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USE OF THE "MURA" TRANSFORMATION TO GENERATE THE FIELDS AND CALCULATE THE MOTION OF PROTONS IN THE DESIGNED ARGONNE MINI-ASPUN FFAG SPIRAL SECTOR ACCELERATOR

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Introduction

As a long range goal for the production of high intensity neutrons, Argonne National Laboratory has proposed the construction of a 1.5 GeV FFAG Spiral Sector Accelerator called ASPUN.¹ The 300-MeV injector for this proposed accelerator is a smaller FFAG Spiral Sector Accelerator named Mini-ASPUN. Until such a time as the larger machine could be built, it was planned that Mini-ASPUN would replace the present RCS now being used for the IPNS program at Argonne.

In order to obtain an accurate estimation of the orbits and betatron oscillations in such a machine, it is necessary that realistic field values be used in the equations of motion. Obtaining these fields from 3-dimensional relaxation calculations is both time consuming and costly. However, because of the required scaling of the machine, the field-generating potential of three variables can be separated into a known function of the radius and a function of two variables. The second order differential equation satisfied by this function can be solved by ordinary relaxation methods. The fields generated from a mesh of values for this function will be accurate except for the extreme inside and outside orbits, which will be affected by the necessary termination of the inside and outside ends of the magnet.

Generation of the Field Mesh

The vertical component of the midplane magnetic field for a scaling FFAG Spiral Accelerator has the form

$$B_z = \bar{B}_0 \left(\frac{r}{r_0} \right)^k \left[1 + \sum_n f_n \cos n\theta - \tan\phi \ln \frac{r}{r_0} \right] \quad (1)$$

where

r is the radius measured from the machine center,

$\bar{B}_0$ is the average field at the reference orbit r_0 ,

f_n are the harmonic components of the azimuthally varying field,

N is the number of identical sector magnets,

ϕ is the spiral angle, and

k is the mean field index.

In the regions outside the iron and the coils, the field components can be represented by

$$B_z = \frac{\partial}{\partial z} V(r, \theta, z) \quad (2)$$

$$B_r = \frac{\partial}{\partial r} V(r, \theta, z)$$

$$B_\theta = \frac{1}{r} \frac{\partial}{\partial \theta} V(r, \theta, z)$$

where the potential function V satisfies the Laplace equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad (3)$$

with boundary conditions determined by the location of the coils and the iron.

In the late 1950's, the accelerator group at MURA discovered that the potential function V could be separated into the form²

$$V = V_0 \left(\frac{r}{r_0} \right)^{k+1} G(\xi, \eta) \quad (4)$$

where

$$\xi = N(\theta - \tan\phi \ln \frac{r}{r_0}), \text{ and}$$

$$\eta = \frac{N}{1 + \tan^2\phi} \frac{z}{r} \approx \frac{NKz}{r}$$

The equation satisfied by the function G is

$$\frac{\partial^2 G}{\partial \xi^2} + \left(1 + \left(\frac{\eta}{NK} \right)^2 \right) \frac{\partial^2 G}{\partial \eta^2} + \frac{2 \tan\phi \eta}{NK^2} \frac{\partial^2 G}{\partial \xi \partial \eta} - \frac{2(k+1) \tan\phi}{NK^2} \frac{\partial G}{\partial \xi} - \frac{(2k+1)\eta}{NK^2} \frac{\partial G}{\partial \eta} + \left(\frac{1+k}{NK} \right)^2 G = 0 \quad (5)$$

Equation (5) was solved by relaxation for Mini-ASPUN using a rectangular mesh. The design values are $k = 8$, $\phi = 57^\circ$, $N = 16$, and $r_0 = 8.448$ m. The results and boundary conditions are shown in Fig. 1. There are 233 mesh spaces in the region from $\xi = -0.7735$ to $\xi = 0.8375$ π . The magnet iron extends from $\xi = -0.1505$ to $\xi = 0.1505$ π . The half gap for the Mini-ASPUN magnets is 0.09 m at $r = r_0$. The value of η at the pole face is 0.3129. The vertical mesh size

Fig. 1. Equipotentials from the relaxed value of $G(\xi, \eta)$ for Mini-ASPUN. One sector runs from $-\pi$ to $+\pi$.

