**Covariance Statistics of
Turbulence Velocity Components
for Wind Energy Conversion
System Design-Homogeneous,
Isotropic Case**

G. H. Fichtl

September 1983

**Prepared for Pacific Northwest Laboratory
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COVARIANCE STATISTICS OF TURBULENCE VELOCITY
COMPONENTS FOR WIND ENERGY CONVERSION SYSTEM
DESIGN-HOMOGENEOUS, ISOTROPIC CASE

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PREFACE

Wind turbines are nearly unique among machines in that they must be dynamic, light, and yet strong enough to withstand substantial and continuing turbulent wind force in the atmospheric boundary layer. A program to optimize the cost of energy generated under such stringent conditions requires attention to the effects of wind in considerable detail during the design, testing and operation of the wind turbine.

This report was prepared under the supervision of the Design and Performing Evaluation Group of the Wind Characteristics Program Element of the Federal Wind Energy Program. The group is responsible for characterizing atmospheric properties in ways that are useful for designing and analyzing wind turbine response. This research includes the following components:

1. Restructuring and interpreting published conventional meteorological data for application to Wind Energy Conversion Systems (WECS)
2. Developing new characterizations of wind properties.
3. Describing wind characteristics at each important, distinctly different class of wind turbine site.

The ultimate objective of technical reports from work done in the Wind Characteristics Program Element is to provide practical information in a form easily interpreted and used by the engineering designer of WECS. The reports may have several different orientations. For example, some reports contain meteorological, mathematical and/or measurement detail or elaboration that provide advanced understanding of wind characteristics. These reports will subsequently be condensed and combined with other related reports to provide a more practical and general reference. Other reports represent multidisciplinary efforts, e.g., of engineers, atmospheric scientists, physicists and mathemat-

icians. Still other reports provide advanced understanding of wind characteristics and revelation of unsuspected features of machine-wind interactions that should be considered by the wind energy community in design of machines. By quickly making this information known to users, we anticipate that research and improvement of the information will occur, and that some potential problems of machine-wind interactions will be brought to the attention of designers. We expect that in some cases there will be a need for later reports to condense this information and incorporate related work into more practical reference documents.

In brief, a diversity of topics and their treatment ranging from practical engineering design curves to rather fuller scientific expositions of the subject matter are to be found in our publications. All reports are directed toward the ultimate, but not distant, goal of having accurate and representative characteristics of the wind-turbine layer of the atmosphere for use in the design and evaluation of performance of WECS.

This report gives a modern theoretical indication of the significance of rotational sampling of the wind by a wind turbine blade spinning about a horizontal axis. The description of the process in the atmosphere and the graphical presentation of the modification of wind spectra that may be expected are to verify mathematically for the design engineer the concept of frequency shifting. This concept has been described by Connell (1980) using rotationally composited winds measured at the Pacific Northwest Laboratory (PNL) vertical plane array and more recently by Connell (1981) by theory and new observations. The detailed mathematical derivations and generalizations in this report are intended for engineers and scientists who wish to apply such theory in its present or improved form to WECS design. Present indications are such that the additional measurement and theoretical work now taking place on the rotational sampling of the wind by wind turbines with either horizontal or vertical axes will provide a firm basis for engineering decisions that will lead to improved design of WECS. The theoretical work reported here provided independent evidence of the concept

and of the magnitude of the effect observed in the wind measurements at PNL. This evidence was important before undertaking several studies now started, which should provide practical, quantitative information for use in design and lifetime assessment.

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SUMMARY

When designing a wind energy conversion system (WECS), it may be necessary to take into account the distribution of wind across the disc of rotation. The specific engineering applications include structural strength, fatigue, and control. This wind distribution consists of two parts, namely that associated with the mean wind profile and that associated with the turbulence velocity fluctuation field. The work reported herein is aimed at the latter, namely the distribution of turbulence velocity fluctuations across the WECS disk of rotation

A theory is developed for the two-point, two-time covariance matrix for turbulence velocity vector components for wind energy conversion system (WECS) design. The theory is developed for homogeneous and isotropic turbulence with the assumption that Taylor's hypothesis is valid. The Eulerian turbulence velocity vector field is expanded about the hub of the WECS. Formulae are developed for the turbulence velocity vector component covariance matrix following the WECS blade elements. It is shown that upon specification of the turbulence energy spectrum function and the WECS rotation rate, the two-point, two-time covariance matrix of the turbulent flow relative to the WECS blade elements is determined. This covariance matrix is represented as the sum of nonstationary and stationary contributions. Generalized power spectral methods are used to obtain two-point, double frequency power spectral density functions for the turbulent flow following the blade elements. The Dryden turbulence model is used to demonstrate the theory. A discussion of linear system response analysis is provided to show how the double frequency turbulence spectra might be used to calculate response spectra of a WECS to turbulent flow. Finally the spectrum of the component of turbulence normal to the WECS disc of rotation, following the blade elements, is compared with experimental results.

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INTRODUCTION

The purpose of this document is to develop a model that can be used to estimate the two-point, two-time covariance statistics of atmospheric turbulence velocity fluctuations following the blade elements of a wind energy conversion system (WECS). The flow in the atmospheric boundary layer can be represented in terms of a mean wind field (obtained by averaging wind records over a one hour period, say) and a departure from this mean. The departures, termed turbulence velocity fluctuations, constitute a three-dimensional, time dependent stochastic field. The WECS blade elements experience a highly time-dependent flow as a result of these turbulence velocity fluctuations. Because of the stochastic nature of these turbulence velocity fluctuations it is useful to describe the fluctuations with two-point, two-time covariance statistics, i.e., the average product between a turbulence velocity component at a specified location on a given WECS blade at a specified time with a turbulence velocity component at another specified location on a given WECS blade at another specified time. If the locations, times, and velocity components are the same then the covariance is the variance of the velocity component in question. If the velocity components in question are different we obtain a measure of the degree to which the velocity components are related, or correlated, at a fixed point on the WECS at a fixed time. If the locations and times are different we then obtain a measure of the degree to which the velocity components (same or different) are related between blade elements and between time points.

Consideration of the covariance statistics herein is limited to the homogeneous and isotropic case. Homogeneous turbulent flow corresponds to a flow in which all statistics (means, covariances, etc.) associated with velocity fluctuations at a set of arbitrary points in space and time do not vary upon spatial translation of the set of points keeping the relative positions of the points the same. Isotropic turbulence corresponds to the case in which the turbulence statistics do not vary upon imposing coordinate system rotations or coordinate axes reflections. Atmospheric boundary layer flow is vertically nonhomogeneous. Horizontal homogeneity depends on the degree of uniformity of the terrain roughness and heat

transfer characteristics. The symmetry properties of atmospheric boundary layer turbulence depends upon the scale of the eddies. Thus, the relatively long wavelength or energy containing eddies, i.e., eddies with horizontal scale comparable to the integral scale of turbulence are anisotropic, while the relatively short wavelength eddies with horizontal scale comparable to or less than the height of the eddy above the surface of the earth tend to be isotropic. The homogeneous and isotropic case permits mathematical simplicity and at the same time retains much of the physical process we wish to describe. The homogeneous and isotropic turbulence model permits the calculation of the two-point, two-time covariance statistics following the WECS blade elements upon specification of the turbulence energy spectrum function in the sense of Batchelor (1960). In the nonhomogeneous and anisotropic case a single defining function, like the turbulence energy spectrum function, for calculation of all covariances following the WECS blade elements is not available. The turbulence energy spectrum function permits the calculation of the Eulerian covariance function matrix. The elements of the Eulerian covariance matrix are composed of the two-point, two-time velocity component covariances in a frame of reference attached to the earth. In lieu of the availability of a turbulence energy spectrum function in the nonhomogeneous and anisotropic case one must directly specify the components of the Eulerian covariance matrix to calculate the turbulence covariance statistics following the blade elements of a WECS.

The engineering motivation for determining turbulence covariance statistics following the WECS blade elements is three-fold. First, the turbulence covariance statistics can be used directly in linear response analyses to calculate the covariances of WECS responses. Second, the turbulence covariances can be used to simulate turbulence velocity fluctuations via band-limited white noise and shaping filters for linear and nonlinear WECS operations simulation studies. Finally, turbulence velocity covariances can be used to derive discrete gust inputs. All of the above inputs may be required in the WECS design process. Discrete inputs are typically used for preliminary design, while turbulence space-time series would be used in final design verification studies. Turbulence spectra and covariances are used typically during intermediate and final design studies.

In the analysis that follows we derive the two-point, two-time turbulence statistics for WECS design via a technique first used by Etkin (1960) in the context of airplane dynamics. The starting point for the analysis is the definition of wind fields. We first specify the Eulerian wind field and partition it into a mean flow with a superimposed turbulent contribution. The coordinate system for this representation is at the WECS hub, but does not rotate with WECS blade elements. The Eulerian wind field is expanded via Taylor series expansion about the WECS hub. The Taylor series representation of the Eulerian field is evaluated at a specified blade element. We next proceed to calculate the turbulence velocity covariances following the WECS blade elements with this Taylor series representation of the turbulence field evaluated at the WECS blade elements. The resulting two-point, two-time covariance statistics calculated with the Taylor series expansion of the wind field about the hub results in a series with each term composed of a two-time statistic of the turbulence field, evaluated at the hub, multiplied by a function of the coordinates of the WECS blade elements in question. The analysis herein retains terms in the covariance series to second-order in space coordinates, which requires retention of terms to second-order in the Taylor series expansion of the wind field. Thus, the task of developing the two-point, two-time covariance statistics of turbulence velocity fluctuations for WECS design is reduced to determining the two-time statistics of the coefficients in the spatial Taylor series expansion of the Eulerian wind field about the WECS hub. To do this we assume that the turbulence is homogeneous and isotropic and that Taylor's hypothesis is valid. This permits the calculation of frequency domain spectra of the turbulence velocities and associated gradients at the WECS hub from a single defining function, namely the turbulence energy spectrum function (Batchelor, 1960). The spectra at the WECS hub can be used to calculate the two-time statistics of the coefficients in the spatial Taylor series expansion of the Eulerian field noted above. The Dryden turbulence model is used to calculate these frequency domain spectra. We next turn our attention to linear systems to discuss the application of the model developed herein. This leads to the development of nonstationary spectra of turbulence

velocity fluctuations following the blade elements.

Appendices are provided which outline: 1) a procedure for the exact calculation of two-point, two-time velocity covariance functions following the WECS blade elements, and 2) a procedure for the calculation of the same covariance functions for the vertically nonhomogeneous atmospheric boundary layer using the concept of Taylor series expansion of the wind field about the hub.

WIND FIELDS

In this section we calculate wind fluctuations following a WECS blade element. We shall use a Cartesian Coordinate system like that shown in Figure 1. The x_1 - and x_2 -axes are parallel to the surface of the earth and the x_3 -axis is directed along the vertical. The disk of rotation of the WECS lies in the (x_2, x_3) -plane with the hub at the origin of the coordinate system.

EULERIAN WIND FIELD

The x_i -component of the wind velocity vector is denoted by u_i . The quantity u_i is a stochastic function of x_1, x_2, x_3 , and t . The wind field is assumed to be statistically steady and horizontally homogeneous.

The ensemble average of the wind velocity vector is assumed to be given by

$$\langle u_i \rangle = \bar{u}(x_3) \delta_{1i} \quad (1)$$

where

$$\begin{aligned} \langle \rangle &= \text{ensemble average} \\ \delta_{ij} &= \text{Kronecker delta function} \\ \bar{u}(x_3) &= \text{mean wind profile.} \end{aligned}$$

The mean flow is directed along the x_1 -axis and is a function of x_3 only, which is consistent with the assumption of statistical steadiness and horizontal homogeneity. The turbulence velocity fluctuations are obtained by subtracting the ensemble average velocity vector field from the wind field, i.e.,

$$v_i' = u_i - \langle u_i \rangle \quad (2)$$

The turbulence velocity fluctuations are stochastic functions of x_1, x_2, x_3 , and t , and are statistically steady and fully homogeneous.

