CONF-910847--1

ATTITUDE CONTROL OF A SPINNING ROCKET
VIA THRUST VECTORING

SAND--90-3230C

DE91 006267

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Two controllers are developed to provide attitude control of a spinning rocket that has a thrust vectoring capability. The first controller has a single-input/single-output design that ignores the gyroscopic coupling between the control channels. The second controller has a multi-input/multi-output structure that is specifically intended to account for the gyroscopic coupling effects. A performance comparison between the two approaches is conducted for a range of roll rates. Each controller is tested for the ability to track step commands, and for the amount of coupling impurity. Both controllers are developed via a linear-quadratic-regulator synthesis procedure, which is motivated by the multi-input/multi-output nature of second controller. Time responses and a singular value analysis are used to evaluate controller performance.

I. Introduction

This paper describes the development and comparison of two controllers that are designed to provide attitude control of a spinning rocket that is equipped with thrust vector control. Both controllers are evaluated as to their ability to track step commands, and for the ability to provide adequate decoupling between the pitch and yaw control channels. The dynamic coupling between the two channels is due to gyroscopic effects when the rocket has a non-zero roll rate. This problem could be of interest for several reasons. For a single nozzle rocket design there may be no direct control of roll attitude and some roll rate may be introduced by disturbances. In the case where roll

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control is available, a failure in the roll control channel might introduce a significant roll rate. Finally, it might be possible to lower the cost and weight of delivering a spin-stabilized payload by spinning-up the booster prior to motor burn-out and payload release.

The first controller is basically a single-input/single-output (SISO) design that ignores the gyroscopic coupling effects and attempts to provide adequate control as if the channels are dynamically uncoupled. The second controller attempts to account for the gyroscopic coupling effects via a multi-input/multi-output (MIMO) design approach. A linear-quadratic-regulator (LQR) control system synthesis procedure is used to design both controllers. This design technique is motivated by the MIMO nature of the second controller, since LQR is a matrix procedure that inherently produces the proper loop inter-connections. Both controllers are shown to be capable of providing steady state tracking (for no CG offsets), although the decoupling characteristics of the MIMO controller are significantly better than those of the SISO design even for relatively modest roll rates. A gain adjustment technique for the MIMO controller is shown to approximately maintain the closed-loop transfer function of a baseline design as a function of actual roll rate, and reduces the MIMO controller to that of the SISO controller as roll rate approaches zero. The stability margins associated with the SISO controller are shown to be decreased relative to the MIMO controller for non-zero roll rates.

The organization of this paper is as follows. Section II derives the nonlinear dynamical equations of motion, which are then simplified to produce the linear dynamics that are the basis for the control system design. Section III develops the state spaces that determine the controller structures, and discusses the LQR synthesis procedure. Section IV contains a brief description of a singular value analysis that is used to evaluate controller stability margins and robustness to parameter uncertainty. Section V discusses the performance of the controllers as determined from a simulation study. Conclusions are presented in Section VI.

II. System Dynamics

A. Nonlinear Dynamics

This section derives the nonlinear equations of motion for a spinning spacecraft system. The vehicle and its body fixed reference frame are shown in Fig. 1, and assumed physical data are given in Table 1. An inertial reference frame is defined to be coincident with the body frame for a zero roll angle. The inertial frame is preferred from a control design perspective [1], since controller commands are most naturally expressed in this frame. The equations of motion used by the simulation are written with respect to the body frame [2], however, since the control torques are

applied in this frame. The vehicle is assumed to be spinning about the body X-axis. Two perturbation masses with radial displacements y and z along the body axes Y and Z will be assumed in this analysis as a means for introducing radial center of mass offsets into the simulation. The center of mass of the system, including control masses, is located at point A for $y=z=0$. Point O is the new location of the center of mass when the control masses are located to a position off of the vehicle centerline. The body axes are located with respect to point A, and the vehicle is assumed to be nominally symmetric with mass moment of inertia of I_x about the X-axis and equal inertias of I about the Y and Z axes. The perturbation masses are assumed to be of equal mass, m .

