Effect of Alpha Drift and Instabilities on Tokamak Plasma Edge Conditions

George H. Miley and Chan K. Choi
Fusion Studies Laboratory
University of Illinois
214 Nuclear Engineering Laboratory
103 S. Goodwin Avenue
Urbana, Illinois 61801

CONF - 830795 - J

CONF-830795--2

DE83 017502

Abstract

As suprathermal fusion products slow down in a Tokamak, their average drift is inward. The effect of this drift on the alpha heating and thermalization profiles is examined. In smaller TFTR-type devices, heating in the outer region can be cut in half. Also, the fusion-product energy-distribution near the plasma edge has a positive slope with increasing energy, representing a possible driving mechanism for micro-instabilities. Another instability that can seriously affect outer plasma conditions and shear Alfvén transport of alphas is also considered.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

MASTER

EAB

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

I. Introduction

Alpha particle heating and "ash" buildup represent important factors in determining outer region conditions in reactor grade plasmas. As a result, limiters and/or divertor requirements as well as plasma-wall interactions are strongly influenced by alpha effects in this region. Several effects specific to tokamak plasmas must be considered. First, during slowing down, alpha particle orbits generally undergo an inward drift, the orbit asymptotically approaching the outer 10% of the plasma radius is depleted of "low" energy (thermal $< E < 3$ MeV) alphas. This reduces outer region heating. Also, "ash" enters the region predominantly by diffusion flow as opposed to direct thermalization. Control of outer region conditions must account for these effects. Potential alpha instabilities must also be considered. Theoretically the inverted energy distribution created by the alpha drift can induce microinstabilities, increasing particle and energy transport in the outer region. This effect can be controlled by developing profiles that maintain alpha densities below (or above) the instability threshold. Alternately, fast startup techniques might be used to induce volumetric velocity-space alpha instabilities, averting drift effects altogether.

II. Alpha Slowing-Down Drift Theory

It is well-known that highly-energetic, charged fusion products (f_p 's) undergo large radial displacements relative to the magnetic flux surface of their birthpoint. For short times the ion energy can be assumed constant, so that the particle trajectories are well-defined. However, for longer times, the magnetic moment and the canonical angular momentum, both taken to be invariant in the constant energy case, undergo sizeable changes during slowing down.

As illustrated in Fig. 1, the interaction between the particle's energy loss and the magnetic configuration of the torus cause drifting of the suprathermal's orbit [1,2]. The solid line represents the initial orbit projected on the $r - \theta$ plane. In Case I, the particle is assumed to lose its energy only in passing through the region designated $P_{\max}$; the orbit evolves as shown with one of the orbital segments anchored at $P_{\max}$. If, on the other hand, the particle's energy were to be lost in the $P_{\min}$ region along the initial orbit, as shown in Case II, the trajectory would develop with a portion of it secured to $P_{\min}$. Finally, in Case III, the suprathermal energy is assumed to be lost uniformly along the orbit; the result is intermediate between the extremes of Cases I and II. In Ref. 1 it is shown that the actual slowing-down trajectories can be accurately approximated by a weighted superposition of the uniform (Case III) and the extrema loss model (Cases I and II), and this approximation is employed in the present study.

Implicit in this analysis is the assumption of a low- β , circular cross-section plasma with a uniform toroidal field (i.e., no magnetic ripple). Although velocity-space scattering occurs during slowing down, its effect on shifting the orbit is relatively small [1].

A schematic diagram for the calculation is shown in Fig. 2. The number of fusion particles born per second in the plasma is divided into small phase-space groups, each of these groups being characterized by initial phase-space coordinates. These coordinates completely define the initial trajectory for all the fp 's produced within that cell (or group). During subsequent energy loss, the fusion particles comprising each of these initial phase-space groups are assumed to move as a unit along prescribed slowing-down trajectories. The code that implements

this Monte Carlo type treatment has been labeled SYMALF, and this designation will be used in later sections to identify results using it.

For tabulation of results, the background plasma is also divided into 50 concentric rings. Then, for a typical Tokamak the width of each ring is of the same order as the fp gyroradius and this is found to be near the optimum for present purposes. The time that a given particle group spends in crossing each ring along its orbit is calculated by knowing its radial drift and the ring thickness. With the energy loss rate for each ring determined from the background temperature and density profiles (as well as the particles' energy), the energy deposited in each of the rings traversed by that group is evaluated using a Sivukhin-type formulation [3].