We expand the quantity v_i' about the origin of the coordinate system in the (x_2, x_3) -plane, so that

$$v_i'(0, x_2, x_3, t) = w_i(t) + p_i(t)x_2 + q_i(t)x_3 + \frac{1}{2}p_i(t)x_2^2 + \frac{1}{2}q_i(t)x_3^2 + M_i(t)x_2x_3 \quad (3)$$

where

$$w_i(t) = v_i'(0, 0, 0, t) \quad (4)$$

Figure 1. Frame of reference

$$p_i(t) = \left(\frac{\partial \hat{v}_i}{\partial x_2} \right)_o \quad (5)$$

$$q_1 = \left(\frac{\partial \hat{v}_1}{\partial x_3} \right)_o \quad (6)$$

$$p_i(t) = \left(\frac{\partial^2 v_i}{\partial x_2^2} \right)_o \quad (7)$$

$$Q_i(t) = \left(\frac{\partial^2 \hat{v}_i}{\partial x_3^2} \right)_o \quad (8)$$

$$M_i(t) = \left(\frac{\partial^2 v_i}{\partial x_2 \partial x} \right)_o \quad (9)$$

and we have truncated the series to include second-order terms. The coefficients in the expansion are evaluated at the origin of the coordinate system and are functions of time only. The expansion of the Eulerian field given by Equation (3) is the basis for the analysis that follows.

WIND FLUCTUATIONS AT A BLADE ELEMENT

We define a reference line located in the WECS plane of rotation. The reference line (see Figure 2) is fixed with respect to the WECS blades. At time $t = 0$ the reference line is coincident with the x_2 -axis. At time $t \neq 0$ the angle between the reference line and the x_2 -axis is $\omega_o t$, where ω_o is the angular rotation rate of the WECS. The angle between the reference line and the WECS blade line is α . The trajectory coordinates of a point located at

Figure 2. WECS blade coordinate system

radial distance r on the blade line are given by

$$X_1(t) = 0 \quad (10)$$

$$X_2(t) = r \cos (\omega_0 t + \alpha) \quad (11)$$

$$X_3(t) = r \sin (\omega_0 t + \alpha) \quad (12)$$

The angle α serves to define the angular location of a WECS blade and radial distance r serves to locate a blade element along the blade line. The number of values that α can take on is equal to the number of blades. Evaluation of the Eulerian wind field at a blade element located at radial distance r along the blade line which in turn is located at angular distance α from the WECS reference line is given by

$$\hat{v}_i(X_1(t), X_2(t), X_3(t), t) = v_i(t; r, \alpha) \quad (13)$$

where

$$\begin{aligned} v_i(t; r, \alpha) = & w_i(t) + p_i(t)r \cos (\omega_0 t + \alpha) \\ & + q_i(t)r \sin (\omega_0 t + \alpha) + \frac{1}{2}P_i(t)r^2 \cos^2 (\omega_0 t + \alpha) \\ & + \frac{1}{2}Q_i(t)r^2 \sin^2 (\omega_0 t + \alpha) \\ & + M_i(t)r^2 \cos (\omega_0 t + \alpha) \sin (\omega_0 t + \alpha) \quad (14) \end{aligned}$$

Equation (14) permits the calculation of turbulence velocity fluctuations at the blade element in question upon specification of the wind processes $w_i(t)$, $p_i(t)$, $q_i(t)$, $P_i(t)$, $Q_i(t)$, and $M_i(t)$ at the WECS hub and the rotation of the blade element ω_0 and position r, α . It should be kept in mind that the turbulence velocity fluctuations in Equation (14) are velocities, following a WECS blade element, in a frame of reference fixed with respect to the ground.

COVARIANCES

In this section we calculate the covariance matrix of the turbulent velocity fluctuations following the WECS blade elements. We consider two blade elements with coordinates (r_1, α_1) and (r_2, α_2) with respect to the WECS reference line as given in Figure 3. Multiplication of v_i evaluated at time $t + \tau$ at blade element located at (r_1, α_1) with v_j evaluated at time t at blade element located at (r_2, α_2) , ensemble averaging the resulting relationship, and retention of terms to second-order in r yields

$$R_{v_i v_j}(\tau; t, r_1, r_2, \alpha_1, \alpha_2) = S_{ij}(\tau; r_1, r_2, \alpha_1, \alpha_2) + N_{ij}(\tau; t, r_1, r_2, \alpha_1, \alpha_2) \quad (15)$$

where

$$\begin{aligned} S_{ij}(\tau; r_1, r_2, \alpha_1, \alpha_2) = & R_{w_i w_j}(\tau) + \frac{r_2^2}{2} \left[R_{w_i p_j}(\tau) + R_{w_i q_j}(\tau) \right] \\ & + \frac{r_1^2}{2} \left[R_{p_i w_j}(\tau) + R_{q_i w_j}(\tau) \right] + \frac{r_1 r_2}{2} \left[R_{p_i p_j}(\tau) + R_{q_i q_j}(\tau) \right] \cdot \\ & \cos(\omega_0 \tau + \alpha_1 - \alpha_2) + \frac{r_1 r_2}{2} \left[R_{q_i p_j}(\tau) - R_{p_i q_j}(\tau) \right] \cdot \\ & \sin(\omega_0 \tau + \alpha_1 - \alpha_2) \end{aligned} \quad (16)$$

Figure 3. Two-point WECS blade geometry

and

$$\begin{aligned}
N_{ij}(\tau; t, r_1, r_2, \alpha_1, \alpha_2) = & r_2 R_{w_i p_j}(\tau) \cos(\omega_0 t + \alpha_2) \\
& + r_2 R_{w_i q_j}(\tau) \sin(\omega_0 t + \alpha_2) \\
& + r_1 R_{p_i w_j}(\tau) \cos(\omega_0 t + \omega_0 \tau + \alpha_1) \\
& + r_1 R_{q_i w_j}(\tau) \sin(\omega_0 t + \omega_0 \tau + \alpha_1) \\
& + \frac{r_1 r_2}{2} \left[R_{p_i p_j}(\tau) - R_{q_i q_j}(\tau) \right] \cos(2\omega_0 t + \omega_0 \tau + \alpha_1 + \alpha_2) \\
& + \frac{r_1 r_2}{2} \left[R_{p_i q_j}(\tau) + R_{q_i p_j}(\tau) \right] \sin(2\omega_0 t + \omega_0 \tau + \alpha_1 + \alpha_2) \\
& + \frac{r_2^2}{4} \left[R_{w_i p_j}(\tau) - R_{w_i q_j}(\tau) \right] \cos(2\omega_0 t + 2\alpha_2) \\
& + \frac{r_1^2}{4} \left[R_{p_i w_j}(\tau) - R_{q_i w_j}(\tau) \right] \cos(2\omega_0 t + 2\omega_0 \tau + 2\alpha_1) \\
& + \frac{r_2^2}{2} R_{w_i M_j}(\tau) \sin(2\omega_0 t + 2\alpha_2) \\
& + \frac{r_1^2}{2} R_{M_i w_j}(\tau) \sin(2\omega_0 t + 2\omega_0 \tau + 2\alpha_1)
\end{aligned} \tag{17}$$

with

$$R_{\xi_i \xi_j}(\tau) = \langle \xi_i(t + \tau) \xi_j(t) \rangle \tag{18}$$

The quantities S_{ij} and N_{ij} represent the stationary and nonstationary contributions to the temporal gust covariance with respect to the blade elements. Both the stationary and nonstationary contributions are constructed from the temporal covariance functions of the stationary processes w , p , q , P , Q , and M at the hub. The results given by Equations (15, 16, 17) apply to inhomogeneous and anisotropic turbulent flow.

HOMOGENEOUS AND ISOTROPIC TURBULENCE

The three-fold wave number-frequency decomposition of $\hat{v}_i(x_1, x_2, x_3, t)$ is given by

$$\hat{v}_i(x_1, x_2, x_3, t) = \int_{-\infty}^{\infty} e^{i(x_j \kappa_j + \omega t)} dZ_i(\kappa_1, \kappa_2, \kappa_3, \omega) \quad (19)$$

(4)

where

κ_j = wavenumber along x_j -axis

ω = radian frequency

dZ_i = Fourier amplitude

$$\int_{-\infty}^{\infty} ( ) dZ_i = \text{fourfold stochastic Fourier integral over wavenumber and frequency domains} \quad (4)$$

$$i = \sqrt{-1}$$

The Einstein summation convention is applicable.

We define

$$\xi_{ikm\nu\lambda}(t) = \left(\frac{\partial^{\nu} + \lambda \hat{v}_i}{\partial x_k^{\nu} \partial x_m^{\lambda}} \right)_o \quad (20)$$

where

$( )_o$ = evaluation at WECS hub.

The Einstein summation convention does not apply to Greek indices. By appropriate selection of the subscripts on ξ we can obtain any of the coefficients in the Taylor series expansion given by Equation (3). Substitution of Equation (19) into Equation (20) yields

$$\xi_{ikm\nu\lambda}(t) = \int_{-\infty}^{\infty} e^{i\omega t} (i\kappa_k)^{\nu} (i\kappa_m)^{\lambda} dZ_i(\kappa_1, \kappa_2, \kappa_3, \omega) \quad (21)$$

(4)

COVARIANCES AND SPECTRA

We construct the following covariance function

$$R_{\xi_{ikm\nu\lambda}\xi_{jsn\beta\gamma}}(\tau) = \int_{-\infty}^{\infty} (i\kappa_k)^\nu (i\kappa_m)^\lambda (-i\kappa_s)^\beta (-i\kappa_n)^\gamma e^{i\omega\tau} \phi_{ij}(\kappa_1, \kappa_2, \kappa_3, \omega) d\kappa_1 d\kappa_2 d\kappa_3 d\omega \quad (4) \quad (22)$$

where

$$\phi_{ij}(\kappa_1, \kappa_2, \kappa_3, \omega) = \text{three-fold wave number-frequency spectral density function of } v_i(x_1, x_2, x_3, t) \text{ and } v_j(x_1, x_2, x_3, t) .$$

Thus, upon specification of $\phi_{ij}(\kappa_1, \kappa_2, \kappa_3, \omega)$ all of the relevant covariance functions in S_{ij} and N_{ij} and hence, in $R_{v_i v_j}$ can be determined.