The nonlinear equations of motion can be derived from

$$\vec{M}_A = \frac{d}{dt} (\vec{h}_A) + (\vec{R}_A \times m_T \vec{a}_A) \quad (1)$$

where the left side of the equation represents the sum of the moments about point A, and the right side is the time rate of change of angular momentum. The only external torques assumed to be acting on the system are due to the rocket thrust vector control moments, which are expressed as $\vec{M}_A = (T l \sin(\delta_y)) \vec{j} + (T l \sin(\delta_z)) \vec{k}$ where T is the thrust of the motor, l is the moment arm between the nozzle and the center of mass, and δ is the deflection of the nozzle. The location of the vehicle center of mass with respect to point A is given by

$$\vec{R}_A = (\mu y) \vec{j} + (\mu z) \vec{k} \quad (2)$$

where $\vec{i}$, $\vec{j}$, $\vec{k}$ are unit vectors along the X, Y, Z axes, and μ is the mass fraction m/m_T . The acceleration of point A is

$$\begin{aligned} \vec{a}_A &= -\frac{d^2}{dt^2} (\vec{R}_A) = -\mu [\vec{R}_A + (\vec{\omega} \times \vec{R}_A) + 2(\vec{\omega} \times \dot{\vec{R}}) + (\vec{\omega} \times (\vec{\omega} \times \vec{R}_A))] \\ &= -\mu [(z\dot{q}_B - y\dot{r}_B + yp_B q_B + zp_B r_B) \vec{i} \\ &\quad + (-zp_B - yp_B^2 + zq_B r_B - yr_B^2) \vec{j} \\ &\quad + (yp_B + yq_B r_B - zp_B^2 - zq_B^2) \vec{k}] \end{aligned} \quad (3)$$

where the angular velocity with respect to the rotating body frame is $\vec{\omega} = p_B \vec{i} + q_B \vec{j} + r_B \vec{k}$, and $\dot{y} = \dot{z} = 0$. Finally

$$\begin{aligned} \vec{R}_A \times m_T \vec{a}_A &= -\mu m [(y^2 \dot{p}_B + y^2 q_B r_B - yz(\dot{p}_B^2 + \dot{q}_B^2) \\ &\quad + z^2 \dot{p}_B + yz p_B^2 - z^2 q_B r_B + yz r_B^2) \vec{i} \\ &\quad + (z^2 \dot{q}_B - yz \dot{r}_B + yz p_B q_B + z^2 p_B r_B) \vec{j} \\ &\quad - (yz \dot{q}_B - y^2 \dot{r}_B + y^2 p_B q_B + yz p_B r_B) \vec{k}] \end{aligned} \quad (4)$$

The angular momentum of the vehicle with respect to A is

$$\vec{h}_A = [\vec{i} \vec{j} \vec{k}] I_A \vec{\omega} - m (\vec{r}_m \times \vec{v}_m) \quad (5)$$

where $\vec{r}_m = x\vec{i} + y\vec{j} + z\vec{k}$ and $\vec{v}_m = \dot{y}\vec{j} + \dot{z}\vec{k} = 0$, and

$$I_A = \begin{bmatrix} I_{xx} & -mxy & -mxz \\ -mxy & I + mz^2 & 0 \\ -mxz & 0 & I + my^2 \end{bmatrix} \quad (6)$$

Hence

$$\begin{aligned} \vec{h}_A &= [(I_{xx} + my^2 + mz^2)p_B - (mxy)q_B - (mxz)r_B] \vec{i} \\ &\quad + [(-mxy)p_B + (I + mz^2)q_B] \vec{j} \\ &\quad + [(-mxz)p_B + (I + my^2)r_B] \vec{k} \end{aligned} \quad (7)$$

