

Using eqn. (4.2) we can also write

$$\bar{w}_{gd} = \bar{w}_g - \bar{\alpha}_g \bar{w}_g - \bar{\alpha}_f \bar{w}_f = \bar{\alpha}_f (\bar{w}_g - \bar{w}_f) . \quad (4.4a)$$

If we multiply eqn. (4.4) by $\bar{\alpha}_g$, then we obtain

$$\bar{\alpha}_g \bar{w}_{gd} = \bar{\alpha}_g \bar{w}_g - \bar{\alpha}_g \bar{w}_v . \quad (4.5)$$

By taking the space average, eqn. (4.5) becomes

$$\langle \bar{\alpha}_g \bar{w}_{gd} \rangle = \langle \bar{\alpha}_g \bar{w}_g \rangle - \langle \bar{\alpha}_g \bar{w}_v \rangle \quad (4.6)$$

and dividing by $\langle \bar{\alpha}_g \rangle$, we get

$$\frac{\langle \bar{\alpha}_g \bar{w}_{gd} \rangle}{\langle \bar{\alpha}_g \rangle} = \frac{\langle \bar{\alpha}_g \bar{w}_g \rangle}{\langle \bar{\alpha}_g \rangle} - \frac{\langle \bar{\alpha}_g \bar{w}_v \rangle}{\langle \bar{\alpha}_g \rangle} \cdot \frac{\langle \bar{w}_v \rangle}{\langle \bar{w}_v \rangle} . \quad (4.7)$$

Using definition eqn. (2.10), we can write eqn. (4.7) as

$$\hat{w}_{gd} = \hat{w}_g - \frac{\langle \bar{\alpha}_g \bar{w}_v \rangle}{\langle \bar{\alpha}_g \rangle} \cdot \frac{\langle \bar{w}_v \rangle}{\langle \bar{w}_v \rangle} , \quad (4.8)$$

and if we define

$$C_o \equiv \frac{\overline{\langle \alpha_g w_v \rangle}}{\overline{\langle \alpha_g \rangle} \overline{\langle w_v \rangle}}, \quad (4.8a)$$

we get

$$\hat{w}_g = C_o \overline{\langle w_v \rangle} + \hat{w}_{gd}, \quad (4.8b)$$

where C_o is the so-called Zuber and Findlay distribution parameter, or the pseudo-slip, and $\hat{w}_{gd}$ is the drift velocity. Equation (4.8a) simply states that the average of a product is not equal to the product of the averages provided that $C_o \neq 1$. If the pseudo-slip C_o and the drift velocity are assumed constant for a given flow regime, eqn. (4.8b) has a simple geometrical interpretation, namely that of a straight line in $(\hat{w}_g, \overline{\langle w_v \rangle})$ coordinates.

An expression for the void fraction can be written as

$$\overline{\langle \alpha_g \rangle} = \frac{\overline{\langle \alpha_g \rangle} \hat{w}_g}{C_o \overline{\langle w_v \rangle} + \hat{w}_{gd}} = \frac{\overline{\langle \alpha_g w_g \rangle}}{C_o \overline{\langle w_v \rangle} + \hat{w}_{gd}}. \quad (4.9a)$$

And using eqn. (2.12), we get

$$\overline{\langle \alpha_g \rangle} = \frac{\psi \langle x \rangle v_g}{C_o \overline{\langle w_v \rangle} + \hat{w}_{gd}}. \quad (4.9b)$$

Hence, the void fraction $\overline{\langle \alpha_g \rangle}$ can be predicted from eqn. (4.9b) using the drift flux model if the pseudo-slip, C_o , and the drift velocity, $\hat{w}_{gd}$ are known. The other parameters such as the max flux ψ , the quality x , the vapor specific volume v_g , and the mixture volumetric velocity $\overline{\langle w_v \rangle}$ can be measured experimentally as we explained in Section II.

The success of the drift flux model can then be checked by a comparison between the predicted and measured values of void fraction, for example. The check can be made also by comparing the hold-up $\langle \bar{\alpha}_f \rangle$ or the slip ratio σ . However, we will show in the following section that the comparison should be done based on the slip ratio σ .

If the flow is homogeneous, one-dimensional, and the velocity is uniform, then $\bar{w}_{gd}=0$ from eqn. (4.4a) which means that $\hat{w}_{gd}=0$; in addition, $C_0=1$ since the velocity is uniform. Therefore, we recover from eqn. (4.9b) the void fraction that corresponds to the uniform-velocity, homogeneous flow case, i.e.,

$$\langle \bar{\alpha}_g \rangle_o = \frac{\psi \langle x \rangle v_g}{\langle w_v \rangle} = \frac{\psi \langle x \rangle v_g}{\psi [\langle x \rangle v_g + (1 - \langle x \rangle) v_f]}$$

or

$$\langle \bar{\alpha}_g \rangle_o = \frac{1}{1 + \frac{1 - \langle x \rangle}{\langle x \rangle} \frac{1}{\gamma}} \quad (4.10)$$

Using eqn. (4.10) we can easily form the ratio

$$\frac{\langle \bar{\alpha}_f \rangle_o}{\langle \bar{\alpha}_g \rangle_o} = \frac{1 - \langle x \rangle}{\langle x \rangle \gamma} \quad (4.11)$$

and eqn. (2.21) can be rearranged as

$$\sigma = \frac{\langle \bar{\alpha}_f \rangle_o / \langle \bar{\alpha}_g \rangle_o}{\langle \bar{\alpha}_f \rangle_o / \langle \bar{\alpha}_g \rangle_o} = \frac{r}{r_o} \quad (4.12)$$

where

$$r = \langle \bar{\alpha}_f \rangle / \langle \bar{\alpha}_g \rangle . \quad (4.12a)$$

If we assume that the uncertainties in an experiment are solely due to the uncertainty in the measurement of hold-up, then it can be seen from eqn. (4.12) that the relative error in the slip ratio σ is proportional to the relative error in r , rather than the relative error in either $\langle \bar{\alpha}_g \rangle$ or $\langle \bar{\alpha}_f \rangle$, i.e.,

$$\frac{\Delta \sigma}{\sigma} = \frac{\Delta r}{r} , \quad (4.13)$$

where Δ is the deviation of the predicted value from its true experimental value. Moreover, from eqn. (4.12a) we can write

$$\ln r = \ln \langle \bar{\alpha}_f \rangle - \ln \langle \bar{\alpha}_g \rangle . \quad (4.14)$$

It can be shown that

$$\frac{\Delta \langle \bar{\alpha}_f \rangle}{\langle \bar{\alpha}_f \rangle} = \langle \bar{\alpha}_g \rangle \frac{\Delta r}{r} , \quad (4.15b)$$

and

$$\frac{\Delta \langle \bar{\alpha}_g \rangle}{\langle \bar{\alpha}_g \rangle} = -\langle \bar{\alpha}_f \rangle \frac{\Delta r}{r} . \quad (4.15b)$$

It can be seen that the magnitudes of the relative deviations in $\langle \bar{\alpha}_g \rangle$ and $\langle \bar{\alpha}_f \rangle$ are smaller than that in r . For example, Hughmark's correlation [81] predicts $\langle \bar{\alpha}_g \rangle = 0.67$ for an experimental void fraction $\langle \bar{\alpha}_g \rangle = 0.75$ for one of the points.

Thus,

$$\% \frac{\Delta \langle \bar{\alpha}_g \rangle}{\langle \bar{\alpha}_g \rangle} = -10.7\% ,$$

and

$$\% \frac{\Delta \langle \bar{\alpha}_f \rangle}{\langle \bar{\alpha}_f \rangle} = 32\% ,$$

but eqn. (4.13) predicts

$$\% \frac{\Delta r}{r} = \% \frac{\Delta \sigma}{\sigma} = 42.7\% .$$

Hence, the relative deviation in r , which is also the relative deviation in the slip ratio, is also a more conservative test than the prediction of deviations in either the void fraction or the hold-up.

From here on, we will drop the overscore and keep in mind that the parameters are averaged in time.