We assume Taylor's hypothesis is valid, so that

$$\omega = k_1 u_0 \quad (23,a)$$

and

$$\phi_{ij}(\kappa_1, \kappa_2, \kappa_3, \omega) = \phi_{ij}(\kappa_1, \kappa_2, \kappa_3) \delta(\kappa_1 u_0 - \omega) \quad (23,b)$$

where

$$\delta(\kappa_1 u_0 - \omega) = \text{Dirac delta function} .$$

This assumes

$$\bar{u}(x_3) = \bar{u}(0) = u_0 \quad (24)$$

i.e., the wind profile is assumed to be constant. Substitution of Equation (23) into Equation (22) yields

$$R_{\xi_{ikm\nu\lambda}\xi_{jsn\beta\gamma}}(\tau) = \int_{-\infty}^{\infty} \psi_{\xi_{ikm\nu\lambda}\xi_{jsn\beta\gamma}}(\omega) e^{i\omega\tau} d\omega \quad (25)$$

where

$$\begin{aligned} \psi_{\xi_{ikm\nu\lambda}\xi_{jsn\beta\gamma}}(\omega) &= \int_{-\infty}^{\infty} (i\kappa_k)^\nu (i\kappa_m)^\lambda (-i\kappa_s)^\beta (-i\kappa_n)^\gamma \cdot \\ (2) & \\ \frac{1}{u_0} \phi_{ij}(\frac{\omega}{u_0}, \kappa_2, \kappa_3) d\kappa_2 d\kappa_3 & \end{aligned} \quad (26)$$

According to Batchelor (1960)

$$\phi_{ij}(\kappa_1, \kappa_2, \kappa_3) = \frac{E(\kappa)}{4\pi\kappa^4} (\kappa^2 \delta_{ij} - \kappa_i \kappa_j) \quad (27)$$

where

$$\kappa = (\kappa_1^2 + \kappa_2^2 + \kappa_3^2)^{1/2}, \quad (28)$$

and $E(\kappa)$ is the energy spectrum function. As stated earlier by proper selection of the subscripts on ψ in Equation (26) we can generate the frequency spectral density function for any coefficient in Equation (14). Thus, if we set $\nu = \lambda = 0$, $\beta = 0$, $\gamma = 1$, $n = 2$ then Equation (26) yields the cross-spectral density function for $w_i(t)$ and $q_j(t)$, namely

$$\begin{aligned} \psi_{w_i q_j}(\omega) &= \int_{-\infty}^{\infty} -\frac{i\kappa_3}{n_0} \phi_{ij}(\frac{\omega}{u_0}, \kappa_2, \kappa_3) d\kappa_2 d\kappa_3 \cdot \\ (2) & \end{aligned} \quad (29)$$

It can be shown that

$$\psi_{w_i p_j}(\omega) = -\psi_{p_i w_j}(\omega) \quad (30)$$

$$\psi_{w_i q_j}(\omega) = -\psi_{q_i w_j}(\omega) \quad (31)$$

$$\psi_{w_i p_j}(\omega) = \psi_{p_i w_j}(\omega) = -\psi_{p_i p_j}(\omega) \quad (32)$$

$$\Psi_{w_i q_j}(\omega) = \Psi_{q_i w_j}(\omega) = -\Psi_{q_i q_j}(\omega) \quad (33)$$

$$\Psi_{w_i m_j}(\omega) = \Psi_{m_i w_j}(\omega) = -\Psi_{q_i p_j}(\omega) = -\Psi_{p_i q_j}(\omega) \quad (34)$$

with similar relationships for corresponding covariances. Thus,

$$\begin{aligned} S_{ij}(\tau; r_1, r_2, \alpha_1, \alpha_2) &= R_{w_i w_j}(\tau) - \left[R_{p_i p_j}(\tau) + R_{q_i q_j}(\tau) \right] \cdot \\ &\quad \frac{1}{2} \left[r_1^2 + r_2^2 - r_1 r_2 \cos(\omega_o \tau + \alpha_1 - \alpha_2) \right] \quad (35) \\ N_{ij}(\tau; t, r_1, r_2, \alpha_1, \alpha_2) &= R_{w_i p_j}(\tau) \left[r_2 \cos(\omega_o t + \alpha_2) - \right. \\ &\quad \left. r_1 \cos(\omega_o t + \omega_o \tau + \alpha_1) \right] + R_{w_i q_j}(\tau) \left[r_2 \sin(\omega_o t + \alpha_2) - \right. \\ &\quad \left. r_1 \sin(\omega_o t + \omega_o \tau + \alpha_1) \right] - \frac{1}{2} R_{q_i p_j}(\tau) \left[r_1^2 \sin(2\omega_o t + 2\omega_o \tau + 2\alpha_1) \right. \\ &\quad \left. + r_2^2 \sin(2\omega_o \tau + 2\alpha_2) - 2r_1 r_2 \sin(2\omega_o t + \omega_o \tau + \alpha_1 + \alpha_2) \right] \\ &\quad + \frac{1}{4} \left[R_{q_i q_j}(\tau) - R_{p_i p_j}(\tau) \right] \left[r_1^2 \cos(2\omega_o t + 2\omega_o \tau + 2\alpha_1) \right. \\ &\quad \left. + r_2^2 \cos(2\omega_o \tau + 2\alpha_2) - 2r_1 r_2 \cos(2\omega_o t + \omega_o \tau + \alpha_1 + \alpha_2) \right] \quad (36) \end{aligned}$$

The defining spectral density functions $\Psi_{w_i w_j}$, $\Psi_{w_i p_j}$, $\Psi_{w_i q_j}$, $\Psi_{q_i q_j}$, $\Psi_{p_i p_j}$, and $\Psi_{q_i p_j}$ can be Fourier transformed according to Equation (25) to yield the relevant covariance functions in Equations (35) and (36). These spectral density functions in terms of the energy function, $E(\kappa)$, are as follows

$$\Psi_{w_1 w_1}(\omega) = \frac{1}{4\pi u_o} \int_{-\infty}^{\infty} \frac{E(\kappa_o)}{\kappa_o^2} (\kappa_2^2 + \kappa_3^2) d\kappa_2 d\kappa_3 \quad (37)$$

$$\Psi_{w_2 w_2}(\omega) = \frac{1}{4\pi u_o} \int_{-\infty}^{\infty} \frac{E(\kappa_o)}{\kappa_o^4} \left(\frac{\omega^2}{u_o^2} + \kappa_3^2 \right) d\kappa_2 d\kappa_3 \quad (38)$$

$$\Psi_{w_3 w_3}(\omega) = \Psi_{w_2 w_2}(\omega) \quad (39)$$

$$\Psi_{w_i w_j}(\omega) = 0, \quad i \neq j \quad (40)$$

$$\Psi_{w_i p_j}(\omega) = -(\delta_{i1}\delta_{j2} + \delta_{i2}\delta_{j1}) \frac{i\omega}{4\pi u_o^2} \int_{-\infty}^{\infty} \frac{\kappa_2^2 E(\kappa_o)}{\kappa_o^4} d\kappa_2 d\kappa_3 \quad (41)$$

$$\Psi_{w_i q_j}(\omega) = -(\delta_{i1}\delta_{j3} + \delta_{i3}\delta_{j1}) \frac{i\omega}{4\pi u_o^2} \int_{-\infty}^{\infty} \frac{\kappa_3^2 E(\kappa_o)}{\kappa_o^4} d\kappa_2 d\kappa_3 \quad (42)$$

$$\Psi_{p_1 p_1}(\omega) = \frac{1}{4\pi u_o} \int_{-\infty}^{\infty} \frac{\kappa_2^2 E(\kappa_o)}{\kappa_o^4} (\kappa_2^2 + \kappa_3^2) d\kappa_2 d\kappa_3 \quad (43)$$

$$\Psi_{p_2 p_2}(\omega) = \frac{1}{4\pi u_o} \int_{-\infty}^{\infty} \frac{\kappa_2^2 E(\kappa_o)}{\kappa_o^4} \left(\frac{\omega^2}{u_o^2} + \kappa_3^2 \right) d\kappa_2 d\kappa_3 \quad (44)$$

$$\Psi_{p_3 p_3}(\omega) = \frac{1}{4\pi u_o} \int_{-\infty}^{\infty} \frac{\kappa_2^2 E(\kappa_o)}{\kappa_o^4} \left(\frac{\omega^2}{u_o^2} + \kappa_2^2 \right) d\kappa_2 d\kappa_3 \quad (45)$$

$$\Psi_{p_i p_j}(\omega) = 0, \quad i \neq j \quad (46)$$

$$\Psi_{q_1 q_1}(\omega) = \frac{1}{4\pi u_o} \int_{-\infty}^{\infty} \frac{\kappa_3^2 E(\kappa_o)}{4 \kappa_o} (\kappa_2^2 + \kappa_3^2) d\kappa_2 d\kappa_3 = \Psi_{p_1 p_1}(\omega) \quad (47)$$

$$\Psi_{q_2 q_2}(\omega) = \frac{1}{4\pi u_o} \int_{-\infty}^{\infty} \frac{\kappa_3^2 E(\kappa_o)}{4 \kappa_o} \left(\frac{\omega^2}{u_o^2} + \kappa_3^2 \right) d\kappa_2 d\kappa_3 = \Psi_{p_3 p_3}(\omega) \quad (48)$$

$$\Psi_{q_3 q_3}(\omega) = \frac{1}{4\pi u_o} \int_{-\infty}^{\infty} \frac{\kappa_3^2 E(\kappa_o)}{4 \kappa_o} \left(\frac{\omega^2}{u_o^2} + \kappa_2^2 \right) d\kappa_2 d\kappa_3 = \Psi_{p_2 p_2}(\omega) \quad (49)$$

$$\Psi_{q_i q_j}(\omega) = 0, \quad i \neq j \quad (50)$$

$$\Psi_{q_i p_j}(\omega) = - \frac{(\delta_{2i} \delta_{3j} + \delta_{3i} \delta_{2j})}{4\pi u_o} \int_{-\infty}^{\infty} \frac{\kappa_2^2 \kappa_3^2}{4 \kappa_o} E(\kappa_o) d\kappa_2 d\kappa_3 \quad (51)$$

with

$$\kappa_o = \left(\frac{\omega^2}{u_o^2} + \kappa_2^2 + \kappa_3^2 \right)^{1/2} \quad (52)$$

If $\omega_o = 0$ the quantity N_{ij} does not depend on time, so that the source of the nonstationary contribution to the turbulence covariance matrix following the WECS blade elements is the variation in time of the orientation of the line which connects the points in the two-point, two-time turbulence covariance matrix. Thus, consider the two points A and B on the x_2 -axis in Figure 4.a. At $\tau = 0$ the statistics $\langle (v_2)_A (v_2)_B \rangle$ and $\langle (v_3)_A (v_3)_B \rangle$ are longitudinal and lateral covariances, respectively. After a clockwise quarter turn the points A and B are located along the x_3 -axis as shown in Figure 4.b. At $\tau = 0$ the statistics $\langle (v_2)_A (v_2)_B \rangle$ and $\langle (v_3)_A (v_3)_B \rangle$ are now lateral and longitudinal covariances, respectively. However, at $\tau = 0$ the covariance $\langle (v_1)_A (v_1)_B \rangle$ does not vary with time because v_1 is normal to the WECS plane of rotation. For $\tau \neq 0$, all components in the covariance matrix $R_{v_i v_j}$ vary with time t and, except for

Figure 4. Longitudinal and lateral covariances at $\tau = 0$

selected values of t and τ , are neither longitudinal nor lateral covariances.

Appendix A provides a discussion of the exact calculation of the two-point, two-point covariance statistics for isotropic and homogeneous turbulence. Appendix B provides a discussion of the calculation of the two-point, two-time covariance statistics for the nonhomogeneous case using the formalism developed in the previous section and a generalization of the procedures developed in this section.

A TRANSFORMATION

It may be useful to reference the components of the turbulence velocity vector to a frame of reference (x'_1, x'_2, x'_3) which rotates with the WECS blade elements, such that the x'_1 -axis is parallel to the x_1 -axis, the x'_2 -axis is parallel to the WECS blade line in Figure 5 and the x'_3 -axis lies in the plane of rotation and is normal to the WECS blade line. This coordinate system has the advantage that at $\tau = 0$ the covariances $\langle (v'_2)_A (v'_2)_B \rangle$ and $\langle (v'_3)_A (v'_3)_B \rangle$ are time invariant, where v'_i denotes the component of velocity along the x'_i -axis. The relationship between v'_i and v_k is

$$v'_i = T_{ik} v_k \quad (53)$$

where the transformation matrix T_{ik} is given by

$$T_{ik} = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_o t + \alpha) & \sin(\omega_o t + \alpha) \\ 0 & \sin(\omega_o t + \alpha) & -\cos(\omega_o t + \alpha) \end{Bmatrix} \quad (54)$$

The covariances in the primed frame are related to the ones in the unprimed frame through the following relationship

$$R_{v'_i v'_j}(\tau; t) = T_{ik}(t + \tau) T_{jm}(t) R_{v_k v_m}(\tau; t) \quad (55)$$

ENERGY SPECTRUM FUNCTIONS

The most widely used functions for $E(\kappa)$ are those that correspond to the von Karman and Dryden turbulence models (Lumley and Panofsky, 1964, and Etkin,

Figure 5. Relationship between coordinates (prime) following blade element and stationary coordinates (unprimed)

1972), namely,

von Karman:

$$E(\kappa) = \frac{55}{9} \frac{\sigma^2 L}{\pi} \frac{(1.339 L \kappa)^4}{\left[1 + (1.339 L \kappa)^2\right]^{17/6}} \quad (56)$$

Dryden:

$$E(\kappa) = \frac{8\sigma^2 L}{\pi} \frac{(L\kappa)^4}{\left[1 + (L\kappa)^2\right]^3} \quad (57)$$

where L and σ denote the integral scale and standard deviation of turbulence.