The derivative of $\vec{h}_A$ is written as

$$\frac{d}{dt} (\vec{h}_A) = \dot{\vec{h}}_A + (\vec{\omega} \times \vec{h}_A) \quad (8)$$

where the two terms on the right hand side are given by

$$\begin{aligned} \dot{\vec{h}}_A = & [I_x \dot{p}_B + my^2 \dot{p}_B + mz^2 \dot{p}_B - mxy \dot{q}_B - mxz \dot{t}_B] \vec{i} \\ & + [I \dot{q}_B - mxy \dot{p}_B + 2mx^2 \dot{q}_B + mz^2 \dot{q}_B] \vec{j} \\ & + [I \dot{t}_B - mxz \dot{p}_B + 2mx^2 \dot{t}_B + my^2 \dot{r}_B] \vec{k} \end{aligned} \quad (9)$$

$$\begin{aligned} (\vec{\omega} \times \vec{h}_A) = & m [(y^2 - z^2) q_B r_B + xyp_B r_B - xzp_B q_B] \vec{i} \\ & + [(I_x - I)p_B r_B + m((z^2 - 2x^2)p_B r_B - xyq_B r_B - xz(r_B^2 + p_B^2))] \vec{j} \\ & + [(I - I_x)p_B q_B + m((2x^2 - y^2)p_B q_B - xy(p_B^2 - q_B^2) - xzq_B r_B)] \vec{k} \end{aligned} \quad (10)$$

Substitution of (9) and (10) into (4), and then (4) and (8) into (1) produces

$$\begin{aligned} \begin{bmatrix} C_p & -mxy & -mxz \\ -mxy & C_q & \mu myz \\ -mxz & \mu myz & C_r \end{bmatrix} \begin{bmatrix} \dot{p}_B \\ \dot{q}_B \\ \dot{t}_B \end{bmatrix} &= \begin{bmatrix} 0 & -myz & myz \\ mxz & 0 & mxz \\ mxy & -mxy & 0 \end{bmatrix} \begin{bmatrix} p_B^2 \\ q_B^2 \\ r_B^2 \end{bmatrix} \\ &+ \begin{bmatrix} mxz & -m(1-\mu)(y^2 - z^2) & -mxy \\ \mu myz & mxy & (I - I_x) \\ (I_x - I) & -mxz & -m(1-\mu)z^2 \\ +m(1-\mu)y^2 & & -\mu myz \end{bmatrix} \begin{bmatrix} p_B q_B \\ q_B r_B \\ r_B p_B \end{bmatrix} \end{aligned}$$

$$+ \begin{bmatrix} 0 & 0 \\ Tl_s & 0 \\ 0 & Tl_s \end{bmatrix} \begin{bmatrix} \sin(\delta_y) \\ \sin(\delta_z) \end{bmatrix} \quad (11)$$

where $C_p = I_{xx} + m(1-\mu)(y^2+z^2)$, $C_q = I + m(1-\mu)z^2$,
 $C_r = I + m(1-\mu)y^2$, and $l_s^2 = l^2 + (\mu y)^2 + (\mu z)^2$. Nonlinear
differential equations in p_B , q_B , and r_B are obtained by
inverting the matrix on the left hand side of (11) and
premultiplying the right hand side. This inverse is

$$\frac{1}{D} \begin{bmatrix} C_q C_r - \mu^2 m^2 y^2 z^2 & C_r m x y - \mu m^2 x y z^2 & -\mu m^2 x y^2 z + C_q m x z \\ C_r m x y - \mu m^2 x y z^2 & C_p C_r - m^2 x^2 z^2 & -C_p \mu m y z + m^2 x^2 y z \\ -\mu m^2 x y^2 z + C_q m x z & -C_p \mu m y z + m^2 x^2 y z & C_p C_q - m^2 x^2 y^2 \end{bmatrix} \quad (12)$$

where $D = C_p C_q C_r + m^3 \mu x^2 y^2 z^2 - C_p \mu^2 m^2 y^2 z^2 - C_q m^2 x^2 z^2$.