Five energy segments divide up the slowing down process for the particle groups between initial and "thermalization" energies. Each of these intervals (say, the k^{th}) is represented by the time-averaged energy, $\bar{E}^{(k)}$. For D-T produced alphas, the slowing down interval ranges from 3500 keV to 500 keV. Since five energy intervals were used in the present calculations, the width, ΔE , of each interval is 600 keV---with $\bar{E}^{(1)} \approx 3200$ keV, $\bar{E}^{(2)} \approx 2600$ keV, etc.). Taking smaller energy intervals does not appear to appreciably alter the results.

After a particle has lost 600 keV along its initial orbit, it drops into the next lower energy interval, a new orbit is determined, and the group energy is apportioned along that new orbit. This process continues down to about 500 keV (when pitch-angle scattering becomes important) and is repeated until all the particle groups have been dealt with. Each group's contribution in the calculation of a particular fusion product

property is totaled and the results (which are, in effect, poloidally-averaged) are printed out.

III. Profile Results

Figure 3 compares an *in situ* deposition profile with SYMALF and a single-orbit approximation (SOA) for a TFTR-like device with parameters outlined in Table I.* (The SOA curve assumes the initial 3.5-MeV orbit is unchanged during the slowing down time.) Most striking is the large difference between the profile calculated under an *in situ* approximation and the profiles including orbit effects. The reasons are two-fold. The decreased amplitudes of the latter curves are due to alpha-particle losses from the plasma, not included in the *in situ* case. A broadening of the profile is also obvious, which is caused by the large-orbit excursions that the alpha particles undergo. The distinction between SYMALF and SOA is more subtle. SYMALF indicates that $\sim 10\%$ more energy is deposited in the central plasma region; this is attributed to the net inward drift of the orbit as the alpha particles lose energy.

With all the above, it was assumed that density and temperature profiles were more or less "standard" (i.e., monotonically decreasing to the plasma boundary). A neutral gas blanket, however, may cause an

*To illustrate these effects, two tokamak design concepts are considered here: the "small" TFTR and the "large" UWMak-I. The ρ_0/a for UWMak-I is $\sim 1/20$ that of TFTR, implying much better f_p confinement. The functional forms for background temperature, density, and current density assumed for each is indicated in Table I. In the majority of cases, the profiles are parabolic.

In addition, an extension of the TFTR using a neutral-gas blanket design is considered, designated TFTR-(GB). It employs neutral D-T fuel as a blanket around the plasma in order to provide fresh fuel and protect the plasma from impurities originating at the first wall. As a result, the plasma density itself increases with radial distance from the central region, then rapidly falls off at the boundary.

TABLE 1. TOKAMAK PARAMETERS

Device	TFTR	TFTR-(GB)	UWMAK-1
$R_0(m)$	2.5	2.5	13.0
$a(m)$	0.5	0.5	5.0
$I_p(MA)$	1	1	21
$B_0(KG)$	45	45	38.2
$T_i(keV)$	$10(1-0.8\hat{r}^2)$	$10 \exp\left(\frac{\hat{r}^2}{0.214}\right)$ for $\hat{r} \leq 0.7$ ----- 1 for $\hat{r} \geq 0.7$	$11.1(1-\hat{r}^3)$
$n_i(\frac{1}{m^3} \times 10^{19})$	$8(1-0.8\hat{r}^2)$	$8(1-\exp(\frac{\hat{r}-0.7^2}{0.4}))$ for $\hat{r} \leq 0.7$ ----- $16(1-\frac{\hat{r}-0.7^2}{0.09})$ for $\hat{r} > 0.7$	$8(1-\hat{r}^3)$
Current density profile	$(1-\hat{r}^2)$	$(1-\hat{r}^2)$	$(1-\hat{r}^2)$
Driving	Beam	Beam	Thermonuclear

In Table II each of the three models (i.e., "in situ," SOA, and SYMALF) has been used to calculate the average power deposition $\bar{P}$ in the outer 1/6 of the plasma for the various reactor cases.