CALCULATION OF SPECTRAL FUNCTIONS

Substitution of these functions into Equations (37) through (51) yields the auto- and cross-spectral density functions of w , p , and q . We shall use the Dryden model because it leads to closed form algebraic expressions for these spectra. Because of the finite extent of the WECS the integrals in Equations (37) through (51) must be truncated so as not to include Fourier components with wave numbers $\kappa_2, \kappa_3 > 2\pi/b$ which are inconsistent with the truncated Taylor series approximation, Equation (3). The quantity b is a characteristic length such that upon removing Fourier components with $\kappa_2, \kappa_3 > 2\pi/b$ from the wind field a turbulence process which satisfies the truncated Taylor series approximation Equation (3) is obtained, i.e., the third- and higher-order terms are negligibly small. Later we shall obtain an estimate of the quantity b when we examine the convergence properties of the Taylor series representation of the turbulence velocity field.

Substitution of the energy spectrum function for the Dryden model into Equation (37) through (51) yields

$$\frac{2\omega \Psi_{w_1 w_1}(\omega)}{\sigma^2} = G(\Omega) = \frac{2}{\pi} \left\{ \frac{1}{1 + \Omega^2} - \frac{1 + \Omega^2 + 2\Omega_c^2}{(1 + \Omega^2 + \Omega_c^2)^2} \right\} \Omega \quad (58)$$

$$\frac{2\omega^\Psi w_2 w_2^{(\omega)}}{\sigma^2} = \frac{2\omega^\Psi w_3 w_3^{(\omega)}}{\sigma^2} = \frac{1}{\pi} \left\{ \frac{1 + 3\Omega^2}{(1 + \Omega^2)^2} - \frac{1 + 3\Omega^2 + 2\Omega_c^2}{(1 + \Omega^2 + \Omega_c^2)^2} \right\} \Omega \quad (59)$$

$$\frac{2\omega^\Psi w_i w_j^{(\omega)}}{\sigma^2} = 0, i \neq j \quad (60)$$

$$\frac{2L\omega^\Psi w_i p_j^{(\omega)}}{\sigma^2} = - (\delta_{i1}\delta_{j2} + \delta_{i2}\delta_{j1}) i \frac{\Omega G(\Omega)}{2} \quad (61)$$

$$\frac{2L\omega^\Psi w_i q_j^{(\omega)}}{\sigma^2} = - (\delta_{i1}\delta_{j3} + \delta_{i3}\delta_{j1}) i \frac{\Omega G(\Omega)}{2} \quad (62)$$

$$\begin{aligned} \frac{2L^2\omega^\Psi p_1 p_1^{(\omega)}}{\sigma^2} = \frac{2L^2\omega^\Psi q_1 q_1^{(\omega)}}{\sigma^2} = H(\Omega) = \frac{2}{\pi} \left\{ \frac{1 + \Omega^2}{1 + \Omega^2 + \Omega_c^2} \right. \\ \left. + \log \left(\frac{1 + \Omega^2 + \Omega_c^2}{1 + \Omega^2} \right) - \frac{1}{2} \frac{\Omega_c^4}{(1 + \Omega^2 + \Omega_c^2)^2} - 1 \right\} \Omega \end{aligned} \quad (63)$$

$$\begin{aligned} \frac{2L^2\omega^\Psi p_2 p_2^{(\omega)}}{\sigma^2} = \frac{2L^2\omega^\Psi q_3 q_3^{(\omega)}}{\sigma^2} = \frac{\Omega}{\pi} \left\{ \frac{\Omega^2}{1 + \Omega^2} - \right. \\ \left. \frac{\Omega^2(1 + \Omega^2 + 2\Omega_c^2)}{(1 + \Omega^2 + \Omega_c^2)^2} \right\} + \frac{1}{4} H(\Omega) \end{aligned} \quad (64)$$

$$\frac{2L^2_{\omega\psi} p_3 p_3 (\omega)}{\sigma^2} = \frac{2L^2_{\omega\psi} q_2 q_2 (\omega)}{\sigma^2}$$

$$= \frac{\Omega}{\pi} \left\{ \frac{\Omega^2}{1 + \Omega^2} - \frac{\Omega^2(1 + \Omega^2 + 2\Omega_c^2)}{(1 + \Omega^2 + \Omega_c^2)^2} \right\} + \frac{3}{4} H(\Omega) \quad (65)$$

$$\frac{2L^2_{\omega\psi} p_i p_j (\omega)}{\sigma^2} = \frac{2L^2_{\omega\psi} q_i q_j (\omega)}{\sigma^2} = 0, \quad i \neq j \quad (66)$$

$$\frac{2L^2_{\omega\psi} q_i p_j (\omega)}{\sigma^2} = - (\delta_{2i} \delta_{3j} + \delta_{3i} \delta_{2j}) \frac{H(\Omega)}{4} \quad (67)$$

where

$$\Omega = \frac{\omega L}{u_o} \quad (68)$$

$$\Omega_c = \frac{2\pi L}{b} \quad (69)$$

These spectra are sufficient for calculation of the second moment statistics of turbulence velocity fluctuations, following the WECS blade elements, for the truncated Taylor series expansion Equation (3) of the velocity field. Later we shall discuss the nonstationary spectra of the gust velocity components following the blade elements.

Figures 6 through 10 provides plots of the non-zero functions contained in Equations (58 through 67). The range of the parameter Ω_c is sufficiently wide to encompass the full range of WECS'S anticipated to be constructed in the future, i.e., diameters ranging between 10m to 300m.

Figure 6. Longitudinal velocity component frequency spectra and function $G(\Omega)$.

Figure 7. Lateral and vertical velocity component frequency spectra.

Figure 8. p_1 and q_1 frequency spectra and function $H(\Omega)$.

Figure 9. p_2 and q_3 frequency spectra

Figure 10. p_3 and q_2 frequency spectra

LINEAR SYSTEMS

In this section we indicate the manner in which two-point, two-time covariance statistics might be used in WECS design. The discussion is not intended to be an exhaustive treatment of linear structural dynamics, but rather is intended to show how the covariance statistics might enter a response analysis. A typical approach in system response analysis is to partition a structure, airfoil, etc., into nodes or strips whereby aerodynamic forces are applied and equations of motion are written for each node or strip. Furthermore, the equations are oftentimes linearized and dynamic responses are calculated with spectral density techniques. The linearized equations of motion might then look like

$$L_{ik}\xi_k(t) = A_{im}^{(s)}v_s(t; r_m, \alpha_m) \quad (70)$$

where $\xi_k(t)$ is the k^{th} dynamic variable of the system, and L_{ik} is a linear operator of the form

$$L_{ik} = E_{ik}^{(v)} \frac{d^v}{dt^v} + E_{ik}^{(v-1)} \frac{d^{v-1}}{dt^{v-1}} + \dots + E_{ik}^{(0)} \quad (71)$$

The matrices $E_{ik}^{(v)}$, $v = 1, 2, \dots$ and $A_{im}^{(s)}$ with $s = 1, 2, 3$ depend on the structural mass, stiffness, damping and aerodynamic characteristics. The operator L_{ik} is a square $N \times N$ matrix operator with N equal to the number of dynamic variables of the system, i.e., q_k , $k = 1, 2, \dots, N$. For each value of s the matrix $A_{im}^{(s)}$ is a $N \times M$ matrix with M equal to the number of nodes or strips.

FOURIER TRANSFORMS

Let us assume that a Fourier transform exists for Equation (70), so that

$$\hat{L}_{ik}(\omega) dZ_{\xi_k}(\omega) = A_{im}^{(p)} dZ_{v_p}(\omega; r_m, \alpha_m) \quad (72)$$

where the ξ_k and v_k are related to the Fourier amplitudes (dZ) as follows:

$$x(t) = \int_{-\infty}^{\infty} e^{i\omega t} dZ_{\chi}^{(\omega)} \quad (73)$$

and

$$\hat{L}_{ik}(\omega) = E_{ik}^{(v)}(i\omega)^v + E_{ik}^{(v-1)}(i\omega)^{v-1} + \dots + E_{ik}^{(0)} \quad (74)$$

Multiplication of Equation (72) evaluated at frequency ω_1 by the complex conjugate of Equation (72) evaluated at ω_2 yields

$$\begin{aligned} \hat{L}_{ik}(\omega_1) \hat{L}_{jn}^*(\omega_2) \Psi_{\xi_k \xi_n}(\omega_1, \omega_2) = \\ A_{ik}^{(s)} A_{jn}^{(m)} \Psi_{v_s v_m}(\omega_1, \omega_2; r_k, r_n, \alpha_k, \alpha_n) \end{aligned} \quad (75)$$

where

$$\Psi_{AB}(\omega_1, \omega_2) d\omega_1 d\omega_2 = \langle dZ_A(\omega_1) dZ_B^*(\omega_2) \rangle \quad (76)$$

The quantity $\Psi_{AB}(\omega_1, \omega_2)$ is the double frequency (generalized) power spectral density function.

It can be shown that

$$R_{AB}(t_1, t_2) = \int_{-\infty}^{\infty} \Psi_{AB}(\omega_1, \omega_2) e^{-i(\omega_1 t_1 - \omega_2 t_2)} d\omega_1 d\omega_2 \quad (77)$$

(2)

where

$$R_{AB}(t_1, t_2) = \langle A(t_1) B(t_2) \rangle \quad (78)$$

Furthermore,

$$\Psi_{AB}(\omega_1, \omega_2) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} R_{AB}(t_1, t_2) e^{i(\omega_1 t_1 - \omega_2 t_2)} dt_1 dt_2 \quad (79)$$

These equations are analogous to the Wiener-Khinchine Theorem in the stationary case. Note that $\Psi_{AB}(\omega_1, \omega_2)$ exists if $R_{AB}(t_1, t_2)$ is absolutely integrable. For further details see Bendat and Piersol (1971) or Lin (1967).

We now set $t_1 = t$ and $t_2 = t + \tau$, so that Equation (77) can be written as

$$R_{AB}(\tau; t) = \int_{-\infty}^{\infty} \Psi_{AB}(\omega_1, \omega_2) e^{-i(\omega_1 - \omega_2)t} e^{i\omega_2 \tau} d\omega_1 d\omega_2 \quad (80)$$

where the covariance function is now a function of τ and t , i.e., we construct the covariance of A at time t and B at time $t + \tau$. For stationary processes the covariance does not depend on time t . The only way this can be the case is for the double frequency power spectral density function to be of the form

$$\Psi_{AB}(\omega_1, \omega_2) = \Psi_{AB}(\omega_2) \delta(\omega_2 - \omega_1) \quad (81)$$

i.e., zero spectral density for $\omega_1 \neq \omega_2$. However, if we are dealing with nonstationary turbulence inputs and associated nonstationary responses, $\Psi_{AB}(\omega_1, \omega_2)$ can take on nonzero values for $\omega_1 \neq \omega_2$.

