235
3-7-80

FEBRUARY 1980

AD-801
PPPL-1637
UC-205,d,g

EFFECTS OF TOROIDAL FIELD RIPPLE
ON SUPRA-THERMAL IONS
IN TOKAMAK PLASMAS

BY

R. J. GOLDSTON AND H. H. TOWNER

PLASMA PHYSICS
LABORATORY

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

This work was supported by the U. S. Department of Energy
Contract No. EY-76-C-02-3073. Reproduction, translation,
publication, use and disposal, in whole or in part, by or
for the United States Government is permitted.

Effects of toroidal field ripple on suprathermal ions in tokamak plasmas

R. J. GOLDSTON AND H. H. TOWNER

Plasma Physics Laboratory, Princeton University
Princeton, NJ 08544 USA

Analytic calculations of three important effects of toroidal field ripple on suprathermal ions in tokamak plasmas are presented. In the first process, collisional ripple-trapping, beam ions become trapped in local magnetic wells near their banana tips due to pitch-angle scattering as they traverse the ripple on barely unripple-trapped orbits. In the second process, collisionless ripple-trapping, near-perpendicular untrapped ions are captured (again near a banana tip) due to their finite orbits, which carry them out into regions of higher ripple. In the third process, banana-drift diffusion, fast-ion banana orbits fail to close precisely, due to a ripple-induced "variable lingering period" near the banana tips. These three mechanisms lead to substantial radial transport of banana-trapped, neutral-beam-injected ions when the quantity $\alpha^* \equiv \epsilon |\sin \theta| / Nq\delta$ is of order unity or smaller.

1. Introduction

A considerable number of large and expensive computer codes (Dei-Cas and Marty 1974; Lister, Post, and Goldston 1976; Fowler et al. 1978; Tani et al. 1978) have been recently developed to investigate the behavior of supra-thermal ions in nonaxisymmetric tokamak plasmas. These codes each integrate the guiding

DISCLAIMER

This book is prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily imply its use or recommendation, endorsement, or favoring by the United States Government, or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

24

center orbit equations for energetic ions (with various levels of approximation) and most also follow the fast-ion thermalization process using Monte-Carlo techniques to simulate energy loss and pitch-angle scattering. In general, the codes have been used to predict (Dei-Cas and Marty 1974; Lister, Post, and Goldston 1976; Fowler et al. 1978; Tani et al. 1978) and to analyze (Equipe TFR 1978) the effectiveness of neutral beam heating experiments in tokamaks with substantial toroidal field ripple. If, instead, one closely examines a large number of the individual ion trajectories these codes calculate, a host of surprising -- and unexpected -- features emerge. In particular, one finds that fast ions show a perverse tendency to trap preferentially on the side of the horizontal midplane in the direction of the V_B drift, thus allowing themselves short super-banana trajectories to the plasma surface. In addition, one finds that even banana orbits which do not become localized in magnetic wells show very substantial radial diffusion and can be easily lost from the plasma. If collisions are suppressed, this "banana-drift diffusion" persists, and more surprisingly, local ripple-trapping is also still observed, but at a somewhat reduced rate, and in a much changed form. Without collisions, one finds that for most geometries ripple-trapping occurs exclusively on the "unfavorable" side of the horizontal midplane, and that fast ions are never detrapped before leaving the plasma. In some geometries with very little vertical gradient in the ripple intensity, however, in the absence of collisions ions trap only on the "favorable side" (away from the V_B drift), travel across the horizontal midplane

as super-bananas, and then startlingly detrap at their original minor radius, resuming very nearly their initial banana orbits.

Needless to say, we feel that these intriguing numerical results merit analytic investigation. The purpose of the studies described here is to provide insight into the mechanisms behind the radial transport of fast ions in rippled tokamak fields, so that a general understanding of these effects can lead to more informed use and interpretation of the numerical codes. In addition, we would hope that the fast ion transport rates calculated here will eventually be coupled into Fokker-Planck treatments (Rome et al. 1976; Cordey 1976; Mirin et al. 1977) or sped-up axisymmetric beam-orbit codes, in order to provide considerably more expedition computations of neutral-beam heating.