The differences can become quite sizeable. Thus, Table II shows, $\bar{P}_{\text{SYMALF}}$ differs from $\bar{P}_{\text{SOA}}$ in this region by about a factor of two for TFTR and TFTR-(GB). The energy-loss drift has, in effect, depleted the outer region of half the heating power predicted by SOA (and the "in situ" model). In general, for the TFTR case we find that the drift effects included in SYMALF result in a reduced heating power in the outer region compared to predictions using the approximate SOA or *in-situ* models.

Interestingly enough, as the table shows, $\bar{P}_{\text{in situ}} \approx \bar{P}_{\text{SOA}}$ for TFTR and TFTR-(GB). This result occurs due to two cancelling effects. Alpha particles which are born in outer region, but which have orbits lying outside the plasma, are quickly lost, and, therefore, do not contribute to the heating of this region. However, the resulting loss of heating is roughly compensated by the heating power derived from alphas born in the inner 5/6 of the plasma but having parts of their orbits lying in the outer region.

For the reactors with larger plasma currents (UWMAK-I is used to illustrate this case) the ratio of $\bar{P}_{\text{in situ}}/\bar{P}_{\text{SYMALF}}$ for the outer region is generally less than unity, indicating that in this case the approximate methods underestimate the heating. This rather surprising result occurs because, for the larger device, fast particle losses in the outer region is much less serious than for the lower plasma current designs. Concurrently, the heating from orbits originated in the inner region, noted previously, still occurs. In fact, this latter effect more than compensates for particle losses from the region.

Table II: Average Power Deposition Density for the Outer One-Sixth of the Plasma

Device	$\bar{P}_{\text{in situ}}$ (watts/m ³)	$\bar{P}_{\text{SYMALF}}$ (watts/m ³)	$\bar{P}_{\text{SOA}}$ (watts/m ³)	$\frac{\bar{P}_{\text{in situ}}}{\bar{P}_{\text{SYMALF}}}$	$\frac{\bar{P}_{\text{in situ}}}{\bar{P}_{\text{SOA}}}$
TRTR-I	1.04×10^5	0.493×10^5	0.962×10^5	2.1	1.1
TRTR-(GB)	0.47×10^5	0.24×10^5	0.50×10^5	2.0	0.94
UWMAK-I	1.22×10^2	2.02×10^2	2.02×10^2	0.60	0.60

Since the heating of the plasma in the outer region also depends on the thermal energy transport from the hotter core region, changes in edge temperature will vary with the alpha power deposition but will not be directly proportional to it. Further, small changes in temperature at the outer edge of the plasma can be quite crucial. For example, the efficiencies of the neutral beam and pellet injection are sensitive to outer region temperatures, especially with respect to charge-exchange reactions. Wall erosion via sputtering by charge-exchange neutrals is also strongly dependent on the edge temperature.

In Fig. 5 we once again compare these three models, this time for the alpha thermalization source distribution. (Since the SYMALF model is generally valid only for energies in excess of 500 keV, the curves shown correspond to a slowing down distribution for 500-keV alpha particles. This we have somewhat liberally defined as the thermalization source.) As above, the contrast between *in situ* and the others is sharp. However, the disparity between SOA and SYMALF profiles is also marked. The more flattened SYMALF curve arises out of the fact that the minimum point of an orbit can drift either toward or away from the plasma center. For example, if $P_{\min}$ were initially located at $r = 0$, $P_{\min}$ would actually recede from the plasma center as slowing down occurred. Since the thermalization function depends only on the orbit at $E = 500$ keV, not on where it has been previously (as does the energy deposition calculation), the thermalization function can indeed be flatter for SYMALF than SOA in the vicinity of the origin. As we mentioned in the introductory comments, this effect can have implications on the buildup of alpha-particle "ash" in the plasma. Modest differences in the shape

of the thermalization source profile can dramatically change the thermal alpha-particle diffusion currents, sometimes even reversing directions.⁴

IV. Energy Distribution Functions

The shape of the fp energy-distribution function is of interest for predicting the possible occurrence of microinstabilities. For example, if the slope of the velocity distribution is positive with respect to energy, free energy is available which can drive microinstabilities.

