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An Efficient Technique for Calculating
Multiple Solutions
of Electric-Field Problems
Part I

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Rensselaer Polytechnic Institute
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Foreword to Part I

The computer program described herein was written for the PRIME 500 system then in existence at the RPI Engineering Center. It will not work on the IBM 3033 system also in existence at RPI.

The problem of translation from the PRIME to the IBM System, as well as the correction of a few errors, were undertaken as a master's project in electric power engineering by George G. Theodossiou. His project report is issued as Part II of this final report on this contract.

Prospective users of either program are advised to study both parts of the report.

The principal investigator on this project has some reservation regarding how universal this program version is with regard to geometry. That is, although the program ran successfully on the Prime system for the three examples given in the report, he does not think that it will run properly for some geometric situations on all Prime installations. Errors discovered in the translation to the IBM system are the cause of this lack of certitude.

On the other hand, the IBM system version presented in Part II is believed to be of much more general applicability.

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I. Purpose

The purpose of this report is to present a digital computer program capable of calculating the electrostatic field in an arbitrary two-dimensional configuration. The program was developed as a preliminary result in a project aimed at producing a three-dimensional program. Thus, this report represents an interim report on the entire project.

A. Calculation in practical configurations

The electrostatic field is known to satisfy Laplace' equation in charge-free regions.

$$\nabla^2 \phi = 0$$

The partial differential equation is solved numerically, on a digital computer, in the dielectrics of a given system, subject to given or known boundary conditions. The boundary conditions, given by the operator, will consist of the geometry of the problem surfaces. That is, he will give the location and shape of conducting electrodes, dielectric interfaces and bounding surfaces. He will also give known potentials of electrodes. Among the boundary conditions known from theory to require satisfaction are that a conductor will have a single potential, that cut bounding surfaces have no potential gradient normal to them, and at the interface between two different dielectrics, that the normal components of displacement field are equal and the tangential components of electric field are equal.

It was the aim of the project that no geometric restrictions would be placed on the configuration. This was imperfectly realized. For instance, it was found that boundaries involving small acute angles cause problems. Such restrictions will be discussed in a later part of this report.

B. What is now available?

Several, maybe many, digital programs have been developed to perform this calculation. Some are available commercially. For example, a program written by Prof. O. W. Andersen of Norway is available from him. Other commercial versions of such programs are available from organizations such as Swanson Analysis Systems, Inc. and the General Electric Company Research and Development Center.

The limitations of these programs are in the number of simultaneous equations which can be solved for an acceptable cost. Greater accuracy through smaller divisions (finer grids) results in a larger system of equations. Therefore run costs are greater.

Acceptable costs are defined by the user. Many near-repetitive runs in a trial-and-error design process would require a much lower cost per run than a final design check run.

Run costs depend strongly on the programmer's skill and inventiveness as well as the innovations in large scale computer hardware. It is not unusual to get run costs in the \$5 to \$20 range for grids with 200 to 600 nodes. Although the programs cited can accommodate more than 2000 nodes in reasonably large hardware configurations, the run costs escalate and can be a few hundred dollars for grid sizes around 3000 nodes.

C. Improvements possible

The set of simultaneous equations, resulting from the finite element formulation of a problem, which must be solved, bear a strong resemblance to the equations of large electrical networks. Such networks have been studied extensively and efficient techniques for solving their systems of equations have been developed.

One such technique is used when a system is too large to be solved in one run. The concept is to separate the system into two sub-systems: one sub-system, call it A, is to be solved in detail because solution values are critical in its region; the other sub-system, called B, must be considered in its effect on the A solution, but its detailed internal solution need not be considered. The equations for the nodes in B are solved in terms of variables at those nodes interconnecting B with A, i.e. an "equivalent" of B as viewed from A is formed. This equivalent is much smaller than the total B system. Then the equivalent can be combined with a system for A and solved for the variables in A.

Use of this technique permits separation of a large problem containing many thousand nodes into a "window" and a "discardable" region. The window would correspond to region A and region B is discardable.

For design variations only within the window, region B need be computed only once. Thus the repetitive design runs are limited to the number of equations contained in the window. The result is a lowering of run costs for the repetitive runs, but requires a larger run to establish the equivalent initially.

It should be noted that equivalents, or partial solutions, of parts of complex structures can be formed and retained for future use.

Therefore, it is expected that larger systems can be solved with this technique than can be economically solved by considering the entire region at one time. This permits finer grid subdivisions and therefore greater accuracy if run costs are held constant. On the other hand, if grid size is constant, run costs should decrease for multiple runs.

D. Summary

The program described herein is used to solve Laplace' equation in two dimensions through use of the finite element method. The set of equations

formed by use of this method is seen to be analogous to that of an extensive electrical network of resistive branches. Such networks can be divided into several regions by the method of "diakoptics" as enunciated by G. Kron. Representation of each sub-region in terms of variables at its boundaries with other regions has been described by H. E. Brown. These equivalents at sub-region boundaries permit placing a "window" on a very large problem. Solution within the window is done for a much smaller set of equations than if the total problem were attempted in one computer run.

. Theory

A. The Finite Element Method

1. The variational formulation

We are seeking a technique of calculating the electric field in a given geometry. It is known that an electric field is described by Laplace' equation in a charge-free region:

$$\nabla^2 V = 0 \quad (2.1)$$

The solution of this partial differential equation is known to satisfy a minimization of a functional described by the Euler-Lagrange equation. Texts on variational methods are common. One adequate description of the statement above is found in The Variational Method in Engineering by R. S. Schechter, published by McGraw-Hill 1967.

J. Heuser, in NASA Technical Note D-7274, November 1973, has given the variational form which applies to heat conduction problems in two and three dimensions. Heat conduction is also described by Laplace' equation in regions where no internal heat generation is present. The equations below are Heuser's results with temperature replaced by potential; conductivity replaced by permittivity; boundary conditions simplified to allow only fixed potentials and considerations of internal generation, surface flux, radiation, convection and heat storage are omitted.

In two dimensions Laplace' equation is

$$\frac{\partial}{\partial x}(\epsilon_x \frac{\partial V}{\partial x}) + \frac{\partial}{\partial y}(\epsilon_y \frac{\partial V}{\partial y}) = 0 \quad (2.2)$$

where ϵ is the dielectric permittivity which is allowed to be non-isotropic in the most general case.

The functional, the minimization of which constitutes the solution sought is

$$U = \iint_A [\frac{\epsilon_x}{2} (\frac{\partial V}{\partial x})^2 + \frac{\epsilon_y}{2} (\frac{\partial V}{\partial y})^2] dA \quad (2.3)$$

2. Division into small elements

The process will then be to divide the entire range of the problem into smaller elements. In each small element, there will be assumed a variation of potential. This variation will be introduced into the functional expression (2.3). The functional will be minimized with respect to the assumed variational parameters. The result is a relation among the assumed variational parameters which represents the solution to Laplace' equation.

3. Choice of element shape

The most popular choice of element shape has been the triangle in two dimensions. One advantage of such a choice is that it is easy to automate division of a problem domain into such elements. For example, O. W. Andersen uses such a technique in "Laplacian Electrostatic Field Calculations by Finite Elements with Automatic Grid Generation," IEEE Transactions on Power Apparatus and Systems, vol. PAS-92, no. 5, Sept./Oct. 1973, pages 1485-1492.

Another advantage of choosing triangles is that each element has a minimum number of nodes, thus simplifying algebraic results.

However, elements may be chosen to be of any shape which fills the problem space. Of course, shapes with larger numbers of nodes per element require more algebra per calculation. The advantage may be that elements with more nodes can be larger than simpler elements, and therefore use fewer elements to fill the field.

In that which follows the two-dimensional triangles were chosen for the following reasons:

1. the literature shows greater familiarity with manipulating the simpler elements;
2. results using these shapes are available in several studies;
3. accuracy of the results of such studies has been evaluated and published.

4. Choice of potential variation within elements

The analytical technique is to divide the entire region of the problem into small elements in which the potential variation can be approximately described. For example, it is desirable for simplicity of calculation that, within each element, the potential be assumed to vary linearly with each dimension. This is not necessary, but it makes the calculation of an element easier.

The accuracy of the result depends directly upon how well the assumption of linearity corresponds to the actual field. In this respect, intuition suggests that smaller elements give better approximations to linearity and thus more accurate results.

Some investigators have suggested larger elements with more complex potential variations, such as quadratic, cubic or even higher order polynomials, with distance. The advantage would appear to be that more complicated variational assumptions would permit larger, and therefore fewer, elements within the problem range. Even though more nodes per element might be required, it is possible that the total calculation required might be less than that required with simpler, smaller, more numerous elements.

In that which follows, the linear potential variation is assumed in the smaller elements. This was done because:

1. more familiar results are available and
2. it was easily seen that admittance matrix techniques apply.

B. Results of Choices

1. The results of choosing linear potential variation in a triangular element are given by Heuser⁽¹⁾ to be:

a. $V = a + bx + cy$ (2.5)

b. Using subscripts to denote quantities at the nodes

$$\begin{bmatrix} V_i \\ V_j \\ V_k \end{bmatrix} = \begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad (2.6)$$

c. Inversion of (2.6) gives abc in terms of the node potentials

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix}^{-1} \begin{bmatrix} V_i \\ V_j \\ V_k \end{bmatrix}$$

$$= \frac{1}{2A} \begin{bmatrix} a_i & a_j & a_k \\ b_i & b_j & b_k \\ c_i & c_j & c_k \end{bmatrix} \begin{bmatrix} V_i \\ V_j \\ V_k \end{bmatrix} \quad (2.7)$$

where $2A = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} = \begin{matrix} 2 \text{ times the area} \\ \text{of the triangle} \end{matrix}$ (2.8)

$$a_i = + \begin{vmatrix} x_j & y_j \\ x_k & y_k \end{vmatrix} \quad (2.9)$$

etc.

Expressions for all the abc values for the subscripts i, j and k are given in appendix 1. Note that the results in some cases have signs differing from Heuser's results.

d. Since equation (2.5) may be written in the matrix form

$$V = \begin{bmatrix} 1 & x & y \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

equation (2.7) may be used in this to give

$$V = \frac{1}{2A} \begin{bmatrix} 1 & x & y \end{bmatrix} \begin{bmatrix} a_i & a_j & a_k \\ b_i & b_j & b_k \\ c_i & c_j & c_k \end{bmatrix} \begin{bmatrix} V_i \\ V_j \\ V_k \end{bmatrix} \quad (2.10)$$

Equation (2.10) expresses the potential at any location (x, y) within the triangle in terms of the potentials at the vertices of the triangle.

The method now requires use of equation (2.10) in the functional expression, equation (2.3), and then minimization of (2.3) with respect to each of the node potentials. This process is carried out in appendix 2. The result, in matrix form is, when $\epsilon_x = \epsilon_y = \epsilon_z = \epsilon$:

$$\begin{bmatrix} \partial U_e / \partial V_i \\ \partial U_e / \partial V_j \\ \partial U_e / \partial V_k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \frac{\epsilon}{4A} \begin{bmatrix} b_i & c_i \\ b_j & c_j \\ b_k & c_k \end{bmatrix} \begin{bmatrix} b_i & b_j & b_k \\ c_i & c_j & c_k \end{bmatrix} \begin{bmatrix} V_i \\ V_j \\ V_k \end{bmatrix} \quad (2.11)$$

e. Equation (2.11) is seen to be of the form

$$YV = 0 \quad (2.12)$$

in which V is a (3x1) column of vertex potentials, 0 is a (3x1) null matrix and Y is a (3x3) matrix whose elements are given by

$$Y(r,s) = \frac{\epsilon}{4A}(b_r b_s + c_r c_s) \quad (2.13)$$

where r and s take on the values of i, j and k. This is seen to be symmetric, that is $Y(r,s) = Y(s,r)$.

It is easily established, by direct computation from the expressions in appendix 1, that

$$\begin{aligned} b_i + b_j + b_k &= 0 \\ c_i + c_j + c_k &= 0 \end{aligned} \quad (2.14)$$

As a consequence, the sum of the elements in any row of (2.12) is zero.

Thus, minimization of the functional in the tetrahedron has resulted in a relation among the vertex potentials given by equation (2.12). The elements of the Y matrix are given by (2.13). The matrix is symmetric and any row, or column, sums to zero.

C. Equivalent Electric Network

Thus far there has been obtained the matrix relation

$$YV = 0 \quad (2.15)$$

Where the matrix elements are functions of geometry alone, they are symmetric and the sum of any row, or column, is zero. The Y matrix is (3x3) for the triangular, two-dimensional element. The V matrix is a column of three vertex potentials in two dimensions.

The result was obtained by considering the triangular element with known vertex locations. Within each element, a linear variation of potential with distance was assumed. Introduction of this assumption into the Euler-Lagrange functional and minimizing it with respect to the vertex potentials produced the relation among vertex potentials which satisfied Laplace' equation within the element subject to the assumption of linear variation.

Equation (2.15) must be written for every element into which the field has been divided, suitable boundary potentials assigned to certain vertices and then the entire assemblage of equations must be solved for the unknown potentials.

But equation (2.15) also is characteristic of an electrical network in which nodes located at the element vertices are interconnected by admittance given by

$$y(r,s) = -Y(r,s) \quad (2.16)$$

Since the sum of elements in any row, or column, is zero, no admittance is connected node to ground at any vertex.

The electric network representation is valuable because network analysis has been so widely studied in the past. For instance, the network representing one element is easily combined with the network of an adjacent element merely by adding matrix elements between common nodes. Also, the imposition of fixed potentials from external sources to boundary nodes is easily accomplished and solution for the unknown node potentials is illustrated by

$$\begin{bmatrix} UU & UK \\ KU & KK \end{bmatrix} \begin{bmatrix} V_u \\ V_k \end{bmatrix} = \begin{bmatrix} 0 \\ I_u \end{bmatrix} \quad (2.17)$$

In this equation u means unknown, k means known, V means node potential and I means current required from the external source required to produce the known potentials. Normally I_u is not required in the field problem. Thus only the upper partition of (2.17) is required

$$UU V_u + UK V_k = 0$$

$$V_u = -UU^{-1}UK V_k \quad (2.18)$$

Solutions of this type are a common procedure in network analysis.

Further, the electric network analog facilitates treatment of equipotential nodes, i.e., those nodes on conductor surfaces not connected to fixed potentials ("floating"). In the electric net, all such nodes are combined and treated as one node. Admittances from any node not on the surface to any surface node are combined into one admittance by addition.

D. Specialization to Rectangular Grids

1. Two dimensions

Figure 2.1 illustrates a uniform rectangular grid. Node numbers have been assigned arbitrarily and the six triangles surrounding node 7 have been designated by Roman numerals. It is desired to determine the admittance branches within the elementary triangles. Triangle I in figure 2.1 is re-drawn in figure 2.2. Direct calculation of the relations in appendix 1 and their use in equation (2.13) give the results

Figure 2.1 Regular Rectangular Grid in Two Dimensions

Figure 2.2 Triangle I from the Regular Grid

Figure 2.3 Admittance Network Representing Triangle

$$Y(7,4) = -\epsilon\rho/2 \quad Y(7,6) = 0 \quad Y(4,6) = -\epsilon/2\rho \quad (2.19)$$

$$\text{where } \rho = \Delta y/\Delta x \quad (2.20)$$

This means that triangle I can be represented by the electrical network of admittances shown in figure 2.3.

It is intuitively evident, and proved by calculation, that every triangle in figure 2.1 has an admittance network similar to figure 2.3. That is, there is admittance $\epsilon\rho/2$ from the right-angle vertex horizontally and admittance $\epsilon/2\rho$ vertically. No admittance is connected diagonally.

2. Two dimensions with axial symmetry

Figure 2.4 shows the regular, rectangular, two-dimensional grid with axial symmetry in which the axis of symmetry is horizontal and the radius from that axis is shown vertical. Again, nodes are numbered arbitrarily and six triangles surrounding node 7 are given Roman numeral names.

Figure 2.5 shows triangle I with $i = 7$, $j = 4$ and $k = 6$. The two-dimensional relations in appendix 1 are applicable provided that results for each triangle are weighted by the radius of the centroid of the triangle. The centroid has the radius

$$r_c = (r_i + r_j + r_k)/3 \quad (2.21)$$

Therefore the modified results of appendix 1 for triangle I in figure 2.6 are

$$Y(7,4) = -(R + \frac{\Delta r}{3})\epsilon\rho/2$$

$$Y(4,6) = -(R + \frac{\Delta r}{3})\epsilon/2\rho \quad Y(7,6) = 0 \quad (2.22)$$

$$\text{where } \rho = \Delta r/\Delta z \quad (2.23)$$

Matrix elements with these values are represented by the electrical network in figure 2.6.

E. Solution of $YV = I$

1. Relation of boundary conditions to the network equations

It has been shown that the field problem has been transformed by use of the finite element method to an equivalent, lumped parameter network problem. The network is specified in terms of its admittance matrix.

Thus the problem is now to solve $YV = I$ subject to the existing boundary conditions:

Figure 2.4 Regular Rectangular Grid in Axi-Symmetric Two Dimensions

Figure 2.5 Triangle I from the Axi-Symmetric Grid

Figure 2.6 Admittance Network Representing Axi-Symmetric Triangle

- a. Electrodes with known potentials are represented in the matrix by external sources which apply the known voltages from reference to the nodes. The resulting current which flows in these external sources is unknown.
- b. Cut surfaces, due to limitations on problem extent or due to symmetry in the geometry, are represented by non-connection of the network across the cut surfaces.
- c. Floating electrodes, with constant but unknown potential, are represented by one network node to which are connected all network branches of all element nodes located on the electrode surface.
- d. Changes in permittivity are automatically included in the branch admittance values. It is important that each element consist of only one material, that is, that no element extend across a material boundary.

2. Matrix expression

The problem is now expressed as

$$\begin{array}{|c|c|} \hline A & B \\ \hline C & D \\ \hline \end{array} \begin{array}{|c|} \hline V_u \\ \hline V_k \\ \hline \end{array} = \begin{array}{|c|} \hline I_k=0 \\ \hline I_u \\ \hline \end{array}$$

$\begin{array}{cc} Y & V & I \end{array}$

The matrix equation has been partitioned between the known and unknown voltages. As stated above, the currents are unknown into nodes with known voltages. On the other hand, the unknown voltages to be determined by solution are known to have zero current being injected into them by external sources.

The solution desired is seen to be

$$AV_u = -BV_k$$

since the unknown source currents are not required. The actual solution process will be postponed until further characteristics of the partitioned matrix, A, are described.

The main limitation on such problems is the size of the A matrix which can be contained in existing hardware and run costs.

F. Splitting the problem

1. Reduction of matrix size

In order to accommodate larger matrices, the problem (network) can be split into two parts. In one part (b) total solution is not necessary. It is not necessary to know the unknown potentials within this part. Thus the matrix equation can be partitioned further to separate the a and b regions.

$$\begin{array}{|c|c|c|} \hline E & F & G \\ \hline H & I & J \\ \hline K & L & M \\ \hline \end{array}
 \begin{array}{|c|} \hline V_{ua} \\ \hline V_{ub} \\ \hline V_k \\ \hline \end{array}
 =
 \begin{array}{|c|} \hline I_a=0 \\ \hline I_b=0 \\ \hline I_u \\ \hline \end{array}$$

Since I_u is not required, the bottom partition is ignored. The top two partitions become

$$EV_{ua} + FV_{ub} + GV_k = 0$$

$$HV_{ua} + IV_{ub} + JV_k = 0$$

Elimination of V_{ub} between these two equations gives the result

$$(E - FI^{-1}H)V_{ua} + (G - FI^{-1}J)V_k = 0$$

The solution of this equation for V_{ua} is desired. It is seen to be reduced. That is, it is of order E which is much smaller than E, F, H and I together. This is referred to as a Kron reduction.

One price to be paid for reducing the problem size is the necessity for inverting the matrix I. Another cost is that the matrix started out sparse and banded but in the reduction process it becomes full.

2. H. E. Brown's technique

H. E. Brown in Solutions of Large Networks by Matrix Methods, Wiley 1975 has described a technique for developing the equivalent matrix one step at a time. This is in contrast to the implied method given above where the entire matrix was formed and then reduced. Brown's technique is illustrated for an impedance matrix rather than for an admittance matrix. The transition is trivial.

The procedure is as follows:

- a. Form the admittance matrix for partial networks, one branch at a time, for nodes lying in region b (discardable) of the problem. This is easily done. For any branch being added to the net, the admittance of the branch is added to the diagonal positions in the matrix corresponding to the nodes at the ends of the branch; and the branch admittance is also subtracted from the off-diagonal positions in the matrix corresponding to the intersections of the branch end nodes.
- b. When all branches connected to one specific node, n , are incorporated in the network, the result is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V \\ V_n \end{bmatrix} = \begin{bmatrix} I \\ I_n = 0 \end{bmatrix}$$

In this expression, it is assumed that node n is the node to which all admittance branches have been completely assembled. It is further assumed that n is not on an electrode.

- c. Since it is in the discardable region of the problem, the partitioned equations can be written

$$AV + BV_n = I$$

$$CV + DV_n = 0$$

V_n can be eliminated from these two equations

$$(A - BD^{-1}C)V = I$$

This is seen to be a single-axis Kron reduction. It is easily performed because D is one element only.

- d. The process is continued, adding admittance branches one-by-one and eliminating unwanted nodes in the discardable region as their incorporation in the Y-matrix is completed. Only nodes which are on electrodes are retained.

- e. It should be noted that nodes connecting region b (discardable) to region a (the window), i.e. nodes on the window boundary, are not fully connected to the Y-matrix by admittance branches in b only. Therefore they are not discarded.
- f. Completion of this process in region b, produces a matrix which contains only nodes which are on electrodes or on the surface separating b from the window region, a. Thus, it is an "equivalent" matrix which represents b as seen from a. The effect of the nodes discarded are still present in the matrix. However, it is not possible to solve for potentials at those discarded nodes directly from the equivalent matrix.

The advantage of the building process is that the matrix increases in size only to a certain limit which is considerably smaller than if all nodes in the discardable region were incorporated before any reduction is performed. The final equivalent matrix is somewhat smaller than the working matrix limit.

The disadvantage of the process is that the reduction fills up the matrix. That is, the sparse character of typical admittance matrices is eliminated.

Once the equivalent matrix has been formed, it can be stored for future use in combination with solutions of various configurations within the window. One restriction is that conditions within the window must agree with conditions in the discardable region at the separation surface between them.

Thus, when desired, an admittance matrix for the interior of the window is formed with no reductions. Then it is combined with the equivalent matrix to give the final problem formulation.

G. Solution of the combined matrices

The solution can be accomplished by some form of iteration or relaxation; or alternatively and directly by some form of elimination. Specifically, in this program, successive over relaxation is the technique employed.

I. The Program

A listing of the program is included as appendix 3.

A. Program Philosophy

1. Two-pass program

It was decided to write a "two-pass" program rather than to write two separate programs. That is, it was decided to write a program which, under proper input data control, was capable of being run:

- a. to form an equivalent admittance matrix at the "window" boundaries for a region outside the window; or
- b. to form the admittance matrix within the window, combine it with an equivalent, if present, and solve the resulting equation set.

This was done because there are large areas of the same programming in the two program parts, equivalent formation and window solution.

The common areas are those in which a regular grid is defined, body contours are described, the grid is distorted to match the contours and admittance branch values are calculated.