4.1.2 Prediction of Slip Based on the Zuber and Findlay Method

Zuber and Findlay [37] suggested plotting data in $(\hat{w}_g, \langle w_v \rangle)$ coordinates. Upon inspection of the distribution of the experimental points on such a graph, a linear relationship between $\hat{w}_g$ and $\langle w_v \rangle$ appears to hold. This means that the slope C_o should be constant, as well as the intercept $\hat{w}_{gd}$, for the range covered by the experiments. In addition, they proposed void fraction and mixture volumetric velocity profiles that yield a pseudo-slip C_o in terms of power exponents of the proposed profiles. Specifically, their proposed profiles are of the form

$$\langle \alpha_g \rangle \sim 1 - \left(\frac{r}{r_o} \right)^n ,$$

and

$$\langle w_v \rangle \sim 1 - \left(\frac{r}{r_o} \right)^m ,$$

where r is the distance from the center of the pipe toward the wall; m and n are to be selected on the basis of measured void fraction profiles. Following the original work of Bankoff [35], they assume various laminar and turbulent profiles by adjusting the exponents m and n . They also claim that when m and n are equal to 7, the profiles are flat as in single-phase turbulent flows. At this point we should not confuse their assumed profiles with the well known one-seventh power law for turbulent flows [80] for the following reasons. On the one hand, the one-seventh power law predicts an infinite slope in the velocity profile at the wall, or practically a large slope which is typical of turbulent flow (not for strong adverse pressure gradients). On the other hand, the velocity profiles proposed by Zuber et al. [37] predict a slope at the wall proportional to $(-m/r_0)$, and that is not typical of flat turbulent velocity profiles for $m = 7$, and finite pipe radius.

In the following, series 3-A and 4 are examined to determine the slip based on the drift flux model. Figures 4.5 and 4.6 show the experimental points in $(\hat{w}_g, \langle w_v \rangle)$ coordinates for series 3-A and 4, respectively. Clearly, fig. 4.5 does not indicate any change from a "slug-churn" flow to an "annular" flow. Except for points below $\langle w_v \rangle = 2$ m/s in fig. 4.6, which correspond to a "bubbly" flow, it is striking that all experimental points fall quite well on a straight line. Using the method of least-squares for straight lines [82], eqn. (4.8b) predicts

$$C_o = 1.02 \pm 0.01 ,$$

and

$$\hat{w}_{gd} = 1.00 \pm 0.17 \text{ m/s} ,$$

for series 3-A, and

and

$$C_o = 1.05 \pm 0.01$$

$$\hat{w}_{gd} = 0.55 \pm 0.06 \text{ m/s}$$

for series 4.

Zuber et al. [37] suggest a value of $C_o=1$ for an "annular" regime. The value of pseudo-slip being very close to unity suggests that the average of the product of $\bar{\alpha}_g$ and $\bar{w}_v$ is nearly equal to the product of the averages of $\bar{\alpha}_g$ and $\bar{w}_v$ separately.

If we substitute eqn. (4.9b) in eqn. (2.21), the slip can be written in terms of C_o and $\hat{w}_{gd}$ as

$$\sigma = C_o + (C_o - 1) \frac{x}{1-x} \gamma + \frac{\hat{w}_{gd}}{(1-x) \psi v_f} \quad (4.16)$$

For constant C_o and $\hat{w}_{gd}$, the slip decreases as the mass flux increases at a constant quality x or a constant reduced volume v_f . Also, at constant flux ψ the slip increases with the quality x or v_f . To test the accuracy in the prediction of slip based on the drift flux model, the values found for C_o and $\hat{w}_{gd}$ are used in eqn. (4.16). Figures 4.7 and 4.8 show the predicted slip vs. the experimental slip for series 3-A and 4, respectively. We find that the average relative deviation for the slip ratio is $0.115 \pm 58\%$ for series 3-A, and $0.092 \pm 89\%$ for series 4.

The large scattering in the slip ratio might indeed be attributed to changes in C_o . But we should note first that in a frozen flow, such as with air-water mixtures, the mixture volumetric velocity $\langle w_v \rangle$ remains constant at all cross-sections of a constant diameter test section for steady adiabatic flows because

$$\langle w_v \rangle = \psi [x v_g + (1 - x) v_f] \quad (4.17)$$

is constant since the quality x is constant for a frozen-flow. Moreover, if the mean void fraction profile is fully developed in the flow direction, then eqn. (4.8a) predicts a constant C_o . On the other hand, in single-substance two-phase flow, mass transfer is constantly taking place in the flow direction. In that sense, the flow is never fully developed and a constantly varying pseudo-slip in the flow direction might be expected. To see the variation in C_o , eqn. (4.8b) is written as

$$C_o = \frac{\langle \alpha_g \rangle \hat{w}_g - \langle \alpha_g \rangle \hat{w}_{gd}}{\langle \alpha_g \rangle \langle w_v \rangle} = \frac{\psi x v_g - \langle \alpha_g \rangle \hat{w}_{gd}}{\langle \alpha_g \rangle \langle w_v \rangle} \quad (4.18)$$

Using the values found for $\hat{w}_{gd}$ and the experimental values of the other parameters that appear in eqn. (4.18), it is expected that most values of C_o would vary within its standard deviation of 0.01. The results of eqn. (4.18) are shown in figs. 4.9 and 4.10 for series 3-A and 4, respectively. It can be seen that C_o does vary with $\langle w_v \rangle$ and most values are not even within the standard deviation of C_o . It is interesting to observe that the pseudo-slip C_o and the slip ratio behave qualitatively in exactly the same way by comparing figs. 2.1 and 4.9, and figs. 2.2 and 4.10, for example. Hence, the behavior of slip ratio also indicates the behavior of C_o . When σ reaches a maximum in fig. 2.1, C_o reaches also a maximum in fig. 4.9, and when σ exhibits a large scatter in fig. 2.2, C_o exhibits also a large scatter in fig. 4.10. So we can conclude that whenever we treat the slip ratio as a constant, the pseudo-slip has to be a constant and vice-versa.

It appears that presenting the experimental data in $(\hat{w}_g, \langle w_v \rangle)$ coordinates is probably not the best method for the following reasons. Let us consider series 3-A for example. In this case the vapor superficial velocity $\langle \alpha_g \rangle \hat{w}_g$ varies between 3.8 m/s and 15.1 m/s, whereas the liquid superficial velocity $\langle \alpha_f \rangle \hat{w}_f$ varies between only 0.60 m/s and 0.67 m/s. This is actually the case of an "annular" flow where most of the liquid is dispersed and moves with the vapor in the vapor core. Therefore,

$$\langle \alpha_g \rangle \hat{w}_g \gg \langle \alpha_f \rangle \hat{w}_f$$

and

$$\langle w_v \rangle \approx \langle \alpha_g \rangle \hat{w}_g .$$

Moreover, if the void fraction $\langle \alpha_g \rangle$ is close to 1, then

$$\langle w_v \rangle \approx \hat{w}_g$$

and

$$C_o = 1 .$$

Therefore, a different method of presenting the experimental data should be sought that allows us to determine whether or not $\hat{w}_g$ and $\langle w_v \rangle$ are linearly dependent. The new method is based on writing eqn. (4.8b) in the form

$$\hat{w}_g - \langle w_v \rangle = (C_o - 1) \langle w_v \rangle + \hat{w}_{gd} . \quad (4.19)$$

Clearly, if the function given by eqn. (4.8b) is truly close to a straight line with a slope C_o , then eqn. (4.19) should also be a straight line with a slope equal to $C_o - 1$.

If we normalize eqn. (4.19) with respect to the velocity at flashing w_f^* , we get

$$\frac{\hat{w}_{gv}}{w_f^*} = (C_o - 1) \frac{\langle w_v \rangle}{w_f^*} + \frac{\hat{w}_{gd}}{w_f^*} \quad (4.20)$$

where $\hat{w}_{gv}$ is simply the relative velocity of the vapor phase with respect to the mixture volumetric velocity, $\hat{w}_g - \langle w_v \rangle$. It should be noted that the relative velocity of liquid with respect to $\langle w_v \rangle$ is related to $\hat{w}_{gv}$. In other words, it can be shown that

$$\frac{\hat{w}_{fv}}{w_f^*} \equiv \frac{\hat{w}_f - \langle w_v \rangle}{w_f^*} = - \frac{\hat{w}_{gv}}{w_f^*} \cdot \frac{\langle \alpha_g \rangle}{\langle \alpha_f \rangle} \quad (4.21)$$

Figures 4.11 and 4.12 show the experimental data of $\hat{w}_{gv}/w_f^*$ and $\hat{w}_{fv}/w_f^*$ versus $\langle w_v \rangle/w_f^*$ for series 3-A and 4, respectively. First of all, on these figures the solid line at the origin O corresponds to homogeneous flow. Second, the relative velocity between the two phases, i.e.,

$$\hat{w}_r = \hat{w}_g - \hat{w}_f = \hat{w}_{gv} - \hat{w}_{fv} \quad (4.22)$$

can easily be seen to increase with $\langle w_v \rangle/w_f^*$. Third and most important, it can be seen, especially in fig. 4.11, that $(C_o - 1)$ is continuously changing with $\langle w_v \rangle$ due to mass transfer between the phases; this fact is obscured in fig. 4.5. Moreover, in some regions $(C_o - 1) < 0$ and in others $(C_o - 1) > 0$ and this contradicts the claim of Zuber et al. [37] that $C_o > 1$ in an adiabatic pipe. Again, the similarity between σ , C_o , and $\hat{w}_{gv}$ should be noted in figs. 2.1, 4.9, and 4.11, respectively.