Thus, in this study, we are especially interested in the effect of drifts on the energy distribution. For reference, we first note an approximate treatment that neglects drift effects. By assuming that the spatial and velocity components are separable, that the energetic ions slow down in place (cf "in situ"), and that $V_i \ll V \ll V_e$, we write the (unnormalized) speed distribution function as⁵:

$$f_{\alpha}(\hat{r}, V) = \frac{S(\hat{r})}{4\pi v_{\alpha e}} \left(\frac{V_{\alpha 0}^3 + V_c^3}{V^3 + V_c^3} \right); \quad \begin{cases} V_c < V < V_{\alpha 0} \\ V_i \ll V \ll V_e \end{cases} \quad (1)$$

Here $V_{\alpha 0}$ is the initial alpha speed, V_c is the speed at which its energy loss rate to background ions and to electrons are equal, V_i is the ion thermal speed, V_e is the electron thermal speed, $S(\hat{r})$ is the source density of alpha particles generated at position $\hat{r}$, and $v_{\alpha e}$ is the alpha particle-electron collision rate. For later comparisons with SYMALF, this result is rewritten in terms of the suprathermal alpha density, $n(\hat{r}, k)$, for particles in the k^{th} slowing-down energy-group:

$$n_{\alpha}(r, k) = \frac{S(\hat{r})}{3v_{\alpha e}} \ln \left(\frac{E_u(k)^{3/2} + E_c^{3/2}}{E_L(k)^{3/2} + E_c^{3/2}} \right); \quad E_c < E_L(k) \leq E_{\alpha 0} \quad (2)$$

Here $E_l^{(k)}$ and $E_u^{(k)}$ are the upper and lower energy bounds of the k^{th} energy group, $E_c \equiv \frac{1}{2} M_\alpha V_c^2$, $E_{\alpha 0} = E_{\alpha 0} = \frac{1}{2} M_\alpha V_{\alpha 0}^2$, and M_α is the alpha particle mass.

For comparison, the SYMALF code was used to calculate the alpha energy distribution as a function of radial position. These results then include the change in shape and drift of the orbits as the alphas slow down. Since the time required by a particle in a given phase-space group to traverse a ring is known, the density of suprathermals in a given plasma ring can be calculated. Thus, for any radial position in the plasma, the number density of suprathermal alphas at $\hat{r}$ and in energy interval k can be found. Then, with $n(\hat{r}, k)$ known, the normalized suprathermal energy distribution function is calculated as follows:

$$\hat{f}(\hat{r}, \bar{E}^{(k)}) = \frac{n(\hat{r}, k)}{\sum_{k=1} n(\hat{r}, k)} \quad (3)$$

where $\bar{E}^{(k)}$ is the average energy ($\approx$ midpoint energy) of the k^{th} energy-group.

In this evaluation, each of the five energy intervals was held constant. Regirously speaking, a histogram can be plotted in this manner. We have assumed, however, smooth variations and have superimposed approximate $n(\hat{r}, k)$ curves such that the area under each curve is equal to the corresponding area under the histogram. The center point energy $\bar{E}^{(k)}$ for each group is plotted on the axis while the energy interval involved is shown in brackets. Since, to first order in $\Delta E / \bar{E}^{(k)}$, $f(\hat{r}, \bar{E}_k)$ is proportional to $n(\hat{r}, k)$, the particle density can be obtained from the graphs via Eq. (3).

Comparisons are made for TFTR in Fig. 6 for several radial positions. Again, due to first bounce losses, the suprathermal density calculated via SYMALF is significantly less than that predicted by Eq. (1), here designated by the (dashed) "in situ" curves. In fact, using Eq. (2), we find that $f_{\text{SYM}}(\hat{r}, \bar{E}^{(k)}) \approx 0.73 f_{\text{IS}}(\hat{r}, \bar{E}^{(k)})$ for $\hat{r} \lesssim 0.7$, where f_{SYM} and f_{IS} are the SYMALF and "in situ" distribution functions, respectively.