The result is illustrated in Flow Chart III.A.1. The subroutines indicated there are described briefly below and in more detail in Section III.B.

2. The "window" terminology

Because of its appearance, the region to be retained for final solution is called the "window". See Figure III.A.1. in which Region I is the window. Also in the figure Region II is part of the overall problem, the influence of which on final solution is required, but in which detailed results are not necessary.

It is important to note that the grid definitions specified by data input in Section 3. below must be consistent for the equivalent Y-matrix of Region II and for the window Y-matrix of Region I. That is, the positions of the regular grid lines must be the same for the two runs: pass 1 = equivalent formation and pass 2 = window solution.

3. Subroutine CODING

This subroutine is common to both the formation of the equivalent and to the solution of the window.

- a. The function of the subroutine is to read enough control data so that the proper first pass or second pass run is performed. It also

Flow Chart III.A.1

MAIN Program

Decomposition into Two Regions

Figure III.A.1

reads data which are used commonly for the two passes. These data are the regular grid definitions and the geometric descriptions of the problem bodies (contours).

Flow chart III.A.2. shows the basic organization.

The data input format used is that existing in a program (FLD6) left at RPI by Prof. O. W. Andersen in the 1970's. First, a regular grid definition is entered by specifying, along each of the two axes, locations at which grid spacing changes and also the sequence number of the grid line at that location. Thus, on each axis, there are specified a number of zones and how many equal spaces are in each zone. Second the shapes of the bodies in the problem are described by closed polygons composed of straight line segments connected at vertices. The input data for these contours are a series of (x,y) co-ordinates for the vertex locations in the order met as the contour is traversed. Closed contours are entered by entering the last vertex equal to the first. In addition to the vertices, each contour (body) has entered for it the value of permittivity if it is a dielectric body, or the voltage value for a fixed potential electrode, or the fact that the electrode is "floating".

b. After problem definition data entry, the subroutine moves grid nodes (as defined by the regular grid definition) in the vicinity of the body contours to the contours. That is, the regular grid is distorted to the given geometric contours.

First, regular grid nodes closest to the vertices are moved to the vertex locations for all vertices entered. Then, for each line segment between successive vertices, the regular grid nodes closest to the lines are moved perpendicular to the line until they lie on the line.

c. As the regular grid nodes are being moved to their distorted positions, a code value is assigned to each node position. The node is designated by its undistorted position in the regular grid. To each of these positions is assigned a value in the MVERT array.

Each value in MVERT has 5 digits, call them a b c d e. Position a is used to indicate whether the node (vertex) can be eliminated during computation of the equivalent. It can take two values: 0 for reduceable, and 1 for not reduceable. Only electrodes will have a = 1 assigned.

Positions b c indicate the contour (body) to which the node belongs. Contours are numbered in sequence as they are read.

Position d is the irregularity code and indicates whether the node has been moved during grid distortion. Two values are possible: 0 for not moved and 1 for a moved node.

Position e is the availability code, as originally used in the FLD6 program. It is used in the line fitting stages - 0 for available, 1 for unavailable in some program parts, and 2 for absolutely unavailable.

Flow Chart III.A.2

Subroutine CODING

d. It is shown in the theory part of this report that the admittance branches for any triangular element depend on the permittivity and the geometry of the element. In an element where none of the three triangle vertices have been moved from their regular grid positions, it is found by direct computation that the y-branch representing the hypotenuse of the right triangle has a zero value. Furthermore, the other two branches retain their values no matter how the element is rotated, and also even if the element is reversed as by reflection in a mirror.

Therefore, for regular elements, the y-branches need be computed only once for each regular zone in both axes. This is done in the CODING subroutine and the geometric factors of the y-branches are retained for future use during Y-matrix formation.

4. Subroutine EQUIV

The purpose of this subroutine is to form the admittance matrix representing the discardable area of the problem (Region II in Figure III.A.1) as an equivalent matrix at its common nodes interconnecting it with the window. That is, it is an equivalent Y-matrix representing Region II as seen from Region I.

Flow chart III.A.3. shows the basic organization. The process is as follows:

a. The grid is considered block by block for the entire program area. The start is at the block closest to the origin. Next block is closest along the x-axis until an x-row is formed. Then y is incremented, x is reduced to initial value and the second row is formed. This proceeds until all rows have been included.

b. As each block is considered in turn, it must be divided into triangles. This is done either by a diagonal from upper left to lower right (UL-LR) or from lower left to upper right (LL-UR). The choice of which diagonal is used is on the basis of which divides the block into triangles of most nearly equal area.

c. When the triangles for a block have been specified, the admittance branch values are calculated for the geometric locations of the triangle vertices by using the expressions for the b and c variables derived in the theory section of this report. If a triangle is regular, in the sense that none of its vertices have been distorted from their regular grid positions, it is only necessary to recall the standard admittance branch geometric factors from storage.

The permittivity of a triangle is determined by finding within which body the centroid of the triangle is located. Once determined, the y-branches are modified by this factor.

d. When the y-branches for a block have been calculated, they are added to the total Y-matrix of the equivalent. This is done by standard

Flow Chart III.A.3

Subroutine EQUIV

modification algorithms for adding branches and nodes to a network. The resulting Y elements are stored in array COEFF1. Since Y is symmetric, it is only necessary to retain the lower triangle and principal diagonal terms. Therefore, COEFF1 is a linear array representation of those elements of the Y-matrix.

e. When the branches of a block have been added to Y in this manner, it is frequently found that one or more of the block corners are now complete in the sense that all y-branches connecting them to the remainder of the network have been included in Y.

When this occurs, if the node is not important for the final window solution, the Y-axis corresponding to that node can be "reduced out" of Y by a Kron reduction as explained in the theory section of this report.

All internal dielectric nodes in the discardable region fall in this category of "not important for final window solution". The only nodes which must be retained are those on the window boundary and those which are on electrodes.

f. The final result of this process is a multi-port Norton equivalent Y-matrix for the discardable region as seen from the window. Its elements are stored in the linear array COEFF1. These are read out to a data file, DATA2.

Data required to combine the equivalent Y with a future window Y are read out to a data file, DATA1. Such data are:

the size of the matrix (number of window boundary nodes plus number of electrodes in the discardable region);

the node number associated with each axis in the Y-equivalent (the axis identifications);

the voltages known for each of the electrode nodes.

5. Subroutine SOLUTN

The purposes of this subroutine are to form the admittance matrix for the window area of the problem, to combine it with the Y-matrix equivalent of the discardable area of the problem and to solve the resultant combination for potentials at nodes where voltages are not specified.

Flow chart III.A.4, shows the basic organization. The process is as follows:

a. Window blocks are considered one by one in a scheme similar to that used in EQUIV. That is, the block considered first is the one closest to the origin. Then blocks along the x-direction are considered until a row is formed. Then an adjacent row in the y-direction is considered block by block in the x-direction until all window blocks have been included.

Flow Chart III.A.4

Subroutine SOLUTN

b. As each block is considered, the grid nodes at its corners have assigned to them a voltage which is the node's known voltage, if specified by the problem boundary conditions, or an initial guess at the node's final solution value. Since the solution is iterative, solution time is shortened by more accurate guesses at final values. In the absence of other values, the program assigns the input value of AVEVOL to every node having an unknown voltage value. This is the starting point for the iterative solution.

c. Then, for each block, the steps described for EQUIV in Sections 4.b, c and d are performed. That is, the block is divided into triangles, the admittance branch values are computed for both triangles and the Y-matrix elements are formed.

d. In this case, the Y-matrix elements are stored in the two-dimensional array, COEFF. Although the Y-matrix for the equivalent is very full as a result of the reduction process, the Y-matrix for the window elements alone is sparse and tightly banded. Since any window node can be connected only to the eight surrounding nodes, there are possible only nine distinct Y-elements in each row of the matrix: the diagonal term and eight off-diagonal terms in the lower triangle. Therefore only these nine possible values are stored in COEFF. An array, ITRI, is used to indicate which other nodes are connected to the diagonal node in question.

e. When the window Y-matrix is complete, the Y-matrix of the equivalent is read back from DATA1 and DATA2 with the Y-elements stored in TEMP.

f. Solution of the combined system is by successive over relaxation (S.O.R.) for potentials at nodes where the voltages have not been fixed by boundary conditions of the problem.

g. When node potentials have been found, it is seen that each triangular element has a uniform electric field value, due to the assumption of linear space variation of potential in an element. Therefore, each triangle in the window is examined, its electric field strength calculated, and the maximum field for each different dielectric is retained along with the location of the maximum field.

h. Finally, the (x,y) co-ordinates and potential at each window node are written as output.

6. Subroutine PLOTY

The purpose of this subroutine is to use the output of Section 5.h. and geometric input information to plot the problem body contours and equi-potential lines. This subroutine is hardware dependent on the specific computer installation and must be re-written for any given computer configuration. The program given is for a Prime 500 installation using a Dynagraphics plot package.

B. Program Organization

1. Division into sub-programs

There are twelve sub-programs as indicated in the table below. Also indicated are the number of lines in the listing, the page of listing on which the sub-program begins and the page of listing on which the sub-program ends.

<u>Sub-Program Name</u>	<u>Number of Lines</u>	<u>Starts on Page</u>	<u>Ends on Page</u>
MAIN	30	1	1
CODING	608	1	12
EQUIV	333	12	18
SOLUTN	409	18	25
PLOTY	221	25	29
PERTRI	30	29	30
FSTR	20	30	30
INTER	33	30	31
DIGIT	82	31	32
SUBD	4	32	32
NODFT	5	32	32
NBOUFT	11	32	32

The listing includes only those sub-programs written for this program. Other sub-program calls, e.g. for plotting purposes, which are device dependent are not included here.

Table III.2 indicates the location in the program and from which sub-program the various sub-programs are called.

2. Sub-Program MAIN

Flow chart III.A.1 gives details.

This contains only calls to the major subroutines, CODING, SOLUTN, EQUIV, and PLOTY. No executable instructions are included in it.

Subroutine CODING reads the input data, fits the grids to the contours, and assigns the codes to the vertices on the grid.

Subroutine EQUIV computes the equivalent at the boundary of the window.

MAIN (p.1)

p.1 - CODING (p.1)

p.2 - DIGIT (p.31)

p.3 - DIGIT (p.31)

p.4 - DIGIT (p.31)

p.11 - INTER (p.30)

p.1 - EQUIV (p.12)

p.13 - NBOUFT (p.32)

p.15 - INTER (p.30)

p.16 - NODFT (p.32) 6 times

p.16 - NBOUFT (p.32) 3 times

p.17 - NODFT (p.32) 5 times

p.18 - NODFT (p.32)

p.1 - SOLUTN (p.18)

p.21 - INTER (p.30)

p.21 - PERTRI (p.29)

p.24 - FSTR (p.30)

_____ p.30 - SUBD (p.32)

p.1 - PLOTY (p.25)

Table III.2

Location of Sub-Program Calls

Subroutine SOLUTN computes the potential at every grid point of the window, and the maximum field strength for every permittivity present.

Subroutine PLOTY draws the equipotential lines on the screen of the terminal.

3. Sub-Program CODING

Flow chart III.A.2 gives an over all picture of the process.

- a. This subroutine performs five basic duties. The following is a description of those duties.
 - 1) Reads all of the input data for the program, except the coordinates of the regular grid points. This is done by subroutine DIGIT, and will be explained later.
 - 2) Positions each one of the grid lines.
 - 3) Computes the standard admittances for each one of the different grid densities.
 - 4) Adds the minimum radius to all the input points.
 - 5) Fits the grid to the contours, and assigns the proper code to each one of the vertices in the grid. This is performed in four steps:
 - a) When a point used in the definition of a contour lies in the path of a straight segment joining two points used in definition of a second contour, the first point is made part of the set of points defining the second contour, thus the segment to which reference was made will be broken down into two segments.
 - b) The contours are taken one at a time, and the grid is made to coincide with the points defining the contours.
 - c) Once all the defining points have been fitted, they are joined by imaginary segments, then the grid nodes closest to these segments are forced to lie on the path of the segments.
 - d) In stages b and c, all the vertices involved are coded properly; the fourth stage assigns the proper codes to the vertices internal to the closed contours.
- b. This subroutine was originally developed by O. W. Andersen, and used in the FLD6 program. All the algorithms and most of the

variables are still the same. Changes were made to accommodate a new coding system and to include the computation of the standard admittances. These changes are explained next. Their locations in the final program are indicated.

c. Computation of the standard admittances

Referring to Fig. III.B.3.1, there are different spacings in both directions defining the equally spaced zones indicated by the parentheses. At the intersection of each zone a special type of rectangle is defined. For each type of rectangle standard admittances are computed.

Figure III.B.3.1

Vertical and Horizontal Zones

For a given vertical zone, all the horizontal zones are scanned, and the corresponding admittances computed. (See Flow chart III.B.3.1)

d. Vertex Coding

The array MVERT contains a code for each grid node. The array is two dimensional; the first subscript indicates the horizontal line number; the second indicates the vertical line number. Thus each vertex is specified as the intersection of two lines of the regular grid as if the vertex had not been moved (even though it may have been moved).

The contents of each code entry is a five digit number in which the various digits have meanings as specified in section III.A.3.c.

e. Irregular points

The position of the vertical lines is defined by their X co-ordinate, and the position of the horizontal lines is defined by their Y co-ordinate. The arrays storing those positions are XPOS and YPOS respectively.

A point, defined by the intersection of two lines will have the same X co-ordinate as the vertical line, and the same Y co-ordinate as the horizontal line. Therefore the storage of the co-ordinates for every vertex in the grid is unnecessary. Only the co-ordinates of those vertices moved from their original positions during the contour fitting process have to be stored, along with a code that relates them to the storage position.

Arrays AUX and MAUX contain the (X,Y) co-ordinates and the grid vertex number, as calculated from

$$NVM = (N-1) * LNVER + M$$

where N is the horizontal line number

and M is the vertical line number of the vertex that has been moved.

4. Sub-Program EQUIV

Flow chart III.A.3 gives an overall picture of the process.

When computing the equivalent the grid is processed from left to right and from the bottom up; rectangles are considered one at a time.

Three major blocks can be identified in this subroutine:

- a. computation of the admittances inside the rectangle;
- b. location of the admittances in their proper matrix places;
- c. elimination of those nodes that need not be retained.

Each one of these blocks is subdivided into several "action" blocks.

- a.1) Code decompositions for each one of the four vertices in the rectangle;
- a.2) Assignment of electric network node number;
- a.3) Division of the block into triangles;
- a.4) Assignment of the proper permittivity to each one of the resulting triangles;
- a.5) Computation of the admittances inside the triangles.
- b.1) Identification of the proper matrix position for the electric nodes involved;
- b.2) Summation of the six sub-matrix elements involved.

- c.1) Computation of Kron reduction;
- c.2) Single axis elimination;
- c.3) Matrix rearrangement.

d. Finally, two files are created: DATA1 and DATA2.

DATA1 contains the information necessary for the proper connection of the equivalent to the window.

DATA2 contains the matrix elements representing the network connections of the equivalent.

Each of these action blocks will be explained in detail and a flow chart will be shown for each one of them. The arrays and variables to which reference is made appear in the proper lists.

a.1) Code decomposition

Fig. III.B.4.1 shows two "boxes." A) is regular and B) is irregular. Both have the corners numerated counterclockwise. A four cycle "DO" loop finds the grid position (M,N), the contours to which it belongs, and whether it has the original co-ordinates or not, for each vertex.

Whenever a box is inside the window, the program skips it.

a.2) Assignment of the electric network node number

Each subindex of the array NODE is an electric node number and the variable stored is the grid node position to which the electric node was assigned.

If the vertex is on the window boundary its grid position in the window with a minus sign is used to establish the correspondence.

If the vertex is on or in a contour that is an equipotential the electric node assigned to it will be the same as that of the first vertex encountered on the contour.

After the electric node is assigned, the co-ordinates of the vertex are found. If the vertex has not been moved they are obtained from the lines positions, otherwise a search through the moved points is conducted until the proper one is found.

a.3) Division of the block into triangles

The boxes are divided into two triangles by a diagonal. The diagonal is systematically drawn from bottom to top and from left to right (LL-UR). However when a box is irregular the diagonal is drawn in such a way that the resulting areas are as close in magnitude as possible.

REGULAR BOX

A)

IRREGULAR BOX

B)

DIAGONAL TYPE 1

C)

DIAGONAL TYPE 2

D)

E) STORAGE OF
EQUIVALENT
ADMITTANCES

Figure III.B.4.1
Box Definitions

Referring to Fig. III.B.4.1 C,D, the two possible diagonals are shown and the four possible areas also.

In the example shown, areas 1 and 2 give a difference of areas smaller than 3 and 4.

The choice of diagonal is overridden when the diagonal is part of contour definition in which case, it is forced to follow the contour.

a.4) Permittivity assignation

After the triangles are defined, the centroid of the triangle is found. Codes K1, K2 and K3 are filled with the proper corner that defines the lower triangle so the co-ordinates are accordingly chosen.

$$\bar{X} = (X(K1) + X(K2) + X(K3)) / 3$$

$$\bar{Y} = (Y(K1) + Y(K2) + Y(K3)) / 3$$

The codes K1, K2 and K3 are not necessarily coincident with corners 1, 2, 3 and 4. All their possible values are shown in Fig. III.B.4.1 C,D.

The co-ordinates of the centroid are given to subroutine INTER which tells what contour they are internal to, if any, and from that information the permittivity is obtained.

a.5) Computation of the admittances

If the box is regular we search for the precalculated admittances of the grid zone where the box is. Admittance AB connects corners K1 and K2; admittance AD connects corners K1 and K3, and admittance BD connects corners K2 and K3. If the diagonal is from bottom left (LL-UR), then YAB and YAD are the vertical and horizontal admittance branches respectively. When the diagonal is reversed (UL-LR), YAB and YAD are the horizontal and vertical admittance branches respectively.

The irregular box requires a calculation of YAB, YAD and YBD. However, these values are obtained directly from expressions given in the theory section from the appropriate b and c values which are dependent only on the vertex locations.

Codes NOD1, NOD2, NOD3 are the electric nodes associated with each corner of the triangle. They correspond one to one with K1, K2, K3. If any of these nodes is the same, the admittance connected across the corresponding corners is made equal to zero.

If there is axial symmetry, FLAXI > 1.0, the admittances are multiplied by the distance from the center of the triangle to the axis of symmetry.

c.1) Computation of KRON reduction code (NRED)

After a box is processed the node on corner #1 has all of its electrical connections completed, unless it is an electrode or a floating conductor, in which case that assertion is not necessarily true.

Such a node is normally reduced out of the matrix unless one or more of the following conditions is present:

- i) it is on the window boundary;
- ii) it is an electrode;
- iii) it has not completed all of its electrical connections.

Each time one of these conditions is encountered, a code, NRED, is increased by one. So whenever NRED is greater than zero, the node is not reduced.

If the box that has been processed is on the right hand side boundary of the grid, the reduction logic is repeated for the node on corner #2.

If the box is on the top boundary of the grid, the reduction logic is repeated for the node on corner #4.

If the box is the last on the grid, the reduction logic is performed for all the nodes in the box.

c.2) Single axis elimination

The algorithm for the elimination of the axis K from a M x M matrix is:

```
for J = 1,M
for I = 1,M
  (but J ≠ K and I ≠ K)
```

$$Y(J,I) = Y(J,I) - \frac{Y(J,K) * Y(K,I)}{Y(K,K)}$$

However, due to the symmetry of the matrix, it is only necessary to modify up to the diagonal element of each row.

c.3) Matrix re-arrangement

Once a node is eliminated, the nodes of the matrix whose order is greater than that of the reduced, are decreased by one, the elements of the matrix below the reduced row are shifted one position upward and the elements to the right of the reduced column are shifted one position to the left.

Fig. III.B.4.2 explains the procedures graphically.

Matrix before elimination
of node 2

Matrix after elimination
of node 2

Elements to the right are
relocated

Elements from below are
relocated

Re-assignment of the grid
position to the nodes.

ILLUSTRATION OF MATRIX REARRANGEMENT

Figure III.B.4.2

- d) Writing of the equivalent matrix and the information needed by the subroutine SOLUTN

Two files are created, DATA1 and DATA2.

DATA1:

This contains the information necessary for the subroutine SOLUTN to use the equivalent properly. It contains the total number of nodes in the equivalent (JFIN) and the negative of the grid position that the node will have in the window. If the node is one that does not appear in the window then a fictitious position is assigned to it. This fictitious position starts with the total number of grid points (vertices) in the window plus one, and it is increased by one every time a similar node appears. SOLUTN understands this node as being associated with an imposed potential, in fact they are nodes representing electrodes outside the window, and the potential associated with them is provided next in the program.

DATA2:

Contains the elements of the final equivalent matrix, they are written as a linear array following a convention indicated in Figure III.B.4.1.E).

This user has nothing to do with these files; his only duties are to make sure that they belong to the right equivalent. These files appear by subroutine SRCH\$\$, which is a PRIME system low level subroutine. The information listed in Table III.3 is requested by the subroutine.

5. Sub-Program SOLUTN

Flow chart III.A.4 shows the overall process.

a) Summary of differences from EQUIV

The same general procedure of EQUIV is followed here. The main differences are pointed out next.

- 1) Electric network node numbers and grid vertex numbers are one and the same for all the grid points, so no special resemblance between node number and grid position is necessary. The only code assigned is to indicate whether a node (grid point or vertex) is an electrode or not.

- 2) The admittances are computed as in EQUIV. They are stored in the two dimensional array COEFF(900,9), one row for every node and up to nine elements per row.

- 3) The resultant system of equations is solved by an iterative technique (S.O.R. = successive over relaxation).

Action key: k\$ WRIT for writing
 k\$ CLOS. for closing the file

File name: DATA1, DATA2

Length of the name: 5

File Utility Unit: 4,5 (this is equal to FORTRAN logic unit - 4)

Type of file: sequential (0) (numbers are written one after another
 in sequence)

NCOD: error code in the use of the subroutine

(See PRIME FTN Manual, Sec. 20, for more detailed explanation.)

Table III.3

Information requested by PRIME subroutine SRCH\$\$

4) Maximum field strength for every permittivity present in the problem is computed.

b. Potential code assignation

Array NODE(904) contains a code for the 900 possible vertices in the grid plus four (4) electrodes outside of the window. 0 is assigned to a vertex if it is not an electrode and 1 if it is. There is a special code NCOD(4) assigned to each vertex in a box. This code is the negative of the corner number if it is not on a code 3.0 (floating conductor) contour, otherwise the contour number is assigned to it. This code makes possible the handling of floating conductors that are an open contour. (See flow chart III.B.5.1.)

c. Storage of permittivity for the triangles and triangle vertex

This is done for two reasons:

- 1) field strength is computed for each individual triangle, and storing that information speeds up the process;
- 2) equipotential lines are plotted by interpolation on each one of the triangles (see subroutine PLOTY).

d. Computation of admittances for open floating conductors

A special code (NCOD, see Sec. III.B.5.b) is assigned at the beginning of the subroutine to each corner of the box. These codes are compared, and if they match, a permittivity of 9000 is assigned to the admittance connecting the matching nodes.

e. Assignation of the pointers, and storage of matrix elements

In an admittance matrix, the element $Y(i,j)$ for $i \neq j$ is the negative of the branch admittance between i and j .