In concluding this section, the new method of plotting the relative velocity of vapor $\hat{w}_g$ with respect to the mixture volumetric velocity $\langle w_v \rangle$ vs. $\langle w_v \rangle$ is superior to the method of Zuber et al. [37] because it clearly indicates whether or not C_o is a constant. It also appears that at low mass fluxes, the

pseudo-slip C_0 is less dependent on the mixture volumetric velocity. However, if one can accept relative deviations in the slip ratio of up to 30%, then the drift flux model appears to be satisfactory for those engineering applications.

4.2 Other Void Fraction Correlations

Many correlations have been suggested over the past years for the prediction of void fraction. The oldest one is that of Lockhart and Martinelli [4]. Although their correlations were first proposed for isothermal, two-phase, two-component flows in horizontal pipes, they are still being used in engineering designs under completely different conditions. A recent study by Friedel [6,83] of over 9000 measurements of void fractions showed that the correlations for void fractions by Hughmark [81] and Rouhani [84] are better than other correlations regardless of the type of fluid and the flow direction.

Presently, we will only compare hold-up measurements for series 3-A and 4 against the correlations of Lockhart-Martinelli, Hughmark and Rouhani. A short introduction to these correlations is presented without details since these can be found in the literature.

4.2.1 The Lockhart-Martinelli Correlation

The correlations presented by Lockhart and Martinelli [4] predict frictional pressure gradients and hold-up. They are based, as mentioned above, on two-component, two-phase flow in a horizontal pipe without phase change. The phases are assumed to be fully developed. Hence, the acceleration pressure gradients as well as the gravitational pressure gradients are negligible in their correlations. Later on, these correlations were extended for single-component, two-phase flows [3] such as steam-water mixtures. In [3], the independent variable was chosen to be the ratio of the

superficial frictional pressure gradient of the liquid to that of the vapor

$$X_M^2 = \frac{[dP/dz]_{sf}}{[dP/dz]_{sg}} \quad (4.23)$$

The superficial frictional pressure gradients are evaluated for the liquid and vapor phases as if each were flowing alone in a pipe at the same mass flow rates as that of the two-phase mixtures.

If both phases are assumed to be turbulent, then the frictional pressure gradients can be written generally as

$$\left[\frac{dP}{dz} \right]_{sk} = \frac{1}{2} \frac{f_k}{D} \rho_k w_{sk}^2, \quad (4.24)$$

where

$$w_{sk} = \langle \bar{\alpha}_k \bar{w}_k \rangle \quad (4.25)$$

is the superficial velocity of phase k. Using a generalized form of the Blasius friction factor,

$$f_k = \frac{C_k}{Re_k^{m_k}}, \quad (4.26)$$

where

$$\begin{aligned} C_k &= C_g = C_f = 0.046, \\ m_g &= m_f = 0.25, \end{aligned}$$

and

$$Re_k = \frac{\rho_k w_{sk} D}{\mu_k} \quad (4.27)$$

then the square of the Martinelli variable X_M becomes

$$X_M^2 = \left[\frac{\rho_f}{\rho_g} \right]^{0.75} \left[\frac{w_{sf}}{w_{sg}} \right]^{1.75} \left[\frac{\mu_f}{\mu_g} \right]^{0.25} \quad (4.28)$$

Using eqns. (2.12) and (4.25), the foregoing expression for X_M can be written as

$$X_M = \left[\frac{\rho_g}{\rho_f} \right]^{0.5} \left[\frac{\mu_f}{\mu_g} \right]^{0.125} \left[\frac{1-x}{x} \right]^{0.875} \quad (4.29)$$

Using eqn. (2.21) for the slip ratio, eqn. (4.29) can also be written as

$$X_M = \gamma^{0.375} \sigma^{-0.875} \left[\frac{\langle \alpha_f \rangle}{\langle \alpha_g \rangle} \right]^{0.875} \left[\frac{\mu_f}{\mu_g} \right]^{0.125} \quad (4.30)$$

If the hold-up $\langle \alpha_f \rangle$ is plotted versus the variable X_M , it can be seen from eqn. (4.30) that there will be a family of curves corresponding to different slip ratios σ . After plotting the hold-up or void fraction versus X_M , Lockhart and Martinelli [3] correlated the hold-up versus the variable X_M by a single correlation. The numerical values from their correlation can be approximated by the relationship [85]

$$\langle \bar{\alpha}_f \rangle = X_M [1 + 20X_M + X_M^2]^{-1/2} \quad (4.31)$$

Hence, in a two-component, two-phase flow where the temperature and the quality are fixed, eqn. (4.29) shows that X_M is fixed. Hence, if the total mass flow rate is changed under isothermal conditions and if the quality is fixed, it can be concluded that the variable X_M and the hold-up $\langle \alpha_f \rangle$ given by eqn. (4.31) are insensitive to changes in total mass flow rates.

4.2.2 Hughmark's Correlation

To summarize the work of Hughmark [81], suffice it to say that Hughmark's correlation is a modification of an earlier correlation proposed by Bankoff [35],

$$K_A = 0.71 + 0.0001 P, \quad (4.32)$$

where the pressure P in this equation has units of lbf/in². K_A is the so-called Armand coefficient. In his model, Bankoff assumed velocity and void fraction profiles similar to the one-seventh power law for turbulent flows [80], and subsequently showed that the slip ratio σ is given by

$$\sigma = \frac{\langle \alpha_f \rangle}{K_A - \langle \alpha_g \rangle} \quad (4.33)$$

If we set the drift velocity $\hat{w}_{gd} = 0$ in eqn. (4.16), then the slip ratio can be written as

$$\sigma = C_o + [C_o - 1] \frac{x}{1-x} \gamma \quad (4.34)$$

Using eqn. (2.21), this becomes

$$\sigma = \frac{\langle \alpha_f \rangle}{\frac{1}{C_o} - \langle \alpha_g \rangle} \quad (4.35)$$

A comparison between eqns. (4.33) and (4.35) shows that

$$K_A = \frac{1}{C_o} \quad (4.36)$$

Moreover, if we treat the slip ratio σ to consist of two components, one is due to a local slip represented by the drift velocity and the other a pseudo-slip due to velocity and void fraction profiles, we find that in Bankoff's model, the local slip is neglected and the slip ratio σ is solely due to the velocity variation at a cross-section in the pipe.

Using the previously calculated values of C_o in the drift flux model, we find that K_A is 0.980 and 0.952 for series 3-A and 4, respectively.

Hughmark modified eqn. (4.32) and related K_A to a new variable Z defined as

$$Z = \frac{Re^{1/6} Fr^{1/8}}{\langle \alpha_f \rangle_o^{1/4}} \quad (4.37)$$

where

$$Re = \frac{\psi D}{< \alpha_f > \mu_f + < \alpha_g > \mu_g}, \quad (4.38)$$

and

$$Fr = \frac{< w_v >^2}{gD}. \quad (4.39)$$

The hold-up $< \alpha_f >_0$ that corresponds to a homogeneous flow case ($\sigma = 1$) is given by eqn. (4.11).

4.2.3 Rouhani's Correlation

Rouhani [84] proposed the following correlation for void fraction

$$< \alpha_g > = \frac{[x/\rho_g]}{[C v + \frac{\hat{w}_{gd}}{\psi}]}, \quad (4.40)$$

where v is the mixture specific volume given by eqn. (2.14);

$$C = 1 + 0.2 (1 - x) (g D)^{1/4} / \left[\frac{\psi}{\rho_f} \right]^{1/2}. \quad (4.41)$$

The drift velocity $\hat{w}_{gd}$ is given originally by Zuber et al. [37] as

$$\hat{w}_{gd} = 1.18 \left[\sigma_t g \frac{\rho_f - \rho_g}{\rho_f^2} \right]^{1/4} \quad (4.42)$$

for a "churn-turbulent bubbly" flow regime. It is then modified by Rouhani to

$$\hat{w}_{gd} = 1.18 \left[\sigma_t g \frac{\rho_f - \rho_g}{\rho_f^2} \right]^{1/4} [1 - x] \quad (4.43)$$

In order to see clearly the advantages as well as the limitations of the above proposed correlations, we follow the work of [7] and present in the next section these correlations on a single graph of hold-up $\langle \alpha_f \rangle$ versus the Martinelli variable X_M .