The equations of motion with respect to the inertial
frame are needed for the purposes of control system design.
A reasonable assumption is that $y=z=0$. The angular velocity
with respect to the inertial frame is $\vec{\omega}_I = (p_B+p)\vec{i} + q\vec{j} + r\vec{k}$.
Equation (1) can be expanded with respect to the inertial frame
to produce

$$\ddot{\vec{M}}_A = [\vec{i}_I \vec{j}_I \vec{k}_I] I_A \ddot{\vec{\omega}}_I + (\ddot{\vec{\omega}}_I \times [\vec{i}_I \vec{j}_I \vec{k}_I] I_A \vec{\omega}_I) \quad (13)$$

B. Linear Dynamics

The linear dynamical equations of motion for the spinning rocket with respect to a non-rotating (inertial) command frame (X_I, Y_I, Z_I) can be deduced from the nonlinear equations (13) from Section A. The simplifying assumptions are:

- (1) rocket body spin axis, X , coincides with X_I (i.e., $p=0$),
- (2) the body x -axis is an axis of symmetry,
- (3) the rocket has a spin rate of p_B relative to X_I ,
- (4) the inertial frame is not rotating, and
- (5) all external torques are applied by thrust vectoring.

The nonlinear equations of (13) can now be reduced to

$$\dot{p} = 0 \quad (14)$$

$$\dot{q} = - \left(\frac{I_x}{I} \right) p_B r + \left(\frac{T_l}{I} \right) \sin(\delta_y) \quad (15)$$

$$\dot{r} = + \left(\frac{I_x}{I} \right) p_B q + \left(\frac{T_l}{I} \right) \sin(\delta_z) \quad (16)$$

The right hand sides of (15) and (16) represent the gyroscopic coupling terms and the control moments applied by the thrust vector control system. Since the deflection angles are limited to a maximum of 3 degrees, (15) and (16) can further be reduced with the assumption that $\sin(\delta) \approx \delta$.

A linear, second-order actuator model can be defined as

$$\dot{\delta}_i = \int \ddot{\delta}_i dt \quad (17)$$

$$\ddot{\delta}_i = -H_2 \dot{\delta}_i - H_1 \delta_i + H_1 u_i \quad (18)$$

where the i subscript is replaced with y or z depending on which actuator is under consideration. The control variables become the actuator position inputs of u_i .

III. State Space Formulation And LQR Synthesis

A. State Spaces For LQR Controller Synthesis

A general characteristic of control systems is that of driving some steady-state tracking error to zero. It is also generally considered desirable that the zero steady state tracking error be insensitive to unmodelled system dynamics. The system tracking errors are, therefore, natural state variables for the LQR synthesis procedure [3]. The two tracking errors in the spinning rocket problem are

$$E_{\theta} = \theta_C - \theta \quad (19)$$

$$E_{\psi} = \psi_C - \psi \quad (20)$$

where the variables with C subscripts are assumed here to be constant step input commands, and the state variables (θ, ψ) are measured pitch and yaw angles. The differential equations for the tracking error states can be written as

$$\dot{E}_{\theta} = -q \quad (21)$$

$$\dot{E}_{\psi} = -r \quad (22)$$

The state space used with the LQR synthesis procedure is now determined separately for the SISO and MIMO controllers. First consider the SISO pitch angle control problem. The SISO yaw angle controller will have an identical compensator structure, and the same gains if the vehicle is assumed to be symmetrical. This controller is designed to track a step input pitch angle command, but must be able to accomodate the unmodeled effects neglected in the simplified linear model if this objective is to be successful. The vehicle is

assumed for design purposes to have no radial center-of-gravity (CG) offsets. The gyroscopic coupling terms are also neglected, so that (15) reduces to

$$\dot{q} = M_Y \delta_y \quad (23)$$

where M_Y can be defined from (15). A consistent state space for the SISO pitch angle controller can be obtained by combining (21) and (23) to produce

$$\dot{E}_\theta = -q \quad (24)$$

$$\dot{q} = M_Y \delta_y \quad (25)$$

In general the nozzle deflections will not be zero in steady state except for vehicles that are perfectly symmetric about the body X-axis. The above state space will produce a proportional-derivative (PD) compensator structure that will provide steady state tracking only for symmetrical vehicles. This formulation can be interpreted as having an unbounded LQR performance index, since the nozzle deflection state is not zero in steady state. Integral compensation must be included, if steady-state tracking is desired for radial CG offsets. This is generally accomplished via a steering loop that is in turn commanded by the guidance law. Since no guidance/steering is included in this study, integral compensation is added to the compensators. This is accomplished by differentiating the state space of (24-25) and adding a state variable so that the error variable is retained. This step is equivalent to adding an integrator