The most interesting effects occur in the outer plasma layer, for $\hat{r} \gtrsim 0.7$. There the shape of the suprathermal density profile begins to change with energy, first flattening out, and finally decreasing with decreasing energy (e.g., $\hat{r} = 0.96$). As co-circulating (direction relative to the plasma current) alphas with portions of their initial orbits lying in the outer regions of the plasma slow down, they tend to drift farther to the inside of the plasma. No co-circulating alphas can drift into this regions since it lies at the edge of the plasma. Counter-circulating alphas tend to drift outward, and could add to the density of the edge region. However, they are generally lost from the plasma prior to complete thermalization so that the net effect is to deplete lower energy groups at the plasma edge. Neither "in situ" nor SOA can be used to predict such drifts, i.e., a full SYMALF treatment is necessary for such studies.

Since the drift motion during slowing-down is reduced in the higher plasma current Tokamaks, the results for UWMAK-I (Fig. 7) show less dramatic effects. Although the general shapes of the various curves for these cases are similar, there remains a significant difference in the absolute magnitude of the suprathermal alpha density in the outer regions. The reason for this is essentially the same as that cited earlier to explain the enhanced alpha particle heating for UWMAK-I near the plasma

boundary (i.e., decreased particle losses in the outer region due to the increased toroidal current combined with the additional contributions from the inner region).

The TFTR gas blanket case (Fig. 8) shows a trend similar to that for TFTR, in that the superthermal density curve again ultimately changes from a negative to a positive slope as $\hat{r}$ increases. In addition, a new phenomenon arises. While, as in previous cases, $n_{\text{SYM}} \approx \frac{1}{2}n_{\text{IS}}$ near the centerline, the situation is reversed (e.g., $n_{\text{SYM}} \approx 2n_{\text{IS}}$ at $\hat{r} = 0.64$). Eventually n_{IS} once again becomes much greater than n_{SYM} at $r \sim 0.96$. There are two effects involved in this phenomenon. The slowing-down drift again accounts for the positive slopes in the suprathermal densities near the boundary. However, due to the peculiar background density and temperature profiles created by the gas blanket, the energy loss rate peaks around $\hat{r} \approx 0.7$ so that particles confined in this region slow-down relatively quickly. Consequently, alphas born in the surrounding plasma rings, but which spend a part of their time in the high loss-rate region, raise the SYMALF density over that of "in situ."

V. Shear Alfvén Transport

The preceding calculations assume that the fps slow down in a classical fashion, i.e., do not undergo instabilities. We have considered several instability effects that could lead to anomalous slowing down. Potential candidates include the coupling of the alphas to the fast magnetosonic (FMS) modes, and also to the shear Alfvén modes. FMS effects can strongly influence heating profiles, but these effects are not likely to occur unless special techniques are employed to force the alpha density to

exceed threshold requirements.⁶ Thus, in the following we briefly consider the possibility of shear Alfvén transport which does not lend itself to easy control.

V.1 Transport Model

Shear Alfvén transport of fast fusion products has received attention both as an energy loss mechanism and for ash removal.[7-10] We have developed a conceptual model of 1-D radial, fusion product transport due to a shear Alfvén mode. To study the energy loss from the plasma as a function of radius r , widely spaced Alfvén surfaces should result in a driven $\vec{E}_\theta \times \vec{B}_0$ transport mechanism for $\omega = k_\parallel v_A \ll \Omega_i$. However, for overlapping eigen modes [10] the transport mechanism may be diffusion-like.

In our model, the perpendicular diffusion coefficient $D_\perp \approx \frac{(\Delta r)^2}{\Delta \tau} \approx |\langle v_E \rangle|^2 \Delta \tau$, where the velocity $\langle v_E \rangle$ results from the local effective $\vec{E}$ -field fluctuations. An effective radial transport velocity $\langle v \rangle$ is then defined as $\langle \vec{v} \rangle = -D_\perp \nabla \ln(n_\alpha)$.