We have found that three distinct physical processes contribute to the observed complex behavior of fast ion trajectories. In section 2, we calculate the most straightforward of these: fast ions suffering small-angle scattering near their banana tips may be trapped collisionally into the local ripple wells. Here we also introduce two concepts used throughout these studies: the ripple-plateau regime of collisionality due to Boozer (to be published) and the model magnetic field near the banana tip first employed by Stringer (1972). In Section 3, we investigate the collisionless ripple-trapping process, which arises from the fact that while one branch of a banana orbit may be able to pass over the local magnetic ripples and reflect in the toroidal field, in some cases the local maxima are higher on the return branch, and a small fraction of the banana orbits then become nonadiabatically

trapped (Rutherford and Goldston 1977). In Sections 3f and g, we treat the interaction of this collisionless ripple-trapping process with the collisional trapping and detrapping processes calculated in Section 2. In Section 4, we calculate the banana-drift diffusion of energetic ions, which results from the fact that banana orbits encounter the field ripple at different phases at their reflection points. This gives rise to a "variable lingering period" (Goldston and Jassby 1976) near the banana tips and a concomitant radial transport. Finally, in Section 5 our results are briefly summarized, and a simple calculation of ripple loss for tangential neutral beam injection in strongly nonaxisymmetric systems is presented.

2. Collisional ripple-trapping

(a) Physical picture

Figure 1 illustrates the physical process under consideration in this section. Fast ions near their banana tips barely skim over the tops of the local ripple wells and a small amount of pitch-angle scattering causes them to become trapped into "super-banana" orbits. In the absence of pitch-angle scattering, the radial diffusion of the banana orbits that is also seen in figure 1 persists (see Section 2), but collisionless ripple-trapping (see Section 3) has been greatly reduced by assuming the ripple strength to be uniform across the plasma column.

(b) Model magnetic field

The toroidal magnetic field is taken to be of the form

$$B = \frac{B_0 R_0}{R} [1 + \delta(r, \theta) \sin N\phi]$$

For $Nq \gg 1$, the $1/R$ variation of the toroidal field along a magnetic field line is slow compared to the ripple modulation, so B near the banana tip may be approximated as

$$B \approx \frac{B_0 R_0}{R_b} (1 + \alpha^* \delta x + \delta \sin x) \equiv \frac{B_0 R_0}{R_b} \tilde{B}$$

where $x = N\phi$, $\alpha^* = |\sin \theta_b| / Nq \delta$, θ_b is the poloidal angle of the banana tip or "bounce point", and R_b is the major radius of the banana tip.

A number of important quantities defined in figure 2 can be easily calculated for this model field. Δ , the difference in the normalized field strength between successive ripple maxima, is given by

$$\Delta = 2\pi\delta\alpha^*$$

$x_{\max}$, the location of the first maximum, and $x_{\min}$, the location of the minimum are given by

$$x_{\max} = \pi - \cos^{-1} \alpha^* \quad x_{\min} = \pi + \cos^{-1} \alpha^*$$

and the reduced ripple well depth (Stringer 1972) is

$$\delta^* = 2\delta \left(\sqrt{1 - \alpha^{*2}} - \alpha^* \cos^{-1} \alpha^* \right).$$

For $\alpha^* \geq 1$, the local magnetic wells are eliminated. The bounce point for a marginally trapped particle, x_b , is found by

$$\alpha^* x_b + \sin x_b = \alpha^* \left(\pi - \cos^{-1} \alpha^* \right) + \sqrt{1 - \alpha^{*2}} .$$

The solution is shown in figure 3. We may also approximate

$$x_b \approx 2x_{\min} - x_{\max} = \pi + 3\cos^{-1} \alpha^* .$$

(c) Collisionality regime

For all the calculations presented here, we assume that well-defined banana orbits exist, so we require at least that $v_i^* < 1$. This condition is easily satisfied by injected beam ions. Further upper limits on the collisionality are defined in the text. In addition we also require that the phase at which a fast ion encounters the toroidal field ripple at its banana tip is collisionally decorrelated on successive bounces. Since an ion typically passes Nq coils during a bounce, our requirement then is that $v_i^* > 1/(Nq)^2$. This defines the lower limit of the "ripple-plateau" regime, described by Boozer (to be published). Injected ions are generally collisional enough to satisfy this criterion, but 3.5 MeV α -particles and other fusion products typically are not. However, the collisionality requirement is perhaps overly strict because toroidal precession moves fast ion banana orbits rapidly around the torus

$$\tau_b \frac{d\phi}{dt} \approx \frac{\pi v}{\sqrt{\epsilon} \Omega_{ci}} \frac{q^2}{a} \gg \frac{2\pi}{N} .$$

Thus, to provide an essentially random sampling of ripple phases at the banana tips, collisions must only spoil the periodic phase sampling that is established over many bounces.