to the compensator, since the new control is the derivative of the desired control and an integrator must be inserted between the compensator and the actuator for implementation purposes to provide the proper actuator input signal. This integrator can then be pulled through the summing junctions of the compensator to produce integral compensation. The LQR compatible state space is

$$\begin{bmatrix} \dot{E}_\theta \\ \ddot{E}_\theta \\ \ddot{q} \\ \dot{\delta}_Y \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & M_Y \\ 0 & 0 & 0 & -H \end{bmatrix} \begin{bmatrix} E_\theta \\ \dot{E}_\theta \\ \dot{q} \\ \delta_Y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ H \end{bmatrix} \dot{u}_Y \quad (26)$$

where a first order actuator approximation is used, since the higher order actuator state feedbacks are not measured. Alternatively, the second order model of (17-18) could be used, but the resulting gains on the actuator rate state are set to zero. This state space is controllable, does not include any input command dependent terms, and all of the states and controls are zero in steady state for step input commands. This state space is, therefore, suitable for LQR synthesis. The resulting proportional-integral-derivative (PID) compensator structure is shown in Figure 2, where the derivative compensation is implemented as rate feedback.

The MIMO controller LQR compatible state space can be determined in a manner similar to that used for the SISO case. The gyroscopic terms are, however, retained in the linear dynamics. (21) and (22) and their derivatives are

combined with the derivatives of (15) and (16) to produce a consistent state space of the form $\dot{x} = A x + Bv$, where

$x^T = [E_\theta, \dot{E}_\theta, \dot{q}, E_\psi, \dot{E}_\psi, t, \delta_y, \delta_z]^T$ and $v^T = [\dot{u}_y, \dot{u}_z]$, where the A and B matrices are defined, respectively, as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & N_q & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & H & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & H \end{bmatrix} \quad (27a)$$

$$B^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & H & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & H \end{bmatrix}^T \quad (27b)$$

This state space is controllable, has no input command dependent terms, and has zero steady state values for all states and controls for step commands. The resulting MIMO compensator has a proportional-integral-derivative (PI) structure. The controllability of the MIMO state space must be evaluated after the addition of any integrator states to avoid the introduction of uncontrollable modes [3]. The resulting compensator structure is shown in Figure 3.

Since the control problem has been formulated with respect to the inertial reference frame, all feedback to the controllers must be expressed in this frame. The control inputs to the actuators must then be transformed back into the body frame. Both steps require knowledge of roll angle.

B. Linear Quadratic Regulator Procedure

The LQR synthesis problem is that of determining the control that minimizes the performance index

$$J = \int (x^T Q x + v^T R v) dt \quad (28)$$

where $R > 0$ and $Q \geq 0$, subject to the dynamic constraints

$$\dot{x} = A x + B v \quad (29)$$

where the A and B matrices for the spinning rocket thrust vector control loops are defined symbolically in part A of this section. The solution to this optimization problem is a linear controller of the form

$$v = - (R^{-1} B^T P) x = - G x \quad (30)$$

where P is the algebraic matrix Riccati equation solution [5,6]. The constant gain matrix, G, can be determined from any one of several computer-aided control system design software packages which are currently available to solve the LQR optimization problem. Although the above optimization problem is stated in continuous time, the simulation test gains for the spinning rocket thrust vector controllers are based upon determining the discrete form of (29) and the associated discrete gains for a particular sample rate.

The LQR state weighting matrix for both the SISO and MIMO compensators is chosen so as to satisfy several design criteria. In each case Q is chosen to position the closed-loop poles such that they provide acceptable transient response behavior, reasonable system bandwidth, and adequate system parameter variation insensitivity. The R matrix is

chosen as an identity matrix for the MIMO case since there is no physical reason to weight one control over the other.