We begin by assuming that a shear Alfvén mode $\omega = k_\parallel v_A$ exists in the plasma; this mode re-occurs periodically in time; it includes overlapping of Alfvén surfaces of low mode number; this results in diffusion of resonant alphas due to $\vec{E}_\theta \times \vec{B}_0$. Fusion alphas resonant with $\omega = k_\parallel v_A$ result in spatial transport due to $\vec{E}_\theta \times \vec{B}_0$ drifts with a net flux:

$$\vec{\Gamma} = n_{\alpha r} \langle \vec{v} \rangle = -D_\perp \nabla n_{\alpha r}, \quad (4)$$

where

$n_{\alpha r} = f_{\alpha r} n_\alpha$ = number density of resonant alphas,

$f_{\alpha r}$ = fraction of fusion alphas lying in the velocity space resonance region,

$D_{\perp}$ = perpendicular (radial) diffusion coefficient,

$\langle \vec{v} \rangle$ = the effective radial transport velocity $\langle \vec{v} \rangle$.

We estimate $D_{\perp}$ from the classical expression, $D_{\perp} \approx (\Delta r)^2 / \Delta \tau$, and assumed that $\Delta r \approx \langle v_E \rangle \Delta \tau$ where $\langle v_E \rangle$ is the local drift velocity due to $\vec{E}_\theta \times \vec{B}_0$ fluctuations. If the correlation time $\Delta \tau$ is taken $\approx \langle \omega^{-1} \rangle$ consistent with Ref. 10, we have:

$$D_{\perp} \approx \frac{c^2 E_{1 \text{ eff}}^2}{\langle \omega \rangle B_0^2}, \quad (5)$$

where $E_{1 \text{ eff}}$ is the effective E-field level of the fluctuating fields.

For MHD modes and for a given E-field level, the wave energy density w_E is given by:

$$w_E = \epsilon \frac{E^2}{8\pi}, \quad (6)$$

where ϵ is the plasma dielectric constant. Let the quasi-linear velocity space relaxation of fusion alphas feed energy to the E_1 fields and to the wave MHD motion. Further, let there be n such overlapping modes with individual wave energy densities w_{En} so that $w_E = \sum_n w_{En}$. Then for n sinusoidal modes, the net root-mean-square average electric field becomes:

$$|E_{1 \text{ eff}}|^2 \approx \sum_n |E_{1n}|_{\text{RMS}}^2 \approx \frac{4\pi w_E}{\epsilon} \quad (7)$$

which yields a diffusion coefficient:

$$D_{\perp} \approx \frac{c^2}{\langle \omega \rangle B_0^2} \left[\frac{4\pi w_E}{\epsilon} \right] \epsilon \rightarrow c^2 / v_A^2 = \frac{w_E}{\langle \omega \rangle \rho} \quad (8)$$

where ρ is the plasma mass density: $\rho \approx n_0 \langle m_i \rangle$. Then the effective

transport velocity, $\langle \vec{v} \rangle \approx -D_{\perp} \nabla \ln(n_{\alpha R})$, is found to be:

$$\langle \vec{v} \rangle \approx - \frac{4 C^2 w_E}{\langle \omega \rangle B_0^2 \epsilon} \nabla \ln(f_{\alpha R} n_{\alpha}) \quad (9)$$

V.2 An Example

We take reactor grade plasma parameters listed in Table III and use results from our earlier work [6] to estimate magnitudes for diffusion coefficient, $D_{\perp}$ and the effective transport velocity $\langle \vec{v} \rangle$ from Eqs. (8) and (9). Also, assume that, consistent with Ref. 10,

$$q(r) = 1 + (q_s - 1)(r/a)^2 \quad (10a)$$

$$k_{\parallel} = \frac{m-2}{q} \frac{q}{R_0} \text{ with } m = 3, l = 4 \quad (10b)$$

Then, on axis we have: $\langle \omega \rangle \approx 2 \times 10^6$ rad/sec and $V_A \approx 9.76 \times 10^8$ cm/sec.

From Ref. 6 the maximum energy released by a resonant alpha particle

$\delta \langle E_{\alpha r} \rangle \approx 2/5 \langle E_{\alpha r} \rangle_{\text{initial}}$, and for the above parameters $\langle E_{\alpha r} \rangle_{\text{initial}} \approx 2.67$ MeV.