(d) Collision operator

The Fokker-Planck collision operator for pitch-angle scattering is given by

$$\left. \frac{\partial f}{\partial t} \right|_c = \frac{v_{ii}}{2} \frac{\partial}{\partial \xi} (1 - \xi^2) \frac{\partial}{\partial \xi} f ,$$

where $\xi \equiv v_{\parallel}/v$. Expressing this operator in terms of constants of motion and retaining the dominant diffusive term, we have

$$\left. \frac{\partial f}{\partial t} \right|_c = \frac{v_{ii}}{2} \left[1 - \frac{1 - \tilde{B}_m/\tilde{B}}{\xi_m^2} \right] (1 - \xi_m^2) \frac{\partial^2}{\partial \xi_m^2} f ,$$

where ξ_m is $v_{\parallel}/v$ at the ripple well minimum, $x_{\min}$, at which location $\tilde{B} = \tilde{B}_m$. Defining $\eta \equiv \tilde{B} - \tilde{B}_m$, and expanding in $\delta^* \ll 1$, the operator for barely passing ions may be simplified to

$$\frac{v_{ii}}{2} (1 - \eta/\delta^*) \frac{\partial^2}{\partial \xi_m^2} f .$$

Since we wish to study the pitch-angle scattering of nearly trapped ions during a single transit over the ripple well, we bounce-integrate the collision operator:

$$\oint \left. \frac{\partial f}{\partial t} \right|_c dt = \frac{v_{ii} R_b}{2Nv\delta^*} \underbrace{\oint (1 - \eta/\delta^*)^{1/2} dx}_{L(\alpha^*)} \frac{\partial^2 f}{\partial \xi_m^2}.$$

The integral $L(\alpha^*)$ is determined numerically and shown in figure 4. By assuming a sinusoidal well shape, a well length given by $2(x_{\min} - x_{\max})$, and a well depth of δ^* , the analytic approximation $L(\alpha^*) \approx (16/\pi) \cos^{-1} \alpha^*$ may be obtained.

(e) Trapping rate

Since the bounce-integrated scattering operator has been reduced to the usual diffusion operator, we can easily evaluate the fast-ion distribution after a single bounce. To calculate the trapped fraction we assume that the initial distribution is given by

$$f_0 = 0 \quad \text{for } \xi_m < \xi_t = \sqrt{\delta^*}$$

$$f_0 = 1 \quad \text{for } \xi_m > \xi_t,$$

where ξ_t is the cosine of the pitch-angle of a barely-trapped ion. After a single bounce, we find

$$f_1(\xi_m) = \frac{1}{\sqrt{2\pi}\sigma} \int_{\xi_t}^{\infty} \exp \frac{-(\xi_m - \xi')^2}{2\sigma^2} d\xi'$$

where

$$\sigma = \left[\frac{v_{ii} R_b L(\alpha^*)}{N v \sqrt{\delta^*}} \right]^{1/2}$$

and the effective range of ξ_m which has become trapped is given by

$$\Delta \xi_m = \int_{-\infty}^{\xi_t} f_1(\xi') d\xi' = \frac{\sigma}{\sqrt{2\pi}}.$$

Finally, we must evaluate what fraction of all the ions with their banana tips at $\theta = \theta_b$ is represented by $\Delta \xi_m$. The distribution of particles over "ripple phase" may be treated by characterizing orbits with regard to their symmetric bounce points, x_{bs} , where x_{bs} is defined as the location in x of an ion's bounce point in the limit $\delta \rightarrow 0$, with the particle's energy and magnetic moment held fixed. In the "ripple-plateau" regime, collisions and toroidal precession make f independent of x_{bs} . Thus, the trapped fraction, F , is given by

$$F = \frac{\Delta \xi_m}{2\pi} \left| \frac{\partial x_{bs}}{\partial \xi_m} \right|_{\xi_t} = \frac{\Delta \xi_m \sqrt{\delta^*}}{\pi \delta \alpha^*}.$$

For the validity of the approximation of integrating along the marginally trapped orbit, we require $F < \delta^*/\Delta = (\sqrt{1-\alpha^{*2}} - \alpha^* \cos^{-1} \alpha^*)/(\pi \alpha^*)$. This condition, which is generally satisfied by injected beam ions, defines an upper limit to the range of collisionality validly treated by our calculation.

