A BAYES-MAXIMUM ENTROPY METHOD FOR MULTI-SENSOR DATA FUSION*

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ABSTRACT

In this paper we introduce a Bayes-Maximum Entropy formalism for multi-sensor data fusion, and present an application of this methodology to the fusion of ultrasound and visual sensor data as acquired by a mobile robot. In our approach the principle of maximum entropy is applied to the construction of priors and likelihoods from the data. Distances between ultrasound and visual points of interest in a dual representation are used to define Gibbs likelihood distributions. Both one- and two-dimensional likelihoods are presented, and cast into a form which makes explicit their dependence upon the mean. The Bayesian posterior distributions are used to test a null hypothesis, and Maximum Entropy Maps used for navigation are updated using the resulting information from the dual representation.

I. INTRODUCTION

One of the prerequisites for intelligent behavior in robotic systems is the ability to generate internally self-consistent representations of the environment from sensor data. Often, this is impossible to achieve on the basis of either one sensing operations or a single sensor domain, particularly when the environment is unstructured to any appreciable extent. The term "multi-sensor data fusion" denotes the task of combining data and information from more than one sensor location, and/or from more than one sensor, in order to reduce systematic and random errors, and uncertainties.

Two important classes of problems encountered in building representations of the environment are those due to noisy sensor data and to position uncertainties. Multi-sensor data fusion treatments of these problems have been presented in a number of recent papers. Among these are the Kalman and extended Kalman filtering studies by Ayache and Faugeras [1], Bolle and Cooper [3], Chatila and Laumond [5], Crowley [6], Durrant-Whyte, Rao, and Hu [7], and Grandjean and Robert De Saint Vincent [10]. Ayache and Faugeras [1], for example, investigated problems associated with building and fusing 3-D noisy visual maps using passive stereo. In doing so they employed extended Kalman

filtering to combine and register images in the presence of noise, and emphasized the utility of the methodology in reducing geometric position uncertainties (also see the work by Chatila and Laumond [5], and by Smith and Cheeseman [14] on position uncertainties).

Another important class of errors are those we term "systematic." This class of errors encompasses all errors which are not random in character. These errors are produced whenever there are incorrect and/or inconsistent interpretations of data. They arise in automated, or robotic, systems whenever there is insufficient sensor information to correctly interpret or match the data, and when assumptions concerning the nature of the environment are violated. Only a few investigations of this class of errors have been reported to date. Among these are the studies of ultrasound data mapping by Beckerman and Obrow [2], the investigation of ultrasound and infrared data fusion by Flynn [9], and the recent study of map building for mobile robots by Leonard, Cox, and Durrant-Whyte [12]. In the latter paper, Leonard et al. [12] noted that systematic errors are distinct from noise, are difficult to handle, and often require treatments which differ from those used for noise removal.

In this work we present a Bayes-Maximum Entropy formalism for multi-sensor data fusion, and apply the methodology to the fusion of ultrasound and visual sensor data acquired by a mobile robot. The objective of our Maximum Entropy methodology is not the removal of noise or other random errors as is customary (see, for example, the paper by Gull and Daniell [11]), but rather to treat systematic errors.

In our application we extract the positions in the visual image of the vertical edges of the objects of interest, and the locations in the ultrasound viewing plane of the angular positions of the depth discontinuities. The systematic errors treated by us include ultrasound distortions in size, and visual ambiguities in discriminating depth discontinuities from intensity gradients generated by other details in the visual images. However, our formalism is in no way limited to these particular errors or to the particular visual feature points chosen.

In an alternative approach Matthies and Elfes [13] fused ultrasound and stereo range data using heuristic probability distributions with Bayesian updating. In both that work and the study by Elfes [8] on ultrasound data mapping the emphasis was on noise reduction. The primary data representation appearing in those studies was the Certainty Grid reflecting the meaning of the underlying probability distributions. In the present work we introduce both uniform and Gibbs distributions, and the resulting ultrasound data structures are Maximum Entropy Maps.

