TOPICS IN
GAUGE THEORIES AND THE UNIFICATION
OF ELEMENTARY PARTICLE INTERACTIONS

Yogendra N. Srivastava
Michael T. Vaughn

Northeastern University
Boston, Massachusetts 02115

Progress Report
for period January 1, 1991 - January 31, 1992

February 1992

Prepared for
THE U.S. DEPARTMENT OF ENERGY
GRANT NO. DE-FG02-85-ER40233

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Progress Report on Grant DE-FG02-85ER40233

Period Covered: January 1, 1991 - January 31, 1992

Received by OSTI

MAR 16 1992

Abstract

We report on work done by the principal investigators (Y. N. Srivastava and M. T. Vaughn) and their collaborators on

- (1) purely fermionic composite models,
- (2) gravitational diamagnetism,
- (3) dynamical Casimir effect,
- (4) N -particle amplitudes for large N beyond the tree approximation, and
- (5) analysis of classical scalar ϕ^4 field theory.

I. RESEARCH COMPLETED AND IN PROGRESS

1. Purely Fermionic Composite Models: (Y. Srivastava)

While spontaneous symmetry breaking has been extremely successful in the Standard Model, the underlying physical mechanism producing the required non-vanishing vacuum expectation value (vev) for the scalar field is not completely understood. For example, we do not know to this date, whether a non-zero vev is already present in the potential at the classical level (as is commonly assumed) or is it induced by quantum radiative corrections as in reference (1)? Another important question which is related to asymptotic freedom, is whether Higgs is elementary or composite?

To shed some light on the above questions, it is important to begin classically with a Lagrange density which is both scale- and gauge invariant. The usual four-fermionic composite models of the Nambu-Jona-Lasinio (NJL) type⁽²⁾ or in its resuscitated form of the Top Model, of course violate scale-invariance. On the other hand, several years ago, we were able to construct⁽³⁾ a classical fermionic scale-invariant action with gauge invariant currents for the $SU(3) \times SU(2) \times U(1)$ group. In this approach, at the classical level, the only dynamical degrees of freedom are those of the leptons and quarks. After renormalization, such an action is found to be equivalent to the standard model action with gauge and Higgs degrees of freedom but with two extra features.

(i) Before the spontaneous breakdown, the model does possess scale-invariance and is thus of the Coleman-Weinberg type.

(ii) Unlike the standard model, there are relations among the various coupling constants so that it has some predictive power regarding masses.

The results are as follows⁽⁴⁾. The “bare” weak angle has the $SU(5)$ value

$$\sin^2 \theta_W = g'^2 / (g^2 + g'^2) = 3/8. \quad (1)$$

Also, we find that the bare strong coupling constant

$$g_s = g. \quad (2)$$

Thus, Eqs.(1) and (2) say that there is only one independent gauge-coupling constant. These “bare” results can be interpreted in terms of a renormalization (cut-off) mass Λ , so that the three running coupling constants

$$\alpha_1(\Lambda) = (5/3)(g'^2(\Lambda)/4\pi),$$

$$\alpha_2(\Lambda) = (g^2(\Lambda)/4\pi),$$

$$\alpha_3(\Lambda) = (g_s^2(\Lambda)/4\pi),$$

are indeed equal.

After the symmetry breakdown, there is also a remaining formula connecting the vector boson mass to the fermion masses which reads

$$M_W^2 = (1/8) \sum_{k=1}^3 [m_{kl}^2 + 3(m_{ku}^2 + m_{kd}^2)]$$

where the index k refers to the families, index l refers to leptons and as usual u and d refer to the up and down quarks, respectively. Since all masses but that of the top may be ignored, we find approximately for the top mass

$$m_{Top} \approx \sqrt{(8/3)} M_W \approx 132 GeV,$$

a value in good agreement with the EW radiative corrections.

We are presently investigating constraints on the Higgs mass which is more delicate here than the nominal relationship $m_H = 2m_{fermion}$ found in the NJL type models. For the massless $\lambda\Phi^4$ interaction, there are new solutions⁽⁵⁾ to the RG equation for the one-loop potential which are in contrast to those of reference (1). According to the authors of ref.⁽⁵⁾, the mass of the Higgs is about 2.2 TeV and yet it is only weakly coupled. Given the somewhat different structure of our scheme, it would be of interest to apply their technique to obtain results relevant to our case.