If we define τ_b as the time between successive bounces of a banana orbit, we arrive at a characteristic trapping rate:

$$v_{\text{trap}} = F/\tau_b = \frac{\sqrt{N} q \delta^{*1/4}}{\pi^{3/2} \tau_b \epsilon |\sin \theta_b|} \left[\frac{v_{ii} R_b L(\alpha^*)}{2v} \right]^{1/2}.$$

For typical parameters, the trapping time, (ν_{trap}^{-1}) , is only some 10's to 100's of bounce times, which is generally much shorter than the slowing-down time. As a result, ions with $\alpha^* < 1$ at their banana tips will be effectively trapped into local magnetic wells.

(f) Residence time in the ripple wells

It has been previously assumed (Stringer 1972) that in the collisionless limit, characterized by $\nu_{90}/\delta^* = \nu_{\text{eff}} < \nu^2/(2\Omega_{ci} R_a)$, ions which become locally trapped are lost from the system in the short time required to drift across the plasma column. However, if we take into account the fact that highly collisionless ions are likely to become only marginally trapped in the magnetic wells, it is clear that these ions suffer a significant probability of subsequently scattering out of the local wells before escaping the plasma. This process may be modelled by iteratively first allowing the initially ripple-trapped fast ion distribution function to pitch-angle scatter for a ripple bounce time, τ_{br} , and then setting the resulting passing portion, $f(\xi_m > \xi_t)$, to zero after each bounce. Using the Green's function of the diffusion operator, we find after n bounces

$$f_n(\xi_m) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\xi_t} f_{n-1}(\xi') \exp\left[-\frac{(\xi_m - \xi')^2}{2\sigma^2}\right] d\xi',$$

where $f_1(\xi_m)$ is given above. $N_n \equiv \int_{-\infty}^{\xi_t} f_n(\xi_m) d\xi_m$ represents the number of ions remaining in the ripple well after n bounces.

By defining

$$F = \sum_{n=0}^{\infty} f_n(x) y^n ,$$

the iterative equation for f may be transformed to an integral equation for F . Using Wiener-Hopf techniques, we may then solve for N_n directly, giving

$$N_n = \frac{\sigma}{\sqrt{2\pi n}} .$$

The redistribution of fast ions across the plasma cross-section can now be calculated from the fact that the probability of a trapped ion remaining in the ripple wells for exactly n bounces, after initially becoming trapped, is given by $P_n = 1/\sqrt{n+1}$. The probability that an ion becomes detrapped on the n^{th} bounce is then given by $p_n = (\sqrt{n+1} - \sqrt{n})/\sqrt{n(n+1)}$. The mean number of bounces $\langle n \rangle$ and the mean square number of bounces $\langle n^2 \rangle$ are both infinite, however, so the redistribution process cannot be modelled as simple radial diffusion. We now assume that the trapped ions are collisional enough to scatter out of the region of ξ_m very close to ξ_t , where τ_{br} goes logarithmically to infinity. We may then estimate the ripple bounce time by taking the well depth to be δ^* , the well length to be $x_b - x_{\text{max}}$, and by assuming a parabolic well shape. We arrive at

$$\tau_{br} = \frac{\pi R (x_b + \cos^{-1} \alpha^* - \pi)}{Nv\delta^*} .$$

We deduce then that captured fast ions are redistributed with a profile given by p_n , and that their radial excursion is given by

$$\Delta r_n = \frac{n \tau_{br} v^2 \sin \theta_b}{2 \tilde{\Omega}_{ci} R_b} .$$

3. Collisionless ripple-trapping

(a) Physical picture

Cordey, Core, and Gibson (1979) have pointed out that in a tokamak where δ is a function of r , it is possible for a banana orbit to lead collisionlessly into a localized, or super-banana, orbit. Figure 5 illustrates such an orbit in TFR-400. The physical process at work in this case is that on the inner branch of its banana orbit, a fast ion may skim over the ripple wells easily, but on its return trajectory, the ion has travelled out to larger minor radius where the ripple perturbation is stronger, and it may be unable to traverse the first local maximum it encounters. This process tends, unfortunately, to trap ions on the unfavorable side of the horizontal midplane, giving them a short trajectory to the plasma surface. A competing trapping process is caused by toroidal precession, which alters the major radius of a fast ion at the point where it reencounters the first ripple maximum on its return trajectory. For situations where the gradient of the ripple strength is small in the vertical direction, this process may dominate, and in some regions of the plasma it leads to ripple-trapping on the favorable side of the midplane.