Adequate decoupling between the pitch and yaw channels is also a primary design requirement. This is accomplished for the MIMO case primarily with the choice of the weighting on the rate states, but is also a lesser function of the error and integral of the error state weights since the linear dynamics are a very simplified form of the actual system dynamics. The weights on the rate states must add adequate damping to the open loop poles along the $j\omega$ -axis, while maintaining a system bandwidth sufficiently slower than the actuator natural frequency. Sensitivity to parameter variations will occur if the actuator bandwidth is not at least a factor of 4 greater than the system bandwidth. The selected system weighting matrices are shown in Table 2.

The degree of coupling between the control channels is directly related to the vehicle spin rate, as indicated by (15) and (16). If vehicle inertia properties are assumed to be constant, the cross-channel gains in the MIMO compensator can be shown to be directly related to the spin rate of the rocket. These gains can be adjusted with a scale factor that is a ratio of the actual to design spin rate to approximately maintain the closed-loop transfer function of the baseline design. This procedure produces a MIMO controller that simplifies to that of the SISO case as the roll rate approaches zero.

IV. Singular Value Analysis

The robustness of the spinning rocket thrust vector controllers have been investigated with the use of singular value analysis. Singular value analysis is analogous to the SISO gain and phase margin measures of the closeness of the Nyquist plot to the $(-1,0)$ point, in that the measure of this distance for MIMO systems is in terms of the singularity of a matrix. The LQR has been shown in [7-9] to have useful robustness properties. For a diagonal error model, the LQR has simultaneously in each feedback loop a guaranteed minimum gain margin of $(-6, \infty)$ dB and a guaranteed minimum phase margin of ± 60 degrees. Since the higher-order actuator states are not available as feedbacks, a measure of the gain and phase margins of the proposed designs is required. The procedures of [10] are applicable, given the singular values of the return difference matrix

$$D(s) = [I + K(s) G(s)] \quad (31)$$

where $K(s)$ is the compensator transfer matrix and $G(s)$ is the plant transfer matrix given by $G_p(s) = C(sI-A)^{-1}B$ such that $K(s)G_p(s) = G(sI-A)^{-1}B$. These gain and phase margin results are associated with the nominal system. Singular value analysis can also be useful in assessing the robustness of the system with respect to parameter variations and neglected dynamics. The matrix of most interest here is the inverse return difference matrix

$$T(s) = I + (K(s) G(s))^{-1} \quad (32)$$

The central robustness relationship [11] is that

$$\underline{\sigma} (T(s)) > \overline{\sigma} (L(s)) \quad \text{for all } s = j\omega \quad (33)$$

where $\underline{\sigma}$ is the smallest singular value, and $\overline{\sigma}$ is the largest singular value. $L(s)$ is a multiplicative error model, since the perturbed plant, G_p^* , is expressed as

$$G_p^*(s) = G_p(s) [I + L(s)] \quad (34)$$

If condition (33) is satisfied, then stability is guaranteed. This condition is conservative, and failure to meet this condition does not necessarily imply system instability. The recent literature [12] has described efforts at developing less conservative stability tests.

The construction of the error models is important in order to evaluate control system robustness. The error models of most concern here are those associated with the important system parameters in the A and B matrices. The error model associated with parameter variations can be derived with

the use of a matrix identity. Define $\Psi(s) = (sI - A)^{-1}$. A parameter variation can alter both the A and B matrices.

These perturbations are denoted here by ΔA and ΔB , respectively. An error model can be derived [3] as

$$L(s) = G_p^{-1} C^* [\Delta B + \Delta A (I + \Psi \Delta A)^{-1} \Psi B - \Delta A (I + \Psi \Delta A)^{-1} \Psi \Delta B] \quad (35)$$

This error model has been used to evaluate the robustness of both controllers, as is discussed in the next section.