4.2.4 Comparison with Experimental Results and Discussions

The relationship between the hold-up $\langle \alpha_f \rangle$ and the Martinelli variable X_M given by eqn. (4.30) is shown in fig. 4.13 for slip ratios of 1, 3, and 5. In eqn. (4.30) densities and viscosities of liquid and vapor are evaluated at a temperature $T = 7^\circ\text{C}$. This temperature represents a mean value for the two-phase flow mixture at the section between the two quick closing valves, for series 3-A and 4. This is done because it is impossible to include temperature variations on a single graph. To show the effect of temperature change, another curve (broken line) is plotted as a reference line for a temperature $T = 20^\circ\text{C}$ and a slip ratio $\sigma = 1$. Hence, we can see the effect of increasing the temperature by the horizontal shift distance between the constant slip ratio lines (in this case $\sigma = 1$).

The proposed correlation by Lockhart and Martinelli for void fraction, which can be represented by a relationship such as eqn. (4.31), is independent of temperature and mass flow rate variations. This correlation is also shown in the figure, and it can be seen that it follows variable slip ratio paths as the variable X_M is changed.

Now, if we study Hughmark's correlation given by eqns. (4.33) and (4.37-39), we find that the mass flux ψ and the diameter D have to be specified in addition to temperature. Therefore, on a graph such as fig. 4.13, Hughmark's correlation generates a family of curves at a fixed temperature, but for different mass fluxes and pipe geometries. In order to show Hughmark's correlation in terms of $(\langle \alpha_f \rangle, X_M)$ variables, a correlation between Armand coefficient K_A and Hughmark's variable Z must be established first [81]; then, for a given hold-up $\langle \alpha_f \rangle$ the slip ratio can be calculated from eqn. (4.33) and the corresponding variable X_M can be evaluated from eqn. (4.30).

Rouhani's correlation also accounts for variable mass fluxes and pipe geometries, similar to Hughmark's correlation; but it considers in addition the effect of surface tension σ_f . For refrigerant 114 (R-114), the surface tension $\sigma_f = 12 \times 10^{-3}$ N/m at 25°C. In order to present Rouhani's correlation in fig. 4.13, the hold-up $\langle \alpha_f \rangle$ is varied and the quality x is found by solving eqns. (4.40-41), and (4.43) simultaneously; the corresponding value of X_M is found from eqn. (4.29).

The Hughmark and Rouhani correlations are shown also in fig. 4.13 for series 3-A. The results for series 4 are quite close to those for series 3-A, and the curves are not shown on the graph for clarity. In addition, experimental points of hold-up from series 3-A and 4 are shown also. A careful study of the graph shows that measurements from series 3-A appear to behave in a similar way to the slip ratio in fig. 2.1, where the hold-up

exhibits maxima when plotted vs. the Martinelli variable, while those from series 4 follow approximately a constant slip ratio $\sigma = 4$. None of the above correlations predict the experimental values very well.

The effect of temperature variations is considered next; and the above correlations are evaluated for different temperatures. The ratio $r = \langle \alpha_f \rangle / \langle \alpha_g \rangle$ is evaluated and results are presented in tables 3 and 4 for series 3-A and 4, respectively. Upon inspection of the data, we find that the Martinelli correlation is better than that of Hughmark or Rouhani. The overall average percentage deviations from the experimental values are 34.5%, 57%, and 51% for Lockhart-Martinelli, Rouhani, and Hughmark correlations, respectively. This result seems somewhat surprising because the Lockhart-Martinelli void fraction correlation is developed for horizontal flows while those of Hughmark and Rouhani are developed mainly for vertical flows.

In conclusion, the drift flux model seems to be the best among the four case studies in predicting the slip ratio, followed by the Lockhart-Martinelli correlation. However, the drift-flux model requires *a priori* a knowledge of the pseudo-slip C_0 and the drift velocity $\hat{w}_{gd}$ in addition to a knowledge of temperature and quality. Apparently, the drift flux model is difficult to apply because the drift velocity $\hat{w}_{gd}$ is not uniquely specified as our findings here showed. On the other hand, the Lockhart-Martinelli correlation requires a knowledge of temperature and quality only.

V. SIMULATION OF GEOTHERMAL WELLS

5.1 Survey of Well Data

We have collected data on wellflow characteristics from a number of wells in an effort to find wells that could be simulated in our laboratory without a great deal of modification to the existing facility. The data had to include measurements of pressure and temperature as functions of depth under two-phase flowing conditions, as well as information on the casing profile, mass flow rate, and content of noncondensable gases and dissolved solids. From these data, we could select appropriate candidates for laboratory simulation using our theory as reported elsewhere [12].

The wells that we examined are in New Zealand and Costa Rica. These wells included KA-10, KA-16 (Kawerau, N.Z.); NG-3, NG-9, NG-11, NG-12, NG-13 (Ngawha, N.Z.); BR-11, BR-21 (Broadlands/Ohaaki, N.Z.) [86]; and PGM-10 (Miravalles, C.R.) [87]. The pertinent characteristics of the above wells for which data was available and the corresponding conditions required for simulation using R-114 are listed in table 5.

As it turned out, none of these wells was an ideal candidate for simulation. One of the most common reasons was the fact that the temperature required in our facility to simulate that in the well (at the flash point) was too high to be practically achieved. For example, well KA-10 showed a flash temperature of 250°C ; the R-114 in our test section would have to flash at 45°C . While this temperature is not beyond the capability of our heating system, the pressure that would exist because of the R-114 vapor pressure would threaten the strength of some of the instrument fittings.

Another difficulty involved trying to obtain the correct diameter in our test section. Our similarity theory predicts the exact value to be used, but practical considerations lead one to choose commercially available sizes of

tubes rather than custom-made tubes of just the right size. We tried to overcome this obstacle by adjusting the mass flow rate in our facility to give the properly-scaled mass flux, i.e., mass flow rate/unit cross-sectional area.

Since our simulation applies only from the flash point downstream in the two-phase region, we must determine the flash depth in the flowing well fairly accurately in order to set up a valid comparison. From an examination of the pressure and temperature curves as functions of depth, it is usually possible to determine the flash point. Ideally, a pure fluid will flash when the pressure falls to the saturation pressure corresponding to the fluid temperature. In some cases, inaccuracies in the well measurements, systematic errors inherent in the instrumentation, scatter in the data, non-equilibrium effects, and the presence of impurities may make this determination rather uncertain. For example, in the data made available to use from well PGM-10 [87], the flash point appears fairly sharply defined by the temperature profile (abrupt departure from isothermal conditions), but the pressure measured at that point is nearly seven bars below the corresponding saturation pressure. A sizeable (and unexplained) discrepancy persists through the two-phase region. For example, at a point roughly 240 m downstream of the flash point, the measured temperature is about 218°C (with a corresponding saturation pressure of about 22.2 bars), but the measured pressure is only about 16 bars. Pressures lower than saturation usually mean that the water is in the superheated steam phase, but it is clear that the geofluid exists as a mixture of saturated vapor and liquid just downstream of the flash point.

Yet another problem in drawing comparisons between our laboratory simulations and actual flowing wells concerns the amount of contaminants present in the geothermal wells. Our theory is designed to predict similarity between pure working fluids. If significant amounts of impurities are present

in the geofluid (e.g., noncondensable gases, dissolved solids, etc.), then a more elaborate equation of state must be used. In the presence of salt, the boiling point temperature of the geofluid is elevated whereas in the presence of noncondensable gases the actual pressure will be higher than that corresponding to the saturation pressure of pure water. Actually, it never happens that a high temperature geothermal well flows only pure water as a two-phase mixture. For wells with high gas content, particularly carbon dioxide, CO_2 , the ideal gas law is quite inaccurate. Since the perfect gas equation is a basic assumption in [12], comparisons should be confined to relatively clean geofluids such as found in wells KA-16, BR-11, or PGM-10, all of which are low in gas and solids content. Unfortunately, wells KA-16 and BR-11, are characterized by high flash temperatures which cannot be achieved without modification of our measuring instruments; and well PGM-10 exhibited unexplained discrepancies in pressure and temperature measurements as we explained above.

Thus, our survey shows that most of the data available for use in laboratory simulations appropriate to our facility contain noncondensable gases which are about 1.4% by weight of total mass flow rate. Accordingly, we have carried out two series of measurements designed to determine how closely the similarity theory [12] can simulate a geothermal well with noncondensable gases.