II. BAYES-MAXIMUM ENTROPY METHODS

A. BAYES THEOREM FOR MULTI-SENSOR DATA FUSION

The Bayes' formula of interest to us is of the form

$$p(h_\alpha | ab) \sim p(h_\alpha | a)p(b | h_\alpha a) \quad (1)$$

where h_α denotes a disjoint set of hypotheses, and "a" and "b" represent two data sets. This equation is a statement that the posterior probability is proportional to the product of the prior probability and likelihood. In Eq. (1) we have explicitly conditioned the prior probability upon data set "a." In the application the symbols "a" and "b" will denote the ultrasound and visual data, respectively, so that the prior distribution was determined by the ultrasound string data, while the likelihood depended upon ultrasound string information as well as the visual edge data.

B. PRINCIPLE OF MAXIMUM ENTROPY

The principle of maximum entropy provides us with a means for determining appropriate forms for the probabilities. Let $p(v_i)$ denote a probability density over a discrete space $V = (v_1, v_2, \dots, v_n)$. The corresponding Shannon-Jaynes entropy, S , is given by the expression

$$S = - \sum_i p(v_i) \ln p(v_i), \quad i = 1, 2, \dots, n. \quad (2)$$

The probabilities we seek are those which maximize S subject to the normalization condition

$$\sum_i p(v_i) = 1 \quad (3)$$

and the constraint equations

$$\sum_i q_k(v_i) p(v_i) = \langle q_k \rangle, \quad k = 1, \dots, m; \quad m < n. \quad (4)$$

Using the Lagrange multiplier technique we obtain the formal solution to this variational problem, namely,

$$p(v_i) = e^{-\lambda - \sum_k \lambda_k q_k(v_i)} \quad (5)$$

where the constants λ and λ_k are Lagrange multipliers.

C. PROBABILITY DENSITIES FOR DISTANCE MEASURES

The types of errors under consideration are sometimes difficult to model. The error distributions are often not Gaussian, and may not be known with any precision. We will therefore impose only a single constraint specifying the mean value, μ , for a distance variable, v , used to match features in the two sensor domains:

$$\mu = \sum_i v_i p(v_i) \quad (6)$$

The form for the probability density is then determined by demanding consistency with the principle of maximum entropy. The result, from Eq. (5) is

$$p(v_i) = e^{-\lambda - \lambda_1 v_i} \quad (7)$$

This expression is the well-known Gibbs distribution, with v_i playing the role of the Hamiltonian and with a partition function, e^λ , defined by normalization as

$$e^\lambda = \sum_i e^{-\lambda_1 v_i} \quad (8)$$

We now wish to cast Eq. (8) into a form which, upon eliminating both Lagrange multipliers, will make explicit its dependence upon the mean. To obtain the desired form we introduce the density of states, $\Omega(v)$ and rewrite the partition function as a sum over the variable v which takes on the values 0, 1, 2, ... :

$$e^\lambda = \sum_v \Omega(v) e^{-\lambda_1 v} \quad (9)$$

For the present let us assume that $\Omega(v) = 1$. We can then carry out the sum indicated in Eq. (9) to give the result

$$e^{-\lambda} = 1 - e^{-\lambda_1} \quad (10)$$

We next observe that $v e^{-\lambda_1 v}$ can be obtained from $e^{-\lambda_1 v}$ by differentiation with respect to λ_1 . We then write Eq. (6) as

$$\mu = (1 - e^{-\lambda_1}) \sum_v v e^{-\lambda_1 v} \quad (11)$$

and upon differentiation with respect to λ_1 we find that

$$e^{-\lambda_1} = \frac{\mu}{\mu + 1} \quad (12)$$

Combining Eqs. (7), (10), and (12) gives the remarkably simple expression for the probability density, namely

$$p(v) = \left(\frac{1}{\mu + 1}\right) \left(\frac{\mu}{\mu + 1}\right)^v \quad (13)$$

The extension to two dimensions is straightforward. We introduce a two-dimensional metric for the points of interest, (x_0, y_0) :

$$v_i = [(x_0 - x_i)^2 + (y_0 - y_i)^2]^{\frac{1}{2}} \quad (14)$$

For this 2-D case the density of states can be represented by the circumference of a circle of radius v , i.e.,

$$\Omega(v) = 2\pi v \quad (15)$$

Repeating the steps outlined above yields the maximum entropy probability density

$$p(v) = \frac{4}{\mu^2 - 1} v \left(\frac{\mu - 1}{\mu + 1}\right)^v \quad (16)$$

We now apply the Bayes-Maximum Entropy formalism to the fusion of 2-D ultrasound range data with 2-D visual intensity data as acquired by the HERMIES-IIB mobile robot.