References for Part 1.

1. S. Coleman and E. Weinberg, *Phys. Rev.*, **D7**, 1888 (1973).
2. Y. Nambu and G. Jona-Lasinio, *Phys. Rev.*, **122**, 345 (1961).
3. M.H. Friedman and Y. Srivastava, *Phys. Rev.*, **D24**, 1491 (1983).
4. G.Pancheri and Y. Srivastava, "Higgs and Top: Missing Links of the Standard Model", Frascati Preprint (1992). Invited talk by YS at the Ninth Capri Symposium on Fundamental Interactions, Capri, May 1991 (Proceedings to be published).
5. V. Branchina and M. Consoli, "Spontaneous Symmetry Breaking and the Higgs Mass", INFN Preprint 1992, Catania, Italy.

2. Gravitational Diamagnetism: (Y. Srivastava)

The magnetic and gravitational fields (for a uniform applied magnetic intensity H or by a uniform applied electric field E) are found by solving the Einstein-Maxwell field equations exactly⁽¹⁾. A neutral test mass placed in the gravitational field induced by the Maxwell stress tensor is shown to exhibit "diamagnetic properties" of purely gravitational origin, which leads to two important consequences. Gravity confines magnetic fields and all objects possess a universal, gravitationally induced dipole moment equal to $\hbar\sqrt{(G/2c^2)}$ in Gaussian units. (Clearly this result cannot be obtained classically – since the expression is proportional to $\hbar$ – nor in a perturbation expansion in G). We also find that the ratio of the magnetic moment μ to the angular momentum L will be $(constant)\sqrt{G}/c$.

The above result is of obvious interest to an outstanding unsolved problem for large and small objects – that is, the source of confined magnetic fields for a small planet like the earth to large galaxies. As far as we are aware there has simply been no gravitational calculation using exact solutions of standard general relativity relevant to this problem.

From a phenomenological point of view, the above relationship has been advocated for about seventy years. It has variously been called the rotational gravi-mag hypothesis, Bode's law for magnetism and the Blackett relationship ⁽²⁻⁶⁾.

A large amount of data from condensed objects such as neutron stars to the moon, the earth, other planets, the Sun, etc., indeed shows a linear relationship with a constant $\approx \sqrt{G}/2c$. We find it quite encouraging that such a result appears rather naturally from our analysis and goes towards offsetting Russell's discouraging remark ".....lack of any physical justification for such a law"⁴.

A decisive laboratory test of the gravi-mag hypothesis was originally suggested by Blackett ⁽⁶⁾. He predicted that a 100cm bronze sphere when spun at 100Hz should yield an induced magnetic field of $\approx 10^{-8}$ Gauss which is not out of reach of the present day Squid technology. For more complicated geometries and mass distributions in a given experimental setup, the required results can be obtained through the Lagrangian given in ref.(1). Controlled experiments of this type with the subtraction of known background electromagnetic contributions would be of great interest for the results derived in the present work.

We have also investigated cosmic strings through the Weyl metric in general relativity and its Newtonian limit⁽⁷⁾. The hypothesis is explored that the observed filamentary clustering of galaxies is due to localization about Weyl cosmic strings. If the string tension is determined by a grand unification mass, then the magnitude of observed asymptotic velocities of stars in galaxies can be understood without recourse to 'dark matter'.

References for Part 2.

1. Y. Srivastava and A. Widom, "Gravitational Diamagnetism", accepted for publication by Phys. Lett. B.
2. H.A. Wilson, *Proc. R. Soc., A* 104, 451 (1923)
3. P.M.S. Blackett, *Nature*, 159, 658 (1947)
4. C.T. Russell, *Nature*, 272, 147 (1978)
5. S.P. Sirag, *Nature*, 278, 535 (1979)
6. P.M.S. Blackett, *Phil. Trans. R. Soc., A* 245, 309 (1952)
7. A. Widom, Y. Srivastava and N. Reddington, "Weyl Cosmic Strings and their Consequences", submitted for publication.