After reviewing many data, we settled on one set of data obtained in a well identified as NG-11 in New Zealand. The characteristics of well NG-11 are given in table 6. In addition, actual pressure and temperature measurements as a function of depth are given in table 7 (columns 1, 2 and 3). The saturation pressure of pure water that corresponds to the measured temperature is found from steam tables and is shown in column 4 of table 7.

From a comparison of data in columns 3 and 4, we notice that for the data above a depth of 450 m where the flow is a two-phase mixture, the measured pressure is greater than the saturation pressure of pure water. This difference in pressure is due to the presence of CO_2 which, for the pressures involved, turned out to be a non-ideal gas. One might conclude from this that a more elaborate equation of state is needed than the simple ideal gas law equation to account for the presence of noncondensable gases which would necessitate a major modification of the similarity theory.

A thermodynamic analysis of steam-water mixtures in the presence of CO_2 showed that the latter behaves as a solute in a dilute liquid solution (H_2O). For an ideal solution, Henry's law states that the partial-vapor pressure of CO_2 is given by

$$P_{\text{CO}_2} = K_{\text{CO}_2} X_{\text{CO}_2} \quad (5.1)$$

where K_{CO_2} is a proportionality constant and X_{CO_2} is the mole fraction of CO_2 in the liquid phase.

If water vapor in equilibrium with the liquid solution is treated as an ideal gas, then we can apply Raoult's law to liquid water which states that

$$P_{\text{H}_2\text{O},f} = P_{\text{H}_2\text{O},\text{sat}} X_{\text{H}_2\text{O}} \quad (5.2)$$

where $P_{\text{H}_2\text{O},\text{sat}}$ is the vapor pressure of pure water taken to be a function of temperature alone, and $X_{\text{H}_2\text{O}}$ is the mole fraction of water in the liquid phase.

When CO_2 and H_2O exist in equilibrium in a flashing mixture, the concentration of CO_2 in the liquid phase depends on the number of moles of

CO₂ and H₂O in the liquid and gaseous phases. The ratio of moles of CO₂ in the liquid phase to that in the vapor phase can be expressed as,

$$\frac{n_{\text{CO}_2, \text{f}}}{n_{\text{CO}_2, \text{g}}} = A \frac{n_{\text{H}_2\text{O}, \text{f}}}{n_{\text{H}_2\text{O}, \text{g}}}, \quad (5.3)$$

where A is a proportionality coefficient [88].

The number of moles of H₂O in the liquid and vapor phases depend on the quality x. That is,

$$\frac{n_{\text{H}_2\text{O}, \text{f}}}{n_{\text{H}_2\text{O}, \text{g}}} = \frac{1 - x}{x}. \quad (5.4)$$

The quality x is estimated from a simplified energy balance of the mixture given by eqn. (1.1) in the absence of CO₂. The coefficients K_{CO_2} and A are calculated from [88] for water that contains no salt since the molarity of salts was not reported in [86]. The results of this analysis are shown in table 8. The agreement between predicted and measured pressures is very good.

This analysis was applied also to well NG-12 [86] which is characterized by 1.7% noncondensable gas content of the total mass flow rate. Again, the agreement between predicted and measured pressures in the two-phase region was very good (see table 9). Hence, the data of [86] appear to be reliable and can be used for future studies which concern the effect of noncondensable gases on similarity theories.

Next, we will present briefly the pertinent relations of the similarity

theory [12] used in our analysis to simulate geothermal wells of pure steam-water mixture, using R-114 in our facility.

5.2 Summary of Similarity Theory

In order to relate the two-phase flow of water to the flow of R-114, a similarity analysis developed by Maeder et al. [12] is used. Unlike similarity analyses in a single-phase flow, the case is more complex in a two-phase flow. Hence, some basic assumptions were made in [12]; for example, a constant latent heat of vaporization, a constant specific volume of the saturated liquid, a saturated liquid enthalpy that is a linear function of temperature, and a saturated vapor phase that obeys the perfect gas law were assumed. In addition, the fluid is considered to be in thermodynamic equilibrium; hence, the pressure can be found from saturation conditions i.e.,

$$P_{R-114} = P_{sat} (T_{R-114}) \quad (5.5)$$

and

$$P_{H_2O} = P_{sat} (T_{H_2O}) \quad (5.6)$$

The similarity theory indicates that it is not possible to achieve exact similarity, but that a reasonably good approximation should be possible. The pertinent equations needed to predict steam-water characteristics based on pressure and temperature measurements of R-114 two-phase flow can be summarized as

$$T_{H_2O} = T_{R-114} \left[\frac{h_{fg,H_2O}}{h_{fg,R-114}} \right]_r \left[\frac{R_{a,R-114}}{R_{a,H_2O}} \right]_r \quad (5.7)$$

for the prediction of the temperature, and

$$D_{H_2O} = D_{R-114} \left[\frac{h_{fg,H_2O}}{h_{fg,R-114}} \right]_r \left[\frac{c_{f,H_2O}}{c_{f,R-114}} \right]_r \left[\frac{R_{a,R-114}}{R_{a,H_2O}} \right]_r \quad (5.8)$$

for the prediction of pipe diameter. The similarity theory [12] indicates also that the distance from the flashing front to the point where the flow is choked can be scaled by

$$\frac{z_{H_2O}}{z_{R-114}} = \frac{D_{H_2O}}{D_{R-114}} \quad (5.9)$$

If we assume that the flowing two-phase mixture in the well is pure water and that the two phases are in thermodynamic equilibrium, then similarity requires that R-114 must have the flashing conditions indicated in row 2 of table 10. Due to the difficulty of finding a commercial pipe of 0.0271 m it was decided to use a 0.0254 m commercially available copper tube. The deviation of approximately 6% in the diameters resulted in large deviations in the flash Mach number, Ma^* , and the Froude number, Fr^* , as seen in table 10 (row 4). In series 5 (row 3), the mass flow is retained as required by similarity. In an attempt to compensate for the discrepancy between using a 0.0254 m commercial copper tube and a 0.0271 m tube as required by the similarity theory, the mass flux (i.e., mass flow rate/unit cross-sectional area) is scaled rather than the mass flow rate. To clarify this point, we see from the data in table 10 that similarity requires a mass flux of 589 kg/m²s, whereas the actual measured mass flux in the laboratory is 671 kg/m²s in series 5. So, in a 0.0254 m pipe, a mass flux of 589 kg/m²s would

require a mass flow rate of 0.30 kg/s instead of 0.34. Hence, series 6 corrects for the discrepancies in series 5. The parameters of series 6 are shown in row 5 of table 10 and as can be seen, series 6 meets the similarity requirements for mass fluxes rather than mass flow rates.

5.3 Results and Discussions

Figure 5.1 shows the calculated and actual downstream pressure of steam-water mixture as a function of height from the flashing front. The calculated pressure is found from eqns. (5.5-5.7) in which the actual pressure and temperature measurements of R-114 in the test section are used.

A comparison of the actual pressure measurements of R-114 with the saturation pressure at the measured temperature indicated an average deviation of 3% from thermodynamic equilibrium. However, this was not true near flashing and the deviation from thermodynamic equilibrium was approximately 33%. Hence, except for states near the flashing point, thermodynamic equilibrium was established in the laboratory.

We notice in fig. 5.1 that immediately after flashing there is a steep slope in pressure as a function of height for series 5 and 6. This is associated with the acceleration of flow due to phase change caused by the flashing process. However, in the well this effect is very slight as seen in the actual pressure measurements. The reason for this discrepancy is not clear. We may offer this speculation: The presence of CO_2 gas bubbles at a location where the H_2O is just flashing into steam may act as a damper on the acceleration usually associated with the break-out of steam. This effect requires further study.

It is worth noting that the attempt to correct for the diameter discrepancy between the theory and the commercially available tube resulted in

no measurable differences. The two sets of data for series 5 and 6 practically overlap.

It can be seen also from fig. 5.1 that the actual pressure measurements are much higher than the predicted values from the similarity theory. This is due to the presence of noncondensable gases, the partial pressure of which is a large fraction of the total measured pressure, particularly close to the flash point.

It must be understood, however, that the quantity of interest is the pressure gradient starting at the point of flashing. If the pressure in the fluid at the flash point is known through calculation, then the evolution of pressure in the two-phase region of the well can be computed with the use of our similarity theory, if the theory predicts the correct pressure gradient. An examination of fig. 5.2 shows that the similarity theory predicts the actual pressure gradient very closely, i.e., to within about 8%.