Equation (75) can be solved to obtain the generalized power spectral density functions $\Psi_{\xi_k \xi_n}(\omega_1, \omega_2)$ of the response variables in terms of generalized two-point power spectral density functions of the turbulence. Equation (77) can then be used to obtain response covariances.

Another approach to describing spectral structure of nonstationary processes is to Fourier transform Equation (80) with respect to τ , so that

$$\Psi_{AB}(\omega, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{AB}(\tau; t) e^{-i\omega \tau} d\tau \quad (82)$$

Substitution of Equation (80) into Equation (82) yields

$$\phi_{AB}(\omega; t) = \int_{-\infty}^{\infty} \psi_{AB}(\omega_1, \omega_2) \delta(\omega_2 - \omega) e^{-i(\omega_1 - \omega)t} d\omega_1 d\omega_2 \quad (83)$$

or

$$\phi_{AB}(\omega; t) = \int_{-\infty}^{\infty} \psi_{AB}(\omega_1, \omega) e^{-i(\omega_1 - \omega)t} d\omega_1 \quad (84)$$

This defines the frequency-time spectral density function. For stationary processes

$$\psi_{AB}(\omega_1, \omega) = \psi_{AB}(\omega_1) \delta(\omega - \omega_1) \quad (85)$$

so that Equation (84) reduces to

$$\phi_{AB}(\omega, t) = \psi_{AB}(\omega) \quad (86)$$

Thus, the frequency-time spectral density function concept is a generalization of the traditional spectral decomposition theory for stationary processes.

GENERALIZED TURBULENCE SPECTRAL DENSITIES

If the theory enunciated in the previous section is applicable to WECS design, i.e., the WECS can be represented as a linear system, then we require the Fourier transform of Equation (15), so that

$$\begin{aligned} \psi_{v_i v_j}(\omega_1, \omega_2; r_1, r_2, \alpha_1, \alpha_2) &= \psi_{S_{ij}}(\omega_1, \omega_2; r_1, r_2, \alpha_1, \alpha_2) \\ &+ \psi_{N_{ij}}(\omega_1, \omega_2; r_1, r_2, \alpha_1, \alpha_2) \end{aligned} \quad (87)$$

The quantities r_1, r_2, α_1 and α_2 can be replaced with r_k, r_n, α_k and α_n

(see Equation (75)) and the subscripts can take on integer values as discussed in the previous section. Fourier transformation of Equations (35 and 36) yields generalized spectral density functions for the stationary and nonstationary parts of the two-point, two-time turbulence covariance matrix, so that

$$\begin{aligned} \Psi_{S_{ij}}(\omega_1, \omega_2; r_1, r_2, \alpha_1, \alpha_2) = & \left\{ \Psi_{w_i w_j}(\omega_2) - \frac{1}{2}(r_1^2 + r_2^2) \cdot \right. \\ & \left[\Psi_{p_i p_j}(\omega_2) + \Psi_{q_i q_j}(\omega_2) \right] + \frac{r_1 r_2}{2} \left[(\Psi_{p_i p_j}(\omega_2 - \omega_0) + \right. \\ & \Psi_{q_i q_j}(\omega_2 - \omega_0)) e^{i(\alpha_1 - \alpha_2)} + (\Psi_{p_i p_j}(\omega_2 + \omega_0) + \\ & \left. \left. \Psi_{q_i q_j}(\omega_2 + \omega_0)) e^{-i(\alpha_1 - \alpha_2)} \right] \right\} \delta(\omega_2 - \omega_1) \end{aligned} \quad (88)$$

$$\begin{aligned} \Psi_{N_{ij}}(\omega_1, \omega_2; r_1, r_2, \alpha_1, \alpha_2) = & \frac{1}{2} \left\{ r_2 \Psi_{w_i p_j}(\omega_2) e^{i\alpha_2} \right. \\ & - r_1 \Psi_{w_i p_j}(\omega_2 - \omega_0) e^{i\alpha_1} \left\{ \delta(\omega_2 - \omega_1 - \omega_0) + \frac{1}{2} \right\} r_2 \Psi_{w_i p_j}(\omega_2) e^{-i\alpha_2} \\ & - r_1 \Psi_{w_i p_j}(\omega_2 + \omega_0) e^{-i\alpha_1} \left\{ \delta(\omega_2 - \omega_1 + \omega_0) - \frac{i}{2} \right\} r_2 \Psi_{w_i q_j}(\omega_2) e^{i\alpha_2} \\ & - r_1 \Psi_{w_i q_j}(\omega_2 - \omega_0) e^{i\alpha_1} \left\{ \delta(\omega_2 - \omega_1 - \omega_0) + \frac{i}{2} \right\} r_2 \Psi_{w_i q_j}(\omega_2) e^{-i\alpha_2} \\ & \left. - r_1 \Psi_{w_i q_j}(\omega_2 + \omega_0) e^{-i\alpha_1} \left\{ \delta(\omega_2 - \omega_1 + \omega_0) + \frac{i}{4} \right\} r_1^2 \Psi_{q_i p_j}(\omega_2 - 2\omega_0) e^{i2\alpha_1} \right\} \end{aligned} \quad (89)$$

continued

$$\begin{aligned}
& + r_2^2 \psi_{q_i p_j}(\omega_2) e^{i2\alpha_2} - 2r_1 r_2 \psi_{q_i p_j}(\omega_2 - \omega_o) e^{i(\alpha_1 + \alpha_2)} \left\{ \delta(\omega_2 - \omega_1 - 2\omega_o) \right. \\
& - \frac{i}{4} \left\{ r_1^2 \psi_{q_i p_j}(\omega_2 + 2\omega_o) e^{-i2\alpha_1} + r_2^2 \psi_{q_i p_j}(\omega_2) e^{-i2\alpha_2} \right. \\
& - 2r_1 r_2 \psi_{q_i p_j}(\omega_2 + \omega_o) e^{-i(\alpha_1 + \alpha_2)} \left. \right\} \delta(\omega_2 - \omega_1 + 2\omega_o) \\
& + \frac{1}{8} \left\{ r_1^2 \left(\psi_{q_i q_j}(\omega_2 - 2\omega_o) - \psi_{p_i p_j}(\omega_2 - 2\omega_o) \right) e^{i2\alpha_1} \right. \\
& + r_2^2 \left(\psi_{q_i q_j}(\omega_2) - \psi_{p_i p_j}(\omega_2) \right) e^{i2\alpha_2} - 2r_1 r_2 \left(\psi_{q_i q_j}(\omega_2 - \omega_o) \right. \\
& - \psi_{p_i p_j}(\omega_2 - \omega_o) \left. \right) e^{i(\alpha_1 + \alpha_2)} \left. \right\} \delta(\omega_2 - \omega_1 - 2\omega_o) \\
& + \frac{1}{8} \left\{ r_1^2 \left(\psi_{q_i q_j}(\omega_2 + 2\omega_o) - \psi_{p_i p_j}(\omega_2 + 2\omega_o) \right) e^{-i2\alpha_1} \right. \\
& + r_2^2 \left(\psi_{q_i q_j}(\omega_2) - \psi_{p_i p_j}(\omega_2) \right) e^{-i2\alpha_2} \\
& - 2r_1 r_2 \left(\psi_{q_i q_j}(\omega_2 + \omega_o) - \psi_{p_i p_j}(\omega_2 + \omega_o) \right) e^{-i(\alpha_1 + \alpha_2)} \left. \right\} \delta(\omega_2 - \omega_1 + 2\omega_o) .
\end{aligned}
\tag{89}$$

Equations(88 and 89) permit the calculation of the two-point generalized power spectral density of turbulence for WECS linear response analyses upon specification of $E(\kappa)$, $r_1, r_2, \alpha_1, \alpha_2$ and ω_o . In the case of the Dryden spectral model the auto- and cross-spectral density functions given by Equations (58 through 69) can be evaluated at the frequencies $\omega_2, \omega_2 \pm \omega_o$ and $\omega_2 \pm 2\omega_o$ and substituted into Equations (88 and 89) and the spectrum given by Equation (87) is immediately specified.

The various terms in Equations (88 and 89) are nonzero on the lines

$$\omega_2 - \omega_1 - n\omega_0 = 0, n = 0, 1, 2 \quad (90)$$

in the (ω_1, ω_2) -plane (see Figure 11) and vanish everywhere else. As we add more terms to the Taylor series expansion of the turbulence velocity field the number of lines on which the generalized turbulence spectral densities are non-zero increase, i.e., $n = 3, 4, \dots$. At $\omega_0 = 0$ all of the lines collapse onto the line $\omega_2 - \omega_1 = 0$ and the generalized turbulence spectral densities are of the form given Equation (81) appropriate for a stationary process. As ω_0 increases away from zero the lines in Figure 11 separate. The spacing between these lines can be thought of as being a measure of the degree to which the turbulence velocity statistics depart from statistical stationary conditions.

If $i = j$ the coefficient of the delta function in Equation (88) is nonzero. If $\alpha_1 = \alpha_2$ this coefficient is real and if $\alpha_1 \neq \alpha_2$ the coefficient is complex implying a phase shift between the turbulence components at points (r_1, α_1) and (r_2, α_2) . It is interesting to note that this phase shift vanishes when $\alpha_1 = \alpha_2$ for all values of r_1 and r_2 . If $i \neq j$ the cross-spectral density functions on the right side of Equation (88) vanish identically so that $\Psi_{S_{ij}}$ vanishes.

We scale the position coordinates r_1 and r_2 with WECS blade length c . The lowest order term in Equation (88) is c^0 with the next highest order being c^2 . It can be shown that terms above second-order in $\Psi_{S_{ij}}$ form a series involving even ascending powers of c , i.e., c^q , $q = 4, 6, \dots$.

We now turn our attention to the nonstationary part of the generalized turbulence spectral densities. For ease of discussion we number each delta function term in Equation (89) from left to right from 1 through 8. If $i = j$ terms 1 through 6 vanish. In this case the lead term is of order c^2 . Furthermore, it can be shown that the remaining terms form a series involving ascending even powers of c , i.e., c^q , $q = 4, 6, \dots$. If $i \neq j$ terms 7 and 8 vanish and terms 1 and 2 are nonzero for $i = 1$ and $j = 2$, terms 3 and 4 are nonzero for $i = 1$ and $j = 3$, and terms 5 and 6 are nonzero for $i = 2$ and $j = 3$. Thus, for $i = 1$ and $j = 2$ and $i = 1$ and $j = 3$ the lead term in Equation (89) is of order c^1 and for $i = 2$ and $j = 3$ the lead term is of order c^2 . It can be shown for the case of the former the higher order terms form a series involving

Figure 11. The (ω_1, ω_2) -plane. Each line corresponds to a contribution to the generalized turbulence spectral density function. The spacing between lines is a measure of the degree to which the wind process following the WECS blade elements is a nonstationary process.

ascending odd powers of c , i.e., c^q , $q = 3, 5, \dots$ and in the latter the remaining terms form a series involving ascending even powers of c , i.e., c^q , $q = 4, 6, \dots$