By a simple symmetry, in the absence of collisions these orbits detrapp when their vertical drift brings them to their original minor radius, and therefore little radial transport results.

(b) Model magnetic field

Here we use essentially the same model magnetic field employed in Section 1, with the exception that we make explicit use of the variation of B across, as well as along, field lines.

$$\tilde{B} = \frac{R_i}{R_x} (1 + \alpha^* \delta x + \delta \sin x)$$

α^* and δ are taken to be known functions of R, major radius, and y, height above the horizontal midplane. R_i is the initial major radius, when $x=0$, for a particle traveling in this field. R_x is the major radius at which $x=0$ along the field line the particle currently occupies. R_i is therefore a constant, while R_x varies along a trajectory.

The maximum $\tilde{B}$ along a field line is given by

$$\tilde{B}_{\max} = \frac{R_i}{R_x} \left[1 + \alpha^* \delta (\pi - \cos^{-1} \alpha^*) + \delta \sqrt{1 - \alpha^{*2}} \right]$$

It is convenient to now substitute a new variable for α^* , $\beta \equiv \alpha^* \delta = e |\sin \theta_b| / Nq$. β varies on a scale length of the order of the minor radius, while δ varies on a scale length of order R/N . R_x clearly varies on a scale length of order R. We now have

$$\tilde{B}_{\max} = \frac{R_i}{R_x} \left\{ 1 + \beta \left[\pi - \cos^{-1}(\beta/\delta) \right] + \delta \sqrt{1 - (\beta/\delta)^2} \right\} ,$$

and the gradients in $\tilde{B}_{\max}$ are given by:

$$\frac{\partial \tilde{B}_{\max}}{\partial R_x} = - \frac{\tilde{B}_{\max}}{R_x} = - \frac{1}{R_x}$$

$$\frac{\partial \tilde{B}_{\max}}{\partial \delta} = \frac{R_i}{R_x} \sqrt{1 - (\beta/\delta)^2} = \sqrt{1 - (\beta/\delta)^2}$$

$$\frac{\partial \tilde{B}_{\max}}{\partial \beta} = \frac{R_i}{R_x} \left[\pi - \cos^{-1}(\beta/\delta) \right] = \left[\pi - \cos^{-1}(\beta/\delta) \right] .$$

(c) Cross-field drift velocities

Since we are treating ions near their bounce points, we may take $v_{\perp} \approx v$, and the vertical (positive y direction) drift in the symmetric fields becomes

$$\left. \frac{\partial y}{\partial t} \right|_{\text{sym}} = \frac{v^2}{2R\Omega_{ci}} \equiv v_d .$$

There are also drifts due to the gradients in the ripple field:

$$\left. \frac{dy}{dt} \right|_{\text{asym}} = -v_d R \left(\frac{\partial \delta}{\partial R} \right) \sin x$$

$$\left. \frac{\partial R}{\partial t} \right|_{\text{asym}} = v_d R \left(\frac{\partial \delta}{\partial y} \right) \sin x .$$

Finally, we must consider the banana precession, which arises from the implicit dependence of R_x on β . A change in β alters the distance in R an ion must travel to reach $x=0$. If, at constant x , β changes by $d\beta$, then $\tilde{B}(x=0)$ changes by $-x d\beta$. $\tilde{B}(x=0) = R_i/R_x$, so R_x changes by $(xR_x^2/R_i)d\beta$. Thus

$$\frac{\partial R_x}{\partial \beta} = R_b x$$

(d) Variation of $\tilde{B}_{\max}$ along a trajectory

The variation of $\tilde{B}_{\max}$ in time along an ion trajectory is given by

$$\begin{aligned} \frac{d\tilde{B}_{\max}}{dt} = & \left. \frac{\partial y}{\partial t} \right|_{\text{sym}} \frac{\partial \delta}{\partial y} \frac{\partial \tilde{B}_{\max}}{\partial \delta} + \left. \frac{\partial y}{\partial t} \right|_{\text{asym}} \frac{\partial \delta}{\partial y} \frac{\partial \tilde{B}_{\max}}{\partial \delta} \\ & + \left. \frac{\partial R}{\partial t} \right|_{\text{asym}} \frac{\partial \delta}{\partial R} \frac{\partial \tilde{B}_{\max}}{\partial \delta} + \left. \frac{\partial R}{\partial t} \right|_{\text{asym}} \frac{\partial \tilde{B}_{\max}}{\partial R_x} \\ & + \left. \frac{\partial y}{\partial t} \right|_{\text{sym}} \frac{\partial \beta}{\partial y} \frac{\partial \tilde{B}_{\max}}{\partial \beta} + \left. \frac{\partial y}{\partial t} \right|_{\text{sym}} \frac{\partial \beta}{\partial y} \frac{\partial R_x}{\partial \beta} \frac{\partial \tilde{B}_{\max}}{\partial R_x} \end{aligned}$$