3. Dynamical Casimir Effect: (Y. Srivastava)

Over the past several years, we have been involved in a systematic investigation of the Casimir effect as a means to probe QED in a non-perturbative manner⁽¹⁻³⁾. Over the past year, we have enlarged the scope of our analysis by considering physical situations where the geometry of the system is not changed adiabatically. This may justifiably be termed as the dynamical Casimir effect. To illustrate the concept consider the vacuum photons confined in a cavity. The evaluation of the force as the cavity dimensions are adiabatically changed comprises the usual quasi-static Casimir effect. But what happens if we modulate the frequency of the confined photons by modulating the geometry of the cavity in a significant way?

The above problem can be cast in terms of a 1 - *dim* relativistic wave equation with a time-dependent "potential" $V(t)$, which incorporates the spatial boundary variations of the cavity⁽⁴⁾. (For the frequency range of interest, the spatial variations of the photon wave function itself can be safely ignored.) Now the problem reduces to a calculation of the reflection and transmission amplitudes for the "barrier" provided by the potential. Usual machinery of scattering theory in one-dimension may now be employed with two major physical differences. The variable now being time, energy is of course not conserved. Also, the equation being relativistic, incorporates both positive and negative frequencies. So, a positive frequency photon when "reflected" has negative frequency, which in a relativistic theory means particle (in this case, photon) production! The number of produced photons of a given frequency ω may be written in the usual manner in terms of a frequency dependent "noise" temperature $T_{noise}(\omega)$ introduced by Schwinger.

In ref.(4), we were able to show rigorously that for any physical, finite-duration pulse modelled by $V(t)$, the above noise temperature diverges as $1/\omega$ as the frequency ω goes to zero. More explicitly,

$$T_{noise}(\omega \rightarrow 0) = (\hbar/k\tau_c^{2J})(1/\omega),$$

where τ_c is the "characteristic scattering time" for the pulse. This provides a general explanation of the ubiquitous "1/f-noise" found in circuits at low frequencies.

We are presently investigating periodic time modulations which should result in a periodic production of photons with a frequency spectrum far surpassing the confined frequency modes (and ofcourse much larger by several orders of magnitude the modulation frequency itself.) This opens up the exciting possibility of a source of pulsed photons say in the visible from rather modest input frequency modulations of a cavity or a micro-circuit.

References for Part 3.

1. Y. Srivastava, A. Widom and M. H. Friedman, " Microchips as Precision QED Probes", *Phys. Rev. Lett.*, **55**, 2246 (1985).
2. Y. Srivastava and A. Widom, *Physics Reports*, **148**, 1 (1987).
3. O. Panella, Y. Srivastava and A. Widom, "Casimir Effect for Charged Particles", *Phys. Rev.*, **B42**, 9790 (1990)
4. A. Widom, Y. Srivastava and E. Sassaroli, "Dynamical Casimir Effect and $(1/\omega)$ -noise", submitted for publication.

4. n -particle Amplitudes for large n beyond the Tree Approximation (M. Vaughn)

Several recent calculations have dealt the behavior of n -point amplitudes in field theories with bosons when n is large. In a Euclidean-based calculation for the production of n Higgses (or W 's) in the presence of the (constrained) instanton, Ringwald [1] and later Espinosa [2] showed that this amplitude grows like $n!$, so that unitarity is naively violated at a finite energy. In two subsequent studies of ordinary ϕ^4 theory in four dimensions by Cornwall[3] and Goldberg[4], such anomalies were found at tree level, without reference to instantons. A subsequent study in the context of a Schrödinger approach to ϕ^4 [5] at tree level (*i.e.*, the WKB approximation) gave the same result. In [4] the tree graphs for $q\bar{q} \rightarrow nH$ were analyzed, with the result that the amplitude (for $n \sim E/m_H$) was found to grow like $(\sqrt{\lambda}/m_H)^n n!$, implying a cross section in apparent violation of s -wave unitarity when $n \sim 4\pi/\lambda$. It was also noted that there is no obvious connection between these tree level results and the standard asymptotic nature of perturbation expansions [6,7] in field theory. The latter are concerned with the behavior of the expansion coefficients at *high order in perturbation theory* of a Green's function with a *fixed number* of legs. The concern here is the pathological behavior at *tree level* of an amplitude with *many* external legs.