FREQUENCY-TIME SPECTRAL DENSITIES

Application of Equation (84) to Equation (87) yields the frequency-time spectral density function of the turbulence velocity components following the WECS blade elements, namely

$$\begin{aligned} \Phi_{v_i v_j}(\omega; t, r_1, r_2, \alpha_1, \alpha_2) &= \Phi_{S_{ij}}(\omega; t, r_1, r_2, \alpha_1, \alpha_2) \\ &+ \Phi_{N_{ij}}(\omega, t, r_1, r_2, \alpha_1, \alpha_2) \end{aligned} \quad (91)$$

where

$$\begin{aligned} \Phi_{S_{ij}}(\omega; r_1, r_2, \alpha_1, \alpha_2) &= \Psi_{w_i w_j}(\omega) - \frac{1}{2}(r_1^2 + r_2^2) \cdot \\ &\left[\Psi_{p_i p_j}(\omega) + \Psi_{q_i q_j}(\omega) \right] + \frac{r_1 r_2}{2} \left[\left(\Psi_{p_i p_j}(\omega - \omega_0) + \right. \right. \\ &\left. \Psi_{q_i q_j}(\omega - \omega_0) \right) e^{i(\alpha_1 - \alpha_2)} + \left(\Psi_{p_i p_j}(\omega + \omega_0) + \right. \\ &\left. \Psi_{q_i q_j}(\omega + \omega_0) \right) e^{-i(\alpha_1 - \alpha_2)} \left. \right] \end{aligned} \quad (92)$$

and

$$\begin{aligned}
\Phi_{N_{ij}}(\omega; t, r_1, r_2, \alpha_1, \alpha_2) &= r_2 \Psi_{w_i p_j}(\omega) \cos(\omega_0 t + \alpha_2) + r_2 \Psi_{w_i q_j}(\omega) \sin(\omega_0 t + \alpha_2) \\
&- \frac{1}{2} \left\{ r_1 \Psi_{w_i p_j}(\omega - \omega_0) e^{i\alpha_1} e^{i\omega_0 t} + r_1 \Psi_{w_i p_j}(\omega + \omega_0) e^{-i\alpha_1} e^{-i\omega_0 t} \right\} \\
&+ \frac{i}{2} \left\{ r_1 \Psi_{w_i q_j}(\omega - \omega_0) e^{i\alpha_1} e^{i\omega_0 t} + r_1 \Psi_{w_i q_j}(\omega + \omega_0) e^{-i\alpha_1} e^{-i\omega_0 t} \right\} \\
&- 2r_2^2 \Psi_{q_i p_j}(\omega) \sin(2\omega_0 t + \alpha_2) \\
&+ \frac{i}{4} \left\{ r_1^2 \Psi_{q_i p_j}(\omega - \omega_0) e^{i2\alpha_1} e^{i2\omega_0 t} - r_1^2 \Psi_{q_i p_j}(\omega + 2\omega_0) e^{-i2\alpha_1} e^{-i2\omega_0 t} \right\} \\
&- \frac{i}{2} \left\{ r_1 r_2 \Psi_{q_i p_j}(\omega - \omega_0) e^{i(\alpha_1 + \alpha_2)} e^{i2\omega_0 t} - r_1 r_2 \Psi_{q_i p_j}(\omega + \omega_0) e^{-(\alpha_1 + \alpha_2)} e^{-i2\omega_0 t} \right\} \\
&+ \frac{r_2^2}{4} \left(\Psi_{q_i q_j}(\omega) - \Psi_{p_i p_j}(\omega) \right) \cos(2\omega_0 t + 2\alpha_2) \\
&+ 1/8 \left\{ r_1^2 \left(\Psi_{q_i q_j}(\omega - 2\omega_0) - \Psi_{p_i p_j}(\omega - 2\omega_0) \right) e^{i2\alpha_1} e^{i2\omega_0 t} \right. \\
&\quad \left. + r_1^2 \left(\Psi_{q_i q_j}(\omega + 2\omega_0) - \Psi_{p_i p_j}(\omega + 2\omega_0) \right) e^{-i2\alpha_1} e^{-i2\omega_0 t} \right. \\
&- \frac{1}{4} \left\{ r_1 r_2 \left(\Psi_{q_i q_j}(\omega - \omega_0) - \Psi_{p_i p_j}(\omega - \omega_0) \right) e^{i(\alpha_1 + \alpha_2)} e^{i2\omega_0 t} \right. \\
&\quad \left. + r_1 r_2 \left(\Psi_{q_i q_j}(\omega + \omega_0) - \Psi_{p_i p_j}(\omega + \omega_0) \right) e^{-i(\alpha_1 + \alpha_2)} e^{-i2\omega_0 t} \right\} .
\end{aligned}$$

(93)

The function $\phi_{S_{ij}}$ does not depend on time because $S_{v_i v_j}$ is the stationary part of the two-point turbulence velocity component covariance function. The function $\phi_{N_{ij}}$ clearly depends on time. This function contains sinusoidal variations in time with frequencies $\pm\omega_0$ and $\pm2\omega_0$. If terms beyond second-order were included in the original Taylor series of the gust velocity component field then additional terms with sinusoidal variations in time with frequencies $\pm n\omega_0$, $n = 3, 4, \dots$ would be contained in Equation (93).

AN EXAMPLE OF CALCULATED WIND SPECTRUM
EXPERIENCED BY A ROTATING BLADE

To provide an example we shall consider spectra following a single blade element, i.e., $r_1 = r_2 = r$, and $\alpha_1 = \alpha_2 = \alpha$, for $i = j = 1$. This corresponds to the spectrum of turbulence for the component orthogonal to the disk of rotation. We select this case because measurements are available by which we can assess the fidelity of the theory presented herein. In this case the generalized turbulence spectral densities are given by

$$\begin{aligned} \psi_{S_{11}}(\omega_1, \omega_2; r, r, \alpha, \alpha) = & \left\{ \psi_{w_1 w_1}(\omega_2) + r^2 \left[\psi_{p_1 p_1}(\omega_2 - \omega_0) \right. \right. \\ & \left. \left. + \psi_{p_1 p_1}(\omega_2 + \omega_0) - 2\psi_{p_1 p_1}(\omega_2) \right] \right\} \delta(\omega_2 - \omega_1) \end{aligned} \quad (94)$$

$$\psi_{N_{11}}(\omega_1, \omega_2; r, r, \alpha, \alpha) = 0 \quad (95)$$

The corresponding frequency-time spectral density function is given by

$$\begin{aligned} \phi_{S_{11}}(\omega, t; r, r, \alpha, \alpha) = & \psi_{w_1 w_1}(\omega) + r^2 \left[\psi_{p_1 p_1}(\omega - \omega_0) \right. \\ & \left. + \psi_{p_1 p_1}(\omega + \omega_0) - 2\psi_{p_1 p_1}(\omega) \right] \end{aligned} \quad (96)$$

Equation (96) can be written in nondimensional form namely

$$\begin{aligned} 2\omega\phi_{S_{11}}(\omega, t; r, r, \alpha, \alpha) = & \frac{2\omega\psi_{w_1 w_1}(\omega)}{\sigma^2} \\ & + \left(\frac{r}{L}\right)^2 \left\{ \frac{\omega}{\omega - \omega_0} \frac{2L^2(\omega - \omega_0)}{\sigma^2} \phi_{p_1 p_1}(\omega - \omega_0) + \frac{\omega}{\omega + \omega_0} \frac{2L^2(\omega + \omega_0)}{\sigma^2} \phi_{p_1 p_1}(\omega + \omega_0) \right. \\ & \left. - 2 \frac{2L^2\omega\psi_{p_1 p_1}(\omega)}{\sigma^2} \right\} \end{aligned} \quad (97)$$

The nondimensional spectra on the right hand side of Equation (97) are universal functions of Ω and Ω_c and were presented earlier for the Dryden spectral turbulence model. Measured spectra are usually represented in terms of the cyclic frequency

$$n = \frac{\omega}{2\pi} \quad (98)$$

so that for turbulence models characterized by an integral scale L , like in the von Karman and Dryden models,

$$\Omega = \frac{\omega L}{u_o} = 2\pi \frac{L}{z_h} \frac{nz_h}{u_o} \quad (99)$$

where

$$z_h = \text{WECS hub height.}$$

Furthermore,

$$\frac{n\hat{\phi}(n)}{\sigma^2} = \frac{\omega\phi(\omega)}{\sigma^2} \quad (100)$$

where $\hat{\phi}(n)$ and $\phi(\omega)$ are spectra of the same random function in the cyclic and radian frequency domains. Thus, upon specification of L/z_h and Ω_c , which is discussed below, we can calculate the nondimensional spectrum $n\hat{\phi}_{S_{11}}(n, t; r, r, \alpha, \alpha)/\sigma^2$ as a function of nz_h/u_o .

At the WECS hub ($r = 0$) the spectrum is given by the lead term in the Taylor series expansion of the velocity field. If the WECS does not rotate ($\omega_o = 0$) then the second- and higher-order terms also vanish (see Equation (97)). This means the spectrum at the hub can be applied anywhere in the (y, z) -plane for the nonrotating case. In the rotating case away from the WECS hub the second-order terms do not necessarily vanish because the time history of turbulence as "seen" by a WECS blade element will be different from that experienced in the nonrotating case. This results from the fact that the trajectory of a blade element through the mean atmospheric flow state is a helix

To obtain the spectrum of turbulence following a blade element we must select values of Ω_c and r/L . However, the use of a truncated Taylor series of the turbulent flow implies that we must place restrictions on the permissible values of Ω_c and r/L . In the case of the spectrum for $i=j=1$ all of the terms in the Taylor series expansion, Equation (3), which involve odd-order derivatives vanish. To neglect the fourth- and higher-order terms in Equation (3) and hence in the spectrum, Equation (96), we must require

$$\frac{x_2^2}{2} \frac{\partial^2 u_1}{\partial x_2^2}, \frac{x_3^2}{2} \frac{\partial^2 u_1}{\partial x_3^2}, x_2 x_3 \frac{\partial^2 u_1}{\partial x_2 \partial x_3} > > \frac{x_2^4}{24} \frac{\partial^4 u_1}{\partial x_2^4}, \frac{x_3^4}{24} \frac{\partial^4 u_1}{\partial x_3^4}, \frac{x_2^2 x_3^2}{4} \frac{\partial^4 u_1}{\partial x_2^2 \partial x_3^2}. \quad (101)$$

If we estimate the spatial derivatives as $\partial^2 u_1 / \partial x_2^2 \sim \sigma / L^2$, etc, and then form ratios of corresponding fourth-order to second-order terms and demand the ratios to be very small compared to unity this yields a condition on the permissible values of r/L , i.e.

$$\frac{x_2^2 x_3^2}{4} \frac{\partial^4 u_1}{\partial x_2^2 \partial x_3^2} / x_2 x_3 \frac{\partial^2 u_1}{\partial x_2 \partial x_3} \approx \frac{r^4 \sigma}{4L^4} / \frac{r^2 \sigma}{L^2} = \frac{r^2}{4L} \ll 1 \quad (102, a)$$

or

$$\frac{r}{L} \ll 2. \quad (102, b)$$

If we accept values of r/L which are no more than 10% of the right side of this inequality, then we must require

$$\frac{r}{L} \lesssim 0.2. \quad (103)$$

We now derive a condition for Ω_c . In view of truncation of the third- and higher-order terms in the Taylor series of the turbulence velocity field we must restrict the analysis to Fourier components with lateral and vertical wavelengths which are greater than or equal to twice the WECS diameter. This means we must require $\Omega_c \lesssim 2\pi L/4c$, where c is blade length. Let us choose $c/L \approx 0.2$ consistent

with Equation (103), so that we must require

$$\Omega_c \leq 8. \quad (104)$$

We shall use select values of Ω_c to estimate the second-order contributions to the spectrum, i.e. the terms in the curly brackets in Equation (97). However, we shall use $\Omega_c = \infty$ to calculate the lead term in Equation (97), i.e. the spectrum density of the longitudinal component of turbulence of the hub. This will be conservative from an engineering standpoint.