where the last term explicitly represents the banana precession. We have neglected terms due to asymmetric drifts multiplied by gradients of β as being small both in $(N\delta)$ and in $(\alpha^*/N\epsilon)$. The second and third terms in $d\tilde{B}_{\max}/dt$ cancel, and the expression simplifies to

$$\frac{d\tilde{B}_{\max}}{dt} = v_d \frac{\partial \delta}{\partial y} \left[\sqrt{1 - (\beta/\delta)^2} - \sin x \right] + v_d \frac{\partial \beta}{\partial y} \left[\pi - \cos^{-1}(\beta/\delta) - x \right] .$$

To find the change in the amplitude of $\tilde{B}_{\max}$ between successive passes over the last ripple maximum before the bounce, we integrate $\partial \tilde{B}_{\max} / \partial t$ in time along the marginally trapped orbit.

$$\Delta \tilde{B}_{\max} = \frac{v_d}{v} \left(\frac{1}{\delta} \frac{\partial \delta}{\partial y} \right) \frac{R_b}{N} \sqrt{\delta} \underbrace{\int \frac{(1-\alpha^2)^{1/2} - \sin x}{(\delta^*/\delta - \eta/\delta)^{1/2}} dx}_{T(\alpha^*)} - \frac{v_d}{v} \frac{R_b}{N^2 \sqrt{\delta}} \frac{\partial}{\partial y} \left(\frac{\epsilon |\sin \theta_b|}{q} \right) \underbrace{\int \frac{x - \pi + \cos^{-1} \alpha^*}{(\delta^*/\delta - \eta/\delta)^{1/2}} dx}_{P(\alpha^*)} .$$

$T(\alpha^*)$ is a weighted bounce time, and $P(\alpha^*)$ reflects the toroidal precession. Again ignoring details of the well shape, we may make analytic approximations to these two functions, by taking the well depth to be δ^* , and the well length to be $2(x_{\min} - x_{\max})$. We arrive at

$$T(\alpha^*) \approx \frac{16\sqrt{2} \cos^{-1} \alpha^* \sqrt{1 - \alpha^{*2}}}{\pi \sqrt{(1 - \alpha^{*2})^{1/2} - \alpha^* \cos^{-1} \alpha^*}}$$

and

$$P(\alpha^*) = 8\pi (\cos^{-1} \alpha^*)^2 .$$

The numerically computed functions $T(\alpha^*)$ and $P(\alpha^*)$ are shown in figures 6 and 7 along with their analytic approximations.

(e) Trapping rate

In the "ripple-plateau" regime, the trapping rate is again straightforward to calculate. f is evenly distributed over x_{bs} and a range of x_{bs} corresponding to

$$\tilde{B}_{\max} < 1 + \alpha^* \delta x_{bs} < \tilde{B}_{\max} + \Delta \tilde{B}_{\max}$$

becomes trapped in the ripple on each bounce. Thus the fraction trapped is given by

$$F = \frac{\Delta \tilde{B}_{\max}}{\Delta} = \frac{\Delta \tilde{B}_{\max}}{2\pi \delta \alpha^*} .$$

Again for the validity of our approximation, we require $F < \delta^*/\Delta$, or $\Delta \tilde{B}_{\max} < \delta^*$. Table 1 shows the excellent agreement between this calculation of F and the trapped fraction for a representative orbit in TFR, found using the numerical orbit integration code described in Lister, Post, and Goldston (1976). In some pre-compression TFTR cases, where the calculated F varies rapidly with minor radius due to the near cancellation of the T and P terms, the agreement is not so good, but this appears to be explained by the effect of banana-drift diffusion, which alters the location where F is to be evaluated.