In the electric network representation of the field problem as derived from the finite element method, each grid node can be connected by branches to a maximum of eight other nodes. See figure III.B.5.1.A) which shows a central point E surrounded by the eight nodes A to D and F to I. These are the only nodes to which E can be connected by an admittance branch.

Therefore in the matrix row corresponding to node E, there will be only eight, non-zero, off-diagonal elements. The diagonal element is the self-admittance of the node.

In order to conserve storage space, only the nine possible non-zero elements are stored in a row of the array COEFF(900,9). The Y-matrix elements corresponding to the connections of node E to grid nodes A to D and F to I are stored in array positions 1 to 8 which correspond to the numbered locations in figure III.B.5.1.A). The ninth position stores the diagonal element value. See figure III.B.5.1.B).

Possible connections to node "E"

A)

	1	2	3	4	5	6	7	8
ITRI	D	A	B	C	E	I	H	G

	1	2	3	4	5	6	7	8	9
COEFF	YED	YEA	YEB	YEC	YEF	YEJ	YEH	YEG	YEE

POINTER ARRAY ITRI
 STORAGE ARRAY COEFF
 POINTERS FOR NODE "E"

B)

Figure III.B.5.1

ILLUSTRATION OF POINTER ARRANGEMENT

But now a pointer must be stored to indicate that position 1 for node E contains the element connection to node D. Similar values for the remaining seven surrounding nodes are also required. This is done in array ITRI(900,8). See figure III.B.5.1.B).

The pointer and storage arrays are organized by means of a logic table. The nodes interconnected within a triangle are a function of the kind of diagonal drawn in the box. Table III.B.5.1 shows the organization of the pointers, and Fig. III.B.5.2 shows the four possible triangles.

f. Reading of the equivalent matrix

The first file open is DATA1, the total number of nodes in the equivalent (JFIN) is read; next the grid position of these nodes (NODE4). After the nodes are read their polarity is requested.

Negative polarity means that the node is on the boundary and positive polarity that it is outside the window.

Whenever a node is on the boundary and has a potential assigned to it (NODE = 1), its grid position is changed to positive. Otherwise its code (NODE) is set equal to 2. This is the way of knowing that the node has other connections (to the equivalent) than those inside the window.

If the node is outside the window, the total number of nodes present is increased by 1 and the voltage assigned to it is read.

The second file opened is DATA2, from JFIN, the total number of elements is computed

$$\text{NODE } 3 = (\text{JFIN} + 1) * \text{JFIN}/2$$

and then the elements are read from the file.

g. Solution of the system of equations

The array TEMP contains the equivalent elements, and the array COEFF contains the window elements.

As many iterations as specified by NMAX are done unless the tolerance wanted is reached first.

The nodes on the boundary are questioned, the ones that are electrodes are skipped over, otherwise the admittances connected to the node are multiplied by the corresponding potential and accumulated. The storage of the equivalent elements resembles that of Fig. III.B.5.3.A). The admittances are located 1 by 1 using the following process. The subscript of the array NODE4(J) indicates the node number in the equivalent matrix of the window node indicated by NODE4(J), i.e., if window node number is 10 and corresponding equivalent node number is 3, then NODE4(3) = 10. This makes possible locating the admittances

ORIENTATION OF TRIANGLES

Figure III.B.5.2

NODES CONNECTED	ARRAYS' COLUMN NUMBER				ARRAYS' ROW NUMBER
	TRIANGLE I	TRIANGLE II	TRIANGLE I	TRIANGLE II	
K1,K2	7	3	5	1	K1
K1,K3	1	5	7	3	
K2,K1	3	7	1	5	K2
K2,K3	2	6	8	4	
K3,K1	5	1	3	7	K3
K3,K2	6	2	4	8	
DIAGONAL # 1			DIAGONAL # 2		

TABLE III.B.5.1

LOGIC ARRANGEMENT OF THE POINTERS

A)

Additional Admittances

B)

Window Admittances

Equivalent window additional admittances for

Equivalent Node Window Node Boundary Nodes (Mutual) (Self)

J	Node4(J)	TEMP(K)	
1	1	K = 2,4,7,11,16	1
2	4	K = 2,5,8,12,17	3
3	10	K = 4,5,9,13,18	6
4	30	K = 7,8,9,14,19	10
5	7	K = 11,12,13,14,20	15

C)

Correlation Between Additional
Admittances and Boundary Nodes

Figure III.B.5.3

inected to a given boundary node inside the array TEMP, by using $K1 = (J-1) * J/2 + 1$ where the equivalent node number is J. This locates the first element of the row. The search for the remaining elements up to the diagonal is conducted column by column adding one to K1, and from the diagonal until the end of the row, adding the next column number minus one to the last number obtained.

The rest of the nodes are processed in a conventional manner. The eight columns of ITRI are searched for the node number, and the elements in COEFF corresponding to the column scanned are multiplied by the proper potential.

h. Field Strengths

After the potential at every vertex is obtained the field strength inside each triangle is computed and the maximum for each permittivity is stored. Subroutine FSTR computes them. It is discussed in a separate paragraph.

i. Printing of Potentials

Finally the potential, grid position and co-ordinates of each vertex is written out in file WINOUT.

6. Sub-Program PLOTY

Originally subroutine ELB was taken from the program FLD6. No modifications were made to the main algorithms. The changes were only on the calls to the plotting routines. The original calls were to "IBM CALCOMPLITER". Their format was changed for use with "DYNAGRAPHICS" on IMLAC terminals.

To save necessary space, the array COEFF is used to store the information originally encountered in arrays SEGM, XA, YA, VERT(1 & 2). The correspondence is:

	COEFF								VOLTS	IVERT	PERM
1	2	3	4	5	6	7	8	9	1	1,2,3	1
1	2	3	4	5	1	1	1	1	3	1,2,3	1
	SEGM				XA	YA	VERT		VERT	ITRI	TRI

Table III.B.6.1

Array Concordance Between ELB and PLOTY

The subroutine now plots the equipotential lines, problem identification, maximum potential, minimum potential and percent volts per line.

7. Sub-Program PERTRI

The calling parameters for this subroutine are:

K1ODG: kind of diagonal (1 or 2)
EPSI: permittivity
M,N,L: grid node numbers of triangle vertices

The triangle vertex numbers are stored in a particular way so that they can be handled by subroutine PLOTY. IVERT is the array where this information resides. Fig. III.B.5.2 shows the four types of triangles, and Table III.B.7.1 shows the logic table for the assignment of the vertex numbers to the locations in ITRI.

Permittivity is stored in array PERM.

Node Number	Location in Array IVERT			
	Triangle I	Triangle II	Triangle I	Triangle II
K1	2	3	2	2
K2	3	1	3	3
K3	1	2	1	1
	Diagonal #1		Diagonal #2	

Table III.B.7.1

Logic Table for Array IVERT

I3 is the triangle number and is increased by one every time the subroutine is called.

8. Sub-Program FSTR

This sub-program is taken from FLD6 (same name there).

It requests the triangle number, and returns the electric field in the x direction, in the y direction and the total electric field magnitude.

9. Sub-Program SUBD

This sub-program is taken from FLD6 (same name there).

It computes the area of a triangle when the co-ordinates are given.

10. Sub-Program INTER

This sub-program is taken from FLD6 (same name there).

It determines whether a point is internal to a closed contour or not.

The call parameters are:

I1 : the number of points in the contour;
LPNT: pointer to the place in array LINAC where the co-ordinates of the I1 points start;
X : X co-ordinate of the point in question;
Y : Y co-ordinate of the point in question.

The return value is:

INTOR: code, 1 for internal, 0 for not internal.

11. Sub-Program DIGIT

This sub-program is taken from program FLD6 (same name there).

It reads the (x,y) co-ordinates for input points.

The call parameters are:

NPNTS: number of x, y pairs to be read;
IN : FORTRAN logic unit number for input;
SORIG: scale of the co-ordinates being read.

12. Function NODFT

Computes the correct position of the (i,j) element, when only the lower half of the matrix is stored as a linear array, proceeding by rows.

The call parameters are the row and column numbers of the element, (i and j respectively).

The formula is:

$$POS = (i-1) * i/2 + j \quad \text{for } i \geq j$$

If j is greater than i, the wrong element is chosen, then

$$POS = (j-1) * j/2 + i$$

13. Function NBOUFT

Determines whether a grid point is on the boundary of the window.

Returns 0 if no, and 1 if yes.

Call parameters are:

M: vertical line number defining the vertex;
N: horizontal line number defining the vertex;
LHF, LHL, LVF and LVL: grid line numbers defining the
window.

Descriptions of Arrays

ARRAY NAME	CONTENTS GENERATED IN	DESCRIPTION OF CONTENTS
ADMAB(15)	CODING	Standard, vertically connected admittances.
ADMAD(15)	CODING	Standard, horizontally connected admittances.
AREA(4)	SOLUTN EQUIV	Area of each one of the four possible triangles inside a box.
AUX(500,2)	CODING	(x,y) co-ordinates of vertices which have been moved.
COEFF(900,9)	SOLUTN PLOTY	Elements of the matrix representing the window. Later used for co-ordinates, and equipotential segments in PLOTY.
COEFF1(18145)	EQUIV	Elements of the matrix during computation of the equivalent.
DATLN(70,3)	CODING	Permittivity, potential and code of each contour.
DIGLN(140)	DIGIT	Co-ordinates of the input lines and contour points.
IDENT(20)	CODING	Identification of the problem.
ILN(70)	CODING	Accumulated number of input points for the contours.
ITRI(900,8)	SOLUTN	Pointers indicating the connection of one node to others in the window.
IVERT(2000,3)	SOLUTN	Triangle vertices counterclockwise.
JVERT(6,4)	SOLUTN	Pointer that locates the correct connecting node into ITRI.
LINC(800)	CODING	Co-ordinates of the input points describing the contours.
MAUX(500)	CODING	Grid position number of the vertices which have been moved.

..VERT(79)	CODING	Vertex codes.
NCOD(4)	SOLUTN EQUIV	Special code used to identify the same electrodes in EQUIV and open floating conductors in SOLUTN where the admittances are computed.
NDATLN(70,2)	EQUIV	Grid position of the first point encountered on a conductive contour.
NIRR(4)	SOLUTN EQUIV	Irregularity code for each corner of the grid boxes.
NOD(4)	SOLUTN EQUIV	Electric node associated with each one of the four corners of a grid box.
NODE(900)	EQUIV SOLUTN	In EQUIV, it contains the new node number of the matrix being built. In SOLUTN, a code telling whether the node is an electrode or not.
NODE4(120)	SOLUTN	Position in the equivalent matrix of the boundary nodes.
NTRI(8)	INTER	Codes for deciding whether a point is internal to a closed contour.
NZH(15)	CODING	Last line number of a set of vertical lines equally spaced.
NZON(4)	SOLUTN EQUIV	Contours to which each corner of a grid box belongs.
NZV(15)	CODING	Last line number of a set of horizontal lines equally spaced.
PCHH(16)	CODING	Horizontal lines where new spacing begins.
PCHV(16)	CODING	Vertical lines where new spacing begins.
PERM(2000)	SOLUTN	Triangle permittivity.
POINT(400)	CODING	Grid position of input points for the contours.
TEMP(7260)	SOLUTN	Elements of the matrix representing the equivalent.
VOLTS(904)	EQUIV SOLUTN	Potential at nodes in the window.

X(4)	SOLUTN EQUIV	X co-ordinate of each of the four corners of a grid box.
XPOS(70)	CODING	X co-ordinates of the vertical grid lines.
Y(4)	SOLUTN EQUIV	Y co-ordinate of each of the four corners of a grid block.
YPOS(100)	CODING	Y co-ordinates of the horizontal grid lines.

Equivalence between arrays:

TEMP	7260	Positions	COEFF	8100	Positions	Perm. 2000 Positions
COEFF1			18145		Positions	

LIST OF VARIABLES

AVEVOL: Initial estimate of the solution; later becomes dummy variable.

CODE: Indicates type of body.

DELX: Grid spacing in X-direction.

DELY: Grid spacing in Y-direction.

EJ: Dummy variable, indicates distance.

EPS: Permittivity of a given contour.

EPSO: Relative permittivity if not specified otherwise.

EQLIN: Number of equipotential lines desired in output plot.

FLAXI: Code indicating flat or axial symmetry.

GAMMA: Acceleration factor in the iterative solution.

HLF: First horizontal line of the window.

HLL: Last horizontal line of the window.

IN: Logical unit for reading.

I6=ITOT: Number of vertices (grid points) on the grid.

JLAST: Highest electric node number reached.

JP: Number of points which no longer are at the original co-ordinates.

LINAC: Number of the contour in process.

LINTO: Total number of contours.

LNHOR: Number of horizontal lines in the grid.

LNVER: Number of vertical lines of the grid.

MAC: Dummy variable for the availability code.

MAXI: Maximum number of iterations in solution.

MIR: Dummy variable for the irregularity code.

NOCOL: Number of columns in the grid.

JLIN: Number of contours.
 NOROW: Number of rows in the grid.
 NPNTS: Number of points.
 NZHT: Number of equi-spaced zones from bottom to top.
 NZVT: Number of equi-spaced zones from left to right.
 OUT: Logical unit for writing.
 PI: 3.1416
 PI02: $\text{PI}/2$
 POT: Potential of a given contour.
 RMIN: Minimum radius of field.
 RMIX: Minimum radius parallel to X axis.
 RMIY: Minimum radius parallel to Y axis.
 SORIG: Scale of the original drawing.
 TOL: Convergence criterion for the iterative solution.
 TOLR: Tolerance allowed between input points describing the contours.
 VLF: First vertical line of the window.
 VLL: Last vertical line of the window.
 WARN: Warning format number.
 WINDAT: Name of the input file.
 WINOUT: Name of the output file.
 WITHN: Code indicating use of the equivalent or not.
 XCORR: Spacing between axis of symmetry and the first grid line parallel to the X-axis.
 YCORR: Spacing between axis of symmetry and the first grid line parallel to the Y-axis.

C. Users' Manual

1. Introduction

The program, WINDOW, computes electric potential distribution and maximum electric field stress in each one of the different permittivities present in a portion of the whole region representative of the problem.

The portion solved for is called the "window." It is a rectangular sub-region of the larger rectangular region representative of the problem.

An equivalent of the region outside the "window" at the boundary of the latter is computed to assure completeness of the solution.

The first run of the program is used to compute the equivalent and subsequent runs to compute the electric potential distribution within the "window", corresponding to different design alternatives.

Two sets of input data are prepared, corresponding to the "equivalent" and the "window" respectively. Both follow the same format and are differentiated by proper coding.

The program is written in FORTRAN IV. Information regarding size, loading mode, etc., is provided at the end of this users' manual.

2. Input Data

The list below shows the order in which input data are read. The first column is for the equivalent formation run. The second column is for the window solution run.

<u>Equivalent Formation</u>	<u>Window Solution</u>	<u>Format</u>
IDENT	IDENT	20A4
FLAXI, RMIN, SORIG	FLAXI, RMIN, SORIG	3F10.0
EQLIN, EPSO, WITHN, NOLIN	EQLIN, EPSO, WITHN, NOLIN	4F10.2
HLF, HLL, VLF, VLL	MAXI, TOL, GAMMA, AVEVOL	4F10.2
PCHV	PCHV	8F10.0/8F10.0
DIGLIN(a)	DIGLIN(a)	10F7.0 repeated
PCHH	PCHH	8F10.0/8F10.0
DIGLIN(b)	DIGLIN(b)	10F7.0 repeated
PNTS, EPS, POT, CODE	PNTS, EPS, POT, CODE	4F10.0
DIGLIN(c)	DIGLIN(c)	10F7.0 repeated

A brief description of each variable is given in the GLOSSARY which follows immediately. A fuller description is given in following sections.

3. Run identification and over-all geometry (IDENT, FLAXI, RMIN, SORIG)

IDENT can have up to 78 alpha-numeric characters at the user's discretion. It is used as a run identifier for output.

FLAXI and RMIN - Electric field problems for flat two-dimensional geometry and for axi-symmetric three-dimensional geometry can be solved. The distance from the edge of the region closer to the axis of symmetry is specified as the minimum radius. For flat geometries it is zero. Proper coding is used to indicate the type of geometry and the axis of symmetry. Table III.C.1 indicates all the information needed.

Code/Variable Name	Possible Values	Meaning	Location on Input Sheet	Format
FLAXI	1.0	Flat Geometry	2nd Card 1st Position	FORTRAN F 10.0
	2.0	X-Axis Symmetry		
	3.0	Y-Axis Symmetry		
RMIN	0.0	Flat Geometry	2nd Card	FORTRAN F 10.0
	≥ 0.0	Symmetry	2nd Position	

Table III.C.1
FLAXI and RMIN

SORIG is a value which divides all dimensions read in during a run. It can be thought of as the scale of an original drawing from which dimensions in millimeters are measured to provide dimensional information. Thus if a drawing has a scale of 1 cm representing 1 meter so that its scale is 1 to 100 or .01, then the program adjusts each given dimension by dividing it by .01. Thus a drawing measurement of 30 mm will be adjusted to $30/.01 = 3000 \text{ mm} = 3.0 \text{ m}$.

Dimensions are treated as identical within $0.5/\text{SORIG}$ during computer processing. That is, if two dimensions are given which are within 0.5 of the input data units, they will be treated as equal.

GLOSSARY

IDENT (20):	alphanumeric array where the run identification is stored.
FLAXI:	index for specifying flat field or axi-symmetric field.
RMIN:	distance from the axis of symmetry to the closest grid edge.
SORIG:	scale of the drawing.
EQLIN:	index for specifying equivalent formation and number of equipotential lines.
EPSO:	permittivity if not specified otherwise.
WITHN:	index for use of the equivalent when solving for the potentials in the window.
NOLIN:	total number of contours to be read.
HLF:	lower horizontal line number of the window.
HLL:	upper horizontal line number of the window.
VLF:	left vertical line number of the window.
VLL:	right vertical line number of the window.
MAXI:	maximum number of iterations to be used in window solution.
TOL:	tolerance permissible in window solution.
ALPHA:	acceleration factor for iterative window solution.
AVEVOL:	initial estimate of the potentials for the window solution.
PCHV(16):	vertical grid line numbers where grid density changes.
PCHH(16):	similar for the horizontal lines.
PNTS:	number of points that describe the following contour.
EPS:	permittivity of material inside the contour
POT:	known potential of an electrode (0.0 if code $\neq$ 2.0).
CODE:	indicates whether the contour is a dielectric, an electrode or a floating conductor. (0., 2., 3.)
DIGLN(140):	temporary storage for several pairs of (x, y) co-ordinates.

Code Name	Possible Values	Meaning	Location on Input Sheet	Format
SORIG	>0	Scale of the drawing	Card 2 Position 3	F 10.0

4. EQLIN, EPSO, WITHN, NOLIN

EQLIN is an index which specifies the number of equi-potential lines to be plotted, if a window solution is being run. If it is zero, an equivalent formation is run.

EPSO is the base permittivity, assumed to apply outside all contours. It may be either absolute or relative permittivity.

WITHN is an index set to 0.0 if an equivalent formed previously is to be used in this run. It is set to 1.0 if a previously formed equivalent is not to be used.

NOLIN is the number of contours which will be read in as input data later.

Code Name	Possible Values	Meaning	Location on Input Sheet	Format
EQLIN	0.0	Compute Equivalent	Card 3	F 10.0
	>0.	Solve Window with EQLIN no. of equipoten- tial lines	Position 1	
EPSO	<u>≥</u> 0.	Base permittivity	Card 3 Position 2	F 10.0
WITHN	0.	Use previous equivalent	Card 3 Position 3	F 10.0
	1.	Do not use equivalent		
NOLIN	1. to 70.	Number of contours to be entered	Card 3 Position 4	F 10.0

5. Window location (for equivalent formation runs only)
(HLF, HLL, VLF, VLL)

A regular grid must first be defined. See section 6. on Grid Definition.

Once the grid has been defined, the window is chosen, just by selecting the grid line numbers that frame it. These lines are two horizontal and two vertical. The window can be anywhere in the grid as long as the number of grid points in it does not exceed 900.

Variable Name	Possible Values	Meaning	Location on Input Sheet	Format
HLF	1 to 100	Lower Horiz. Line	Card 4 Position 1	F 10.0
HLL	1 to 100	Upper Horiz. Line	Card 4 Position 2	F 10.0
VLF	1 to 70	Left Vertical Line	Card 4 Position 3	F 10.0
VLL	1 to 70	Right Vertical Line	Card 4 Position 4	F 10.0

The total number of nodes present at any one time during equivalent formation cannot exceed 190, including those on the window boundaries. When the equivalent is finally formed, only 120 nodes may be included, divided into 116 on the window boundary (30x30 window grid) and four electrodes outside the window.

6. Grid definition
(PCHV(16), DIGLN, PCHH(16), DIGLN)

The whole region is overlaid with a uniform rectangular grid. The grid is made up of vertical and horizontal lines; the intersection of any two of such lines defines a "grid point" which may eventually be called a "node."

Spacing between the lines, in either direction, does not have to be equal; the spacing is determined by the number of lines in a given distance. Only the line number where the new spacing begins has to be specified. Also the coordinates of those lines are specified (see the example at the end of this users' manual).

A maximum of 70 vertical lines and of 100 horizontal lines is accepted. both can be used simultaneously, yielding a maximum of 7,000 grid points.

A maximum of 16 grid density changes in each direction is accepted.

How close the lines should be depends on the degree of accuracy desired. A spacing of 0.5 millimeters between consecutive lines is the minimum allowed.

The first line number is always 1, and has to be specified. The last line number must also be specified.

In PCHV(16) and PCHH(16) there are 16 input fields (Format F 10.0) allotted to each set of line numbers defining the grid; those that are not needed must be filled out with zeros.

In DIGLN the co-ordinates of each line given in the grid definition are specified as an x-y pair. The coordinate x is irrelevant for the definition of horizontal lines and coordinate y is irrelevant for vertical lines. However, their input fields must be filled out with zeros.

There are 32 input fields (Format F 7.0) allotted to each set of coordinates defining the lines. Co-ordinates are given only for the line numbers different from zero.

Array Name	Possible Values	Meaning	Location on Input Sheet	Format
PCHV(16)	1 to 70	Vertical line numbers where horizontal spacing changes	Cards 5 and 6 Positions 1 to 8	F 10.0
DIGLN(140)	≥ 0 .	Co-ordinates of the lines in the form $(x,y) = (x,0.)$	Follows card 6	F 7.0
PCHH(16)	1 to 100	Horizontal line numbers where vertical spacing changes	Follows vertical co-ordinates	F 10.0
DIGLN(140)	≥ 0 .	Co-ordinates of the lines in the form $(x,y) = (0.,y)$	Follows horizontal lines	F 7.0

Table III.C.2

Grid Co-Ordinate Data

7. Contour specification (PNTS, EPS, POT, CODE, DIGLN)

The geometry is represented by two dimensional contours. The contours are formed piecewise by straight segments; the segments originate when joining a series of consecutive points that define a shape of interest.

To define a contour the necessary number of points (PNTS) and the coordinates (DIGLN) of each point are given.

To indicate a closed contour the first and last point defining it should be the same, otherwise it is considered open.

These contours can represent electrodes, floating conductors and bodies of different permittivities. They are selected by proper coding (CODE).

Open contours can represent only electrodes and floating conductors; a permittivity (EPS) other than 0.0 cannot be associated with them.

The potential (POT) and the permittivity (EPS) specified in the contour definition is assigned to all the points inside it only if it is closed.