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was adjusted for 15 mesh spaces between the midplane and the pole of the magnets. The boundary condition for the coil varies linearly from 1.0 at $z = 0.175$ m to 0 at $z = 0.375$ m. The mesh extends to $z = 0.594$ m in the regions outside the pole face.

The equipotential lines are shown in Fig. 1. It is apparent from the flattening of the 0.01 contours that the actual field never quite reaches zero in the region between the sector magnets. The field values are very small, however, and would not have very much effect on the orbit calculation. This "cross talk" effect could be included in the boundary conditions for G by requiring the continuity of G at $\xi = \pm\pi$.

The field values in terms of G and its derivatives are given by

$$B_z = \frac{V_0 N K}{r_0} \left(\frac{r}{r_0}\right)^k \frac{\partial G}{\partial \eta} \quad (6)$$

$$B_r = \frac{V_0}{r_0} \left(\frac{r}{r_0}\right)^k \left[(1+k)G - \frac{N K z}{r} \frac{\partial G}{\partial \eta} - N \tan \psi \frac{\partial G}{\partial \xi} \right]$$

$$B_\theta = \frac{V_0 N}{r_0} \left(\frac{r}{r_0}\right)^k \frac{\partial G}{\partial \xi}$$

In the final form, G exists in a computer file as a matrix of values $G(i,j)$ with i ranging from 1 at $\xi = -0.7735$ to 234 at $\xi = 0.8575$ and j ranging from 1 on the midplane, to 16 at the pole face for the region under the pole, and to 100 at the uppermost boundary for the regions outside the pole. On the boundary, the vertical component of the field may be written as

$$B_z(z=0) = \frac{B_0}{\Delta G} \left(\frac{r}{r_0}\right)^k \Delta G_1 \quad (7)$$

where $\Delta G_1 = G(i,2)$ and V_0 has been adjusted to give $B_z(z=0) = B_0$ for $r = r_0$ at the center of the magnet where $\Delta G_1 = \Delta G_c$.

Figure 2 shows a plot of the azimuthal variation of $B_z(z=0)$ along a constant radius normalized to unity at the center of the magnet.

Fig. 2. Variation of the midplane vertical magnetic field component through one sector of Mini-ASFUN.

Because of the spiral angle ψ , the field is not symmetrical about the center of the magnet. The first sixteen harmonic components of the field are tabulated in Table I.

Table I
First 16 Harmonic Coefficients of the
Midplane Field at Constant Radius

k	cos Nkθ	sin Nkθ
0	0.25560	-
1	0.41296	0.02218
2	0.23661	0.01165
3	0.11247	0.00159
4	0.03701	-0.00108
5	-0.00213	-0.00336
6	-0.01540	-0.00191
7	-0.01747	-0.00159
8	-0.01306	0.00067
9	-0.00790	0.00003
10	-0.00342	-0.00006
11	-0.00304	0.00042
12	0.00105	0.00009
13	0.00158	0.00020
14	0.00155	0.00010
15	0.00103	-0.00003
16	0.00048	0.00007

Charged Particle Motion through the PFAC Field

The equations of motion of a charged particle in a magnetic field are

$$\ddot{r} = r\dot{\phi}^2 + \frac{q}{mc} [z \dot{B}_\theta - r \dot{B}_r] \quad (8)$$

$$\ddot{z} = \frac{q}{mc} [r \dot{B}_r - z \dot{B}_\theta]$$

$$v^2 = \dot{r}^2 + r^2 \dot{\theta}^2 + \dot{z}^2 = \text{constant}$$

Because the field values are given by the $G(i,j)$ mesh at equal ξ intervals, it is convenient to choose ξ as the independent variable. One has

$$v^2 = \xi^2 (\dot{r}^2 + r^2 \dot{\theta}^2 + \dot{z}^2)$$

where $(\dot{})$ indicates derivatives with respect to ξ . The equation of motion may then be written as

$$r' = \frac{r \sin \phi_x}{N(\cos \phi_x - \tan \psi \sin \phi_x)} \quad (9)$$

$$\theta' = \frac{1}{N} + \frac{r' \tan \psi}{r}$$

$$\phi_z' = \frac{e}{pc \cos \phi_x} [r \dot{\theta}' B_r - r' \dot{\theta} B_\theta]$$

$$\phi_x' = \phi_z' \tan \phi_x \tan \phi_z + \theta' + \frac{e}{pc} \frac{(z' B_\theta - r' B_z)}{\cos \phi_x \cos \phi_z}$$

where

$$\tan \phi_x = z'/r\theta',$$

$$\tan \phi_y = z'/\sqrt{r^2 + r^2 \theta'^2}.$$

The field values are obtained from equations (6) and second order interpolation of the mesh values. In

integrating equation (9), one normally uses many integration steps per mesh space.