Thus, in order to predict the pressure itself as a function of height from the flash point, it is necessary to specify: (1) reservoir temperature (i.e., the temperature of the geofluid at flash), (2) the amount of noncondensable gases (usually assumed as CO_2), (3) well casing profile, and (4) total mass flow rate. From this information, in conjunction with the governing equations, namely (1.1) and (5.1-5.4), it is possible to calculate also the temperature evolution in the well.

In the special case where there are no noncondensable gases in the geofluid, it can be seen from fig. 5.1 that the simulated pressures and actual pressures would essentially overlap (to within about 8%) and that the temperature would simply be the saturation value for the corresponding pressure.

5.4 Conclusion

We have shown that the similarity theory of Macder et al. [12] can be used to predict accurately the pressure gradient in the two-phase region of a geothermal well based on laboratory measurements using R-114. The theory also can be used when the well contains significant amounts of noncondensable gases when corrections are made to account for the gas partial pressure.

VI. SUMMARY

A Misty Core/Liquid Film model has been developed to explore the physics of two-phase flow whether it is in the "slug-churn" or "annular" regime. An approach similar to the Karman-Pohlhausen velocity approximation allowed as to describe the local flow reversal in the turbulent liquid film. None of previous models, to my knowledge, were capable of showing these phenomena when the film is turbulent.

Although the above approach is an approximation, it has many advantages. First of all, with an assumed velocity profile in the turbulent liquid film, we were able to predict parameters such as the fraction of liquid flowing in the film, the film thickness, the interfacial friction coefficient, the universal constant, and other parameters, based on measurements of temperature, total pressure drop, and hold-up. Secondly, results indicate definite trends which suggest correlations between various parameters. Having made such a decision for a large data bank of mass flow rates and flashing parameters, one choice would be the correlation between the universal constant and the liquid quality. Such a correlation can be used in turn in the expression of the interfacial friction coefficient which is the proportionality constant in the interfacial shear stress closure.

The proposed expression for the interfacial friction coefficient can be used best if the liquid film is turbulent and if the velocity profile in the core is quite flat. If the liquid film is laminar, then the boundary layer in the core just at the interface may not be completely turbulent and this should be accounted for in the proposed expression for the interfacial friction coefficient. It should be noted that this expression is the first to be derived as a function of the universal constant in addition to the film thickness.

In this study the effect of the flashing Reynolds number, in particular, was investigated and its effect on the liquid film was indicated. Previous investigations [33] did not consider the effect of such a parameter.

Finally, the prediction of the slip ratio by other models and correlations were compared to measured values. The study shows that a comparison based on the slip ratio is a better choice than that based on either the void fraction or the hold-up.

In the drift flux model, emphasis was placed upon the pseudo-slip or the Zuber and Findlay distribution parameter. This study shows that the pseudo-slip varies qualitatively similar to the slip ratio, which is continuously changing with the reduced volume or the mixture volumetric velocity. Hence, in two-phase flow with mass transfer, the pseudo-slip is always varying in the flow direction. Moreover, the plotting method of Zuber and Findlay appears to hide the variation of the pseudo-slip for an axisymmetric vertical flow with mass transfer. A new method of presenting the experimental data shows a curvature in the pseudo-slip particularly at the highest mass flow rate considered in this study.

Among the other correlations examined for the prediction of slip ratio, the Lockhart-Martinelli correlation appeared to be satisfactory. However, this conclusion is limited to the two series investigated. Unfortunately, no suggestions could be made to improve any of the examined correlations because of the limited number of experiments and the nature of these correlations.

Finally, the simulation of an actual geothermal well based on laboratory measurements using refrigerant 114 indicated that the similarity theory can be used to predict accurately the pressure gradient in the two-phase region of a well that flows a steam-water mixture with noncondensable gases.

APPENDIX A

A theoretical analysis is carried out to see why the temperature of a two-phase mixture is higher in the adiabatic glass test section than that in a diathermal test section. No effort is made here to develop the conservation laws that govern a two-phase flow, and the two-fluid model equations developed in [33,89] are considered instead.

The equation of motion is written as

$$-\frac{1}{\rho} \frac{dP}{dz} = \frac{1}{2} \frac{f}{D} w_v w_{cm} + w_v \frac{dw_{cm}}{dz} + \frac{\rho_m}{\rho} g \sin A . \quad (A.1)$$

The energy equation is written as

$$\frac{dh}{dz} = -\frac{d}{dz} \left[\frac{w_{rms}^2}{2} \right] - g \sin A + \frac{1}{m} \frac{d\dot{q}}{dz} , \quad (A.2)$$

and the entropy equation is given by

$$T \frac{ds}{dz} = \frac{1}{2} \frac{f_D}{D} w_{rms}^2 + \frac{1}{m} \frac{d\dot{q}}{dz} . \quad (A.3)$$

The rate of heat transfer gradient along the test section is given by

$$\frac{d\dot{q}}{dz} = \pi D_o h_c [T_\infty - T_{b,i}] , \quad (A.4)$$

where

$$T_{b,i} = \frac{1}{2} [T_\infty + T] . \quad (A.5)$$

For long vertical cylinders, the heat transfer coefficient for external natural convection is given in [90] as:

$$h_c = 0.5 k \left[\frac{\Delta T}{L} \frac{g\beta}{\nu^2} Pr \right]^{1/4} \quad (A.6a)$$

$$\text{for } Pr > 0.5 \text{ and } 10^4 < Gr Pr < 10^9; \quad (A.6b)$$

or

$$h_c = 0.12 \frac{k}{L} [Gr Pr]^{1/3} \quad (A.7a)$$

$$\text{for } 10^9 < Gr Pr < 10^{12}. \quad (A.7b)$$

Radiation and conduction heat transfer are neglected.

In these equations

$$\rho_m = \alpha_g \rho_g + \alpha_f \rho_f \quad (A.8)$$

is the "in place" density;

$$\rho = x \rho_g + (1-x) \rho_f \quad (A.9)$$

is the mixture density;

$$w_v = \alpha_g w_g + \alpha_f w_f \quad (A.10)$$

is the mixture volumetric velocity;

$$w_{cm} = x w_g + [1 - x] w_f \quad (A.11)$$

is the center of mass velocity; and

$$w_{rms} = \left[x w_g^2 + (1 - x) w_f^2 \right]^{1/2} \quad (A.12)$$

is the root-mean-square velocity.

The set of equations (A.1) to (A.12) are integrated numerically from flashing to choking for the case of a homogeneous flow where the average velocities of the individual phases are equal. In this case the "in place" density ρ_m becomes also equal to the mixture density ρ ; and the mixture volumetric velocity, center of mass velocity, and root-mean-square velocity become equal to each other.

The room temperature T_∞ is taken to be 22°C and the dissipation factor f_D is 1.5%. The downstream temperature of R-114 mixture as a function of height from the flashing front is shown in fig. A.1 for two different mass flow rates and stagnation conditions as indicated. For each case, the upper curve corresponds to an adiabatic test section and the lower curve corresponds to a diathermal test section. It can be seen that at a given cross section in the pipe, the temperature of the fluid is higher for an adiabatic pipe than for a diathermal one. We can also see that whether or not the pipe is adiabatic has a pronounced effect on the choking length at a low mass flow rate. For a high mass flow rate, the choking length varies within 2 m only if the pipe is adiabatic; however, for a lower mass flow rate the choking length exceeds the length predicted by the diathermal case by as much as 20 m.

The reason for predicting a higher temperature in the adiabatic case can be illustrated qualitatively on a h-s diagram as shown in fig. A.2. First of all, we know that the pressure decreases along the test section since the

pressure difference between the accumulator and the condenser is the driving force. Secondly, the entropy increases since the process is irreversible. So let us consider the energy equation (A.2) for a control volume between the stagnation point (say, point 1) and any point along the test section (say, point 2). Let us assume that the flow is fully developed. Hence, the third term in eqn. (A.1) and the second term in eqn. (A.2) will drop out. For the adiabatic pipe, we see from eqn. (A.2) that the enthalpy gradient changes by a value of $(-g)$ as the flow changes states from point 1 to point 2. This change is shown in fig. A.2 as process A. In the diathermal pipe, we see from eqn. (A.2) that the enthalpy gradient changes by a value of

$$-g + \frac{1}{\dot{m}} \frac{d\dot{q}}{dz}$$

Hence, the enthalpy at point 2 has a higher value in the diathermal pipe than that in the adiabatic pipe. In addition and as a result of the convective heat added to the control volume, the quality and the velocity of the mixture will be greater at point 2 in the diathermal pipe than in the adiabatic pipe. Therefore, for a greater velocity we can see from eqn. (A.2) that the pressure drop from point 1 to point 2 will be greater in the diathermal pipe than in the adiabatic pipe. This is shown in fig. A.2 as process B. We can easily see that the temperature of state 2 of process B is lower than that of process A.