A maximum of 70 contours, with up to 70 points per contour, is allowed. However, the total number of points may not exceed 400.

If two points are spaced 0.5 millimeters or less they are considered to be physically the same.

Floating conductors that are a closed contour are specified as if they were a body with a relative permittivity of 9000. The special code for floating conductors should be saved for open lines only unless the equivalent is to be computed, in which case it should be 3.0 for closed and open lines.

Points describing a contour cannot be closer than the local spacing of the grid, i.e., there must be at least as many grid points available as there are contour vertices in a region, otherwise the program aborts and an error message is printed.

Two different contours can have common lines. The order in which the contours are entered is important. Electrodes should be entered first; next, the floating conductors, and last the remaining type.

The information needed for each contour is:

- a) number of points that describe it (PNTS);
- b) relative permittivity (0.0 if contour is open or if describing an electrode) (EPS);
- c) potential in volts (0.0 if other than electrode) (POT);

- d) code: 2.0 for electrodes
 3.0 for floating conductors
 0.0 for dielectric

- e) co-ordinates for the points in a) (DIGLN)

Table III.C.3 gives information on items a) to d). Item e), the point co-ordinates are given in x-y pairs, 5 pairs per line, in 10F7.0 format. The dimensions are assumed to be in millimeters.

Figure III.C.1 shows a rectangular dielectric shape with a poor approximation to a circle resting on one end. Input data required for these two shapes are:

5.0 2.0 0.0 0.0 (4F10.0 format)

0051. 0000. 0071. 0000. 0071. 0051.
 A B C

0051. 0051. 0051. 0000.
 D A

All on one
 card
 (10F7.0 format)

7.0 0.0 100.0 2.0 (4F10.0 format)

0051. 0051. 0071. 0051. 0081. 0068. 0071. 0086.
 D C E F

0051. 0086. 0041. 0068. 0051. 0051.
 G H D

All on one
 card
 (10F7.0 format)

Figure III.C.1
Input Data Sample

Code Name	Possible Values	Meaning	Format
NPTS	1 to 70	Number of points in a contour	F 10.0
EPSI	ϵ_R	Relative permittivity of material inside the contour	F 10.0
POT	Any	Potential of the contour	F 10.0
CODE	0 2 3	Dielectric Electrode Floating Conductor	F 10.0

Table III.C.3

8. Computation of the equivalent

Use of the proper code causes the program to compute the equivalent; the input data is that for the whole grid.

When computing the equivalent, each electrode or floating conductor is considered as one node.

The information regarding the equivalent is stored in two files which are opened automatically by the program.

The only out-put the user receives is in the file "WINOUT" and contains a repeat of the input data, and any error messages.

9. Computation of field strengths

New input data are produced; this time the window is considered the whole grid observing all the restrictions already mentioned. Use of the proper code will cause the program to compute the voltage distribution within the window.

Additional information regarding the solution of a system of equations must be specified. (MAXI, TOL, GAMMA, AVEVOL)

Code/Variable Name	Possible Values	Meaning	Location on Input Sheet	Format
MAXI	>0	Maximum No. of iterations	Card 4 Position 1	F 10.0
TOL	>0	Tolerance	Card 4 Position 2	F 10.0
GAMMA	0→2	Acceleration factor	Card 4 Position 3	F 10.0
AVEVOL	ϵ_R	Initial estimate	Card 4 Position 4	F 10.0

The solution of the linear system of equations is done by the successive overrelaxation (S.O.R.) method. The acceleration factor (GAMMA) is a number between 0. and 2.; overrelaxation occurs when it is greater than one, and underrelaxation when less than one. The number of iterations needed for a certain convergence depends on the number of grid-points in the windows and the initial estimate of the solution. There is no definite way of best proceeding, but as a first approximation the maximum number of iterations (MAXI) should be equal to or larger than the number of grid points in the window, the acceleration factor (GAMMA) to 1.4, or in any case, overrelaxed, and the initial voltage value (AVEVOL) to the weighted average of what the operator feels is going to be the final potential distribution. The average of the maximum and minimum potential will do if no better estimate can be provided. All of the above instructions are for a tolerance of 0.1%, where tolerance (TOL) means the maximum change of potential for one given grid-point between two consecutive iterations.

The output consists of two parts:

- a) a graphic display;
- b) a repetition of the input data, the maximum field strengths for each permittivity present and the potential at every grid-point.

A graphic display of the equipotentials is always part of the program's output. It is programmed to produce the largest picture possible and also shows the identification of the problem and the percent volts per line.

The graphics package is "Dynagraphics," as implemented on the "IMLAC" terminals.

A file with the following items is provided:

- 1) identification;
- 2) grid line numbers and their co-ordinates;
- 3) repetition of the input data, regarding the contours (shapes);
- 4) warnings or errors if any occur during the execution of the program;
- 5) maximum field strength for each permittivity, and the co-ordinates and grid numbers of the vertices of the triangles where they happened;
- 6) potential in volts at every vertex, as well as their grid numbers and co-ordinates.

10. Input and Output files

The program was implemented in a "PRIME 500" computer system. Advantage was taken of its file management system, and the program is run interactively from a terminal.

The input data is written in a file by the name of "WINDAT." This file is automatically opened by the program and the information is read as it is requested. When the program is finished reading, this file is closed.

The output is handled in a similar way. When building the equivalent the program generates two output files, "DATA 1" and "DATA 2." "DATA 1" contains the grid number of the vertices on the window boundary that have connections to the equivalent, the assigned number of the electrodes that were on the equivalent and its potentials. "DATA 2" contains the equivalent matrix, stored in a linear array.

These two files must exist when the solution in the window is required if the equivalent matrix is to be used. The program opens and closes them automatically.

When solving for the potentials in the window the output is written out in a file called "WINOUT" which is also opened and closed automatically.

Refer to the PRIME FTN MANUAL for instructions on how to get a hard-copy of the files.

11. Error messages

The program aborts and prints out one or more of the following error messages, if it is necessary to modify the input:

1. The number of horizontal grid lines is more than 100; the number of vertical grid lines is more than 70.
2. The number of nodes in the grid is more than 7000; the number of nodes in the window is more than 900.
3. There are more than 70 contour lines.
4. A contour line is described by more than 70 points.
5. The total number of contour points exceeds 400.
6. The permittivity is given different from zero, and the last point on the line is not equal to the first.
7. The line fitting was interrupted for line segment-near-. (Co-ordinates define the line segment and the point where the line fitting got stuck.)
8. This point is too close to the preceding point-. (The point is defined by its co-ordinates.)
9. The program has moved vertex - from one line to another line which is too close. (The vertex point is defined by its number and its co-ordinates.)
10. The area came out negative or zero for triangle with-. (The triangle is defined by its vertex numbers and co-ordinates.) Change line sequence or move grid lines.
12. Compiling, loading and running the program

The program has the following memory requirements

for the compiled program:	PROCEDURE	12010 words
for variables and array storage:	DATA	<u>67681</u>
	Total	79782

The program is compiled in the virtual mode (FTN WINDOW -64V). It loads in the segmented mode (SEG). The Dynagraphic's Library and the FORTRAN library must be loaded also. The order is important and Dynagraphic's is first.

Once the program is properly loaded, it is run in the segmented (virtual) mode. (SEG # WINDOW.)

The data files and the output files are opened and closed from within the program; to obtain a listing of these files the command "SPOOL" is used, i.e. for a listing of "WINOUT" we type:

SPOOL WINOUT - FTN.

The FTN option tells the computer to execute FORTRAN carriage control commands. It should not be used for listing of data files. A typical session in one of the terminals would appear as follows; the user types the underlined words.

13. Example

The problem consists of the determination of the best shape of the end of a semiconducting sheath on a cable joint, as shown in Fig. III.C.2. It is desired to choose a shape so as to reduce the field stress on the surface of contact of the materials with permittivities ϵ_1 and ϵ_2 .

The problem was simplified to an arrangement of electrodes and dielectrics, as shown in Fig. III.C.3.

The grid line changes for the whole problem and for the window are shown in Fig. III.C.4.

Fig. III.C.5.a shows the input data file for the computation of the equivalent and Fig. III.C.5.b shows the same for the solution in the window.

Fig. III.C.6 shows graphic output of the program in the window region.

SAMPLE SESSION

OK, LOGIN VARELA LAUBER
VARELA (23) LOGGED IN AT 14'12 D92179
ENTER LOGIN PASSWORD: #####

THE MACHINE RESPONDS WITH SYSTEM IDENTIFICATION,
TIME OF LOGIN AND DATE OF LAST LOGOUT.

OK, FTN WINDOW -64V

GO

0000 ERRORS [<.MAIN.>FIN-REV16.4]
0000 ERRORS [<CODING>FIN-REVI6.4]
0000 ERRORS [<SOLUTN>FIN-REV16.4]
0000 ERRORS [<PERTRI>FIN-REV16.4]
0000 ERRORS [<FSTR >FIN-REV16.4]
0000 ERRORS [<PLOTY >FIN-REV16.4]
0000 ERRORS [<EQUIV >FIN-REV16.4]
0000 ERRORS [<SUBD >FIN-REV16.4]
0000 ERRORS [<DIGIT >FIN-REV16.4]
0000 ERRORS [<INTER >FIN-REV16.4]
0000 ERRORS [<NODFT >FIN-REV16.4]
0000 ERRORS [<NBOUFT>FIN-REV16.4]

OK, SEG

GO

VLOAD #WINDOW

\$ LO B_WINDOW

\$ LI HGPIFV

\$ LI

LOAD COMPLETE

\$ SAVE

\$ EXECUTE

GO

OK, SPOOL WINOUT -FTN

Cable Splice

Figure III.C.2

Schematic of the Problem
(Dimensions in Millimeters)

Figure III.C.3

Lines Indicating Grid Density Changes and the Window
(Dimensions in Millimeters)

Figure III.C.4

CABLE SPLICE CONICAL EQUIVALENT.

2.0	11.3	1.						
0.0	3.0	0.0	4.0					
1.0	19.0	16.0	35.0					
1.	16.	35.	60.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	0.	
0000.0	0000.0	0035.0	0000.0	0044.5	0000.0	0109.2	0000.0	
1.	2.	12.	20.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	0.	
0000.0	0000.0	0000.0	0001.7	0000.0	0009.7	0000.0	0014.0	
6.	0.	100.	2.					
0000.0	0006.7	0039.4	0006.7	0039.4	0004.5	0014.0	0004.5	0014.0
0109.2	0000.0							
3.	0.	0.	2.					
0000.0	0014.1	0032.1	0014.1	0109.2	0007.0			
5.0	0.0	0.0	2.0					
0096.5	0004.5	0109.2	0004.5	0109.2	0005.2	0096.5	0005.2	0096.5
5.0	2.5	0.0	0.0					
0014.0	0000.0	0109.2	0000.0	0109.2	0004.5	0014.0	0004.5	0014.0

Equivalent Input Data

Figure III.C.5.a

VIEW OF CORNER IN CABLE JOINT.

2.0	11.3	1.						
19.9	3.0	0.0	3.0					
240.0	0.001	1.34	50.0					
1.	20.	0.	0.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	0.	
0035.0	0000.0	0044.5	0000.0					
1.	2.	12.	19.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	0.	
0000.0	0000.0	0000.0	0001.7	0000.0	0009.7	0000.0	0011.9	
6.0	0.0	100.0	2.0					
0035.0	0005.7	0000.0	0001.0	0006.0	0002.2	0037.2	0004.5	0000.0
0035.0	0004.5							
2.0	0.0	100.0	2.0					
0035.0	0000.0	0044.5	0000.0					
5.0	2.5	0.0	0.0					
0035.0	0000.0	0044.5	0000.0	0044.5	0004.5	0035.0	0004.5	0035.0

Window Input Data

Figure III.C.5.b

Window Solution of Cable Joint

Figure III.C.6

7. Program Verification

A. Summary

Results for the electrostatic field in three different examples have been obtained.

1. a cable joint with two different radii and permittivities;
2. a shield for a bushing throat on an air blast circuit breaker;
3. a portion of a transformer winding.

The results were compared to those obtained using the Andersen program and found to be numerically the same. Cost of forming the equivalent matrix at the retained window surface was found to be comparable to the cost of the Andersen program result for the entire problem. Cost of each succeeding run for the retained window combined with the equivalent was significantly less than the Andersen program cost.

1. A Cable Splice

Figure IV.1 shows an axi-symmetric problem where a smaller conductor with insulation ϵ_1 and a semi-conducting outer sheath enter from the right.

A larger conductor enters from the left. It has an inner semi-conducting sheath and insulation ϵ_2 as well as an outer conducting sheath.

The outer sheath of the smaller conductor is shown cut back. The larger conductor is shown under-cut from its inner sheath. The two are then butted inside the large conductor, inner sheath. The splice is insulated by filling the conical space with insulation ϵ_2 and wrapping the entire joint with a conducting sheath.

The field problem is to determine maximum electric stresses in each of the dielectrics. Potential = 100% is specified on both conductor surfaces and on the inner semi-conducting layer of the large conductor. Potential = 0 is specified on the outer conical wrapping and its extensions and also on the outer semi-conducting sheath of the small conductor.

For illustrative purposes, the window was located over the end of the large conductor, inner sheath extension as indicated in Figure IV.1. Another possible location of large stress is at the end of the small conductor, outer sheath, but this case was not run for this report.

The question to be illustrated for this report is the effect of the shape of the end of the sheath extension, i.e. is the ramp cut shown optimal or would a square cut or quarter-round end be better?

The grid used was the same for two solutions: an Andersen program solution and the new window solutions. The entire problem had 20 x 60 grid lines giving 1200 nodes. The window had 19 x 20 grid lines giving 380 nodes.

Figure IV.1 Cable Splice

Figure IV.2 shows the Andersen program solution for this example. Note that right and left have been reversed and that the end of the inner semi-conducting sheath has a quarter-round shape. Figure IV.3 is the solution of the window region using the new program. Figure IV.4 is an enlargement of the Andersen solution in the area of the window so that a closer comparison can be made. It is evident that the solutions are the same.

Figure IV.5 shows two other runs made in the window region on the new program. They illustrate the fact that the window geometry can be varied without repeating the "first pass" run which forms the equivalent at the window surface.

2. Circuit Breaker Bushing Throat Shield

Figure IV.6 is a picture taken from a manufacturer's instruction book illustrating an interrupter head for a circuit breaker. Figure IV.7 is a simplification of the throat region of the horizontal bushing. The field problem is to determine electric stress near the throat and, in particular, to determine the length of the shield located there (part 29).

Again, runs were made to compare the Andersen solution to the window solution. Figure IV.8 shows the complete Andersen solution. Figure IV.9 shows the window solution. The window was located over the end of the shield as indicated in Figure IV.7. Total grid was $64 \times 33 = 2112$ nodes. Window grid was $39 \times 23 = 897$ nodes. Figure IV.10 is an enlargement of the Andersen solution in the shield end region. Comparison of Figures 9 and 10 shows that the same result was obtained. Figure IV.11 shows two other runs of the window program, without repeating the first-pass equivalent formation used in Figure IV.9. This figure illustrates varying the shield end location within the window.

3. Transformer Turns

Figure IV.12 shows an approximation of a transformer winding which is the lower half of a high voltage winding wound symmetrically from the center to the ends. The left and bottom surfaces are at zero potential representing the low voltage winding and the bottom core yoke respectively. The upper surface is a cut surface representing symmetry with the upper half winding. The right surface is also a cut surface. The top turn, representing the line connection is held at 100% potential. All other conductors are floating. Thus, this solution is an approximation to impulse response. The top two sections are paper wrapped, and spaced farther than the other turns which are uniform.

Two solutions were obtained: the complete Andersen solution and the window solution. The window was located over the top three turns as shown in Figure IV.12. Total grid was $32 \times 17 = 544$ nodes. The Andersen solution of the complete problem is shown in Figure IV.13. Figure IV.14 shows the window solution. Figure IV.15 shows an enlargement of the Andersen solution. Again, the results are the same.

B. Cost Comparisons

The table given below gives approximate costs of computer runs plus hard copy graphic results on the Prime 500 system.

	Splice	Shield	Transformer
Andersen Solution	\$10.00	\$12.50	\$8.00
Equivalent Formation	\$5.00	\$16.00	\$2.20
Window Solution	\$4.00	\$12.00	\$5.00

The Andersen solutions were also run on the IBM3033. Costs were approximately \$1.00 less than the Prime 500 costs given in the table.

Costs for the splice and transformer examples are very favorable for the window solution versus the Andersen solution. For the shield example, difficulty was encountered in convergence of the matrix solutions. Thus extra cost was incurred.

Figure IV.2 Andersen Solution of Cable Joint

Figure IV.3 Window Solution of Cable Joint

Figure IV.4 Enlargement of Andersen Solution in Window Region of Cable Splice

Figure IV.5a Window Solution for Square-Cut End

Figure IV.5b Window Solution for Ramp-Cut End

Figure IV.6 Interrupter Tank Assembly

Figure IV.7 Circuit Breaker Bushing Throat Detail

Figure IV.3 Andersen Solution of Circuit Breaker Bushing Throat

Figure IV.9 Window Solution of Circuit Breaker Bushing Throat

Figure IV.10 Enlargement of Andersen Solution in Window
Region of Circuit Breaker Shield

Figure IV.11 Shield Variations

Figure IV.12 Transformer Winding

Figure IV.13 Andersen Solution for
Transformer Winding

Figure IV.14 Window Solution for Transformer Winding End

Figure IV.15 Enlargement of Andersen
Solution in End Region
of Transformer Winding

Program Limitations

A. Many of the limitations of the program given are due to the fact that the technique described was grafted onto an existing program (FLD6 due to O. W. Andersen). That program is commercially useful and can be applied in practical problems now.

On the other hand, it was developed in the late 1960's and early 1970's. Thus, it does not take advantage of the dramatic changes in computational technology in the last ten years. For example, the use of graphic input/output is rather cumbersome.

B. Grid size limitations as well as limits on numbers of curves and points are dictated by the computer installation at which the program will run. For example, when some of the sample problems were run on the Prime 500 system, they were found to require write-out and read-back from tape to accommodate large matrix processing. This was found to be a rather costly process on Prime. On the other hand, when the identical problem was run on the IBM3033, the fast access memory was large enough to accommodate the large matrix sizes and operational cost was reduced.

Compromise sizes of arrays are given in the program for use on the Prime system. They can be adjusted for any given installation.

C. Some limitations are due to the geometric technique introduced by Andersen and followed in this program. For example, the technique of describing contours as series of straight line segments by entering successive vertex coordinates, and then distorting a regular grid to fit these contours creates at least one limitation.

The problem occurs when a small acute angle exists in a contour. After grid distortion fits the vertex and after one side of the angle has been fitted by the line distortion process, it is sometimes found that attempts to fit the other side of the angle result in attempts to move grid nodes which have already been moved.

It was concluded that the difficulty arises when the angle is less than that of the aspect ratio of the regular grid definition near the vertex. Thus, when the program error message gives such an indication, the problem can be corrected by making the grid finer in the vertex region. Such a solution may not be feasible.

Another possibility is to blunt the acute angle to an appropriate value.

D. Inefficiencies in algorithm selection and programming can be identified. Unfortunately, they add to program run-time and storage requirements. For example, the Kron reduction process can make use of more efficient algorithms described by H. E. Brown.

Also, the technique of shifting data within matrices, rather than using a

pointer system for empty axes, adds to run costs with no accompanying improvements.

E. Finally, it should be stated that this program was not intended as a polished result, useful in itself. It was intended solely to show that the "window" technique was feasible in that it gave the same results for much lower run costs. Thus, it was intended to be one step in the development of a three-dimensional program.

Appendix 1
a b c Constants for Triangles

$$\frac{1}{2A} \begin{vmatrix} a_i & a_j & a_k \\ b_i & b_j & b_k \\ c_i & c_j & c_k \end{vmatrix} = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix}^{-1}$$

$$a_i = + \begin{vmatrix} x_j & y_j \\ x_k & y_k \end{vmatrix} \quad a_j = + \begin{vmatrix} x_k & y_k \\ x_i & y_i \end{vmatrix} \quad a_k = + \begin{vmatrix} x_i & y_i \\ x_j & y_j \end{vmatrix}$$

$$b_i = - \begin{vmatrix} 1 & y_j \\ 1 & y_k \end{vmatrix} \quad b_j = - \begin{vmatrix} 1 & y_k \\ 1 & y_i \end{vmatrix} \quad b_k = - \begin{vmatrix} 1 & y_i \\ 1 & y_j \end{vmatrix}$$

$$c_i = + \begin{vmatrix} 1 & x_j \\ 1 & x_k \end{vmatrix} \quad c_j = + \begin{vmatrix} 1 & x_k \\ 1 & x_i \end{vmatrix} \quad c_k = + \begin{vmatrix} 1 & x_i \\ 1 & x_j \end{vmatrix}$$

Appendix 2

Minimization of the Functional

The functional to be minimized within the triangular area is equation (2.3) using potential in terms of the vertex potentials given by equation (2.10).

Denote the functional in the elementary tetrahedral volume by U_e . Then minimization with respect to V_i requires $\partial U_e / \partial V_i = 0$. There are two other simultaneous relations, similar in form, for the potentials V_j and V_k . These can be expressed in matrix form as

$$\begin{bmatrix} \partial U_e / \partial V_i \\ \partial U_e / \partial V_j \\ \partial U_e / \partial V_k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (A2.1)$$

The partial derivative, $\partial U_e / \partial V_i$, can be found from equation (2.3) to be

$$\frac{\partial U_e}{\partial V_i} = \iint_{A_e} [\epsilon_x \frac{\partial V}{\partial x} \frac{\partial}{\partial V_i} (\frac{\partial V}{\partial x}) + \epsilon_y \frac{\partial V}{\partial y} \frac{\partial}{\partial V_i} (\frac{\partial V}{\partial y}) + \epsilon_z \frac{\partial V}{\partial z} \frac{\partial}{\partial V_i} (\frac{\partial V}{\partial z})] dA$$

In matrix form

$$\frac{\partial U_e}{\partial V_i} = \iint_{A_e} \frac{\partial}{\partial V_i} \left(\begin{bmatrix} \frac{\partial V}{\partial x} & \frac{\partial V}{\partial y} \end{bmatrix} \right) \begin{bmatrix} e_x & 0 \\ 0 & e_y \end{bmatrix} \begin{bmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial V}{\partial y} \end{bmatrix} dA \quad (A2.2)$$

From equation (2.10), the partial derivatives are

$$\begin{bmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial V}{\partial y} \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} b_i & b_j & b_k \\ c_i & c_j & c_k \end{bmatrix} \begin{bmatrix} V_i \\ V_j \\ V_k \end{bmatrix} \quad (A2.3)$$

From (A2.3) it is seen that

$$\frac{\partial}{\partial V_i} \begin{bmatrix} \partial V / \partial x \\ \partial V / \partial y \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} b_i \\ c_i \end{bmatrix} \quad (A2.4)$$

with similar results for $\partial / \partial V_j$, etc.