III. ULTRASOUND AND VISUAL SENSOR DATA

HERMIES-IIB, is one of a series of mobile robot prototypes developed at Oak Ridge National Laboratory's Center for Engineering Systems Advanced Research (CESAR). The hardware for this robot includes a mobile platform, dual manipulator arms, and a sensor turret equipped with an array of twenty-four ultrasonic range sensors and several CCD cameras. One of these cameras and the ultrasound sensor array provided the data for the present study.

In more detail, the visual sensor used in the study consisted of a single CCD camera equipped with a wide-angle, 4.8mm lens. This camera provided 256×256 pixels of 8-bit grey-level intensity values from a 60 degree field-of-view. The twenty-four ultrasonic sensors were grouped into six phased-arrays of four units each. One phased-array was oriented in the forward direction for reflexive collision avoidance. The five remaining phased-arrays were mounted on the sensor turret to form a partial ring in the horizontal plane. In the present study the sensor turret was panned to provide 120 samples of range data in 3 deg steps.

Although the 2-D planes in the ultrasound and visual sensor domains are different, the former referring to a horizontal scanning plane and the latter denoting a vertical image

plane, the two sensor domains share a common angular coordinate. This overlap enables us to use the ultrasound information to reduce systematic errors in the visual domain, and vice versa.

As mentioned in the Introduction the information we wished to extract from the data consisted of the positions in the visual image of the vertical edges of objects, and the locations in the ultrasound viewing plane of the occupied and empty regions of space, and also the angular positions of the depth discontinuities. These particular features were chosen for their utility with respect to robotic navigation. The Bayes-Maximum Entropy distance measure was used to match the correct set of vertical edges to the corresponding ultrasound range data structure, thereby identifying the physical object of interest in the environment.

IV. WORLD MODELING AND FUSION

A. ULTRASOUND STRINGS

In the initial stage of processing the ultrasound scan information, the 120 sample of range data was transformed into 240 samples of range data spanning 360 deg in 1.5 deg steps. This was done to enable the ray-tracing algorithm used in the mapping stage to cover the appropriate region of the space. Two types of data structures, Maximum Entropy Maps and strings, were built in the second stage of the ultrasound data processing. The Maximum Entropy Maps are global, rectilinear representation of occupied and empty regions of space.

The term "strings" was introduced in [2] to describe a sequence of returns of similar range from neighboring scan angles, bounded at both ends by depth, or range, discontinuities. Strings are the interesting features in the ultrasound scene, marking the possible locations of object surfaces. This second type of data structure is a robot-centered, polar representation of the data, containing information on

- (i) the robot coordinates
- (ii) the range and angle of each element
- (iii) the (x, y) map coordinates of each element

To build this data structure returns from adjacent angles within a 360 degree scan were compared to one another. If their range differences were less than the maximum amount consistent with noise, accumulation of string information was initiated. A string was terminated when the range difference next exceeded the noise limit, signaling the onset of a depth discontinuity.

Strings from different scan positions were used in [2] to remove errors. These data structures are similar in function to the parameterized line segments introduced by

Crowley [6]. In the present work we map these data into the visual domain, in order to fuse, or match them, with the vertical edge sequences.

B. DUAL VISUAL DOMAIN

Shown in Figs. 1 and 2 are the results of using a first derivative, Canny-like [4] operator to find the vertical edges in two grey-scale CCD images. The vertical edges of interest in these figures correspond to an object (box) tall enough to intercept the ultrasound viewing plane when placed on the floor of the lab. For our environment we have depth discontinuities which characterize the object, and we also have intensity gradients generated by variations in the background and floor markings, by the overhead lights and air-conditioning ducts, and by shadows and glare. The number of vertical edges in the processed images, in fact, far exceeds the number of actual depth discontinuities forming boundaries of the physically interesting objects.