It appears also that in the case of natural convective heat transfer the term

$$\frac{1}{\dot{m}} \frac{d\dot{q}}{dz} \ll \left| \frac{d}{dz} \left[\frac{w_{rms}^2}{2} \right] + g \sin A \right|$$

for high mass flow rate. Hence, above mass flow rates of 2 kg/s, it was not necessary to insulate the test section.

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TABLE 1

BASIC PARAMETERS FOR
DATA SERIES 3-A, 4, 5 and 6

	Series 3-A	Series 4	Series 5	Series 6
<u>Test Section:</u>				
Material	glass pipe	glass pipe	copper tube	copper tube
I.D. (mm)	(50.8)	(50.8)	(25.4)	(25.4)
insulation	none	fiberglass	insultube	insultube
<u>Orifice Plate:</u>				
Material	aluminum	aluminum	aluminum	aluminum
No. of holes	41	1	1	1
Effective dia. (mm)	12.7	9.32	5.10	5.10
$\dot{m}$ (kg/s)	2.10	1.05	0.340	0.292
% Uncertainty	± 0.95	± 1.5	± 0.50	± 0.99
T^* (°K)	291.63	293.08	304.36	304.03
% Uncertainty	± 0.04	± 0.07	± 0.13	± 0.02
h_o (kJ/kg)	18.13	19.60	31.05	30.72
% Uncertainty	± 0.63	± 1.0	± 1.3	± 0.24
Ma^*	0.330	0.159	0.156	0.134
% Uncertainty	± 0.94	± 1.4	± 1.0	± 1.0
Fr^*	0.997	0.500	0.940	0.801
% Uncertainty	± 0.79	± 1.5	± 0.51	± 1.0
$Re^* \times 10^{-3}$	142.73	72.580	53.650	45.460
% Uncertainty	± 0.78	± 1.5	± 0.70	± 0.70

TABLE 2
QUALITY, HOLD-UP MEASUREMENTS FOR
EXPERIMENTAL SERIES 3-A AND 4

Series 3-A			Series 4		
$x \pm 0.0007$	α_f	% Uncertainty in α_f	$x \pm 0.0010$	α_f	% Uncertainty in α_f
0.0420	0.2793	± 1.05	0.0020 ± 0.0005	0.9970	± 0.00
0.0448	0.2948	± 0.39	0.0142 ± 0.0005	0.7400	± 1.00
0.0559	0.2701	± 0.44	0.0356	0.3142	± 1.50
0.0676	0.2232	± 0.42	0.0382	0.3795	± 1.00
0.0734	0.1547	± 1.21	0.0459	0.3218	± 1.90
0.0744	0.1719	± 0.72	0.0482	0.3238	± 2.50
0.0816	0.1448	± 1.20	0.0532	0.2847	± 0.97
0.0862	0.1770	± 1.48	0.0536	0.2994	± 0.06
0.0863	0.1620	± 1.31	0.0569	0.2505	± 1.70
0.0892	0.1670	± 0.90	0.0594	0.2856	± 0.83
0.0912	0.1573	± 1.40	0.0638	0.2700	± 0.77
0.0924	0.1690	± 0.87	0.0692	0.2379	± 2.42
0.0982	0.1443	± 0.93	0.0754	0.2132	± 2.10
0.0990	0.1557	± 0.98	0.0812	0.2176	± 1.80
0.1040	0.1193	± 0.38	0.0867	0.1983	± 1.06
0.1042	0.1215	± 0.30	0.0881	0.1712	± 3.00
0.1043	0.1313	± 0.06	0.0953	0.1828	± 3.00
0.1046	0.1247	± 0.49	0.1070	0.1426	± 1.04
0.1066	0.1247	± 0.42	0.1092	0.1479	± 1.02
0.1092	0.1267	± 0.31	0.1115	0.1544	± 2.07
0.1100	0.1010	± 0.20	0.1124	0.1680	± 1.60
0.1102	0.0967	± 0.15	0.1160	0.1200	± 1.01
0.1106	0.1030	± 0.62	0.1243	0.1223	± 0.62
0.1112	0.1033	± 0.21	0.1259	0.1450	± 0.88
			0.1299	0.1230	± 0.75
			0.1312	0.1280	± 1.40
			0.1355	0.1082	± 0.47

TABLE 3

COMPARISON OF MARTINELLI, ROUHANI, AND HUGHMARK
VOID FRACTION CORRELATIONS WITH EXPERIMENTAL VALUES

SERIES 3-A

T °C	Experimental r	Martinelli r	Rouhani r	Hughmark r	% deviation from experimental r		
					Martinelli	Rouhani	Hughmark
2.86	0.1071	0.2171	0.2348	0.2426	102	119	127
3.06	0.1123	0.2178	0.2353	0.2434	94	109	117
3.07	0.1148	0.2173	0.2353	0.2432	89	105	112
3.25	0.1451	0.2194	0.2390	0.2463	51	65	70
3.29	0.1152	0.2171	0.2370	0.2442	88	106	112
3.81	0.1425	0.2235	0.2392	0.2493	57	68	75
3.92	0.1383	0.2267	0.2420	0.2528	64	75	83
4.00	0.1511	0.2278	0.2427	0.2539	51	61	68
4.08	0.1355	0.2283	0.2425	0.2543	69	79	88
4.64	0.1844	0.2369	0.2480	0.2624	28	34	42
4.90	0.1686	0.2385	0.2504	0.2646	41	48	57
5.06	0.1425	0.2277	0.2417	0.2533	60	70	78
5.56	0.2034	0.2494	0.2567	0.2746	22	26	35
5.72	0.1867	0.2528	0.2607	0.2784	35	40	49
6.05	0.2005	0.2563	0.2626	0.2817	28	31	41
6.50	0.2151	0.2628	0.2681	0.2883	22	25	34
6.68	0.1933	0.2640	0.2688	0.2896	37	39	50
7.27	0.1693	0.2745	0.2779	0.3000	62	64	77
8.36	0.1830	0.2973	0.2941	0.3214	62	61	76
9.28	0.2076	0.3113	0.3057	0.3355	50	47	62
10.01	0.2873	0.3323	0.3238	0.3558	16	13	24
11.04	0.3701	0.3620	0.3512	0.3855	-2	-5	4
13.04	0.3875	0.4473	0.4327	0.4727	15	12	22
13.10	0.4180	0.4216	0.4058	0.4455	1	-3	7

TABLE 4

COMPARISON OF MARTINELLI, ROUHANI, AND HUGHMARK
VOID FRACTION CORRELATIONS WITH EXPERIMENTAL VALUES

SERIES 4

T °C	Experimental	Martinelli	Rouhani	Hughmark	% deviation from experimental r		
	r	r	r	r	Martinelli	Rouhani	Hughmark
1.06	0.1213	0.1878	0.2945	0.2429	55	143	100
1.80	0.1468	0.1935	0.3005	0.2491	32	105	70
1.91	0.1403	0.1949	0.2993	0.2498	39	113	78
2.62	0.1696	0.2004	0.3046	0.2556	18	80	51
3.40	0.1393	0.2046	0.3099	0.2608	47	122	87
3.70	0.1364	0.2135	0.3181	0.2685	57	133	97
4.08	0.2019	0.2181	0.3211	0.2725	8	59	35
4.58	0.1826	0.2207	0.3218	0.2751	21	76	51
5.12	0.1663	0.2279	0.3305	0.2822	37	99	70
5.65	0.1736	0.2327	0.3342	0.2868	34	93	65
6.70	0.2237	0.2492	0.3495	0.3023	11	56	35
7.82	0.2066	0.2642	0.3629	0.3161	28	76	53
8.44	0.2473	0.2687	0.3618	0.3199	9	46	29
8.84	0.2781	0.2804	0.3771	0.3310	1	36	19
9.70	0.2710	0.2974	0.3922	0.3467	10	45	28
10.66	0.3122	0.3159	0.4136	0.3648	1	32	17
11.71	0.3699	0.3349	0.4289	0.3830	-9	16	4
12.10	0.3998	0.3507	0.4493	0.3983	-12	12	0.4
12.22	0.3342	0.3618	0.4616	0.4086	8	38	22
12.58	0.4273	0.3801	0.4830	0.4266	-11	13	-0.2
12.62	0.3980	0.3756	0.4760	0.4219	-6	20	6
13.60	0.4789	0.4047	0.5129	0.4523	-15	7	-6
13.82	0.4745	0.4168	0.5252	0.4642	-12	11	-2
14.94	0.6116	0.4664	0.5793	0.5423	-24	-5	-11
15.34	0.4582	0.4964	0.6227	0.5826	8	36	27
18.10	2.8462	0.9011	1.2442	1.2163	-68	-56	-57