V. System Performance

The performance of the SISO and MIMO controllers have been evaluated from a singular value analysis, and from time response behavior against a dynamic simulation based on (11)-(12). Controller gains and the associated weighting matrices and system eigenvalues are given in Table 2. Simulated transient responses are given in Figs. 4-13. The despun system response for a 10 degree yaw command is shown in Figs 4-5. The time response of the two controllers is nearly identical for this situation. Figs. 6-7, 8-9, 10-11, and 12-13 show the behavior for roll rates of 0.25, 0.5, 1.0, and 2.0 Hz, respectively. These plots are polar plots of pitch angle versus yaw angle to emphasize the coupling between the control channels. In each case the decoupling performance of the MIMO controller is better than that of the SISO controller. This difference in performance is particularly obvious if the roll rate is increased to a level that stresses the frequency response capability of the actuator. For the actuator parameters given in Table 1, the SISO controller will go unstable for a roll rate between 2 and 3 Hz, while the MIMO controller is stable out to 5 Hz. If the actuator is slower than that used for these results, then the difference in decoupling performance is exaggerated at lower frequencies. Off-nominal responses due to parameter variation in the nozzle deflection effectiveness are also shown in Figs. 4-13. The effectiveness parameters are known only as well as the axial CG location, which can vary

significantly during the burn of a large rocket. The actual vehicle roll rate for scaling the baseline MIMO gains is another parameter of interest, but reasonable variations in the measured roll rate does not appreciably degrade performance. The off-nominal transients include variations of $\pm \text{TBD}\%$ in the effectiveness parameters for the MIMO controller, and $\pm \text{TBD}\%$ for the SISO controller.

A singular value plot of the return difference matrix (31) for the MIMO controller is given in Fig. 14. This plot shows a minimum of $\pm \text{TBD}$ dB. Since an LQR design with full state feedback has a minimum of 0.0 dB, there is a penalty in stability margin for the lack of full state feedback (no actuator rate feedback). A similar plot is shown in Fig. 15 for the SISO controller. This plot shows a minimum of $\pm \text{TBD}$ dB. The gain and phase margins can be determined from [10] as a gain margin of $(-\text{TBD}, +\text{TBD})$ dB and a phase margin of $\pm \text{TBD}$ degrees for the MIMO controller, and $(-\text{TBD}, +\text{TBD})$ and $\pm \text{TBD}$ degrees for the SISO controller. Fig. 16 shows the singular values of the inverse return difference matrix (32) for the MIMO controller, plotted against the error model (35) for the parameter variations of $\pm \text{TBD}\%$ in the effectiveness parameters. Fig. 17 shows a similar plot for the SISO controller for a variation of $\pm \text{TBD}\%$. Since $\bar{\sigma}(L) < \underline{\sigma}(T)$, stability is guaranteed for these variations. Simulations and stability analysis both show that the MIMO controller is more robust to variations in this parameter.

VI. Conclusions

Two controllers for the thrust vector control of a spinning rocket are described and compared. One of the controllers is a single-input/single-output (SISO) design that assumes that the vehicle is not spinning, so that there is no gyroscopic coupling between the pitch and yaw control channels. The second controller formulation includes terms that account for the gyroscopic coupling when the rocket motor is spinning. The multiple-input/multiple-output (MIMO) nature of the latter case has motivated the use of a linear-quadratic-regulator (LQR) synthesis procedure. The choice of the state space upon which the LQR synthesis is to be applied is analogous to the selection of the compensator structure in classical control theory. The state spaces are selected to ensure zero steady-state tracking for constant angle step commands, by the addition of states which are equivalent to integrators in the compensator. Both controllers have a proportional-integral-derivative structure. Integral compensation was included for the convenience of this study. A guidance/steering outer loop would normally provide the same effect, such that integral compensation would not be required in the control loops.

The weighting matrices selection for the LQR performance index is analogous to the choice of gains in a classical approach. The performance index weights are chosen here by a trial and error approach to achieve the primary goal of the compensator, which is to provide adequate tracking of

pitch and yaw attitude commands. For a spinning vehicle, adequate decoupling between the two channels is also desirable. The quality of the decoupling is a function of the vehicle inertias, spin rate, and actuator capabilities, as well as the structure and gains of the compensator. A simple technique has been proposed and tested for adjustment of the MIMO controller gains as a function of the actual spin rate of the vehicle to properly account for the gyroscopic coupling between the control channels. As the roll rate decreases to zero, the MIMO compensator structure reduces to the uncoupled SISO structure.