In order to understand if this problem could be remedied by going beyond the tree approximation, Goldberg and I studied [8] the behavior of the n -point function in a zero-dimensional scalar theory with Lagrangian

$$\mathcal{L}(\phi) = -\frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 + J\phi$$

in the presence of an external source J . The generating functional for this theory is given by

$$Z[J] = \int_{-\infty}^{\infty} e^{\mathcal{L}(\phi)} d\phi .$$

$Z[J]$ generates the connected Green functions of the theory, which are given by

$$G_c^{(n)} = \frac{\partial^n}{\partial J^n} \ln Z[J] .$$

For fixed n , the perturbation series in λ for $G_c^{(n)}$ counts the number of diagrams in each order, weighted by the appropriate symmetry factor for the diagram.

We find that the usual $n! (\sqrt{\lambda})^n$ behavior is obtained in saddle-point (tree) approximation. Here, however, we can also show that a similar behavior $n! [f(\lambda)]^n$ persists in the full theory, where $f \sim \sqrt{\lambda}$ for $\lambda \rightarrow 0$, $f \sim \lambda^{-1/4}$ for $\lambda \gg 1$. The difficulty in the full theory can be traced to the existence of zeros in the complex J -plane of the generating functional $Z[J]$. The criterion for the existence of these zeros is shown to be established by the asymptotic behavior in J of $Z[J]$ calculated in the saddle-point approximation, although the actual locations of the zeros must be determined numerically (they not well-determined in the saddle-point approximation). None of these calculations involve perturbative expansions in λ .

We are continuing to study the large N behavior of N -point functions in more realistic theories to understand if this behavior persists. See, in particular, the cubic model described in the current proposal.

References for Part 4.

1. A. Ringwald, Nucl.Phys. **330**, 1 (1989).
2. O. Espinosa, Nucl.Phys. **343**, 310 (1990).
3. J. M. Cornwall, Phys. Lett. **243**, 271 (1990).
4. H. Goldberg, Phys. Lett. **246**, 445 (1990).
5. M. B. Voloshin, TPI preprint TPI-MINN-90/54-T (1990).
6. L. N. Lipatov, Sov. Phys. JETP **45** (2), 216 (1977).
7. E. Brezin, J.C. Le Guillou, and J. Zinn-Justin, Phys. Rev. D **15**, 1544 (1977).
8. H. Goldberg and M. T. Vaughn, Phys. Rev. Letters **66**, 1267 (1991).

5. Analysis of Classical Scalar ϕ^4 Field Theory (M. Vaughn)

From the analysis described in the previous section, it seems likely that high-energy scattering leading to large numbers of bosons in the final state may not be amenable to an ordinary perturbative treatment. Thus we (H. Goldberg, D. Nash, and myself) were led to a study of the classical $\lambda\phi^4$ theory; in particular, to a study of the flow of energy in the classical system between normal modes of the free field. If we start from initial conditions in which the energy resides in a few high-frequency modes of the free field (somewhat analogous to a field configuration which might be produced in a high-energy collision), then we can follow the energy to see whether it is dissipated into many ‘quanta’ of the low-frequency modes.

We have done extensive numerical studies on both the $(1 + 1)$ -dimensional theory and a special class of spherically symmetric solutions in the $(3 + 1)$ -dimensional theory (with $m^2 > 0$), and found some rather striking behavior, which is rather puzzling to us, and no sign of cascading of the energy into low-frequency modes. For weak coupling, the energy tends to stay in the initial modes, with some excursion into nearby modes; we can understand this as a motion of the field in quasi-normal modes which have been slightly distorted from the normal modes of the free field by the interaction. As the coupling strength is increased, the energy disperses more widely, but nevertheless rather slowly on a scale of the frequency of the field as a whole (since we are confining the field to a finite region of size L , the lowest natural frequency of the field is $\omega_0 \sim \sqrt{(\pi/L)^2 + m^2}$ with corresponding time scale $T \sim 2\pi/\omega_0$). In the $(1 + 1)$ -dimensional theory, there is some evidence for quasi-periodic motion, though not a full return to the initial modes, over time spans $\sim 1000 - 2000 T$, up to relatively large coupling constants.