(A2.3) and (A2.4) are factors in the integrand of (A2.2). They are not functions of x , y and z ; the variables of integration. Therefore they can be moved outside the integral. Also when $\epsilon_x = \epsilon_y = \epsilon_z = \epsilon$, the isotropic case, a factor, ϵ , can be moved outside. That which remains is the area integral of the element. Thus these changes and use of (A2.3) and (A2.4) cause (A2.2) to become

$$\frac{\partial U_e}{\partial V_i} = \frac{\epsilon}{4A} \begin{bmatrix} b_i & c_i \end{bmatrix} \begin{bmatrix} b_i & b_j & b_k \\ c_i & c_j & c_k \end{bmatrix} \begin{bmatrix} V_i \\ V_j \\ V_k \end{bmatrix} \quad (A2.5)$$

Results for $\partial U_e / \partial V_j$ and $\partial U_e / \partial V_k$ are similar except that the subscripts in the row matrix in (A2.5) are, respectively; j and k .

When partial derivatives with respect to each of the vertex potentials are assembled in an array, the matrix relation shown in equation (2.11) results.

Appendix 3

Program Listing


```

C --- WINDOW PROGRAM, PRIME 500 FORTRAN IV
C --- MAIN CALL PROGRAM
C --- SUBROUTINES CODING AND PLOTY, MODIFIED BY C. VARELA
C --- SUBROUTINES SOLUTN AND PERTRI, ORIGINAL WORK OF C. VARELA
C --- SUBROUTINE EQUIVALENT, ORIGINAL WORK OF C. VARELA
C --- FUNCTIONS NODFT AND NBOUFT, ORIGINAL WORK OF C. VARELA
C --- ALL OTHER SUBROUTINES, TAKEN FROM PROGRAM FIELD6 BY O.W.ANDERSEN.
REAL LINC(800), IDENT(20), MAXI
DIMENSION XPOS(70), YPOS(100), ILN(70), AUX(500,2), MAUX(500)
1, LNS(4,4), NZV(15), NZH(15), ADMAB(15,15), ADMAD(15,15)
INTEGER*2 MVERT(70,100)
COMMON/BLOCK1/XPOS, YPOS, LNVER, LNHOR, IDENT, EQLIN,
116, 15, FLAXI, EPS0/BLOCK2/LINC, NZV, NZH, ADMAB
2, ADMAD, DATLN(70,3)
COMMON/BLOCK3/MVERT/BLOCK4/DIGLN(140)/BLOCK5/AUX, MAUX
COMMON/BLOCK6/NTRI(8)/BLOCK7/LINTO, ILN
COMMON/BLOCK8/VOLTS(904), NODE(900), JP, NZHT, NZVT
COMMON/BLOCK9/COEFF1(18145)
COMMON/BLOCK11/ITRI(900,8), IVERT(2000,3)
COMMON/BLOCK12/WITHN, MAXI, TOL, GAMMA, AVEVOL, HLF, HLL
1, VLF, VLL
CALL CODING
IF(EQLIN.NE.0.)GO TO 10
CALL EQUIV
CALL EXIT
10 CALL SOLUTN
CALL PLOTY
CALL EXIT
END
SUBROUTINE CODING
SINSERT SYSCOM KEYS.F
C --- WINDOW PROGRAM, PRIME 500 FORTRAN IV, CODIFICATION
REAL LINC(800), NOLIN, LINPR, IDENT(20), MAXI
INTEGER OUT, POINT(400), WARN
DIMENSION XPOS(70), YPOS(100), PCHV(16), PCHH(16), DIGLN(140),
1NTRI(8), AUX(500,2), ILN(70), NZV(15), NZH(15), MVERT(70,100)
2, DATLN(70,3), LNS(4,4), MAUX(500), ADMAB(15,15), ADMAD(15,15)
COMMON/BLOCK1/XPOS, YPOS, LNVER, LNEOR, IDENT, EQLIN,
116, 15, FLAXI, EPS0/BLOCK2/LINC, NZV, NZH, ADMAB
2, ADMAD, DATLN
COMMON/BLOCK3/MVERT/BLOCK4/DIGLN/BLOCK5/AUX, MAUX/BLOCK6/NTRI
COMMON/BLOCK7/LINTO, ILN/BLOCK8/VOLTS(904), NODE(900), JP, NZHT
1, NZVT/BLOCK12/WITHN, MAXI, TOL, GAMMA, AVEVOL, HLF, HLL, VLF, VLL
1 FORMAT(20A4/3F10.0)
2 FORMAT(SF10.0/SF10.0)
3 FORMAT(4F10.2)
4 FORMAT(4F10.0)
5 FORMAT(33HIRENSSELAER POLYTECHNIC INSTITUTE,7X,14HPROGRAM WINDOW,
17X,29HELAPLACIAN ELECTROSTATIC FIELD/1H0,20A4//)
6 FORMAT(23H FLAT, TWO DIMENSIONAL FIELD)
7 FORMAT(20H AXI-SYMMETRIC FIELD/15H MINIMUM RADIUS,F7.1/
123H AXIS OF SYMMETRI:- X -)
75 FORMAT(20H AXI-SYMMETRIC FIELD/15H MINIMUM RADIUS,F7.1/
123H AXIS OF SYMMETRI:- Y -)
8 FORMAT(24H SCALE, ORIGINAL DRAWING,F6.3/
126H0POSITIONS, VERTICAL LINES)