To map the ultrasound string elements into the visual domain we use the string angular coordinate to locate the corresponding x -coordinates in the image. The mapping of the floor coordinates to the corresponding pixel positions in the lower half of the visual image is not linear. Instead, there are distortions due to perspective geometry and the wide-angle lens of the CCD camera. To identify the y -coordinates in the visual image a look-up table was constructed from detailed floor calibration data. This information was used together with the range data to locate the bottoms of the objects in the image.

Results of this mapping are shown in Figs. 3 and 4. In these figures we observe the vertical edge segments after the weak thresholding with respect to segment length and intensity gradient magnitude. Also shown in the figures are a series of small squares. These squares are spaced in 1.5 degree intervals, and denote the ultrasound string elements falling within the field-of-view of the CCD camera. The y -coordinates of these squares denoted the location of the ground (floor) at the appropriate distance from the CCD camera.

The effective beam width [2] of the ultrasound sensors depends upon the radiated power, the surface properties of the surfaces scanned, and the amplifier threshold sensitivity. Measurements were done to establish the value for the effective beam width, and to quantify the maximum distortion errors possible. The results of these measurements are presented in Fig. 5 in terms of length distortions. The error bars in the figure show the imprecision due to the size of the pixels in the maps used to represent the data. The solid line appearing in this plot was generated using the expression

$$\text{distortion} = 2 * \text{distance} * \tan(\Theta_{eff}/2)$$

This expression represents the (maximum) linear distortion to be expected when the object surface is normal to the beam axis. As can be seen in Fig. 5 we obtain reasonable agreement for all but the lowest data point when using an 18 deg effective beam width.

C. MAXIMUM ENTROPY MAPS

Maps corresponding to the visual scene shown in Figs. 1 and 2, and produced at the termination of the initial processing, are displayed in Figs. 6 and 7. Each cell of the maps denotes a 15 cm $\times$ 15 cm region of space, and is labeled as either occupied, empty or unknown. In constructing the map shown in Figs. 6 and 7 all possible cells representing the scattering source at the given range were labeled as occupied. For each return a ray-tracing algorithm was used to find and label as empty (unoccupied) the pixels denoting regions between the robot and the sources of the echos. All remaining pixels, that is, those representing regions which were either occluded by objects or were located beyond the maximum range of the sensors, were labeled as unknown.

The collections of cells which are located in the upper center of the maps, and are labeled as being occupied, represent the locations of the exposed surface of the box seen in the visual images. The corresponding distortion errors are shown in the error plot, Fig. 5. In labeling the cells of the map in the manner described above we are using uniform probability distributions. These distributions represent the most conservative choice possible, consistent with the principle of maximum entropy for situations where there are no constraints other than normalization. For this reason we term our Cartesian representation a Maximum Entropy (MaxEnt) map. These maps are analogous to the MaxEnt maps introduced in, say, x-ray and radio astronomy [11].

D. FUSION

1) Prior Distribution: The index "i" appearing in Eqs. (2)–(5) is a short-hand notation for the i_{th} pixel (x, y) coordinate in the dual representation. The appropriate null hypothesis, h_0 , is that the pixel under consideration identifies, or is consistent with the location of, an ultrasound scattering source boundary and a corresponding visual depth discontinuity. The ultrasound string data in Figs. 3 and 4 define the horizontal limits for the location of the scattering source left and right boundaries. The most conservative estimate possible for the prior distribution is obtained from the principle of maximum entropy assuming no constraints other than normalization. We therefore take as the prior the uniform distribution

$$p(h_0 | a) = \begin{cases} \frac{1}{\Omega_0}, & x_{\min} \leq x_i \leq x_{\max} \\ 0, & \text{elsewhere} \end{cases} \quad (17)$$

In Eq. (17) $x_{\min}$ and $x_{\max}$ denote the string endpoints, and gives the number of pixels within that range.