Two striking features of the results in the $(3 + 1)$ -dimensional theory can be noted:

- (i) The time for the energy in the initial modes to decrease to $\sim 40\%$ of its initial value is strictly proportional to $1/\lambda$ over a wide range of λ (almost two orders of magnitude), although for very weak coupling, the energy in the initial modes never falls that low before starting to return to the initial modes.

(ii) If we define the number of 'quanta' J_n in the n^{th} normal mode as E_n/ω_n , where E_n is the energy, and ω_n the frequency of the n^{th} normal mode, the the total number of 'quanta' $N \equiv \sum_{\text{modes}} J_n$ is approximately conserved over times as long as $5000 T$, even when the coupling constant $\lambda/4\pi \sim 1.0$. This number is not exactly conserved – it fluctuates over short time scales, but by much less than a simulated distribution obtained by distributing energy randomly among a relatively narrow band of modes. Nevertheless, it is highly suggestive that there may be a hitherto unrecognized (partial) integrability in the classical system.

We note that the recurrent behavior in a system of coupled oscillator first discovered numerically by Fermi, Pasta and Ulam[1] is not relevant here – that system is known to reduce to a completely integrable system in the continuum limit, while the $\lambda\phi^4$ theory (with $m^2 > 0$) has no such rich structure. The standard works [2-4] on classical systems with few degrees of freedom provide us with some ideas, but no results directly applicable to the field theoretic system. While the applicability of our results to the quantum problem is unclear at the present, we believe the results are of intrinsic interest in the classical context.

We are writing up our results [5] on the $(3 + 1)$ -dimensional system, and continuing to explore the $(1 + 1)$ -dimensional case.

References for Part 5.

1. E. Fermi, J. Pasta and S. Ulam, *Studies of Non-linear Problems*, Los Alamos report LA-1940 (1955), reprinted in *Collected Works of Enrico Fermi*, vol. II, p. 978, University of Chicago Press (Chicago, 1965).
2. V. I. Arnold, *Mathematical Methods of Classical Mechanics*, Springer-Verlag (New York, 1978).
3. A. J. Lichtenberg and M. A. Lieberman, *Regular and Stochastic Motion*, Springer-Verlag (New York, 1983).
4. M. C. Gutzwiller, *Chaos in Classical and Quantum Mechanics*, Springer-Verlag (New York, 1990).
5. H. Goldberg, D. Nash and M. T. Vaughn, "Classical $\lambda\phi^4$ Theory in $3 + 1$ Dimensions", to be submitted to Phys. Rev. D.

II. PUBLICATIONS SUPPORTED BY THIS GRANT

1. Y. Srivastava et al., "Casimir Effect for Charged Particles", *Phys. Rev.*, **B42**, 9790 (1990).
2. Y. Srivastava et al., " $Z^0\gamma^*$ Production at Hadron Colliders", Preprint *LNF – 91/026* (accepted for publication by Phys. Rev.).
3. Y. Srivastava et al., "Gravitational Diamagnetism", accepted for publication by Physics Letters B.
4. Y. Srivastava et al., "Higgs and Top: Missing Links of the Standard Model", Two Invited Talks at the Ninth Capri Conference on Fundamental Interactions, Capri 1991. (Proceedings to be published).

5. Y. Srivastava et al., "Dynamical Casimir Effect and $(1/\omega)$ -noise", submitted for publication.
6. Y. Srivastava et al., "Weyl Cosmic Strings and Their Consequences", to be submitted for publication.
7. "Tree and Non-tree Multiparticle Amplitudes" (with H. Goldberg), Phys. Rev. Letters **66**, 1267 (1991).
8. "N-Particle Amplitudes for Large N", Northeastern University preprint NUB 3030 (1991), to appear in the proceedings of the conference PASCOS '91 held in Boston, March 1991.
9. "Classical $\lambda\phi^4$ Theory in 3 + 1 Dimensions" (with H. Goldberg and D. Nash), to be submitted to Phys. Rev. D.

III. PERSONNEL SUPPORTED BY THIS GRANT

1. Y. N. Srivastava (Co-Principal Investigator)
2. M. T. Vaughn (Co-Principal Investigator)
3. Orlando Panella (Graduate Research Assistant, Fall 1988-Fall 1991)
4. Vladan Lucic (Graduate Research Assistant, Winter 1992-)

END

**DATE
FILMED**

4 / 22 / 92