TABLE 5

SIMILARITY CONDITIONS FOR SELECTED GEOTHERMAL WELLS⁽¹⁾

Well No.	Geofluid					Refrigerant-114				
	D cm	T °C	m kg/s	Ma*	Fr*	D cm	T °C	m kg/s	Ma*	Fr*
KA-10	15.04	250	42.0	0.105	2.437	2.00	45.4	0.201	0.107	1.037
KA-16	15.04	248	19.0	0.049	1.098	2.01	44.3	0.092	0.049	0.469
NG-3	15.04	228	20.0	0.068	1.117	2.04	33.7	0.104	0.069	0.496
NG-3	15.04	228	24.4	0.083	1.362	2.04	33.7	0.127	0.084	0.605
NG-11	19.88	224	64.4	0.133	1.779	2.71	31.6	0.340	0.134	0.797
NG-11	19.88	224	68.8	0.142	1.895	2.71	31.6	0.362	0.142	0.849
NG-12	19.88	228	13.3	0.026	0.370	2.70	33.7	0.069	0.026	0.164
NG-13	22.05	225	19.4	0.032	0.414	3.00	32.1	0.102	0.032	0.185
NG-13	22.05	225	53.0	0.088	1.131	3.00	32.1	0.280	0.088	0.506
BR-11	19.88	240	62.2	0.102	1.765	2.68	40.1	0.311	0.103	0.765
BR-21	19.88	260	21.9	0.028	0.645	2.61	50.5	0.100	0.028	0.270
BR-21	19.88	260	26.1	0.033	0.769	2.61	50.5	0.120	0.033	0.321
PGM-10	22.05	235	24.0	0.035	0.524	2.97	37.46	0.122	0.035	0.229

⁽¹⁾Data is from Ref. [86], except for PGM-10 which is from Ref. [87].

TABLE 6

CHARACTERISTICS OF WELL NG-11 [86]

Reservoir temperature, $T(^{\circ}\text{C})$	224.0
Reservoir pressure, $P_{\text{res}}(\text{bar})$	93.0
Wellhead pressure, $P_{\text{wh}}(\text{bar})$	21.2
Stagnation enthalpy (kJ/kg)	965
Mass flow rate, $\dot{m}(\text{kg/s})$	64.4
Noncondensable gas content, NGC/% total flow (by mass)	1.4

TABLE 7

MEASURED PRESSURE AND TEMPERATURE AS A
FUNCTION OF DEPTH OF WELL NG-11 [86]

Depth	T _{meas}	P _{meas}	P _{H₂O,sat} (T _{meas})
m	°C	bar,a	bar,a
0	206	20.0	17.6
100	211.5	23.5	19.6
200	215	27.5	21.0
300	218.5	32.0	22.5
350	--	34.9	--
400	220	38.0	23.2
450	--	41.6	--
500	222.5	44.8	24.2
550	--	49.0	--
600	223.5	52.9	24.7
700	224	61.1	25.0

*Calculated from Steam Tables at the measured temperature

TABLE 8

PREDICTED PARTIAL-VAPOR PRESSURES
AND MEASURED TOTAL PRESSURE
AS A FUNCTION OF DEPTH OF WELL NG-11 [86]

Depth (m)	0	100	200	300	400
Partial pressure of CO ₂ (bar,a) (Henry's law)	2.70	4.25	6.24	10.59	13.92
Partial pressure of H ₂ O (bar,a) (Raoult's law)	17.6	19.7	21.1	22.5	23.1
Total predicted pressure (bar,a)	20.3	24.0	27.3	33.1	37.0
Measured pressure (bar,a)	20.0	23.50	27.5	32.0	38.0

TABLE 9

PREDICTED PARTIAL-VAPOR PRESSURES
AND MEASURED TOTAL PRESSURE
AS A FUNCTION OF DEPTH OF WELL NG-12 [86]

Depth (m)	0	50	100	200
Partial pressure of CO ₂ (bar,a) (Henry's law)	4.30	6.30	6.59	11.10
Partial pressure of H ₂ O (bar,a) (Raoult's law)	19.87	21.51	21.71	23.37
Total predicted pressure (bar,a)	24.17	27.81	28.30	34.47
Measured pressure (bar,a)	25.10	26.80	28.90	33.50

TABLE 10
SIMILARITY BETWEEN WATER AND R-114

	$T(^{\circ}\text{K})$	$\dot{m} \text{ (kg/s)}$	$D \text{ (m)}$	$\psi \text{ (kg/m}^2\text{s)}$	Ma^*	Fr^*	$\text{Re}^* \times 10^{-3}$
Water ⁽¹⁾	497.16	64.4	0.1988	2075	0.134	1.779	3461.0
R-114 ⁽²⁾	304.72	0.340	0.0271	589	0.134	0.797	50.071
R-114 Test (Series 5)	304.36	0.340	0.0254	671	0.156	0.940	53.650
% Deviation Between Rows 2 & 3	0.12	0	- 6.3	13.9	16.4	17.9	7.1
R-114 Test (Series 6)	304.03	0.292	0.0254	576	0.134	0.801	45.460
% Deviation Between Rows 2 & 5	- 0.23	- 14.0	- 6.3	- 0.02	0.0	0.50	- 9.2

⁽¹⁾Data taken from well NG-11 [86].

⁽²⁾According to similarity theory of Maeder et al. [12].

FIGURE 1.1 [32]

FIGURE 1.2 [33]

FIGURE 1.3 [32]

NO.	PART NO.	DESCRIPTION	QTY.	PRICE	WORTH	UNIT	MATERIAL	REMARKS
1	1000	COVER PLATE	1	1.00	1.00	1.00	STEEL	✓
2	1001	O-RING	1	0.50	0.50	0.50	NBR	
3	1002	PLATE	1	1.00	1.00	1.00	ALUMINUM	
4	1003	FITTING	1	0.50	0.50	0.50	BRASS	
5	1004	ROD	1	1.00	1.00	1.00	STEEL	
6	1005	CASING	1	1.00	1.00	1.00	BRASS	
7	1006	AIR CYLINDER	1	1.00	1.00	1.00	BRASS	
TOTAL								
APPROVED BY: [Signature]								
DATE: 10/1/11								
DRAWN BY: [Signature]								
CHECKED BY: [Signature]								
APPROVED BY: [Signature]								
DATE: 10/1/11								
DRAWN BY: [Signature]								
CHECKED BY: [Signature]								
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FIGURE 1.5 (Photo by R. DiPippo)

FIGURE 1.6 (Photo by R. DiPippo)

FIGURE 1.7 (Photo by R. DiPippo)

FIGURE 1.8 (Photo by R. DiPippo)

FIGURE 1.9

FIGURE 2.1

FIGURE 2.2

FIGURE 2.3

FIGURE 3.1

FIGURE 3.2

FIGURE 3.3

FIGURE 3.4

FIGURE 3.5

Case a

Case b

Case c

Case d

Case e

Case f

Case g

FIGURE 3.6.

FIGURE 3.7

FIGURE 3.8

FIGURE 3.9

FIGURE 3.10

FIGURE 3.11

FIGURE 3.12

FIGURE 3.13

FIGURE 3.14

FIGURE 3.15

$\hat{W}_1 / W_f^*$
 FIGURE 3.16

FIGURE 3.17

FIGURE 3.18

FIGURE 3.19

FIGURE 3.20

FIGURE 3.21

FIGURE 3.22

Reduced volume V_r
FIGURE 3.24

FIGURE 3.25

FIGURE 3.26

FIGURE 3.27

FIGURE 3.28

FIGURE 3.29

FIGURE 3.30

FIGURE 3.31

Liquid quality $1-x$

FIGURE 3.32

FIGURE 4.1

FIGURE 4.2

FIGURE 4.3

FIGURE 4.4

FIGURE 4.5

FIGURE 4.6

FIGURE 4.7

FIGURE 4.8

FIGURE 4.9

FIGURE 4.10

FIGURE 4.11

FIGURE 4.12

FIGURE 4.13

FIGURE 5.1

FIGURE 5.2

FIGURE A.1

FIGURE A.2