Finally, the trapping rate is given by

$$v_{\text{trap}} = F/(2\tau_b) = \tau_b^{-1} \frac{v_d}{v} \frac{q R_b \sqrt{\delta}}{4\pi \epsilon |\sin \theta_b|} \left[\frac{T(\alpha^*)}{\delta} \frac{\partial \delta}{\partial y} - \frac{P(\alpha^*)}{N\delta} \frac{\partial}{\partial y} \frac{\epsilon |\sin \theta_b|}{q} \right].$$

Note that we have assumed v_d to be in the +y direction, so a positive trapping rate above the horizontal midplane indicates unfavorable trapping, while a negative rate above the midplane indicates trapping below, followed by inward drift. The term associated with $\partial \delta / \partial y$ will always give rise to unfavorable trapping since δ increases away from the plasma midplane. In regions where $\partial B_\theta / \partial r$ is positive, however, or where $\sin \theta \approx 0$, the precession term gives rise to favorable trapping. In conditions where collisional ripple trapping is relatively slow and $\partial \delta / \partial y$ is small, this effect can provide protection against ripple loss for a substantial region near the plasma center. For typical tokamak injection experiment parameters, however, and in conditions where the precession term is relatively small, as α^* becomes less than unity the collisionless trapping time rapidly falls to 10's to 100's of bounce times, comparable to the collisional trapping times in these same conditions. This indicates that a fairly sharp boundary exists between devices with good and poor confinement of banana-trapped suprathermal ions and that the transition is marked by the appearance of large "symmetric trapping regions" of $\alpha^* > 1$. This hypothesis is supported by the Monte-Carlo beam-orbit code results shown in table 2.

Near-perpendicular injection into a low density precompression plasma in TFTR is calculated to result in substantial ripple loss when the plasma is positioned at 310 cm, such that the peak to peak ripple at the outer plasma surface is 6%, and the "symmetric trapping regions" are small. If the plasma center is moved inward to 285 cm, the ripple at the surface is reduced to 2%, the symmetric trapping regions envelop most of the plasma, and the orbit loss becomes negligible compared to the total trapped beam power.

(f) Interaction of collisional and collisionless ripple trapping

The interaction of collisional and collisionless ripple-trapping may be investigated using the approach developed in Section 2(d-f). The collisionless ripple trapping process can be seen as shifting the location of ξ_t after each bounce. For positive values of $\Delta \tilde{B}_{\max}$ the shift traps additional ions into the ripple wells, beyond those captured through pitch-angle scattering alone, while for negative values of $\Delta \tilde{B}_{\max}$, the trapping boundary recedes away from the spreading distribution function, and fewer ions are captured. Defining $F_{c'al}$ as the trapped fraction due to collisional effects [calculated in Section 3(e)] and $F_{c'less}$ as the fraction of ions trapped collisionlessly, we can express the relative strength of the latter process in terms of a parameter,

$$\rho = \frac{F_{c'less}}{\sqrt{2\pi} F_{c'al}} .$$

Note again that ρ can be either positive or negative, depending upon the sign of $\tilde{\Delta B}_{\max}$. $f_1(\xi_m)$ is defined as in Section 2(e), but the trapped range of ξ_m is now given by

$$\Delta \xi_m = \int_{-\infty}^{\xi_t + \rho \sigma} f_1(\xi') d\xi' = \frac{\rho \sigma}{2} \left[1 + \operatorname{erf}\left(\frac{\rho}{\sqrt{2}}\right) \right] + \frac{\sigma e^{-\rho^2/2}}{\sqrt{2\pi}},$$

with the result that

$$F_{\text{total}} = F_{c,al} \left\{ \rho \sqrt{\frac{\pi}{2}} \left[1 + \operatorname{erf}\left(\frac{\rho}{\sqrt{2}}\right) \right] + e^{-\rho^2/2} \right\}.$$

(g) Residence time in the ripple wells

In the absence of collisions, the residence time for collisionally trapped fast ions is simply determined by the evolution of $\tilde{\Delta B}_{\max}$ along the super-banana trajectory. Typically for ions trapped on the side of the midplane in the direction of the VB drift (the "unfavorable" side), $\tilde{\Delta B}_{\max}$ remains positive along the orbit, and the ion becomes more and more deeply trapped as it drifts out of the plasma. For ions trapped on the opposite side of the midplane (the "favorable" side), however, the picture is quite different. $\tilde{\Delta B}_{\max}$ has odd symmetry in the y direction, with the result that although the ions become more deeply trapped as they approach $y=0$, after they cross the midplane the process is symmetrically reversed by the negative $\tilde{\Delta B}_{\max}$ found on the opposite side. Finally, when the super-banana orbit reaches the minor radius at which it originated, it detraps back onto very nearly the same banana orbit from which it began. This startling

process has been observed frequently in the numerical orbit integration code in cases where the vertical gradient of the ripple is small, and collisions are neglected.