```

```

10  FORMAT(1H ,F6.1,I3,9(F7.1,I3))
11  FORMAT(28H0POSITIONS, HORIZONTAL LINES)
12  FORMAT(30H NUMBER OF EQUIPOTENTIAL LINES,F5.1/
246H RELATIVE PERMITTIVITY, IF NOT GIVEN OTHERWISE,F7.2)
13  FORMAT(22H0RELATIVE PERMITTIVITY,F7.2/17H POTENTIAL, VOLTS,F9.1/
15H CODE,F4.1)
14  FORMAT(4X,1HX,7X,1HY)
16  FORMAT(1H ,F6.1,F8.1)
45  FORMAT(16H0ERRONEOUS INPUT)
46  FORMAT(29H0THERE ARE MORE THAN 70 LINES)
47  FORMAT(45H0THE LINE IS DESCRIBED BY MORE THAN 70 POINTS)
48  FORMAT(39H0THE TOTAL NUMBER OF POINTS EXCEEDS 400)
49  FORMAT(104H0THE PERMITTIVITY IS GIVEN DIFFERENT FROM ZERO, AND THE
1 LAST POINT ON THE LINE IS NOT EQUAL TO THE FIRST)
50  FORMAT(50H0THE LINE FITTING WAS INTERRUPTED FOR LINE SEGMENT/
114H X Y /1H ,F6.1,F8.1/1H ,F6.1,F8.1/5H NEAR/
21H ,F6.1,F8.1)
51  FORMAT(47H0THIS POINT IS TOO CLOSE TO THE PRECEDING POINT/
114H X Y /1H ,F6.1,F8.1)
52  FORMAT(13H LONGEST SIDE,F9.2/15H SMALLEST DEPTH,F7.2)
53  FORMAT(35H WARNING: THE RATIO IS MORE THAN 10)
54  FORMAT(87H0CHECK THE ACCURACY OF THE FIELD STRENGTHS BY COMPARING
1VALUES IN NEIGHBORING TRIANGLES)
55  FORMAT(29H0THE PROGRAM HAS MOVED VERTEX,15)
56  FORMAT(49H FROM ONE LINE TO ANOTHER LINE WHICH IS TOO CLOSE)
57  FORMAT(56H0THE NUMBER OF HORIZONTAL OR VERTICAL GRID LINES IS MORE
1 ,/,44H THAN ALLOWED. 100 HORIZONTAL . 700VERTICAL )
58  FORMAT(50H0THE NUMBER OF NODES IN THE GRID IS MORE THAN 900)
585  FORMAT(50H0THE NUMBER OF NODES IN THE GRID IS MORE THAN 7000)
WARN=0
PI=3.1415927
PI02=PI/2.0
C --- LOGICAL UNIT NUMBERS
IN=5
OUT=6
C --- OPEN INPUT FILE "WINDAT"
CALL SRCHGS(KSREAD,'WINDAT',6,1,0,NCOD)
C --- OPEN OUTPUT FILE "WINOUT"
CALL SRCHGS(KSWRITE,'WINOUT',6,2,0,NCOD)
C --- READ CARDS 1 TO 4
READ(IN,1) IDENT,FLAXI,RMIN,SORIG
READ(IN,3)EQLIN,EPS0,WITHN,NOLIN
IF(EQLIN.NE.0.)GO TO 1151
READ(IN,3)HLF,HLL,VLF,VLL
GO TO 1152
1151 READ(IN,3)MAXI,TOL,GAMMA,AVEVOL
C --- READ CARDS 5 TO 9
1152 READ(IN,2) PCHV
TOLR=0.5/SORIG
C --- ASSIGN THE PROPER VALUES TO RMIX AND RMIY
RMIX=0.0
RMIY=0.0
IF(FLAXI.EQ.2.0)RMIY=RMIN
IF(FLAXI.EQ.3.0)RMIX=RMIN
C --- WRITE FIRST PART OF INPUT
WRITE(OUT,5) IDENT

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      I=FLAXI
      GO TO (1001,1002,1003), I
1001  WRITE(OUT,6)
      GO TO 1004
1002  WRITE(OUT,7) RMIN
      GO TO 1004
1003  WRITE(OUT,75) RMIN
1004  WRITE(OUT,8) SORIG
      NPNTS=16
102   IF (PCHV(NPNTS).GT.0.5) GO TO 103
      NPNTS=NPNTS-1
      IF (NPNTS.GE.2) GO TO 102
C --- NUMBER OF VERTICAL LINES
103   VERLN=PCHV(NPNTS)
      LNVER=VERLN
      IF (LNVER.GT.70) WARN=57
      IF (LNVER.GT.70) GO TO 999
      NOCOL=LNVER-1
      CALL DIGIT(NPNTS, IN, SORIG)
C --- POSITIONS, VERTICAL LINES
      XPOS(1)=DIGLN(1)
      I=1
      J=2
104   IF (I.GE.NPNTS) GO TO 107
      K=PCHV(I+1)
      NZV(I)=K
C --- SPACING BETWEEN LINES
      E1=(DIGLN(2*I+1)-DIGLN(2*I-1))/(PCHV(I+1)-PCHV(I))
105   XPOS(J)=XPOS(J-1)+E1
      J=J+1
      IF (J.GT.K) GO TO 106
      GO TO 105
106   NZVT=I
      I=I+1
      GO TO 104
C --- HORIZONTAL LINES
107   READ(IN,2) PCHH
      NPNTS=16
108   IF (PCHH(NPNTS).GT.0.5) GO TO 109
      NPNTS=NPNTS-1
      IF (NPNTS.GE.2) GO TO 108
109   HORLN=PCHH(NPNTS)
      LNEOR=HORLN
      IF (LNEOR.GT.100) WARN=57
      IF (LNEOR.GT.100) GO TO 999
      NOROW=LNEOR-1
      CALL DIGIT(NPNTS, IN, SORIG)
      YPOS(1)=DIGLN(2)
      I=1
      J=2
110  IF (I.GE.NPNTS) GO TO 113
      K=PCHH(I+1)
      NZH(I)=K
      E1=(DIGLN(2*I+2)-DIGLN(2*I))/(PCHH(I+1)-PCHH(I))
111  YPOS(J)=YPOS(J-1)+E1
      J=J+1

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      IF (J.GT.K) GO TO 112
      GO TO 111
112  NZHT=I
      I=I+1
      GO TO 110
113  XCORR=RMIX-XPOS(1)
      DO 114 I=1, LNVER
114  XPOS(I)=XPOS(I)+XCORR
      YCORR=RMIY-YPOS(1)
      DO 115 I=1, LNHOR
      YPOS(I)=YPOS(I)+YCORR
115  CONTINUE
C --- LINE CHECK
      LINTO=NOLIN
      IF (LINTO.GT.70) WARN=46
      IF (LINTO.GT.70) GO TO 999
C --- WRITE SECOND PART OF INPUT
      J=1
      K=10
116  IF (K.GT.LNVER) K=LNVER
      WRITE(OUT,10)(XPOS(I), I, I=J, K)
      IF (K.EQ.LNVER) GO TO 117
      J=J+10
      K=K+10
      GO TO 116
117  WRITE(OUT,11)
      J=1
      K=10
118  IF (K.GT.LNHOR) K=LNHOR
      WRITE(OUT,10)(YPOS(I), I, I=J, K)
      IF (K.EQ.LNHOR) GO TO 119
      J=J+10
      K=K+10
      GO TO 118
119  WRITE(OUT,12)EQLIN, EPSO
C --- NUMBER OF VERTICES
      I6=LNHOR*LNVER
      IF (EQLIN.EQ.0.) GO TO 486
      IF (I6.GT.900) WARN=58
      IF (I6.GT.900) GO TO 999
      GO TO 487
486  IF (I6.GT.7000) WARN=585
      IF (I6.GT.7000) GO TO 999
C --- THE STANDARD ADMITTANCES FOR EVERY ZONE COMPUTED HERE.
487  DO 3000 J=1, NZVT
      N1=NZV(J)
      DELX=XPOS(N1)-XPOS(N1-1)
      IF (DELX.EQ.0.) GO TO 3100
      DO 3001 I=1, NZHT
      N2=NZH(I)
      DELY=YPOS(N2)-YPOS(N2-1)
      IF (DELY.EQ.0.) GO TO 3100
      ADMAB(J, I)=-(DELX/DELY)
      ADMAD(J, I)=-(DELY/DELX)
3001  CONTINUE
      GO TO 3000

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3100 WRITE(OUT,76)
76  FORMAT(32H ATTEMP TO DIVIDE BY ZERO IN THE
    1/39H COMPUTATION OF THE STANDARD ADMITTANCES
    2/23H CHECK YOUR INPUT DATA.)
    WRITE(1,3110)DELX,DELY
3110 FORMAT('DELX ',F8.4,' DELY ',F8.4)
    GO TO 612
3000 CONTINUE
C --- INITIALIZE MVERT
    DO 1100 M=1,LNHOR
    DO 1101 N=1,LNVER
    MVERT(M,N)=0
1101 CONTINUE
1100 CONTINUE
    JCV=0
    LINAC=1
    LPNT=0
C --- READ LINE
143  READ(IN,4) PNTS, EPS, POT, CODE
    NPNTS=PNTS
    CALL DIGIT(NPNTS, IN, SORIG)
    IF (NPNTS.GT.70) WARN=47
    IF (NPNTS.GT.70) GO TO 999
    DO 144 I=1,NPNTS
    DIGLN(2*I-1)=DIGLN(2*I-1)+XCORR
    IF (DIGLN(2*I-1).LT.RMIX+TOLR) DIGLN(2*I-1)=RMIX
    IF (DIGLN(2*I-1).GT.XPOS(LNVER)-TOLR) DIGLN(2*I-1)=XPOS(LNVER)
    DIGLN(2*I)=DIGLN(2*I)+YCORR
    IF (DIGLN(2*I).LT.RMIY+TOLR) DIGLN(2*I)=RMIY
    IF (DIGLN(2*I).GT.YPOS(LNHOR)-TOLR) DIGLN(2*I)=YPOS(LNHOR)
144  CONTINUE
C --- IF A POINT IS WITHIN 0.5 MM OF A PREVIOUS POINT, SET COORDINATES
C --- EQUAL TO THOSE OF THE PREVIOUS POINT
    DO 151 K1=1,NPNTS
    L=NPNTS+1-K1
    M=L-1
    IF (M.LE.0) GO TO 146
    DO 145 N=1,M
    E1=SQRT((DIGLN(2*L-1)-DIGLN(2*N-1))**2+(DIGLN(2*L)-DIGLN(2*N))**2)
    IF (E1.GT.TOLR) GO TO 145
    DIGLN(2*L-1)=DIGLN(2*N-1)
    DIGLN(2*L)=DIGLN(2*N)
145  CONTINUE
146  IF (LINAC.LE.1) GO TO 152
    I1=LINAC-1
    DO 150 J=1,I1
    I2=1
    IF (J.GT.1) I2=ILN(J-1)+1
    I3=ILN(J)
    DO 147 I=I2,I3
    E1=SQRT((DIGLN(2*L-1)-LINC(2*I-1))**2+(DIGLN(2*L)-LINC(2*I))**2)
    IF (E1.GT.TOLR) GO TO 147
    DIGLN(2*L-1)=LINC(2*I-1)
    DIGLN(2*L)=LINC(2*I)
147  CONTINUE
150  CONTINUE

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151 CONTINUE
C --- TRANSFER TO ARRAYS DATLN, ILN AND LINC
152 DATLN(LINAC,1)=EPS
    DATLN(LINAC,2)=POT
    DATLN(LINAC,3)=CODE.
    ILN(LINAC)=NPNTS
    IF (LINAC.GE.2) ILN(LINAC)=ILN(LINAC-1)+NPNTS
    IF (ILN(LINAC).GT.400) WARN=48
    IF (ILN(LINAC).GT.400) GO TO 999
    K=0
    IF (LINAC.GE.2) K=ILN(LINAC-1)
    DO 401 I=1,NPNTS
        J=I+K
        LINC(2*J-1)=DIGLN(2*I-1)
401  LINC(2*J)=DIGLN(2*I)
        LINAC=LINAC+1
        IF (LINAC.LE.LINTO) GO TO 143
C --- INSERT NEW POINTS, WHERE REQUIRED
C --- POINTS ON LINE I ARE CHECKED AGAINST SEGMENTS IN LINE I1
        I=1
        J=1
402  IF (I.GE.2) J=ILN(I-1)+1
        K=ILN(I)
C --- POINT ON LINE I
        L=J
403  X=LINC(2*L-1)
        Y=LINC(2*L)
C --- LINE I1
        I1=1
406  IF (I1.EQ.I) GO TO 413
        I2=1
        IF (I1.GE.2) I2=ILN(I1-1)+1
        I3=ILN(I1)
C --- FIRST AND LAST POINT IN SEGMENT ON LINE I1
        M=I2
407  N=M+1
        XF=LINC(2*M-1)
        YF=LINC(2*M)
        XL=LINC(2*N-1)
        YL=LINC(2*N)
        IF ((X.EQ.XF.AND.Y.EQ.YF).OR.(X.EQ.XL.AND.Y.EQ.YL)) GO TO 413
        E1=AMIN1(XF,XL)
        E2=AMAX1(XF,XL)
        E3=AMIN1(YF,YL)
        E4=AMAX1(YF,YL)
C --- ANGLE OF LINE SEGMENT XF,YF - XL,YL
        ALPHA=PI02
        IF (YL.LT.YF) ALPHA=-PI02
        IF (XL.NE.XF) ALPHA=ATAN((YF-YL)/(XL-XF))
        IF (XL.LT.XF) ALPHA=ALPHA+PI
C --- ANGLE OF LINE XF,YF - X,Y
        E5=PI02
        IF (Y.LT.YF) E5=-PI02
        IF (X.NE.XF) E5=ATAN((YF-Y)/(X-XF))
        IF (X.LT.XF) E5=E5+PI
C --- LENGTH OF LINE XF,YF - X,Y

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      E6=SQRT((X-XF)**2+(Y-YF)**2)
C --- DISTANCE OF X,Y FROM LINE SEGMENT
      E7=E6*SIN(E5-ALPHA)
      IF (ABS(E7).GT.TOLR) GO TO 412
C --- PROJECTED POINT
      XPL=X+E7*SIN(ALPHA)
      YPL=Y-E7*COS(ALPHA)
      IF (XPL.LT.E1-0.01.OR.XPL.GT.E2+0.01.OR.YPL.LT.E3-0.01.OR.
1YPL.GT.E4+0.01) GO TO 412
C --- INSERT NEW POINT
      I4=ILN(LINTO)
      IF (I4.GE.400) WARN=48
      IF (I4.GE.400) GO TO 999
      DO 408 I3=N,I4
      I9=I4+N-18
      LINC(2*I9+1)=LINC(2*I9-1)
408   LINC(2*I9+2)=LINC(2*I9)
      LINC(2*N-1)=X
      LINC(2*N)=Y
      DO 411 I4=I1,LINTO
411   ILN(I4)=ILN(I4)+1
      GO TO 402
C --- SEARCH AGAINST NEXT SEGMENT
412   M=M+1
      IF (M.LT.I3) GO TO 407
C --- SEARCH AGAINST NEXT LINE
413   I1=I1+1
      IF (I1.LE.LINTO) GO TO 406
C --- TAKE NEXT POINT ON LINE I
      L=L+1
      IF (L.LE.K) GO TO 403
C --- TAKE POINTS FROM NEXT LINE
      I=I+1
      IF (I.LE.LINTO) GO TO 402
C --- TRANSFER LINES BACK INTO ARRAY DIGLN
      LINAC=1
      I=1
      JP=1
416   IF (LINAC.GE.2) I=ILN(LINAC-1)+1
      J=ILN(LINAC)
      NPNTS=J-I+1
      DO 417 K=1,NPNTS
      L=I+K-1
      DIGLN(2*K-1)=LINC(2*L-1)
417   DIGLN(2*K)=LINC(2*L)
      EPS=DATLN(LINAC,1)
      POT=DATLN(LINAC,2)
      CODE=DATLN(LINAC,3)
C --- WRITE LINE
C --- MOVE THE CLOSEST POINTS IN THE GRID TO THE POINTS ON THE LINE
161   LASTI=0
      DO 148 L=1,NPNTS
      DELX=9.9E9
      DELY=9.9E9
      DO 153 M=1,LNVER
      E1=ABS(DIGLN(2*L-1)-XPOS(M))

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      IF (E1.GT.DELX) GO TO 154
      DELX=E1
      IF (M.EQ.LNVER) GO TO 155
153  CONTINUE
154  M=M-1
155  DO 156 N=1,LNHOR
      E1=ABS(DIGLN(2*L)-YPOS(N))
      IF (E1.GT.DELY) GO TO 157
      DELY=E1
      IF (N.EQ.LNHOR) GO TO 158
156  CONTINUE
157  N=N-1
158  I=M+LNVER*(N-1)
      IF (I.EQ.LASTI) WARN=51
      IF (I.EQ.LASTI) WRITE(OUT,51) DIGLN(2*L-1),DIGLN(2*L)
      IF (I.EQ.LASTI) GO TO 1621
      MAC=MVERT(N,M)-(MVERT(N,M)/10)*10
      IF (MAC.EQ.1) GO TO 1621
      IF (XPCS(M).EQ.DIGLN(2*L-1).AND.YPOS(N).EQ.DIGLN(2*L))
160  GO TO 1622
      WARN=55
      WRITE(OUT,55) I
      WRITE(OUT,14)
      WRITE(OUT,16) XPOS(M),YPOS(N)
      WRITE(OUT,56)
      MVERT(N,M)=MVERT(N,M)+10
      AUX(JP,1)=DIGLN(2*L-1)
      AUX(JP,2)=DIGLN(2*L)
      MAUX(JP)=1
      JP=JP+1
1622 MVERT(N,M)=LINAC*100+1+MVERT(N,M)
      IF (MAC.EQ.2) MVERT(N,M)=MVERT(N,M)-1
      IF (CODE.EQ.2.0) MVERT(N,M)=MVERT(N,M)+10000
1621 LASTI=I
      JBP01=LPNT+L
148  POINT(JBP01)=1
C --- FIT GRID TO LINE SEGMENTS
      IF (NPNTS.LE.1) GO TO 185
      I1=LINAC-1
      I2=NPNTS-1
      DO 173 I=1,I2
      XF=DIGLN(2*I-1)
      YF=DIGLN(2*I)
      XL=DIGLN(2*I+1)
      YL=DIGLN(2*I+2)
      IF (LINAC.LE.1) GO TO 166
C --- CHECK IF XF,YF AND XL,YL ARE EQUAL TO SUCCESSIVE PREVIOUS POINTS
      DO 165 J=1,I1
      I3=1
      IF (J.GT.1) I3=ILN(J-1)+1
      I4=ILN(J)
      DO 164 K=I3,I4
      IF (XF.NE.LINC(2*K-1).OR.YF.NE.LINC(2*K)) GO TO 164
      IF (K.LE.I3) GO TO 163
      IF (XL.EQ.LINC(2*K-3).AND.YL.EQ.LINC(2*K-2)) GO TO 173
163  IF (K.GE.I4) GO TO 164

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      IF (XL.EQ.LINC(2*K+1).AND.YL.EQ.LINC(2*K+2)) GO TO 173
164  CONTINUE
165  CONTINUE
C --- FIT GRID TO LINE SEGMENT XF,YF - XL,YL
166  X=XF
      Y=YF
      K1=LPNT+1
      J=POINT(K1)
      JF=0
      DO 1673 M=1, LNHOR
      DO 1671 N=1, LNVER
      JF=JF+1
      IF(JF.EQ.J) GO TO 1672
1671 CONTINUE
1673 CONTINUE
1672 SXL=1.0
      IF (XL.LT.XF) SXL=-1.0
      SYL=1.0
      IF (YL.LT.YF) SYL=-1.0
C --- ANGLE OF LINE SEGMENT
      ALPHA=PI02
      IF (YL.LT.YF) ALPHA=-PI02
      IF (XL.NE.XF) ALPHA=ATAN((YL-YF)/(XL-XF))
      IF (XL.LT.XF) ALPHA=ALPHA+PI
      SALPH=SIN(ALPHA)
      CALPH=COS(ALPHA)
1667 ASINB=1000.0
      K=0
      M1=M-2
      N1=N-2
      DO 1670 JF=1,3
      JM=JF+M1
      IF(JM.LE.0) GO TO 1670
      IF(JM.GT.LNHOR) GO TO 1670
      DO 1668 JF1=1,3
      JN=N1+JF1
      IF(JN.LE.0) GO TO 1668
      IF(JN.GT.LNVER) GO TO 1668
      IF(M.EQ.JM.AND.N.EQ.JN) GO TO 1668
      JCV=0
      MAC=MVERT(JM,JN)-(MVERT(JM,JN)/10)*10
      MIR=(MVERT(JM,JN)/10)-(MVERT(JM,JN)/100)*10
      IF(MAC.GT.1) GO TO 1668
      XPO=XPOS(JN)
      YPO=YPOS(JN)
      IF(MIR.LT.1) GO TO 1669
      NUM=(JM-1)*LNVER+JN
      DO 2020 NENA=1,JP
      IF(AUX(NENA).EQ.NUM) GO TO 2021
2020 CONTINUE
      JCV=0
      GO TO 1669
2021 JCV=NENA
2022 XPO=AUX(JCV,1)
      YPO=AUX(JCV,2)
1669 E1=PI02-ALPHA

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      IF(YPO.LT.Y) E1=E1-PI
C --- ANGLE BETWEEN LINE SEGMENT AND ADJOINING POINT (BETA)
      IF (XPO.NE.X) E1=ATAN((YPO-Y)/(XPO-X))-ALPHA
      IF (XPO.LT.X) E1=E1+PI
      IF (COS(E1).LT.0.001) GO TO 1668
      IF(XPO.EQ.X.AND.YPO.EQ.Y) GO TO 1668
C --- DISTANCE TO ADJOINING POINT
      E2=SQRT((YPO-Y)**2+(XPO-X)**2)
      E3=E2*SIN(E1)
      IF (ABS(E3).GT.ABS(ASINB)) GO TO 1668
      XPL=XPO+E3*SALPH
      YPL=YPO-E3*CALPH
      IF ((XPL-XL)*SKL.GT.0.01) GO TO 1668
      IF ((YPL-YL)*SYL.GT.0.01) GO TO 1668
      ASINB=E3
      JS=JCV
      KMAC=MAC
      KM=JM
      KN=JN
      K=1
1668 CONTINUE
1670 CONTINUE
      IF(K.EQ.0) WARN=50
      IF(K.EQ.0) WRITE(OUT,50) XF,YF,XL,YL,X,Y
      K1=LPNT+1+1
      NOS=(KM-1)*LNVER+KN
      IF (NOS.EQ.POINT(K1)) GO TO 173
      IF(KMAC.LE.0) GO TO 1720
      WARN=55
      WRITE(OUT,55) NOS
      WRITE(OUT,14)
      WRITE(OUT,16) XPOS(KN),YPOS(KM)
      WRITE(OUT,56)
C --- MOVE POINT SHORTEST DISTANCE TO LINE
1720 IF(JS.EQ.0) GO TO 1721
      AUX(JS,1)=AUX(JS,1)+ASINB*SALPH
      AUX(JS,2)=AUX(JS,2)-ASINB*CALPH
      GO TO 1722
1721 ASSAL=ABS(ASINB*SALPH)
      ASCAL=ABS(ASINB*CALPH)
      IF(ABS(ASINB*SALPH).GT.0.01.OR.ABS(ASINB*CALPH).
1GT.0.01) GO TO 1723
      MVERT(KM,KN)=MVERT(KM,KN)+LINAC*100+2
      IF(CODE.EQ.2.0) MVERT(KM,KN)=MVERT(KM,KN)+10000
      GO TO 1724
1723 AUX(JP,1)=XPOS(KN)+ASINB*SALPH
      AUX(JP,2)=YPOS(KN)-ASINB*CALPH
      MAUX(JP)=NOS
      JP=JP+1
1722 MVERT(KM,KN)=0
      MVERT(KM,KN)=12+LINAC*100
      IF(CODE.EQ.2.0) MVERT(KM,KN)=MVERT(KM,KN)+10000
1724 JPM1=JP-1
      M=KM
      N=KN
      GO TO 1667

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173  CONTINUE
      I2=2*LPNT
C --- N2=1 FOR CLOSED CURVE
      N2=1
      K1=I2+2*NPNTS
      IF (LINC(K1-1).NE.LINC(I2+1).OR.LINC(K1).NE.LINC(I2+2).OR.
1NPNTS.LE.2) N2=0
      IF (N2.EQ.0.AND.EPS.GT.0.0) WARN=49
      IF (N2.EQ.0.AND.EPS.GT.0.0) WRITE(OUT,49)
      I1=POINT(LPNT+1)
      I1=NPNTS-1
C --- MIN AND MAX LIMITS
      XMIN=LINC(I2+1)
      XMAX=XMIN
      YMIN=LINC(I2+2)
      YMAX=YMIN
      DO 175 I=2, NPNTS
      K1=I2+2*I
      IF (LINC(K1-1).LT.XMIN) XMIN=LINC(K1-1)
      IF (LINC(K1-1).GT.XMAX) XMAX=LINC(K1-1)
      IF (LINC(K1).LT.YMIN) YMIN=LINC(K1)
      IF (LINC(K1).GT.YMAX) YMAX=LINC(K1)
175  CONTINUE
C --- LIMITS FOR LINE NUMBERS
      N5=1
      DO 177 I=1, LNHOR
      IF (YPOS(I).LT.YMIN) N5=I
      IF (YPOS(I).GT.YMAX) GO TO 180
177  CONTINUE
180  N6=I
      IF (N6.GT.LNHOR) N6=LNHOR
      N7=1
      DO 181 I=1, LNVER
      IF (XPOS(I).LT.XMIN) N7=I
      IF (XPOS(I).GT.XMAX) GO TO 182
181  CONTINUE
182  N8=I
      IF (N8.GT.LNVER) N8=LNVER
C --- SET POTENTIAL AND CODE FOR INTERIOR POINTS
      IF (N2.NE.1) GO TO 184
      DO 184 IROW=N5, N6
      DO 123 ICOL=N7, N8
C --- VERTEX NUMBER
      X=XPOS(ICOL)
      Y=YPOS(IROW)
      MIR=MVERT(IROW, ICOL)/10-(MVERT(IROW, ICOL)/100)
1*10
      IF (MIR.EQ.0) GO TO 1812
      NUM=(IROW-1)*LNVER+ICOL
      DO 2030 NENA=1, JP
      IF (MAUX(NENA).EQ.NUM) GO TO 2031
2030  CONTINUE
      GO TO 1812
2031  JCV=NENA
      X=AUX(JCV, 1)
      Y=AUX(JCV, 2)

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1812 CALL INTER(INTOR,I1,LPNT,X,Y)
      IF (INTOR.EQ.0) GO TO 183
      MVERT(IROW,ICOL)=MVERT(IROW,ICOL)+LINAC*100
      IF(CODE.EQ.2.0) MVERT(IROW,ICOL)=MVERT(IROW,ICOL)+10000
183  CONTINUE
184  CONTINUE
C --- TAKE NEXT LINE
185  LINAC=LINAC+1
      LPNT=LPNT+NPNTS
      IF (LINAC.LE.LINTO) GO TO 416
C --- ABORT
999  WRITE(OUT,45)
      IF(WARN.EQ.46) WRITE(OUT,46)
      IF(WARN.EQ.47) WRITE(OUT,47)
      IF(WARN.EQ.48) WRITE(OUT,48)
      IF(WARN.EQ.57) WRITE(OUT,57)
      IF(WARN.EQ.58) WRITE(OUT,58)
      IF(WARN.EQ.585) WRITE(OUT,585)
      CALL SRCHSS(KSCLOS,'WINDAT',6,1,0,NCOD)
612  RETURN
      END
      SUBROUTINE SOLUTN
GINSERT SYSCOID KEYS.F
      DIMENSION VOLTS(904),NODE(900),NIRR(4),COEFF1(18145)
      1,NZON(4),NDATLN(70,2),NOD(4),X(4),Y(4),AREA(4),ILN(70)
      2,XPOS(70),YPOS(100),AUX(500,2),MAUX(500),NCOD(4),ITRI(900,8)
      3,TEMP(7260),JVERT(6,4),NODE4(120),IVERT(2000,3),PERM(2000)
      4,COEFF(900,9)
      EQUIVALENCE (COEFF1(1),TEMP(1)),(COEFF1(7261),COEFF(1)),
      1(COEFF1(15361),PERM(1))
      REAL LINC(800),IDENT(20),MAXI
      INTEGER*2 MVERT(70,100),OUT
      COMMON/BLOCK1/XPOS,YPOS,LNVER,LNHOR,IDENT,EQLIN,
      1 ITOT,ISQ,FLAXI,EPS0/BLOCK2/LINC,NZV(15),NZE(15),
      1 ADMAB(15,15),ADMAD(15,15),DATLN(70,3)
      COMMON/BLOCK3/MVERT/BLOCK4/DIGLN/BLOCK5/AUX,MAUX/BLOCK6/NTRI
      COMMON/BLOCK7/LINTO,ILN/BLOCK8/VOLTS,NODE,JP,NZHT,NZVT
      COMMON/BLOCK9/COEFF1/BLOCK11/ITRI,IVERT
      COMMON/BLOCK12/WITHN,MAXI,TOL,ALPHA,AVEVOL,HLF,HLL,VLF,VLL
      DATA JVERT(1,1),JVERT(1,2),JVERT(1,3),JVERT(1,4)/7,3,5,1/
      DATA JVERT(2,1),JVERT(2,2),JVERT(2,3),JVERT(2,4)/1,5,7,3/
      DATA JVERT(3,1),JVERT(3,2),JVERT(3,3),JVERT(3,4)/3,7,1,5/
      DATA JVERT(4,1),JVERT(4,2),JVERT(4,3),JVERT(4,4)/2,6,8,4/
      DATA JVERT(5,1),JVERT(5,2),JVERT(5,3),JVERT(5,4)/5,1,3,7/
      DATA JVERT(6,1),JVERT(6,2),JVERT(6,3),JVERT(6,4)/6,2,4,8/
      ICRIPTS(NG,MG)=(NG-1)*LNVER+MG
C --- THE NUMBER OF COLUMNS AND ROWS IS COMPUTED.
      IN=5
      OUT=6
      NOCOL=LNVER-1
      NOROW=LNHOR-1
      MTRI=NOCOL*NOROW*2
      ITOT=LNHOR*LNVER
      NMAX=MAXI
      OMAPLH=1.0-ALPHA
C --- THE ARRAY ITRI IS INITIALIZED

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DO 6010 I=1,ITOT
DO 6011 J=1,8
ITRI(I,J)=0
6011 CONTINUE
6010 CONTINUE
C --- THE ARRAY COEFF. IS INITIALIZED
DO 6012 I=1,ITOT
DO 6013 J=1,9
COEFF(I,J)=0.0
6013 CONTINUE
6012 CONTINUE
C --- THE NUMBER OF SQUARES "ISQ" IS INTIALIZED.
ISQ=0
C --- THE PROCESSING IS DONE ONE COLUMN AT A TIME FOR EVERY ROW.
6020 DO 9001 N=1,NOROW
DO 9000 M=1,NOCOL
MIRT=0
ISQ=ISQ+1
DO 20 K=1,4
NCOD(K)=-K
GO TO (30,40,50,60),K
30 NG=N
MG=M
GO TO 70
40 NG=N
MG=M+1
GO TO 70
50 NG=N+1
MG=M+1
GO TO 70
60 NG=N+1
MG=M
C --- AFTER THE PROPER GRID POSITION IS FOUND,
C --- THE NODE CUALITIES ARE COMPUTED.
70 NIRR(K)=MVERT(NG,MG)/10-(MVERT(NG,MG)/100)*10
NZON(K)=MVERT(NG,MG)/100-(MVERT(NG,MG)/10000)*100
C --- THE RESEMBLANZE OF GRID POINTS AND ELEC.NODES IS ESTABLISHED HERE
I=ICRPTS(NG,MG)
IF(NZON(K).EQ.0)GO TO 71
ID=NZON(K)
IF(DATLN(ID,3).NE.2.0)GO TO 71
VOLTS(I)=DATLN(ID,2)
NOD(K)=I
NODE(I)=1
GO TO 73
71 VOLTS(I)=AVEVOL
NOD(K)=I
NODE(I)=0
IF(DATLN(ID,3).EQ.3.0)NCOD(K)=ID
73 IF(NIRR(K).EQ.0)GO TO 80
DO 90 J=1,JP
IF(I.EQ.MAUX(J))GO TO 100
90 CONTINUE
100 X(K)=AUX(J,1)
Y(K)=AUX(J,2)
GO TO 19

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80      X(K)=XPOS(MC)
      Y(K)=YPOS(NG)
19      MIRT=MIRT+NIRR(K)
20      CONTINUE
C --- REGULAR SQUARES TO INSTRUCTION 79, IRREGULARS TO NEXT.
      KIODG=1
      IF(MIRT.EQ.0)GO TO 79
C --- THE AREA OF THE FOUR POSSIBLE TRIANGLES IS CALCULATED.
      AREA(1)=((X(2)-X(1))*(Y(4)-Y(1))-(Y(2)-Y(1))*(X(4)-X(1)))
      AREA(2)=((X(4)-X(3))*(Y(2)-Y(3))-(Y(4)-Y(3))*(X(2)-X(3)))
      AREA(3)=((X(3)-X(2))*(Y(1)-Y(2))-(Y(3)-Y(2))*(X(1)-X(2)))
      AREA(4)=((X(1)-X(4))*(Y(3)-Y(4))-(Y(1)-Y(4))*(X(3)-X(4)))
      DO 1900 KLING=1,4
      IF(AREA(KLING).LE.0)GO TO 2000
1900    CONTINUE
      GO TO 2001
2000    WRITE(OUT,2002)
2002    FORMAT(35H THE FOLLOWING TRIANGLE RESULTED IN
      1/31H EITHER A NEGATIVE OR ZERO AREA
      2/17H VERTICES NUMBER:)
      GO TO(2003,2004,2005,2006),KLING
2003    WRITE(OUT,2007)NOD(1),NOD(2),NOD(4),X(1),X(2),X(4)
      GO TO 2008
2004    WRITE(OUT,2007)NOD(2),NOD(3),NOD(4),X(2),X(3),X(4)
      GO TO 2008
2005    WRITE(OUT,2007)NOD(1),NOD(2),NOD(3),X(1),X(2),X(3)
      GO TO 2008
2006    WRITE(OUT,2007)NOD(1),NOD(3),NOD(4),X(1),X(3),X(4)
2007    FORMAT(1H3(10X,15)/3(5X,F10.2))
2008    GO TO 3125
2001    AR1=ABS((AREA(1)/AREA(2))-1.0)
      AR2=ABS((AREA(3)/AREA(4))-1.0)
C --- THE KIND OF DIAGNL IS ESTABLISHED: 1- LL,UR,OR 2- UL,LR
      IF(AMIN1(AR1,AR2).EQ.AR1)KIODG=2
79      IF((NZON(1).LT.1).AND.(NZON(3).LT.1))GO TO 77
      IF(NZON(1).NE.NZON(3))GO TO 77
      KIODG=1
      GO TO 10
77      IF((NZON(2).LT.1).AND.(NZON(4).LT.1))GO TO 10
      IF(NZON(2).EQ.NZON(4))KIODG=2
C --- STORE THE KIND OF DIAGONAL (KIODG)
C --- FIND THE PERMITIVITY OF THE FIRST TRIANGLE IN THE SQUARE.
10      DO 190 L=1,2
      GO TO (189,188),L
183      GO TO(187,186),KIODG
186      K1=3
      K2=4
      K3=2
      K4=4
      MAR=2
      GO TO 185
187      K1=4
      K2=1
      K3=3
      K4=2
      MAR=4

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      GO TO 185
189   GO TO(184,183),KIODG
183   K1=1
      K2=2
      K3=4
      K4=3
      MAR=1
      GO TO 185
184   K1=2
      K2=3
      K3=1
      K4=1
      MAR=3
185   XBAR=(X(K1)+X(K2)+X(K3))/3.0
      YBAR=(Y(K1)+Y(K2)+Y(K3))/3.0
      DO 200 LINAC=1,LINTO
      NPTS=ILN(LINAC)
      KI=1
      IF(LINAC.LT.2)GO TO 201
      NPTS=ILN(LINAC)-ILN(LINAC-1)
      KI=ILN(LINAC-1)+1
201   I1=NPTS-1
      KF=ILN(LINAC)
      IF(LINC(2*KI-1).NE.LINC(2*KF-1).OR.LINC(2*KI).NE.LINC(2*KF)
1.OR.NPTS.LE.2)GO TO 200
      LPNT=KI-1
      CALL INTER(INTOR,I1,LPNT,XBAR,YBAR)
      IF(INTOR.NE.0)GO TO 203
200   CONTINUE
      EPSI=EPSO
      GO TO 2041
203   EPSI=DATLN(LINAC,1)
C --- THE VERTEX AND THE PERMT. OF EACH TRIANGLE IS STORED.
2041  CALL PERTRI(KIODG,EPSI,NOD(K1),NOD(K2),NOD(K3))
      IF(MIRT.NE.0)GO TO 140
      DO 150 JM=1,NZVT
      IF(NZV(JM).GT.M)GO TO 160
150   CONTINUE
160   DO 170 JN=1,NZHT
      IF(NZH(JN).GT.N)GO TO 180
170   CONTINUE
C --- THE ADMITANCES FOR REGULAR TRIANGLES ARE ASSIGNED HERE.
180   YAB=ADMAB(JM,JN)*EPSI
      YAD=ADMAD(JM,JN)*EPSI
      IF(KIODG.EQ.1)GO TO 1855
      YAUX=YAD
      YAD=YAB
      YAE=YAUX
1855  YBD=0.0
      GO TO 300
C --- THE ADMITANCES FOR IRREGULAR TRIANGLES ARE ASSIGNED HERE.
140   YAB=((Y(K2)-Y(K3))*(Y(K3)-Y(K1))+(X(K3)-X(K2))*(X(K1)-X(K3)))
1*EPSI/AREA(MAR)
      YAD=((Y(K2)-Y(K3))*(Y(K1)-Y(K2))+(X(K3)-X(K2))*(X(K2)-X(K1)))
1*EPSI/AREA(MAR)
      YBD=((Y(K3)-Y(K1))*(Y(K1)-Y(K2))+(X(K1)-X(K3))*(X(K2)-X(K1)))

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      1*EPSI/AREA(MAR)
C --- FOR FLOATING CONDUCTORS OF ZERO THICKNESS CHANGE Y'S
300  IF(NCOD(K1).EQ.NCOD(K2))YAB=(YAB/EPSI)*9000.
      IF(NCOD(K1).EQ.NCOD(K3))YAD=(YAD/EPSI)*9000.
      IF(NCOD(K2).EQ.NCOD(K3))YBD=(YBD/EPSI)*9000.
C --- IF AXIAL SYMMETRY IS WITH RESPECT TO Y GO TO
      IF(FLAXI.LT.2.)GO TO 310
C --- AXI-SYMMETRIC FIELDS
      IF(FLAXI.GT.2.0)GO TO 311
      YAB=YAB*YBAR
      YAD=YAD*YBAR
      YBD=YBD*YBAR
      GO TO 310
C --- AXI-SYMMETRIC FIELDS
311  YAB=YAB*XBAR
      YAD=YAD*XBAR
      YBD=YBD*XBAR
C --- BOTH TYPES
310  NOD1=NOD(K1)
      NOD2=NOD(K2)
      NOD3=NOD(K3)
C --- THE COEFFICIENTS OF THE 'Y' MATRIX ARE FORMED HERE.
      COEFF(NOD1,9)=COEFF(NOD1,9)-YAB-YAD
      COEFF(NOD2,9)=COEFF(NOD2,9)-YAB-YBD
      COEFF(NOD3,9)=COEFF(NOD3,9)-YAD-YBD
      J1=JVERT(1,K4)
      ITRI(NOD1,J1)=NOD2
      COEFF(NOD1,J1)=COEFF(NOD1,J1)+YAB
      J1=JVERT(2,K4)
      ITRI(NOD1,J1)=NOD3
      COEFF(NOD1,J1)=COEFF(NOD1,J1)+YAD
      J1=JVERT(3,K4)
      ITRI(NOD2,J1)=NOD1
      COEFF(NOD2,J1)=COEFF(NOD2,J1)+YAB
      J1=JVERT(4,K4)
      ITRI(NOD2,J1)=NOD3
      COEFF(NOD2,J1)=COEFF(NOD2,J1)+YBD
      J1=JVERT(5,K4)
      ITRI(NOD3,J1)=NOD1
      COEFF(NOD3,J1)=COEFF(NOD3,J1)+YAD
      J1=JVERT(6,K4)
      ITRI(NOD3,J1)=NOD2
      COEFF(NOD3,J1)=COEFF(NOD3,J1)+YBD
190  CONTINUE
9000 CONTINUE
9001 CONTINUE
      ITOT1=ITOT
      JFIN=1
      NODE4(1)=1
      IF(WITHIN.NE.0.)GO TO 3060
      CALL SRCHSS(KSREAD,'DATA1',5,3,0,NCOD)
      READ(7,3061)JFIN
3061  FORMAT(15)
      DO 3062 J=1,JFIN
      READ(7,3061)NODE4(J)
3062  CONTINUE

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DO 3063 J=1,JFIN
NOD1=NODE4(J)
IF(NOD1.GT.0)GO TO 3064
NOD1=-NOD4(J)
IF(NODE(NOD1).EQ.1)NOD4(J)=NOD1
IF(NODE(NOD1).EQ.0)NOD4(J)=2
GO TO 3063
3064 ITOT1=ITOT1+1
READ(7,3065)VOLTS(NOD1)
3065 FORMAT(E12.5)
3063 CONTINUE
CALL SRCHGS(KSCLOS,'DATA1',5,3,0,NCOD)
CALL SRCHGS(KSREAD,'DATA2',5,5,0,NCOD)
NOD3=(JFIN+1)*JFIN/2
DO 3073 K=1,NOD3
READ(9,3074)TEMP(K)
3074 FORMAT(E12.5)
3073 CONTINUE
CALL SRCHGS(KSCLOS,'DATA2',5,5,0,NCOD)
3060 K4=JFIN-1
DO 3000 I=1,NMAX
DIFF=0.0
DO 3070 J=1,JFIN
IF(NODE4(J).GT.0)GO TO 3070
NOD1=-NOD4(J)
NOD3=0
K1=(J-1)*J/2
K2=J-1
SAVE=VOLTS(NOD1)
RES=0.
IF(J.EQ.1)GO TO 3076
DO 3071 K=1,K2
NOD2=IABS(NODE4(K))
NOD3=K1+K
RES=RES+TEMP(NOD3)*VOLTS(NOD2)
3071 CONTINUE
3076 NOD4=NOD3+1
NOD3=NOD4
IF(J.EQ.JFIN)GO TO 3077
DO 3075 K=J,K4
NOD3=NOD3+K
NOD2=IABS(NODE4(K+1))
RES=RES+TEMP(NOD3)*VOLTS(NOD2)
3075 CONTINUE
3077 DO 3072 K=1,8
NOD2=ITRI(NOD1,K)
IF(NOD2.EQ.0)GO TO 3072
RES=RES+COEFF(NOD1,K)*VOLTS(NOD2)
3072 CONTINUE
AVEVOL=TEMP(NOD4)+COEFF(NOD1,9)
VOLTS(NOD1)=VOLTS(NOD1)*OMAPLH-(ALPHA/AVEVOL)*RES
TOL1=DIFF-ABS(VOLTS(NOD1)-SAVE)
IF(TOL1.LT.0.)DIFF=ABS(VOLTS(NOD1)-SAVE)
3070 CONTINUE
DO 3020 J=1,ITOT
IF(NODE(J).GT.0)GO TO 3020

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      SAVE=VOLTS(J)
      RES=0.0
      DO 3021 K=1,8
      J1=ITRI(J,K)
      IF(J1.EQ.0)GO TO 3021
      RES=RES+COEFF(J,K)*VOLTS(J1)
3021  CONTINUE
      VOLTS(J)=OMAPLE*VOLTS(J)-(ALPHA/COEFF(J,9))*RES
      TOL1=DIFF-ABS(VOLTS(J)-SAVE)
      IF(TOL1.LT.0.)DIFF=ABS(VOLTS(J)-SAVE)
3020  CONTINUE
      IF((TOL-DIFF).GE.0.)GO TO 3040
3000  CONTINUE
      WRITE(OUT,3050)TOL,NMAX
3050  FORMAT('TOLERANCE OF',E12.5,' WAS NOT MET AFTER',I4,' ITERATIONS')
C --- PRINT THE MINIMUM DIFF.REACHED
3040  WRITE(OUT,3051)DIFF,I
3051  FORMAT('MIN. DIFF. OF: ',E12.5,' AFTER ',I4,' ITERATIONS')
C --- TRANSFER THE COORDINATES TO COEFF ARRAY.
      K1=0
      DO 3101 J=1,LNHOR
      DO 3102 K=1,LNVER
      K1=K1+1
      COEFF(K1,8)=XPOS(K)
      COEFF(K1,9)=YPOS(J)
3102  CONTINUE
3101  CONTINUE
C --- TRANSFER ARRAYS AUX 182 TO COEFF.
      JP=JP-1
      IF(JP.LE.0)GO TO 3104
      DO 3103 J=1,JP
      K1=MAUX(J)
      COEFF(K1,8)=AUX(J,1)
      COEFF(K1,9)=AUX(J,2)
3103  CONTINUE
C --- EMPTY THE REMAINING OF THE ARRAY COEFF.
3104  DO 3105 J=1,ITOT
      DO 3106 K=1,7
      COEFF(J,K)=0.
3106  CONTINUE
3105  CONTINUE
C --- CALCULATE MAX.FIELD STRENCH. FOR EACH PERMT.
      ISQ=ISQ*2
      K1=LINT0+1
      DATLN(K1,1)=EPS0
      DO 3041 J=1,K1
      EPSI=DATLN(J,1)
      IF((EPSI.LT.0.001).OR.(EPSI.GT.1000.0))GO TO 3041
      EMAX=0.
      DO 3042 K=1,ISQ
      IF(EPSI.NE.PERM(K))GO TO 3042
      CALL FSTR(K,EX,EY,E,D)
      IF(E.LT.EMAX)GO TO 3042
      EMAX=E
      K2=K
3042  CONTINUE

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      WRITE(OUT,3043)EPSI,EMAX
3043  FORMAT(40H0FOR MATERIAL WITH RELATIVE PERMITIVITTY,F7.2,
137H THE MAXIMUN FIELD STRENGTH, VOLTS/MM,F9.2/
224H OCCURS AT TRIANGLE WITH)
      IF(FLAXI.GT.1.5)WRITE(OUT,3044)
      IF(FLAXI.LT.1.5)WRITE(OUT,3045)
3044  FORMAT(15H VERTEX NUMBERS,7X,1HZ,8X,1HR)
3045  FORMAT(15H VERTEX NUMBERS,7X,1HX,8X,1HY)
      NOD1=IVERT(K2,1)
      NOD2=IVERT(K2,2)
      NOD3=IVERT(K2,3)
      WRITE(OUT,3046)NOD2,COEFF(NOD2,8),COEFF(NOD2,9)
      WRITE(OUT,3046)NOD3,COEFF(NOD3,8),COEFF(NOD3,9)
      WRITE(OUT,3046)NOD1,COEFF(NOD1,8),COEFF(NOD1,9)
3046  FORMAT(6X,14,F15.1,F9.1)
3041  CONTINUE
C --- PRINTOUT OF THE POTENTIALS.
      WRITE(OUT,3100)
3100  FORMAT(27H1VERTEX POTENTIALS IN VOLTS)
      K1=1
      K2=10
3126  IF(K2.GT.ITOT)K2=ITOT
      WRITE(OUT,3111)(K3,K3=K1,K2)
      WRITE(OUT,3112)(COEFF(K3,8),K3=K1,K2)
      WRITE(OUT,3113)(COEFF(K3,9),K3=K1,K2)
      WRITE(OUT,3114)(VOLTS(K3),K3=K1,K2)
3111  FORMAT(9H0VERTEX :,14,9112)
3112  FORMAT(4H X :,F9.1,9(2X,F10.1))
3113  FORMAT(4H Y :,F9.1,9(2X,F10.1))
3114  FORMAT(2H ,E11.3,9(1X,E11.3))
      K1=K2+1
      IF(K1.GT.ITOT)GO TO 3125
      K2=K2+10
      GO TO 3126
3125  CALL SRCSS(KSCLOS,'WIFOUT',6,2,0,NCOD)
      RETURN
      END
      SUBROUTINE PERTRI(KIODG,EPSI,M,N,L)
      DIMENSION IVERT(2000,3),PERM(2000),ITRI(900,8)
      COMMON/BLOCK9/COEFF1(18145)
      COMMON/BLOCK11/ITRI,IVERT
      EQUIVALENCE (COEFF1(15361),PERM(1))
      DATA I2,I3/0,0/
      I2=I2+1
      I3=I3+1
      GO TO (10,20),KIODG
      GO TO (40,50),I2
40  IVERT(I3,1)=L
      IVERT(I3,2)=M
      IVERT(I3,3)=N
      GO TO 60
50  IVERT(I3,1)=N
      IVERT(I3,2)=L
      IVERT(I3,3)=M
      GO TO 60
20  GO TO (70,80),I2

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70  IVERT(13,1)=L
    IVERT(13,2)=M
    IVERT(13,3)=N
    GO TO 60
80  IVERT(13,1)=L
    IVERT(13,2)=M
    IVERT(13,3)=N
60  IF(12.GE.2)12=0
    PERM(13)=EPSI
    RETURN
    END
    SUBROUTINE FSTR(I,EX,EY,E,D)
    DIMENSION COEFF(900,9),PERM(2000),COEFF1(18145)
    COMMON/BLOCK8/VOLTS(904),NODE(900),JP,NZHT,NZVT
    COMMON/BLOCK9/COEFF1/BLOCK11/ITRI(900,8),IVERT(2000,3)
    EQUIVALENCE (COEFF1(7261),COEFF(1)),(COEFF1(15361),PERM(1))
    L=IVERT(1,1)
    M=IVERT(1,2)
    N=IVERT(1,3)
    XL=COEFF(L,8)
    YL=COEFF(L,9)
    XM=COEFF(M,8)
    YM=COEFF(M,9)
    XN=COEFF(N,8)
    YN=COEFF(N,9)
    CALL SUBD(D,XM,YM,XN,YN,XL,YL)
    EX=(VOLTS(L)*((YN-YD+VOLTS(M)*((YL-YN)+VOLTS(N)*((YM-YL))/D
    EY=-((VOLTS(L)*((XN-XD+VOLTS(M)*((XL-XN)+VOLTS(N)*((XM-XL))/D
    E=SQRT(EX*EX+EY*EY)
    RETURN
    END
    SUBROUTINE PLOTY
C --- LAPLACIAN ELECTROSTATIC FIELD, PRIME 500 FORTRAN IV, PART 2
    DIMENSION XPOS(70),YPOS(100),ILN(70),NZV(15),NZH(15)
    1,ADMAB(15,15),ADMAD(15,15),DATLN(70,3),TEMP(7260),COEFF(900,9)
    2,PERM(2000),COEFF1(18145)
    REAL IDENT(20),LINC(800)
    INTEGER*2 IVERT(2000,3),IPRCV(4)
    COMMON/BLOCK1/XPOS,YPOS,LNVER,LNHOR,IDENT,EQLIN,ITOT
    1,ISQ,FLAXI,EPSO
    COMMON/BLOCK2/LINC,NZV,NZH,ADMAB,ADMAD,DATLN
    COMMON/BLOCK7/LINTO,ILN
    COMMON/BLOCK8/VOLTS(904),NODE(900),JP,NZHT,NZVT
    COMMON/BLOCK9/COEFF1
    COMMON/BLOCK11/ITRI(900,8),IVERT
    EQUIVALENCE (COEFF1(1),TEMP(1)),(COEFF1(7261),COEFF(1))
    1,(COEFF1(15361),PERM(1))
    DATA BLANK/' '/
C --- DETERMINE THE VALUES OF POTMX AND POTMN
331  POTMX=-1000.
    POTMN=1000
    DO 332 J=1,ITOT
    IF(VOLTS(J).GT.POTMX)POTMX=VOLTS(J)
    IF(VOLTS(J).LT.POTMN)POTMN=VOLTS(J)
332  CONTINUE
    --- DETERMINE IF OPTIONAL PLOT IS DESIRED...

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      IF(EQLIN.LE.0.0)GO TO 999
C --- DETERMINE SIZE OF REGION TO PLOT...
      E1=XPOS(LNVER)-XPOS(1)
      E2=YPOS(LNHOR)-YPOS(1)
      E5=9.0/E2
      IF(E1.GT.E2)E5=10.0/E1
C --- SELF-DETERMINING SCALING FACTOR...
C --- PERCENT VOLTS PER LINE
      PERCV=0
      IF (EQLIN.GT.0.5) PERCV=100.0/(EQLIN+1.0)
C --- ENTER INTO GRAPHICS MODE...
      CALL GRESET
      CALL ENTGRA
C --- DRAW IDENTIFICATION, SCALE AND PERCENT VOLTS PER LINE
      N=4
      DO 141 M=2,20
      IF (IDENT(M).NE.BLANK) N=N+4
141  CONTINUE
      CALL SLTRZ(0)
      CALL MOVE(-5.0,-5.0)
      CALL TEXT(IDENT,N)
      CALL MOVE(-5.0,-4.7)
      CALL TEXT('PERCENT VOLTS PER LINE:',23)
10  ENCODE(8,10,IPRCV)PERCV
      FORMAT(F8.3)
      CALL MOVE(-1.5,-4.7)
      CALL TEXT(IPRCV,8)
      ENCODE(8,10,IPRCV)POTMX
      CALL TEXT(' P.MAX:',7)
      CALL TEXT(IPRCV,8)
      ENCODE(8,10,IPRCV)POTMN
      CALL TEXT(' P.MIN:',7)
      CALL TEXT(IPRCV,8)
C --- SECTION BOUNDARIES
143  E1=XPOS(1)
      E2=XPOS(LNVER)
      E3=YPOS(1)
      E4=YPOS(LNHOR)
      E6=-4.5
      COEFF(1,6)=-5.0
      COEFF(1,7)=E6
      COEFF(2,6)=-5.0
      COEFF(2,7)=E5*(E4-E3)+E6
      COEFF(3,6)=E5*(E2-E1)-5.0
      COEFF(3,7)=COEFF(2,7)
      COEFF(4,6)=COEFF(3,6)
      COEFF(4,7)=E6
      COEFF(5,6)=-5.0
      COEFF(5,7)=E6
      CALL MOVE(COEFF(1,6),COEFF(1,7))
      DO 1001 JP=2,5
1001  CALL DRAW(COEFF(JP,6),COEFF(JP,7))
C --- ELECTRODES AND DIELECTRIC MATERIALS
148  DO 242 I=1,LINTO
      LPNT=0
      IF (I.GT.1) LPNT=ILN(I-1)

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```

N=ILN(I)-LPNT
IF (N.LE.1) GO TO 242
DO 241 J=1,N
N1=2*(LPNT+J)
COEFF(J,6)=E5*(LINC(N1-1)-E1)-5.0
241 COEFF(J,7)=E5*(LINC(N1)-E3)+E6
COEFF(N+1,6)=-5.0
COEFF(N+1,7)=-4.5
COEFF(N+2,6)=-5.0
COEFF(N+2,7)=-4.5
CALL MOVE(COEFF(1,6),COEFF(1,7))
DO 1002 JP=2,N
1002 CALL DRAW(COEFF(JP,6),COEFF(JP,7))
242 CONTINUE
C --- CHANGE COORDINATES
DO 243 I=1,ITOT
COEFF(I,8)=E5*(COEFF(I,8)-E1)-5.0
243 COEFF(I,9)=E5*(COEFF(I,9)-E3)+E6
C --- EQUIPOTENTIAL LINES
NLINE=EQLIN
XEND=0
YEND=0
E1=(EQLIN+1.0)/(POTMX-POTMN)
DO 602 I=1,ITOT
602 VOLTS(I)=E1*(VOLTS(I)-POTMN)
DO 610 N1=1,NLINE
J=0
E1=FLOAT(N1)
DO 609 I=1,ISQ
I10=0
K=IVERT(I,1)
L=IVERT(I,2)
M=1
603 IF (VOLTS(K).GT.VOLTS(L)) GO TO 604
IMIN=K
IMAX=L
POTMN=VOLTS(K)
POTMX=VOLTS(L)
GO TO 605
604 IMIN=L
IMAX=K
POTMN=VOLTS(L)
POTMX=VOLTS(K)
605 IF (E1.LT.POTMN.OR.E1.GT.POTMX) GO TO 607
E2=0
IF (POTMN.LT.POTMX) E2=(E1-POTMN)/(POTMX-POTMN)
IF (I10.EQ.0) J=J+1
DO 606 I11=8,9
K1=I10+I11-7
606 COEFF(J,K1)=COEFF(IMIN,I11)+E2*(COEFF(IMAX,I11)-COEFF(IMIN,I11))
COEFF(J,5)=0
I10=2
607 IF (M.GE.2) GO TO 608
K=IVERT(I,2)
L=IVERT(I,3)
M=2

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```

GO TO 603
608 IF (M.EQ.3) GO TO 609
K=IVERT(1,3)
L=IVERT(1,1)
M=3
GO TO 603
609 CONTINUE
IF (J.EQ.0) GO TO 610
700 INV=0
C --- FIRST LINE SEGMENT INTO ARRAYS XA AND YA
DO 701 I=1,2
COEFF(1,6)=COEFF(1,2*I-1)
701 COEFF(1,7)=COEFF(1,2*I)
N=2
IF (J.EQ.1) GO TO 708
C --- LOOK FOR CONNECTING LINE SEGMENT
702 DO 706 I=2,J
IF (COEFF(1,5).GT.0.5) GO TO 706
IF (ABS(ABS(COEFF(N,6))-COEFF(1,1)).LT.0.00001.AND.
1ABS(COEFF(N,7)-COEFF(1,2)).LT.0.00001) GO TO 703
IF (ABS(ABS(COEFF(N,6))-COEFF(1,3)).LT.0.00001.AND.
1ABS(COEFF(N,7)-COEFF(1,4)).LT.0.00001) GO TO 704
GO TO 706
703 N=N+1
COEFF(N,6)=COEFF(1,3)
COEFF(N,7)=COEFF(1,4)
COEFF(1,5)=1
GO TO 705
704 N=N+1
COEFF(N,6)=COEFF(1,1)
COEFF(N,7)=COEFF(1,2)
COEFF(1,5)=1
705 IF (N.LE.J) GO TO 702
GO TO 708
706 CONTINUE
C --- INVERT LINE AND TRY AGAIN
IF (INV.EQ.1) GO TO 708
IEXIT=1
GO TO 714
712 INV=1
GO TO 702
C --- DRAW LINE
703 INV=0
IEXIT=2
IF((ABS(COEFF(1,6))-XEND)**2+(COEFF(1,7)-YEND)**2.GT.(ABS(COEFF(N
1,6))-XEND)**2+(COEFF(N,7)-YEND)**2) GO TO 714
713 XEND=ABS(COEFF(N,6))
YEND=COEFF(N,7)
COEFF(N+1,6)=-5.
COEFF(N+1,7)=-5.
COEFF(N+2,6)=-4.
COEFF(N+2,7)=-4.
CALL MOVE(COEFF(1,6),COEFF(1,7))
DO 1003 JP=2,N
1003 CALL DRAW(COEFF(JP,6),COEFF(JP,7))
IF (J.GE.2) GO TO 709

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      GO TO 610
C --- REMAINING LINE SEGMENTS
709  M=0
      DO 711 I=2,J
      IF (COEFF(I,5).GT.0.5) GO TO 711
      M=M+1
      DO 710 K=1,5
710  COEFF(M,K)=COEFF(I,K)
711  CONTINUE
      J=M
      IF (J.GE.1) GO TO 709
      GO TO 610
C --- INVERT LINE
714  M=N/2
      DO 707 I=1,M
      E1=COEFF(I,6)
      E2=COEFF(I,7)
      K=N-I+1
      COEFF(I,6)=COEFF(K,6)
      COEFF(I,7)=COEFF(K,7)
      COEFF(K,6)=E1
707  COEFF(K,7)=E2
      GO TO (712,713), IEXIT
610  CONTINUE
C      RETURN TO ALPHA MODE...
      CALL EXITGR
999  RETURN
      END
      SUBROUTINE EQUIV
SINSERT SYSCOM KEYS.F
C --- THIS SUBROUTINE CALCULATES THE MATRIX THAT REPRESENTS
C --- THE BOUNDARY CONDITIONS FOR THOSE GRID POINTS ON THE
C --- BORDER LINE OF THE REGION WHERE CHANGES ARE TO BE DONE.
      DIMENSION VOLTS(904), NODE(900), NIRR(4), COEFF1(13145)
      1, NZON(4), NDATLN(70,2), NOD(4), X(4), Y(4), AREA(4), ILN(70)
      2, XPCS(70), YPCS(100), AUX(500,2), MAUX(500)
      REAL LINC(800), MAXI, IDENT(20)
      INTEGER*2 MVERT(70,100), STEP, OUT
      COMMON/BLOCK1/XPOS, YPOS, LNVER, LNHOR, IDENT, EQLIN,
      1 I6, I5, FLAXI, EPSO/BLOCK2/LINC, NZV(15), NZH(15),
      1 ADMAB(15,15), ADMAD(15,15), DATLN(70,3)
      COMMON/BLOCK3/MVERT/BLOCK4/DIGLN/BLOCK5/AUX, MAUX/BLOCK6/NTRI
      COMMON/BLOCK7/LINTO, ILN/BLOCK8/VOLTS, NODE, JP, NZHT, NZVT
      COMMON/BLOCK9/COEFF1
      COMMON/BLOCK12/WITHN, MAXI, TOL, ALPHA, AVEVOL, HLF, HLL, VLF, VLL
      IGRPTS(NG, MG) = (NG-1)*LNVER+MG
C --- THE NUMBER OF COLUMNS AND ROWS IS COMPUTED.
      IN=5
      OUT=6
      NCCOL=LNVER-1
      NOROW=LNHOR-1
C --- THE WINDOW LINE NUMBERS ARE DEFINED.
      LHF=HLF
      LEL=HLL
      LVF=VLF
      LVL=VLL

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C --- THE NUMBER OF RETAINED NODES IS INITIALIZED.
      JLAST=1
C --- THE PROCESSING IS DONE ONE COLUMN AT A TIME FOR EVERY ROW.
      DO 9001 N=1,NOROW
      DO 9000 M=1,NOCOL
C --- IF THE SQUARE IS INSIDE THE KEPT REGION, IT IS SKIPPED.
      IF(((N.GE.LHF).AND.(N.LT.LHL)).AND.((M.GE.LVF).AND.(M.LT.LVL))
*)GO TO 9000
C --- THE ELEMENT OF AREA IS ANALYZED
      DO 20 K=1,4
      GO TO (30,40,50,60),K
30      NG=N
      MG=M
      GO TO 70
40      NG=N
      MG=M+1
      GO TO 70
50      NG=N+1
      MG=M+1
      GO TO 70
60      NG=N+1
      MG=M
C --- AFTER THE PROPER GRID POSITION IS FOUND,
C --- THE NODE CUALITIES ARE CALCULATED.
70      NIRR(K)=MVERT(NG,MG)/10-(MVERT(NG,MG)/100)*10
      NZON(K)=MVERT(NG,MG)/100-(MVERT(NG,MG)/10000)*100
C --- THE RESEMBLANZE OF GRID POINTS AND ELEC.NODES IS ESTABLISHED HERE
      ID=NZON(K)
      ID1=NBOUFT(MG,NG,LHF,LHL,LVF,LVL)
      IF(ID1.EQ.1)GO TO 721
      IF(NZON(K).EQ.0)GO TO 71
      IF(DATLN(ID,3).LT.2.0)GO TO 71
      IF(NDATLN(ID,1).NE.0)GO TO 72
      NDATLN(ID,1)=NG
      NDATLN(ID,2)=MG
      NODE(JLAST)=IGRPTS(NG,MG)
      VOLTS(JLAST)=DATLN(ID,2)
      NOD(K)=JLAST
      JLAST=JLAST+1
      GO TO 73
72      I=IGRPTS(NDATLN(ID,1),NDATLN(ID,2))
      GO TO 76
721      I=-((NG-LHF)*(LVL-LVF+1)+MG-LVF+1)
      GO TO 76
71      I=IGRPTS(NG,MG)
76      JF=JLAST-1
      DO 74 J=1,JF
      IF(NODE(J).EQ.1)GO TO 75
74      CONTINUE
      NODE(JLAST)=I
      VOLTS(JLAST)=0.0
      J=JLAST
      JLAST=JLAST+1
75      NOD(K)=J
73      IF(NIRR(K).EQ.0)GO TO 80
      I=IGRPTS(NG,MG)

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DO 90 J=1,JP
  IF(I.EQ.MAUX(J))GO TO 100
90  CONTINUE
100  X(K)=AUX(J,1)
    Y(K)=AUX(J,2)
    GO TO 19
80  X(K)=XPOS(MG)
    Y(K)=YPOS(NG)
19  MIRT=MIRT+NIRR(K)
20  CONTINUE
C --- REGULAR SQUARES TO INSTRUCTION 79,IRREGULARS TO NEXT.
    KIODG=1
    IF(MIRT.EQ.0)GO TO 79
C --- THE AREA OF THE FOUR POSSIBLE TRIANGLES IS CALCULATED.
    AREA(1)=((X(2)-X(1))*(Y(4)-Y(1))-(Y(2)-Y(1))*(X(4)-X(1)))
    AREA(2)=((X(4)-X(3))*(Y(2)-Y(3))-(Y(4)-Y(3))*(X(2)-X(3)))
    AREA(3)=((X(3)-X(2))*(Y(1)-Y(2))-(Y(3)-Y(2))*(X(1)-X(2)))
    AREA(4)=((X(1)-X(4))*(Y(3)-Y(4))-(Y(1)-Y(4))*(X(3)-X(4)))
DO 1900 KLING=1,4
  IF(AREA(KLING).EQ.0)GO TO 2000
1900 CONTINUE
    GO TO 2001
2000 WRITE(1,2002)KLING,AREA(KLING)
2002 FORMAT('AREA(',I4,')',E12.5)
    PAUSE 1
2001 AR1=ABS(AREA(1)-AREA(2))
    AR2=ABS(AREA(3)-AREA(4))
C --- THE KIND OF DIAGNL.IS ESTABLISHED:1- LL,UR,OR 2- UL,LR
    IF(AR1.LT.AR2)KIODG=2
79  IF((NZON(1).LT.1).AND.(NZON(3).LT.1))GO TO 77
    IF(NZON(1).NE.NZON(3))GO TO 77
    KIODG=1
    GO TO 10
77  IF((NZON(2).LT.1).AND.(NZON(4).LT.1))GO TO 10
    IF(NZON(2).EQ.NZON(4))KIODG=2
C --- FIND THE PERMITIVITY OF THE FIRST TRIANGLE IN THE SQUARE.
10  DO 190 L=1,2
    GO TO (189,183),L
183  GO TO(187,186),KIODG
186  K1=3
    K2=4
    K3=2
    MAR=2
    GO TO 185
187  K1=4
    K2=1
    K3=3
    MAR=4
    GO TO 185
189  GO TO(184,183),KIODG
183  K1=1
    K2=2
    K3=4
    MAR=1
    GO TO 185
184  K1=2

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      K2=3
      K3=1
      MAR=3
185   XBAR=(X(K1)+X(K2)+X(K3))/3.0
      YBAR=(Y(K1)+Y(K2)+Y(K3))/3.0
      DO 200 LINAC=1,LINTO
      NPTS=ILN(LINAC)
      KI=1
      IF(LINAC.LT.2)GO TO 201
      NPTS=ILN(LINAC)-ILN(LINAC-1)
      KI=ILN(LINAC-1)+1
201   I1=NPTS-1
      KF=ILN(LINAC)
      IF(LINC(2*KI-1).NE.LINC(2*KF-1).OR.LINC(2*KI).NE.LINC(2*KF)
1.OR.NPTS.LE.2)GO TO 200
      LPNT=KI-1
      CALL INTER(INTOR,I1,LPNT,XBAR,YBAR)
      IF(INTOR.NE.0)GO TO 203
200   CONTINUE
      EPSI=EPS0
      GO TO 2041
203   EPSI=DATLN(LINAC,1)
2041  IF(MIRT.NE.0)GO TO 140
      DO 150 JM=1,NZVT
      IF(NZV(JM).GT.MD)GO TO 160
150   CONTINUE
160   DO 170 JN=1,NZHT
      IF(NZH(JN).GT.N)GO TO 180
170   CONTINUE
C --- THE ADMITANCES FOR REGULAR TRIANGLES ARE ASSIGNED HERE.
180   YAB=ADMAB(JM,JN)*EPSI
      YAD=ADMAD(JM,JN)*EPSI
      IF(KIODG.EQ.1)GO TO 1855
      YAUX=YAD
      YAD=YAB
      YAB=YAUX
1855  YBD=0.0
      GO TO 300
C --- THE ADMITANCES FOR IRREGULAR TRIANGLES ARE ASSIGNED HERE.
140   YAB=((Y(K2)-Y(K3))*(Y(K3)-Y(K1))+(X(K3)-X(K2))*(X(K1)-X(K3)))
1*EPSI/AREA(MAR)
      YAD=((Y(K2)-Y(K3))*(Y(K1)-Y(K2))+(X(K3)-X(K2))*(X(K2)-X(K1)))
1*EPSI/AREA(MAR)
      YBD=((Y(K3)-Y(K1))*(Y(K1)-Y(K2))+(X(K1)-X(K3))*(X(K2)-X(K1)))
1*EPSI/AREA(MAR)
C --- IN CASE OF TWO NODES BEING THE SAME THEY ARE NOT ALTERED.
300   NOD1=NOD(K1)
      NOD2=NOD(K2)
      NOD3=NOD(K3)
      IF(NOD1.EQ.NOD2)YAB=0.0
      IF(NOD1.EQ.NOD3)YAD=0.0
      IF(NOD2.EQ.NOD3)YBD=0.0
      MIRT=0
C --- IF AXIAL SYMMETRY IS PRESENT THE RATIO OF THE CENTER IS ADDED.
      IF(FLAXI.LT.1.5)GO TO 310
      IF(FLAXI.GT.2.0)GO TO 311

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C --- AXI-SYMMETRIC FIELDS (X - SYMMETRI)
YAB=YAB*YBAR
YAD=YAD*YBAR
YBD=YBD*YBAR
GO TO 310
C --- AXI-SYMMETRIC FIELDS (Y - SYMMETRI)
311 YAB=YAB*XBAR
YAD=YAD*XBAR
YBD=YBD*XBAR
C --- BOTH TYPES
C --- THE COEFFICIENTS OF THE EQUIVALENT MATRIX ARE FORMED HERE.
310 ILM=NODFT(NOD1,NOD1)
COEFF1(ILM)=COEFF1(ILM-YAB-YAD)
ILM=NODFT(NOD1,NOD2)
COEFF1(ILM)=COEFF1(ILM+YAB)
ILM=NODFT(NOD1,NOD3)
COEFF1(ILM)=COEFF1(ILM+YAD)
ILM=NODFT(NOD2,NOD2)
COEFF1(ILM)=COEFF1(ILM-YAB-YBD)
ILM=NODFT(NOD2,NOD3)
COEFF1(ILM)=COEFF1(ILM+YBD)
ILM=NODFT(NOD3,NOD3)
COEFF1(ILM)=COEFF1(ILM-YAD-YBD)
190 CONTINUE
C --- KRON REDUCTION LOGIC IS EXECUTED HERE.
STEP=1
KSIG=1
IF(M.EQ.NOCOL)KSIG=KSIG+1
IF(N.EQ.NOROW)KSIG=KSIG+2
IF(KSIG.EQ.3)STEP=2
C --- CHANGE THE ORDER OF THE NODES
NAUX=NOD(3)
NOD(3)=NOD(4)
NOD(4)=NAUX
KTROL=0
C --- KRON REDUCE AS MANY TIMES AS NEEDED
DO 470 L=1,KSIG,STEP
NRED=6
NODE1=NOD(L)-KTROL
GO TO (410,420,430,440),L
410 NRED=MVERT(N,M/10000)
NRED=NRED+NBOUFT(M,N,LHF,LHL,LVF,LVL)
IF((NOD(1).EQ.NOD(2)).OR.(NOD(1).EQ.NOD(3)).OR.(NOD(1).EQ.NOD(4)))
*)NRED=NRED+1
GO TO 460
420 NRED=MVERT(N,M+1)/10000
NRED=NRED+NBOUFT(M+1,N,LHF,LHL,LVF,LVL)
IF((NOD(2).EQ.NOD(3)).OR.(NOD(2).EQ.NOD(4)))NRED=NRED+1
GO TO 460
430 NRED=MVERT(N+1,M/10000)
NRED=NRED+NBOUFT(M,N+1,LHF,LHL,LVF,LVL)
IF((NOD(3).EQ.NOD(4)).OR.(NOD(3).EQ.NOD(2)))NRED=NRED+1
GO TO 460
440 NRED=MVERT(N+1,M+1)/10000
NRED=NRED+NBOUFT(M+1,N+1,LHF,LHL,LVF,LVL)
460 IF(NRED.NE.0)GO TO 470

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C --- EXECUTE THE KRON REDUCTION

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JLAST=JLAST-1
ILM=NODFT(NODE1,NODE1)
IF(COEFF1(ILM).NE.0.)GO TO 4000
WRITE(OUT,4001)L,JLAST,KSIG,M,N,NODE1,(NOD(KLMD,KLM=1,4)
4001 FORMAT('COEFF1(',I3,')',, WHEN L=',I3,
*,',OUT OF ',I3,', NODES',, WITH KSIG=',I2,
*,',AT COLUMN ',I2,', AND ROW ',I2,', WHILE NODE1 = ',I4
*,',AND NOD 1,2,3,84 = ',I4I3)
RETURN
4000 DO 204 J=1,JLAST
IF(J.EQ.NODE1)GO TO 204
ILM2=NODFT(NODE1,J)
DO 205 I=1,J
IF(I.EQ.NODE1)GO TO 205
ILM3=NODFT(NODE1,I)
ILM4=NODFT(J,I)
COEFF1(ILM4)=COEFF1(ILM4)-(COEFF1(ILM2)/COEFF1(ILM))
1*COEFF1(ILM3)
205 CONTINUE
204 CONTINUE
C --- AFTER ELIMINATING ONE NODE ALL THE REMAINING ARE READJUSTED
C --- COEFF. ARE MOVED FROM RIGHT TO LEFT.
NODE2=NODE1+1
DO 208 I=NODE2,JLAST
ILM=NODFT(NODE1,I)
DO 207 J=NODE2,I
ILM2=ILM+1
COEFF1(ILM)=COEFF1(ILM2)
COEFF1(ILM2)=0.
ILM=ILM2
207 CONTINUE
208 CONTINUE
C --- THE COEFF. ARE MOVED FROM BELOW THE REDUCED AXIS.
JFIN=JLAST-1
ILM=NODFT(NODE1-1,NODE1-1)
DO 211 I=NODE1,JFIN
DO 212 J=1,I
ILM=ILM+1
ILM2=ILM+1
COEFF1(ILM)=COEFF1(ILM2)
COEFF1(ILM2)=0.0
212 CONTINUE
211 CONTINUE
C --- THE NODES NUMBER ARE BACKSPACED, AS WELL AS THE VOLTAGE VALUES
DO 210 J=NODE1,JFIN
NODE(J)=NODE(J+1)
VOLTS(J)=VOLTS(J+1)
210 CONTINUE
KTROL=KTROL+1
470 CONTINUE
9000 CONTINUE
9001 CONTINUE
C --- PRINTING OF THE REDUCED MATRIX.
JFIN=JLAST-1
CALL SRCHSS(KSWRT,'DATA1',5,4,0,NCOD)

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      WRITE(8,821)JFIN
821  FORMAT(15)
      ITOT1=(LHL-LHF+1)*(LVL-LVF+1)
      DO 822 J=1,JFIN
      IF(NODE(J).LT.0)GO TO 823
      ITOT1=ITOT1+1
      NODE(J)=ITOT1
823  WRITE(8,821)NODE(J)
822  CONTINUE
      DO 824 J=1,JFIN
      IF(NODE(J).LT.0)GO TO 824
      WRITE(8,825)VOLTS(J)
825  FORMAT(E12.5)
824  CONTINUE
      CALL SRCHSS(KSCLOS,'DATA1',5,4,0,NCOD)
      CALL SRCHSS(KSWRIT,'DATA2',5,5,0,NCOD)
      ILM=NODFT(JFIN,JFIN)
      DO 826 J=1,ILM
      WRITE(9,8271)COEFF1(J)
8271 FORMAT(E12.5)
826  CONTINUE
      CALL SRCHSS(KSCLOS,'DATA2',5,5,0,NCOD)
      RETURN
      END
      SUBROUTINE SUBD(D,XM,YM,XN,YN,XL,YL)
      D=XM*YN-XN*YM+XN*YL-XL*YN+XL*YM-XM*YL
      RETURN
      END
      SUBROUTINE DIGIT(NPNTS,IN,SORIG)
      COMMON/BLOCK4/DIGLN(140)
51  FORMAT(2F7.0)
52  FORMAT(4F7.0)
53  FORMAT(6F7.0)
54  FORMAT(8F7.0)
55  FORMAT(10F7.0)
      I1=NPNTS
      I2=0
801  I3=I1
      IF (I3.GT.5) I3=5
      IF (I3.EQ.1) READ(IN,51) (DIGLN(I4),I4=131,132)
      IF (I3.EQ.2) READ(IN,52) (DIGLN(I4),I4=131,134)
      IF (I3.EQ.3) READ(IN,53) (DIGLN(I4),I4=131,136)
      IF (I3.EQ.4) READ(IN,54) (DIGLN(I4),I4=131,138)
      IF (I3.EQ.5) READ(IN,55) (DIGLN(I4),I4=131,140)
      DO 802 I4=1,10
      K1=I4+I2
802  DIGLN(K1)=DIGLN(I4+130)
      I1=I1-5
      IF (.I1.LT.1) GO TO 803
      I2=I2+10
      GO TO 801
803  DO 806 I1=1,NPNTS
      IF (DIGLN(2*I1-1).EQ.9000.0) GO TO 807
806  CONTINUE
      GO TO 897
807  IF (DIGLN(2*I1).NE.1.0) GO TO 824

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C --- CIRCULAR ARC
C --- NUMBER OF POINTS (GREATER OR EQUAL TO THREE)
      NPARC=DIGLN(2*I1+1)
C --- RADIUS
      RARC=DIGLN(2*I1+2)
C --- CENTER
      XC=DIGLN(2*I1+3)
      YC=DIGLN(2*I1+4)
C --- ANGLE, START AND END
      ANG5=DIGLN(2*I1+5)*3.1415927/180.0
      ANGE=DIGLN(2*I1+6)*3.1415927/180.0
C --- NEW TOTAL NUMBER OF POINTS
      NPNTS=NPNTS+NPARC-4
      IF (NPNTS.GT.70) GO TO 899
      IF (NPNTS.EQ.I1-1+NPARC) GO TO 822
      I2=I1+4
      IF (NPARC.GE.5) GO TO 809
      I5=NPNTS-NPARC+4
      DO 808 I3=I2,I5
      I4=I3+NPARC-4
      DIGLN(2*I4-1)=DIGLN(2*I3-1)
808   DIGLN(2*I4)=DIGLN(2*I3)
      GO TO 822
809   DO 821 I3=I2,NPNTS
      I4=NPNTS+I2-I3
      K1=I4+NPARC-4
      DIGLN(2*K1-1)=DIGLN(2*I4-1)
821   DIGLN(2*K1)=DIGLN(2*I4)
C --- ANGLE BETWEEN POINTS
822   E1=(ANGE-ANG5)/(FLOAT(NPARC)-1.0)
      DO 823 I2=1,NPARC
      I3=I1-1+I2
      E2=ANG5+(FLOAT(I2)-1.0)*E1
      DIGLN(2*I3-1)=XC+RARC*COS(E2)
823   DIGLN(2*I3)=YC+RARC*SIN(E2)
      GO TO 803
824   IF (DIGLN(2).NE.2.0) GO TO 826
C --- RECTANGLES WITH HORIZONTAL AND VERTICAL SIDES
      NPNTS=5
      DIGLN(1)=DIGLN(3)
      DIGLN(2)=DIGLN(4)
      DIGLN(3)=DIGLN(5)
      DIGLN(7)=DIGLN(1)
      DIGLN(8)=DIGLN(6)
      DIGLN(9)=DIGLN(1)
      DIGLN(10)=DIGLN(2)
      GO TO 897
C --- OTHER PREPROGRAMMED LINES CAN BE ADDED HERE
825   CONTINUE
897   I1=2*NPNTS
      DO 898 I4=1,I1
      DIGLN(I4)=DIGLN(I4)/SORIG
898   RETURN
899   END
      SUBROUTINE INTER(INTOR,I1,LPNT,X,Y)
      DIMENSION NZV(15),NZH(15),ADMAB(15,15),ADMAD(15,15)