Figure 12 is a sample calculation of the spectrum, Equation (97), for the Dryden turbulence model for the following parameters:

$$\begin{aligned} z_h &= 36\text{m} \\ \omega_o &= 3.927 \text{ rad sec}^{-1} \\ u_o &= 10\text{m sec}^{-1} \\ L &= 100\text{m} \\ L/z_h &= 2.778 \\ r/L &= 0.25 \\ \Omega_c &= 0, 5, 7, 8, 10 \end{aligned} \quad (105)$$

Our selected value of $r/L = 0.25$ violates our criteria, Equation (103); however, this value of r/L is sufficiently closed to 0.2 such that it is our opinion that the theory herein is still applicable. Our motivation for selecting $r/L = 0.25$ is that experimental data is available by which we can test the theory herein. The solid line corresponds to the lead term in the Taylor series expansion, or the spectrum at the WECS hub. It was obtained by setting $\Omega_c = 0$ in the second-order terms in the turbulence spectrum. The remaining spectra correspond to the spectrum at the WECS blade element located at $r/L = 0.25$ for the various indicated values of Ω_c . We have included the case $\Omega_c = 10$. In this case the spectrum goes negative for a band of frequencies in the neighborhood of $n z_h / u_o \sim 0.2$. The negative values of spectrum can only be corrected by including fourth- and higher-order terms in the Taylor series expansion of the velocity field. We have included the case $\Omega_c = 10$ to show that our criteria for Ω_c appears to work relative to delineating these regions of (κ_2, κ_3) - space where fourth- and higher-order terms are needed in the analysis. The general features of the spectrum for $\Omega_c \neq 0$ are as follows:

Figure 12. Example of turbulence spectrum following a blade element for the component of turbulence normal to the WECS disk of rotation.

1) a relatively small energy deficit relative to the hub spectrum for $0 < nz_h/u_0 \lesssim 0.4$; 2) a substantial energy deficit in the region $0.4 \lesssim nz_h/u_0 \lesssim 0.8$; 3) a pronounced spectral peak at frequency $nz_h/u_0 \simeq 2.25$ corresponding to nondimensional cyclic WECS rotation frequency and 4) a return to the hub spectrum for frequencies larger than the hub frequency.

Connell (1980) has calculated spectra with experimental data acquired at the Pacific Northwest Laboratories, Richland, Washington, for conditions given by Equation (105). Except for the high frequency region beyond the spectral peak associated with the WECS rotation rate the theoretical spectra in this paper and those of Connell are essentially in agreement. The results are in best quantitative agreement with those of Connell for $\Omega_c \simeq 7$. The major discrepancy between these results and those of Connell occur at frequencies beyond $nz_h/u_0 \simeq 2.25$. Connell's observed spectrum increases like n^2 beyond the WECS rotation frequency. We attribute this discrepancy to the failure to retain fourth- and higher-order terms in our analysis. Nevertheless, the truncated Taylor series expansion herein captures most, if not all, of the features observed by Connell for frequencies at and below the WECS rotation frequency. Inclusion of the fourth- and higher-order terms will produce spectral peaks essentially at integral WECS rotation frequencies, i.e., $2\omega_0, 3\omega_0$.

The theory presented herein does not include the contribution due to mean flow wind shear. This contribution to the $i = j = 1$ spectrum is discussed in Appendix C.

CONCLUSIONS AND RECOMMENDATIONS

Specification of covariance wind statistics following WECS blade elements is not straightforward. However, through the use of the simplifying assumptions of isotropic and homogeneous turbulence it has been possible to develop a second-order model of the covariance matrix, such that upon specification of the energy density spectrum function $E(\kappa)$ the nonstationary covariance statistics and associated spectra, for the turbulence velocity vector components following the WECS blade elements, are completely specified, but with accuracy which varies with frequency of the spectral component. The central idea is that the turbulence velocity vector field can be expanded about the WECS hub with the result that the series expansion of the covariance matrix involves coefficients proportional to the covariance statistics of the gust velocity components and associated spatial derivatives to the same order as the highest term retained in the expansion. This expansion given by Equation (15) is general and can be applied to the anisotropic and vertically nonhomogeneous boundary layer as a logical next step. In this case we do not have a single function available, like $E(\kappa)$ in the isotropic and homogeneous case, from which to derive the coefficients in the covariance expansion following the WECS blade elements. To develop a nonstationary gust covariance model for the anisotropic and vertically nonhomogeneous case we require the Eulerian field covariance matrix, namely

$$R_{ij}(\tau, \xi_2, \xi_3; x_3) = \langle v_i'(t + \tau, x_1, x_2 + \xi_2, x_3 + \xi_3) v_j'(t, x_1, x_2, x_3) \rangle \quad (106)$$

to determine the coefficients in the gust covariance matrix following the WECS blade elements. Data is available for $R_{ij}(\tau, 0, \xi_3, x_3)$ and to a lesser degree for $R_{ij}(\tau, \xi_2, 0, x_3)$ for various micrometeorological tower sites. The Battelle Pacific Northwest Laboratories meteorological tower facility for WECS design criteria development provides the unique opportunity to acquire data for $R_{ij}(\tau, \xi_2, \xi_3, x_3)$ for simultaneous nonzero values of τ, ξ_2 , and ξ_3 . In lieu of these gust statistics, working hypotheses might be used to construct the function $R_{ij}(\tau, \xi_2, \xi_3, x_3)$ from presently available Eulerian gust statistics to develop a gust covariance model following the WECS blade elements for the anisotropic and vertically nonhomogeneous boundary layer.

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APPENDIX A

APPROACH FOR DIRECT CALCULATION OF TWO-
TIME, TWO-POINT TURBULENCE STATISTICS

APPROACH FOR DIRECT CALCULATION OF TWO-
TIME, TWO-POINT TURBULENCE STATISTICS

Consider a mean flow parallel to the x_1 -axis with superimposed isotropic turbulence. We assume Taylor hypothesis is valid. Consider the two WECS blade elements #1 and #2 at distances r_1 and r_2 from the WECS hub and both rotating at angular rate ω_o as depicted in Figure 3. We seek the covariance of the i^{th} component of turbulence velocity at blade element #1 at time t and the j^{th} component of turbulence velocity at blade element #2 at time $t + \tau$. Now it can be shown that the Eulerian two-point covariance of $v_j(x_1 + \xi_1, x_2 + \xi_2, x_3 + \xi_3, t + \tau)$ and $v_i(x_1, x_2, x_3, t)$ for homogeneous, isotropic turbulence is given by

$$R_{v_i v_j}(\xi_1 - u_o \tau, \xi_1, \xi_2) = \sigma^2 \left\{ [f(\xi) - g(\xi)] \frac{(\xi_i - \delta_{1i} u_o \tau)(\xi_j - \delta_{1j} u_o \tau)}{\xi^2} + g(\xi) \delta_{ij} \right\} \quad (A-1)$$

where

$$\xi = \left[(\xi_1 - u_o \tau)^2 + \xi_2^2 + \xi_3^2 \right]^{1/2} \quad (A-2)$$

and $f(\xi)$ and $g(\xi)$ is the longitudinal and lateral correlation function. For the points #1 and #2 on the WECS

$$\xi_1 = 0 \quad (A-3)$$

$$\xi_2 = r_2 \cos(\omega_o t + \omega_o \tau + \alpha_2) - r_1 \cos(\omega_o t + \alpha_1) \quad (A-4)$$

$$\xi_3 = r_2 \sin(\omega_o t + \omega_o \tau + \alpha_2) - r_1 \sin(\omega_o t + \alpha_1) \quad (A-5)$$

and

$$\xi = \left\{ u_o^2 \tau^2 + r_1^2 + r_2^2 - 2r_1 r_2 \cos(2\omega_o t + \omega_o \tau + \alpha_1 + \alpha_2) \right\}^{1/2} \quad (A-6)$$

Substitution of (A-3) through (A-6) into (A-1) yields

$$R_{v_i v_j}(\tau; t, r_1, r_2, \alpha_1, \alpha_2) = F_{ij}(\tau; t, r_1, r_2, \alpha_1, \alpha_2) = \\ R_{v_i v_j}(-u_0 \tau, \xi_2(\tau; t, r_1, r_2, \alpha_1, \alpha_2), \xi_3(\tau; t, r_1, r_2, \alpha_1, \alpha_2)) \quad (A-7)$$

where ξ_2 and ξ_3 are functions of the indicated variables. The right side of Equation (A-7) is the Eulerian two-point, two-time velocity component covariance function evaluated at points #1 and #2 on the WECS blade elements which corresponds to the two-point, two-time velocity component covariance function following the WECS blade elements. Specification of the longitudinal and lateral correlation function completely specifies $R_{v_i v_j}(\tau; t, r_1, r_2, \alpha_1, \alpha_2)$.

The generalized power spectral density function corresponding to the two-point, two-time covariance function following the blade elements can be calculated with

$$\Psi_{v_i v_j}(\omega_1, \omega_2) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} F_{ij}(\tau; t) e^{i(\omega_1 - \omega_2)t} e^{-i\omega_2 \tau} d\tau \quad (A-8)$$

The corresponding frequency-time spectral density function is given by

$$\Phi_{v_i v_j}(\omega, t) = \int_{-\infty}^{\infty} \Psi_{v_i v_j}(\omega_1, \omega) e^{-i(\omega_1 - \omega)t} d\omega_1 \quad (A-9)$$

It is understood that F_{ij} , $\Psi_{v_i v_j}$ and $\Phi_{v_i v_j}$ depend on r_1 , r_2 , α_1 and α_2 .

Selection of reasonable functional forms of $f(\xi)$ and $g(\xi)$ for atmospheric turbulence (e.g., Dryden or von Karman models) do not lead to closed form integrals for Equations (A-8) and (A-9), thus, requiring numerical integration of these Fourier integrals. If we require generalized power spectral density and frequency-time spectral density functions for a relatively large number of points (e.g., $r_1, r_2, \alpha_1, \alpha_2$) then the engineering

implementation of Equations (A-8) and (A-9) could be rather complicated. The virtue of using the Taylor series of the wind field about the hub and truncating third- and higher-order terms as discussed in the main body of this report is that one obtains closed forms for these functions in terms of r_1 , r_2 , α_1 and α_2 . However, it should be noted that Equation (A-7) for the covariance function is simpler to implement in comparison to Equations (15) through (17) because numerical integration is required to obtain the back-transforms of the covariance on the right sides of Equations (16) and (17).

APPENDIX B

FORMULATION OF THEORY FOR NONHOMOGENEOUS
TURBULENT FIELD

FORMULATION OF THEORY FOR NONHOMOGENEOUS TURBULENT FIELD

In this appendix we develop a procedure for calculating the velocity component covariance function following the WECS blade elements for a horizontally and statistically stationary boundary layer turbulent flow. The formulation in the section entitled COVARIANCES for the turbulence covariance following the WECS blade elements is general and thus can be applied to a vertically nonhomogeneous boundary layer. We seek the covariances on the right-sides of Equations (16) and (17). The formulation herein follows along the lines in the section entitled HOMOGENEOUS AND ISOTROPIC TURBULENCE. The two-fold wave number-frequency decomposition of $\hat{v}_i(x_1, x_2, x_3, t)$ is given by

$$\hat{v}_i(x_1, x_2, x_3, t) = \int_{-\infty}^{\infty} e^{i(\kappa_1 \kappa_1 + \omega t)} dZ_i(\kappa_1, \kappa_2, x_3, \omega) \quad (B-1)$$

(3)

where the repeated subscripts denote Einstein summation from 1 to 2. We define

$$\frac{\partial^{\nu} + \lambda \hat{v}_i}{\partial x_k^{\nu} \partial x_m^{\lambda}} = \int_{-\infty}^{\infty} e^{i(x_1 \kappa_1 + \omega t)} D_k^{\nu} D_m^{\lambda} dZ_i(\kappa_1, \kappa_2, x_3, \omega) \quad (B-2)$$