2) Likelihood: We begin the specification of the likelihood by defining the variable v_i to be the distance between the ultrasound-determined floor location and the bottom of the visual vertical edge sequence. That is we define

$$v_i = |y_0 - y_i| \quad (18)$$

where y_0 is the floor location, and y_i is the bottom of the i_{th} vertical edge. We observe that in Eq. (18) we have incorporated ultrasound-provided perspective information into the likelihood through the variable v_i .

Let us now return to the specification of the density of states. An examination of Eq. (18) reveals that $\Omega(v) = 2$ for all values of v except $v = 0$ for which $\Omega(v) = 1$. We can therefore take the density of states into account by modifying Eq. (13) to read

$$p(b | h_0\alpha) = p(v) = \begin{cases} \frac{2}{2\mu+1} \left(\frac{\mu}{\mu+1}\right)^2, & v > 0 \\ \frac{1}{2\mu+1}, & v = 0 \end{cases} \quad (19)$$

3) Hypothesis Testing: The hypothesis testing was carried out after fixing the mean, μ , and setting a confidence limit for rejecting the null hypothesis. The edge sequences which survive the testing are identified in Figs. 3 and 4 by the superimposed bright lines. We observe in these figures that all edge sequences except the desired pair have been eliminated by our test.

E. UPDATING THE MaxEnt MAPS

The starting point for the updating of the MaxEnt maps was the dual representations depicted in Figs. 3 and 4. These dual representations were used to identify the nearest string elements to the left and right boundary edges. Denoting the identified string coordinates by $\theta'_{\min}$ and $\theta'_{\max}$ leads to a uniform likelihood of the form

$$p(b | h_0\alpha) = \begin{cases} 1, & \theta'_{\min} < \theta_i < \theta'_{\max} \\ 0, & \text{elsewhere} \end{cases} \quad (20)$$

The prior distribution is that of the original string appearing in the dual representation. Using $\theta_{\min}$ and $\theta_{\max}$ to denote the endpoints of the prior probability density yields a simple posterior distribution defined over the range from

$$\theta''_{\min} = \max(\theta_{\min}, \theta'_{\min}) = \theta'_{\min} \quad (21)$$

to

$$\theta''_{\max} = \min(\theta_{\max}, \theta'_{\max}) = \theta'_{\max}$$

These uniform posterior distributions were used to update the MaxEnt maps. Recall that the string data structures contain the identifying (x, y) coordinates of the corresponding occupied cells of the MaxEnt maps. This information was used to modify the maps to reflect the corrections to the string endpoints. In doing so, cells of the maps located at scan angles lying outside the updated endpoints and initially labeled as occupied and unknown (due to occlusions) were relabeled as empty. Ranges assigned at these angles were based upon data from scan angles lying just beyond the original string endpoints. The results are exhibited in Figs. 8 and 9. Upon comparing pairs of maps, that is, Figs. 6 and 8, and 7 and 9, we find that the exposed surface of the object of interest has been modified in each instance to reflect the visual information, while all other regions of the maps were left unchanged.

V. SUMMARY AND CONCLUDING REMARKS

To summarize, we have introduced a Bayes-Maximum Entropy formalism for multi-sensor data fusion, and have presented an application of this methodology to the fusion of ultrasound and visual sensor data as acquired by a mobile robot. The objective of our Bayes-Maximum Entropy formalism was not to reduce noise or other random errors as is customary, but rather to treat systematic errors which arise in the interpretation of the output of the low level processing stage in the individual ultrasound and visual sensor domains.

In our Bayesian approach all prior probabilities were conditioned on data and as a result were well-defined, while the forms for the probability densities were determined using the principle of maximum entropy. Both uniform and Gibbs distributions, analogous to the microcanonical and canonical ensembles of statistical mechanics, were introduced in this manner.

In the application distances between visual and ultrasound points of interest in a dual visual-ultrasound representation were used to define the appropriate Gibbs likelihoods. Both one- and two-dimensional likelihoods were presented, and cast into a form which made explicit their dependence upon the mean distance. The resulting posterior probability distribution were then used to test suitably chosen null hypotheses. Maximum Entropy Maps useful for navigation purposes were updated using the resulting information from the dual representation.