```

```

REAL LINC(800)
COMMON/BLOCK2/LINC,NZV,NZH,ADMA,ADMAD/BLOCK6/NTRI(8)
INTOR=0
DO 811 I=1,4
811 NTRI(I)=0
DO 815 I=1,I1
N1=2*(LPNT+1)
X1=LINC(N1-1)
Y1=LINC(N1)
X2=LINC(N1+1)
Y2=LINC(N1+2)
IF (Y.LT.AMIN1(Y1,Y2).OR.Y.GT.AMAX1(Y1,Y2).OR.Y1.EQ.Y2) GO TO 812
X3=(Y-Y1)*(X2-X1)/(Y2-Y1)+X1
IF (X.GT.X3-0.01.AND.X.LT.X3+0.01) GO TO 816
IF (Y.EQ.Y1) GO TO 812
IF (X3.LT.X) NTRI(1)=NTRI(1)+1
IF (X3.GT.X) NTRI(2)=NTRI(2)+1
812 IF (X.LT.AMIN1(X1,X2).OR.X.GT.AMAX1(X1,X2).OR.X1.EQ.X2) GO TO 813
Y3=(X-X1)*(Y2-Y1)/(X2-X1)+Y1
IF (Y.GT.Y3-0.01.AND.Y.LT.Y3+0.01) GO TO 816
IF (X.EQ.X1) GO TO 813
IF (Y3.LT.Y) NTRI(3)=NTRI(3)+1
IF (Y3.GT.Y) NTRI(4)=NTRI(4)+1
813 DO 814 N1=1,4
IF (NTRI(N1).EQ.2) NTRI(N1)=0
814 CONTINUE
815 CONTINUE
N1=NTRI(1)+NTRI(2)+NTRI(3)+NTRI(4)
IF (N1.GE.3) INTOR=1
816 RETURN
END
FUNCTION NODFT(NODF,NODL)
NODFT=(NODL-1)*NODL/2+NODF
IF(NODF.GT.NODL) NODFT=(NODF-1)*NODF/2+NODL
RETURN
END
FUNCTION NBOUFT(M,N,LHF,LHL,LVF,LVL)
NBOUFT=0
C --- CHECK THAT YOU ARE NOT ON ANY BOUNDARY
IF((N.EQ.LHF).AND.(M.GE.LVF).AND.(M.LE.LVL)))GO TO 10
IF((N.EQ.LHL).AND.(M.GE.LVF).AND.(M.LE.LVL)))GO TO 10
IF((M.EQ.LVF).AND.(N.GE.LHF).AND.(N.LE.LHL)))GO TO 10
IF((M.EQ.LVL).AND.(N.GE.LHF).AND.(N.LE.LHL)))GO TO 10
RETURN
10 NBOUFT=1
RETURN
END