Tracking of a particle is accomplished by integrating equation (9) after choosing r_0, r_0', r_0'', r_0''' and the desired momentum p . The closed orbit for any energy can be obtained by proper adjustment of r_0, r_0', r_0'', r_0''' . Figure 3 shows the radial position of the 50 MeV equilibrium orbit for the Mini-ASPUN magnet system with the boundary conditions shown in Fig. 1.

The linear transformation matrices for vertical and horizontal oscillations can be obtained as follows. After determining the equilibrium orbit initial condition, one integrates along that orbit. At each integration step one builds the matrix

$$N(\xi + \Delta\xi) = \Delta N N(\xi) \quad (10)$$

where

$$\Delta N = \begin{pmatrix} 1 - \frac{k^2 \Delta s^2}{2} & \Delta s \\ -k^2 \Delta s & 1 - \frac{k^2 \Delta s^2}{2} \end{pmatrix},$$

$$N(0) = I,$$

$$k_x^2 = \frac{1-n}{\rho}, \quad k_y^2 = \frac{n}{\rho},$$

$$\Delta s = \sqrt{r^2 + r\theta'^2} \Delta\xi.$$

r', r , and θ are evaluated on the equilibrium orbit. The local radius of curvature ρ is

$$\frac{1}{\rho} = \frac{B_x(z=0)}{B_0 \rho_0} = \left(\frac{r}{r_0} \right) k \frac{\Delta G(\xi)}{\Delta G_0 \rho_0} \quad (11)$$

The local field index n is defined as

$$n = \frac{-\rho}{B_x(z=0)} \frac{\partial B_x(z=0)}{\partial x} \quad (12)$$

Fig. 3. Position (r) of the 50-MeV orbit through one sector of Mini-ASPUN.

where x is normal to the equilibrium orbit. The normal field gradient is given by

$$\frac{\partial B}{\partial x}(x=0) = \cos \phi \frac{\partial B}{\partial r}(x=0) - \frac{\sin \phi}{r} x \frac{\partial B}{\partial \theta}(x=0) \quad (13)$$

$$= \frac{B_0}{r \Delta G_0} \left(\frac{r}{r_0} \right) k \{ k \Delta G \cos \phi_x - N(\cos \phi_x \tan \phi + \sin \phi_x) \frac{\partial \Delta G}{\partial \theta} \}$$

The total phase advance per sector for each sector and the corresponding Courant Snyder coefficients for one point in the sector are contained in the linear transformation matrices for a complete sector. The phase advances and beta functions for the two planes as a function of distance along the orbit can be obtained from the partial sector transformation matrices. The results for the Mini-ASPUN magnet are shown in Fig. 4.

Fig. 4. The variation of β_x and β_y at 50 MeV as a function of distance through one sector of Mini-ASPUN.

Conclusion

The MURA transformation is very useful in the early design stages of an FFAG accelerator. After examination of the results for a particular set of parameters, one can make changes in the field index, spiral angle, and sector magnet width to arrive at the desired tunes and positions of the orbits as a function of energy. One can then proceed to a 3-dimensional representation of the configuration that preserves as well as possible the field flux shapes of the scaled two dimensional representation.

References

- 1 A 1500-MeV Fixed Field Alternating-Gradient Synchrotron for a Pulsed-Spallation Neutron Source, R. L. Knostom, T. K. Rhoe, and E. A. Crosbie, (U1) this conference.
- 2 Electron Model of a Spiral Sector Accelerator, D. W. Kerst, E. A. Day, H. J. Hausman, R. O. Huxby, L. J. Laslett, F. E. Mills, T. Ohkawa, F. L. Peterson, E. H. Rowe, A. M. Sessler, J. M. Snyder, W. A. Wallenmeyer, Reviews of Scientific Instruments, Vol. 31, 1960, No. 10, pp. 1076, 1106.