Figs. 1 and 2. Vertical edges identified in the two simple visual images.

Figs. 3 and 4. Dual visual-ultrasound representations corresponding to Figs. 1 and 2. Vertical edges remaining after testing of the null hypothesis are shown as bright lines. Small white squares denote the ultrasound string elements.

Fig. 5. Plot of experimental and calculated linear distortions as a function of distance. Data with large error bars were obtained from maps gridded into cells representing 30 cm $\times$ 30 cm regions of space. The data with the small error bar corresponds to the map shown in Fig. 6.

Figs. 6 and 7. MaxEnt maps of the robot's environment. Each cell denotes a 15 cm $\times$ 15 cm region of space. Cells shaded in grey are empty, and those marked in black are unknown. The small white squares denote occupied regions of space. These squares form elements of the strings in 1.5 degree steps.

Figs. 8 and 9. MaxEnt maps after fusion, corresponding to Figs. 6 and 7.

REFERENCES

- [1] Ayache. N. and O. D. Faugeras, "Building, Registering and Fusing Noisy Visual Maps." *Int. J. Robotics. Res.* **6**, 45-65 (1988).
- [2] Beckerman. M. and E. M. Oblow, "Treatment of Systematic Errors in the Processing of Wide Angle Sonar Sensor Data for Robotic Navigation." *IEEE Transactions on Robotics and Automation* **RA-6**, 137-145 (1990).
- [3] Bolle, R. M. and D. B. Cooper, "Optimally Combining Pieces of Information, with Application to Estimating 3-D Complex-Object Position from Range Data." *IEEE Trans. Pattern Anal. Machine Intell.* **PAMI-8**, 619-638 (1986).
- [4] Canny. J., "A Computational Approach to Edge Detection," *IEEE Trans. Pattern Anal. Machine Intell.* **PAMI-8**, 766-778 (1986).
- [5] Chatila. R. and J.-P. Laumond, "Position Referencing and Consistent World Modelling for Mobile Robots," *Proceedings IEEE International Conference on Robotics and Automation*, 138-145 (1985).
- [6] Crowley. J. L., "World Modeling and Position Estimation for a Mobile Robot Using Ultrasonic Ranging," *Proceedings of the IEEE International Conference on Robotics and Automation*, 674-680 (1989).

- [7] Durrant-Whyte, H. F., B. Y. S. Rao and H. Hu, "Toward a Fully Decentralized Architecture for Multi-Sensor Data Fusion," *Proceedings of the IEEE International Conference on Robotics and Automation*, 1331-1336 (1990).
- [8] Elfes, A., "Sonar-Based Real-World Mapping and Navigation," *IEEE Journal of Robotics and Automation*, **RA-3**, 249-265 (1987).
- [9] Flynn, A. M., "Redundant Sensors for Mobile Robot Control," M.S. Thesis, Department of Electrical Engineering and Computer Science, MIT, Cambridge, MA., 1985.
- [10] Grandjean, P. and A. Robert De Saint Vincent, "3-D Modeling of Indoor Scenes by Fusion of Noisy Range and Stereo Data," *Proceedings of the IEEE International Conference on Robotics and Automation*, 681-687 (1989).
- [11] Gull, S. F. and G. J. Daniell, "Image Reconstruction from Incomplete and Noisy Data," *Nature*, **272**, 686-690 (1978).
- [12] Leonard, J. H. Durand-White and I. J. Cox, "Dynamic Map Building for an Autonomous Mobile Robot," *Proceedings IEEE International Workshop on Intelligent Robots and Systems IROS '90*, 89-96 (1990).
- [13] Matthies, L. and A. Elfes, "Integration of Sonar and Stereo Range Data Using a Grid-Based Representation," *Proceedings IEEE International Conference on Robotics and Automation*, 727-733 (1988).
- [14] Smith R. C. and P. Cheeseman, "On the Representation and Estimation of Spatial Uncertainty," *Int. J. Robotics Res.* **5**, 56-68 (1986).

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