(3)

where

$$D_2 = i\kappa_2 \quad (B-3)$$

$$D_3 = \frac{\partial}{\partial x_3} \quad (B-4)$$

Let

$$\xi_{ikm\nu\lambda}(t) = \left(\frac{\partial^{\nu} + \lambda \hat{v}_i}{\partial x_k^{\nu} \partial x_m^{\lambda}} \right)_0 \quad (B-5)$$

where subscript zero denotes evaluation at the WECS hub, so that

$$\xi_{ikm\nu\lambda}(t) = \int_{-\infty}^{\infty} e^{i\omega t} \left[D_k^\nu D_m^\lambda dZ_i(\kappa_1, \kappa_2, x_3, \omega) \right]_0 \quad (B-6)$$

where Einstein summation convention does not apply to Greek subscripts. We construct the following covariance function

$$R_{\xi_{ikm\nu\lambda} \xi_{jsn\beta\gamma}}(\tau) = \int_{-\infty}^{\infty} e^{i\omega\tau} D_k^\nu D_m^\lambda (D_s^{\beta})^* (D_n^{\gamma})^* \cdot \left[\langle dZ_i(\kappa_1, \kappa_2, x_3, \omega) dZ_j^*(\kappa_1', \kappa_2', x_3', \omega') \rangle \right]_{0,0} e^{i(\omega - \omega')t} \quad (B-7)$$

where

$$D_2' = i\kappa_2' \quad (B-8)$$

$$D_3 = \frac{\partial}{\partial x_3} \quad (B-9)$$

and double subscript denotes evaluation at the WECS hub with respect to x_3 and x_3' . Invoking the conditions of statistical horizontal homogeneity and stationary yields

$$R_{\xi_{ikm\nu\lambda} \xi_{jsn\beta\gamma}}(\tau) = \int_{-\infty}^{\infty} e^{i\omega\tau} \left[D_k^\nu D_m^\lambda (D^{\beta})^* (D^{\gamma})^* \cdot \phi_{ij}(\kappa_1, \kappa_2, \omega; x_3, x_3') \right]_{0,0} d\kappa_1 d\kappa_2 d\omega \quad (B-10)$$

where $\phi_{ij}(\kappa_1, \kappa_2, \omega; x_3, x_3')$ is the two-point, two-fold wave number-frequency spectral density of the functions $v_i(x_1, x_2, x_3, t)$ and $v_j(x_1, x_2, x_3', t)$.

Application of Taylor's hypothesis permits us to write

$$\phi_{ij}(\kappa_1, \kappa_2, x_3, \omega) = \phi_{ij}(\kappa_1, \kappa_2, x_3) \delta(\kappa_1 u_0 - \omega) \quad (\text{B-11})$$

so that Equation (B-10) can be written as

$$R_{\xi_{ikm\nu\lambda} \xi_{jsn\beta\gamma}}(\tau) = \int_{-\infty}^{\infty} e^{i\omega\tau} \Psi_{\xi_{ikm\nu\lambda} \xi_{jsn\beta\gamma}}(\omega) d\omega \quad (\text{B-12})$$

where

$$\Psi_{\xi_{ikm\nu\lambda} \xi_{jsn\beta\gamma}}(\omega) = \frac{1}{u_0} \int_{-\infty}^{\infty} \left[D_k^\nu D_m^\lambda (D_s^\beta)^* (D_n^\gamma)^* \cdot \right. \\ \left. \phi_{ij} \left(\frac{\omega}{u_0}, \kappa_2; x_3, x_3' \right) \right]_{0,0} d\kappa_2 \quad (\text{B-13})$$

All of the necessary covariance functions on the right-sides of Equations (16) and (17) can be derived from Equations (B-12) and (B-13) upon specification of $\phi_{ij}(\frac{\omega}{u_0}, \kappa_2; x_3, x_3')$. The necessary Ψ -functions in terms of $\phi_{ij}(\frac{\omega}{u_0}, \kappa_2; x_3, x_3')$ are given by

$$\Psi_{w_i w_j}(\omega) = \frac{1}{u_0} \int_{-\infty}^{\infty} \phi_{ij}(\frac{\omega}{u_0}, \kappa_2, 0, 0) d\kappa_2 \quad (\text{B-14})$$

$$\Psi_{w_i p_j}(\omega) = -\Psi_{p_i w_j}(\omega) = -\frac{i}{u_0} \int_{-\infty}^{\infty} \kappa_2 \phi_{ij}(\frac{\omega}{u_0}, \kappa_2, 0, 0) d\kappa_2 \quad (\text{B-15})$$

$$\Psi_{w_i q_j}(\omega) = \frac{1}{u_0} \left[\frac{\partial}{\partial x_3'} \int_{-\infty}^{\infty} \phi_{ij}(\frac{\omega}{u_0}, \kappa_2; 0, x_3') d\kappa_2 \right]_0 \quad (\text{B-16a})$$

$$\Psi_{q_i w_j}(\omega) = \frac{1}{u_0} \left[\frac{\partial}{\partial x_3} \int_{-\infty}^{\infty} \phi_{ij}(\frac{\omega}{u_0}, \kappa_2; x_3, 0) d\kappa_2 \right]_0 \quad (\text{B-16b})$$

$$\Psi_{W_i P_j}(\omega) = \Psi_{P_i W_j}(\omega) = -\frac{1}{u_0} \int_{-\infty}^{\infty} \kappa_2^2 \phi_{ij}\left(\frac{\omega}{u_0}, \kappa_2, 0, 0\right) d\kappa_2 \quad (B-17)$$

$$\Psi_{W_i Q_j}(\omega) = \frac{1}{u_0} \left[\frac{\partial^2}{\partial x_3^2} \int_{-\infty}^{\infty} \phi_{ij}\left(\frac{\omega}{u_0}, \kappa_2; x_3, x_3'\right) d\kappa_2 \right]_{0,0} \quad (B-18a)$$

$$\Psi_{Q_i W_j}(\omega) = \frac{1}{u_0} \left[\frac{\partial^2}{\partial x_3^2} \int_{-\infty}^{\infty} \phi_{ij}\left(\frac{\omega}{u_0}, \kappa_2; x_3, x_3'\right) d\kappa_2 \right]_{0,0} \quad (B-18b)$$

$$\Psi_{P_i P_j}(\omega) = -\Psi_{W_i P_j}(\omega) \quad (B-19)$$

$$\Psi_{Q_i Q_j}(\omega) = \frac{1}{u_0} \left[\frac{\partial^2}{\partial x_3 \partial x_3'} \int_{-\infty}^{\infty} \phi_{ij}\left(\frac{\omega}{u_0}, \kappa_2; x_3, x_3'\right) d\kappa_2 \right]_{0,0} \quad (B-20)$$

$$\Psi_{W_i M_j}(\omega) = -\frac{i}{u_0} \left[\frac{\partial}{\partial x_3} \int_{-\infty}^{\infty} \kappa_2 \phi_{ij}\left(\frac{\omega}{u_0}, \kappa_2; 0, x_3'\right) d\kappa_2 \right]_0 \quad (B-21a)$$

$$\Psi_{M_i W_j}(\omega) = -\frac{i}{u_0} \left[\frac{\partial}{\partial x_3} \int_{-\infty}^{\infty} \kappa_2 \phi_{ij}\left(\frac{\omega}{u_0}, \kappa_2, x_3, 0\right) d\kappa_2 \right]_0 \quad (B-21b)$$

APPENDIX C

SPECTRAL CONTRIBUTION RESULTING
FROM MEAN FLOW SHEAR

SPECTRAL CONTRIBUTION RESULTING FROM MEAN FLOW

The presence of a mean flow wind shear contributes to the turbulence spectrum for $i = j = 1$. In the most strict sense the theory presented herein does not permit this contribution because the theory requires the mean flow wind profile to be a constant. However, we shall allow the mean flow to vary with height above natural grade to calculate the contribution of mean flow wind shear to the $i = j = 1$ spectral density function. We expand the mean wind profile about the hub, so that

$$\bar{u}(x_3) = u_0 + \left(\frac{\partial \bar{u}}{\partial x_3}\right)_0 x_3 \quad (C-1)$$

where we have truncated second- and higher-order terms. The fluctuations in the mean flow relative to the constant wind profile u_0 as seen by a WECS blade element are thus given by

$$\bar{u}'(x_3) = \left(\frac{\partial \bar{u}}{\partial x_3}\right)_0 r \sin(\omega_0 t + \alpha) \quad (C-2)$$

Evaluation of Equation (C-2) at time $t + \tau$ at blade element located at (r_1, α_1) and at time t at blade element located at (r_2, α_2) , multiplying the resulting relationships, and ensemble averaging yields

$$\langle \bar{u}'(t, \tau; r_1, \alpha_1) \bar{u}'(t, 0; r_2, \alpha_2) \rangle = R_{\bar{u}}(t, \tau; r_1, r_2, \alpha_1, \alpha_2) \quad (C-3)$$

where

$$\begin{aligned} R_{\bar{u}}(t, \tau; r_1, r_2, \alpha_1, \alpha_2) = & \left(\frac{\partial \bar{u}}{\partial x_3}\right)_0^2 \frac{r_1 r_2}{2} \left[\cos(\omega_0 \tau + \alpha_2 - \alpha_1) \right. \\ & \left. - \cos(\omega_0(2t + \tau) + \alpha_1 + \alpha_2) \right] \quad (C-4) \end{aligned}$$

The first term on the right in the brackets is the stationary contribution to

the covariance and the second term is the nonstationary contribution. For the logarithmic profile

$$\left(\frac{\partial \bar{u}}{\partial z}\right)_0 = \frac{u_{*0}}{kz_h} \quad (C-5)$$

where

u_{*0} = surface friction velocity

k = von Karman's constant

(Lumley and Panofsky, 1964), so that the coefficient of the trigonometric terms in Equation (C-4) given by

$$\left(\frac{\partial \bar{u}}{\partial x_3}\right)_0^2 \frac{r_1 r_2}{2} = \frac{u_{*0}^2}{k^2 z_h^2} \frac{r_1 r_2}{2} \quad (C-6)$$

The double frequency generalized power spectral density function associated with Equation (C-4) is given by

$$\begin{aligned} \psi_{\bar{u}}(\omega_1, \omega_2; r_1, r_2, \alpha_1, \alpha_2) = & \left(\frac{\partial \bar{u}}{\partial x_3}\right)_0^2 \frac{r_1 r_2}{2} \left\{ \left[e^{i(\alpha_2 - \alpha_1)\delta(\omega_1 + \omega_0)} \right. \right. \\ & + e^{-i(\alpha_2 - \alpha_1)\delta(\omega_1 - \omega_0)} \left. \right] \delta(\omega_1 - \omega_2) - \left[e^{i(\alpha_1 + \alpha_2)\delta(\omega_1 - \omega_2 + 2\omega_0)} \delta(\omega_1 + \omega_0) \right. \\ & \left. \left. + e^{-i(\alpha_1 + \alpha_2)\delta(\omega_1 - \omega_2 - 2\omega_0)} \delta(\omega_1 - \omega_0) \right] \right\} \quad (C-7) \end{aligned}$$

The frequency-time spectral density function is given by

$$\begin{aligned} \phi_{\bar{u}}(\omega, t; r_1, r_2, \alpha_1, \alpha_2) = & \left(\frac{\partial \bar{u}}{\partial x_3}\right)_0^2 \frac{r_1 r_2}{4} \left\{ e^{i(\alpha_2 - \alpha_1)\delta(\omega + \omega_0)} + e^{-i(\alpha_2 - \alpha_1)\delta(\omega - \omega_0)} \right. \\ & \left. - \left[e^{i(\alpha_1 + \alpha_2)} e^{i(\omega + \omega_0)t} \delta(\omega - \omega_0) + e^{-i(\alpha_1 + \alpha_2)} e^{i(\omega - \omega_0)t} \delta(\omega + \omega_0) \right] \right\} \quad (C-8) \end{aligned}$$

The first two terms in the curly brackets on the right side of Equation (C-8) corresponds to the stationary contribution to the spectrum while the remaining terms correspond to nonstationary contribution. Integration of Equation (C-8) over the domain $-\infty < \omega < \infty$ yields the variance of $\bar{u}$, Equation (C-4) evaluated at $\tau = 0$.

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