```


Appendix 4 - Flow Charts

Table of Key-to-Program-Line* Correspondence

III.B.3.1	(CODING) Standard Admittances Computation
III.B.4.1	(EQUIV) Block Action of EQUIV
III.B.4.2	(EQUIV) Subdivision of Action Block A of EQUIV
III.B.4.3	(EQUIV) Code Decomposition - Block A1
III.B.4.4	(EQUIV) Electric Node Assignment - Block A2
III.B.4.5	(EQUIV) Permittivity Assignment - Blocks A3 and A4
III.B.4.6	(EQUIV) Computation of Admittances - Block A5
III.B.4.7	(EQUIV) Admittance Addition - Block B
III.B.4.8	(EQUIV) Kron Reduction Logic - Block C1
III.B.4.9	(EQUIV) Node Elimination - Block C2
III.B.4.10	(EQUIV) Writing Nodal Information - Block D1
III.B.4.11	(EQUIV) Writing Node Voltages - Block D2
III.B.5.1	(SOLUTN) Potential Code Assignment
III.B.5.4	(SOLUTN) Window Admittance Creation
III.B.5.5	(SOLUTN) Reading Equivalent Matrix
III.B.5.6	(SOLUTN) Solution of System of Equations
III.B.5.7	(SOLUTN) Field Strength
III.B.5.8	(SOLUTN) Writing of Potentials
III.B.6	Subroutine PLOTY
III.B.7	Subroutine PERTRI
III.B.8	Subroutine FSTR
III.B.13	Function NBOUFT

* Program lines are numbered from 1 to 56 for each program page (from 1 to 32) as given in Appendix 3.

Table of Key-to-Program-Line Correspondence

<u>Subroutine</u>	<u>Key</u>	<u>Program</u>	
		<u>Page</u>	<u>Line</u>
CODING	A	4	45
	B	5	8
EQUIV	C	25	27
	D	25	27
	E	26	11
	F	27	23
	G	28	12
	H	28	26
	I	29	2
	J	29	55
	K	30	10
SOLUTN	L	13	37
	M	14	6
	N	13	17
	O	16	19
	P	16	47
	Q	17	20
	R	18	17
	S	19	18

Figure III.B.3.1

Standard Admittances Computation Flowchart (CODING)

Figure III.B.4.1
Block Action of Subroutine EQUIV

Figure III.B.4.2

Subdivision of Action Blocks of Block A of EQUIV

Figure III.B.4.3
Code Decomposition Flow Chart
Subroutine EQUIV
Block A1

Figure III.B.4.4

-Electric Node Number Assignment Flow Chart
Subroutine EQUIV, Block A2

Figure III.B.4.5

Permittivity Assignment Flow Chart
Subroutine EQUIV, Blocks A3 and A4

Figure III.B.4.6

Computation of Admittances Flow Chart
Subroutine EQUIV, Block A5

Figure III.B.4.7

Admittance addition flowchart
Subroutine EQUIV, Block B

Figure III.B.4.8

Kron Reduction Logic Flow Chart
Subroutine EQUIV, Block C1

Figure III.B.4.9
Block action for node elimination
Subroutine EQUIV, Block C2

Figure III.B.4.10
Writing of nodal information
Subroutine EQUIV, Block D1

Figure III.B.4.11
Writing of the node's voltage
Subroutine EQUIV, Block D2

Figure III.B.5.1

Potential Code Assignment Flow Chart
Subroutine SOLUTN

Figure III.B.5.4

Window Admittance Creation Flow Chart
Subroutine SOLUTN

Figure III.B.5.5
Reading of the equivalent matrix, Flowchart (SOLUTN)

Figure III.B.5.6

Solution of the System of Equations Flow Chart
Subroutine SOLUTN

Figure III.B.5.7
Field Strength Flow Chart (SOLUTN)

Figure III.B.5.8
Writing of the Potentials
Subroutine SOLUTN

Figure III.B.6
Subroutine PLOTY, Flowchart

Figure III.B.7

Subroutine PERTRI Flow Chart

Figure III.B.8

Subroutine FSTR Flow Chart

Figure III.B.13
Function NBOUFT Flowchart.