With these values the maximum diffusion coefficient becomes:

$$D_{\perp} |_{\text{max}} \approx \frac{w_E |_{\text{max}}}{\langle \omega \rangle \rho} = \frac{\frac{2}{5} f_{\alpha r} n_T \langle E_{\alpha r} \rangle_{\text{initial}}}{\langle \omega \rangle \langle m_i \rangle} \quad (11)$$

where n_T = alpha threshold fraction. For the present case, this corresponds to $D_{\perp} |_{\text{max}} \approx 2.0 \times 10^5$ cm²/sec which is two orders of magnitude above the

minimum required for overlapping of Alfvén surfaces. [10] With the assumed parabolic density and temperature profiles indicated at $r/a = 0.01$, where

the gradients are relatively small, we find that velocity (for $n_T \sim 5 \times 10^{-5}$):

$|\langle \vec{v} \rangle| \approx 380$ cm/sec where it has been assumed that $n_T \sim 5 \times 10^{-5}$ and $\delta \langle E_{\alpha r} \rangle$

$\sim 40\%$ of its initial value. Even if $\delta \langle E_{\alpha r} \rangle \approx 4\%$ of $\langle E_{\alpha r} \rangle_{\text{initial}}$, we find:

$$|\langle \vec{v} \rangle| \approx 38 \text{ cm/sec} .$$

Thus, even with only one-tenth of the alpha energy which can be released in the quasi-linear relaxation, the diffusion coefficients and effective transport velocities are sufficiently large to ensure complete flattening of the resonant alpha number density profile.

This conclusion must be viewed as approximate without a more detailed treatment. We have not, for example, shown that the alpha threshold fraction is satisfied at every position. Nonetheless, the model proposed does scope alpha energy loss from the central plasma region.

Taking the large diffusion coefficients and transport velocities indicated above as justification, we assume complete flattening of the resonant alpha particle number density over that region of the tokamak for which $0 < f_{\alpha r}$. In the above example, this region is: $0 \leq r \leq 60 \text{ cm}$ (see Fig. 9).

The alphas undergoing shear Alfvén transport (i.e. the resonant fraction $f_{\alpha R}$) rapidly carry their energy from the central core region to the outer plasma volume where, in the present model, they suddenly fall out of resonance. This effect is illustrated in Fig. 10. Here the ratio of the alpha heating rate including shear Alfvén transport to the normal heating rate is plotted as a function of radius for the example case of Table III. (Note that these calculations are not entirely self-consistent in that the background plasma profiles were not adjusted for changes in heating rates. Still, the basic trends remain valid.) The sharp spike in the heating rate ratio occurs at the transition from resonant to non-resonant transport. A larger fraction of resonant alphas occur at the lower field (42 vs 50 kG), causing a larger transport of energy from the core region and a higher spike at the interface.

In summary, an approximate model describing the effect of shear Alfvén has been proposed. It indicates that for cases where such coupling occurs, the resonant alphas (often corresponding to fractions as high as 20%) are rapidly transported out of the central core of the tokamak plasma. This results in an enhanced heating rate (and ash flow) near the outer region of the plasma while flattening central core profiles.

VI. Summary

The calculations of gross heating and alpha thermalization profiles can be reasonably carried out over a range of device sizes using approximate models such as SOA or "in-situ." Indeed, such approximations have normally been used in reactor calculations. On the other hand, none of

Table III. Plasma Parameters for Shear Alfvén Calculations

$n_0 \approx 5 \times 10^{13} \text{ cm}^{-3}$	parabolic profile
$T_{i0} \approx 15 \text{ keV}$	parabolic profile
$T_{e0} \approx 15 \text{ keV}$	parabolic profile
$B_0 \approx 50 \text{ kG}$	(independent of r)
$q_s \approx 2.5 \text{ on-axis}$	
$R_0 \approx 500 \text{ cm}$ (major radius)	
$a \approx 100 \text{ cm}$ (minor radius)	

the approximate models provide an accurate prediction of heating near the outer edge of the plasma where fusion product drift effects become important. Then, only a detailed treatment like SYMALF is adequate. While this effect does not appear to be important for overall energy balances, the details of plasma-wall interaction, which can be vital to successful operation, will be influenced through corresponding changes in the plasma edge temperature.

If, as in the gas blanket version of TFTR, unusual background density-temperature are created, the simple models may fail over a larger volume of the plasma. Consequently, in such cases, use of a full SYMALF-type treatment may be prudent in all heating calculations.