The time response behavior of the two control systems has been tested in a three-degree-of-freedom, nonlinear simulation. The performance of the SISO controller, although generally adequate, is definitely degraded even for moderate roll rates when compared with the MIMO compensator. The differences between the two designs are particularly apparent as the spin rate is increased. The most serious drawback occurs as the spin rate is increased to the limit of the ability of the actuators to respond, in that the SISO controller will go unstable well before the MIMO controller. This is an indication of reduced stability margins for the SISO design relative to the MIMO design, a fact which has been confirmed via a singular value stability analysis.

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TABLE 1: SYSTEM PARAMETERS

$m_t = 1000 \text{ lb}_m$, $m = 10 \text{ lb}_m$
 $I_x = 1400 \text{ lb}_m \text{ ft}^2$, $I = 5500 \text{ lb}_m \text{ ft}^2$
 $R = 2.25 \text{ ft}$, $l = -3.2 \text{ ft}$, $L = 19.0 \text{ ft}$
 $p_B = 6.2832 \text{ rad/sec}$ (Baseline roll rate)
Actuator Model: $f_n = 10 \text{ Hz}$, $\zeta = 0.6$

TABLE 2: CONTROLLER PARAMETERS

SISO Controller

Gains: 0.0316 0.0586 -0.0385 0.0752
Diagonal Weights: $Q(1,1)=Q(2,2)=0.001$,
 $Q(3,3)=0.0005$, $Q(4,4)=0.0$
Open-Loop Eigenvalues: 0.0
0.0 $\pm j1.888$
-62.8
Closed-Loop Eigenvalues: -2.4079
-1.1899 $\pm j0.5075$
-62.7354

MIMO Controller

Gains:
0.0295 0.0554 -0.0378 0.0113 0.0212 0.0006 0.0739 0.0000
-0.0113 -0.0212 -0.0006 0.0295 0.0554 -0.0378 0.0000 0.0739
Diagonal Weights: $Q(1,1)=Q(2,2)=Q(4,4)=Q(5,5)=0.001$,
 $Q(3,3)=Q(6,6)=0.0005$, $Q(7,7)=Q(8,8)=0.0$
Open-Loop Eigenvalues: 0.0
0.0
0.0 $\pm j1.5885$
-62.8
-62.8
Closed-Loop Eigenvalues: -0.9419 $\pm j0.6109$
-1.0239 $\pm j0.3760$
-2.7371 $\pm j1.8267$
-62.7355 $\pm j0.0033$

FIGURE 1

PITCH ANGLE CONTROLLER

FIGURE 2

PITCH/YAW ANGLE CONTROLLER

FIGURE 3

SPINNING ROCKET SIMULATION
SISO TVC CONTROLLER

FIGURE 4: SISO, 0 Hz

SPINNING ROCKET SIMULATION MIMO TVC CONTROLLER

FIGURE 5 : MIMO , 0 Hz

SPINNING ROCKET SIMULATION SISO TVC CONTROLLER

FIGURE 6 : SISO, 0.25 H2

SPINNING ROCKET SIMULATION MIMO TVC CONTROLLER

FIGURE 7: MIMO, 0.25 HZ

SPINNING ROCKET SIMULATION SISO TVC CONTROLLER

FIGURE 8 : SISO , 0.5 Hz

SPINNING ROCKET SIMULATION MIMO TVC CONTROLLER

FIGURE 9: MIMO, 0.5 Hz

SPINNING ROCKET SIMULATION SISO TVC CONTROLLER

FIGURE 10: SISO, 1.0 HZ

SPINNING ROCKET SIMULATION MIMO TVC CONTROLLER

FIGURE 11: MIMO, 1.0 HZ

FIGURE 12: S150, 2.0 Hz

SPINNING ROCKET SIMULATION MIMO TVC CONTROLLER

FIGURE 13: MIMO, 2.0 HZ

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07 / 08 / 91