These calculations all assume classical slowing down of the alphas. However, under special conditions, instabilities may occur which can cause important changes. Two important cases have been considered here.

First, it has been shown that orbit drifting causes some potentially important changes in suprathermal density distribution function as a function of radial position. SYMALF predicts a change-over from negative to positive slope in the distribution functions with respect to energy near the plasma boundary. The positive energy slope of the distribution function for devices like TFTR suggest that a turbulence could be created in the outer region due to kinetic instabilities.

The second example of an instability, coupling to shear Alfvén modes, has been shown to potentially occur over the central portion of tokamak plasmas. However, this instability can also seriously affect the outer plasma region by "dumping" energy and ash at the inner-core outer-region interface.

References

1. T. W. Petrie and G. H. Miley, Proceedings of the ANS Topical Meeting, Tucson (March, 1977), and Nucl. Sci. and Eng., 64 (1977) 151-162.
2. T. W. Petrie and G. H. Miley, IEEE Transactions on Plasma Sciences, PS-8 (1980).
3. D. V. Sivukhim, Reviews of Plasma Physics, 4, Consultants Bureau, New York (1966).
4. W. Downum, T. W. Petrie, C. Choi, and G. H. Miley, Topl. Conf. Controlled Nuclear Fusion, Knoxville, Tennessee (Feb. 1977).
5. A. Neocentini, M. Tessarotto, F. Engelmann, Nucl. Fus., 15 (1975) 359.
6. W. R. Sutton, C. Choi, and G. H. Miley, Fourth Topical Meeting on the Technology of Controlled Nuclear Fusion, King of Prussia, PA (Oct. 1980).
7. V. S. Delikov, et al., Soviet Physics JETP, 39 (1974) 828.
8. D. J. Sigmar, H. C. Chan, Nuclear Fusion, 18 (1978) 1569.
9. D. K. Bahdra, IEEE Transactions on Plasma Science, PS-8 (1980) 2.
10. K. T. Tsang, et al., ORNL/TM-7508 (Dec. 1980).

Fig. 1. Various slowing-down schemes resulting from an initial orbit (solid line) passing through P_{MAX} and P_{MIN} are shown. Case I (alternating dashed curve) assumes particle energy loss to occur only in the immediate vicinity of P_{MAX} . Case II (dotted curve) assumes the energy is lost only in the immediate vicinity of P_{MIN} . Case III (dashed line) assumes approximate uniform energy loss along the orbit.

Fig. 2. Flow chart of SYMALF. Energy loss rates per ring are determined beforehand for each E_k , which represents the characteristic energy at each slowing down energy level. The number of particles for a given phase-space group (characterized by r_0 , θ_0 , ψ_0) are calculated. At A, the orbit is determined by the method outlined in the text; at B, the energy lost in each ring crossed by the orbit is found; at C, this information is stored. If $E_k > 500 \text{ keV}$, the sequence is repeated as shown; if not, a new phase-space group is chosen. This process is continued until all radial, azimuthal, and velocity space possibilities are exhausted.

Fig. 3. The relative energy deposition densities (normalized to the $r = 0$ in situ value) as a function of r are compared for the three models.

Fig. 4. Alpha power deposition density is plotted versus plasma positive $\hat{r}$ for the TFTR neutral gas blanket design. (See Table I.)

Fig. 5. The relative thermalization source functions (normalized to the $r = 0$ in situ value) as a function of r are compared for the three models.

Fig. 6. Suprathermal alpha density is plotted versus energy for various values of $\hat{r}$ in TFTR.

Fig. 7. Suprathermal alpha density is plotted versus energy for various values of $\hat{r}$ in UWMAK-I.

Fig. 8. Suprathermal alpha density is plotted versus energy for various values of $\hat{r}$ in the TFTR gas blanket.

Fig. 9. Average radial frequency $\langle \omega \rangle$ for $\ell = 3, m = 4, q_s = 2.5$ and resonant alpha number fraction $f_{\alpha R}$. Reference 6 contains expressions for $f_{\alpha R}$ curves based upon the same plasma parameters as used in the text.

Fig. 10. Effective alpha heating rate normalized (at each radial position r) to be expected in the absence of shear Alfvén transport.