The formulation of Sections 2(f) and 3(f) may also be extended to calculate the residence time in the ripple wells due to the interactive effects of collisional and collisionless ripple trapping. For $\Delta\tilde{B}_{\max}$ constant along the super-banana trajectory, the recursion relation for f_n is given by

$$f_n(\xi_m) = \int_{-\infty}^{\xi_t + (n-1)\rho\sigma} f_{n-1}(\xi') e^{-(\xi_m - \xi')^2/2\sigma^2} d\xi' ,$$

and then the number of ions remaining trapped after n bounces is given by

$$N_n(\rho) = \int_{-\infty}^{\xi_t + n\rho\sigma} f_n(\xi_m) d\xi_m .$$

The Wiener-Hopf techniques employed in calculating N_n for the special case of $\rho = 0$ may be extended to the $\rho \neq 0$ case, giving the general result:

$$N_n(\rho) = \frac{\rho\sigma}{2} \left[1 + \operatorname{erf}\left(\sqrt{\frac{n}{2}} \rho\right) \right] + \frac{\sigma e^{-n\rho^2/2}}{\sqrt{2\pi n}} .$$

$N_n(\rho)/N_1(\rho)$ is displayed for n up to 100, and for a range of ρ

in figure 8. As expected, for large positive values at ρ , particles once trapped are well confined in the local wells, while for large negative values, the few collisionally captured ions are rapidly ejected from the traps. This constitutes a second protection against ripple loss due to the precession effect.

4. Banana-drift diffusion

(a) Physical picture

In previous calculations (Davidson 1976; Tsang 1977) of banana-drift diffusion, [which derive from an analysis by Krommes and Rutherford (1974) of the effect of tearing-mode magnetic perturbations on neoclassical transport], only the radial component of the ripple field is taken into account. This gives rise to a radial oscillatory motion of both the orbits and the magnetic surface, with

$$\delta r \sim \frac{\delta B_r}{B_0} \frac{R}{N}.$$

Coupled with the magnetic shear, an oscillation in θ along an orbit results:

$$\dot{\theta} = \frac{v_{\theta}}{qR} \quad \delta \dot{\theta} = \dot{\theta} \frac{1}{q} \frac{dq}{dr} \delta r \sim \frac{\dot{\theta}}{N\epsilon} \frac{\delta B_r}{B_0}.$$

The oscillations in θ give rise to small variations in the bounce time, which, when coupled with the usual ∇B drift, cause net excursions across the magnetic surfaces, and failure of the banana orbits to close.

The process under consideration here is rather simpler. We consider only the toroidal component of δB , which causes an oscillation in θ due to the variation of $v_{\parallel}$

$$\dot{\delta\theta} = \frac{\delta v_{\parallel}}{qR} \quad \delta\dot{\theta} = \dot{\theta} \frac{v_{\perp}^2}{2v_{\parallel}^2} \frac{\delta B_{\phi}}{B_0} .$$

Since for trapped particles $v_{\perp}^2/v_{\parallel}^2 \sim 1/\epsilon$, and since $\delta B_{\phi} \sim \delta B_r$, this contribution to $\delta\dot{\theta}$ is $N/2$ larger than the previously considered one, and furthermore it diverges at the banana tips where $v_{\parallel} \rightarrow 0$. This is source of the "variable lingering period" near the banana tip, discussed in Goldston and Jassby (1976). Figures 9 and 10 shown an example of banana-drift diffusion in TFR in the absence of collisions. Strictly speaking, however, collisions are necessary to decorrelate the ripple phase at successive bounces (i.e., to be in the ripple-plateau regime) and thus to cause a diffusive rather than oscillatory motion. However if the collisionality is too high [$v_{\perp}^* > (Nq)^{-1/2}$] collisions even decorrelate successive passes across the ripple periods and our calculation, based on integrating along collisionless banana orbits, will be incorrect.

(b) Radial excursion

Since the dominant component of banana-drift diffusion arises from the region near the banana tip, we calculate the random variation in τ_b and multiply by the radial drift near the tip in order to get the net radial excursion of a particular trajectory:

$$\delta r = \frac{v^2 \sin \theta_b}{2Rq_{ci}} \Delta t .$$

And, using our model field near the banana tip:

$$\Delta t(x_{bs}) = \frac{2R}{NV} \left[\int_{-\infty}^{x_{bs}} \left(1 - \frac{1 + \alpha^* \delta x}{1 + \alpha^* \delta x_{bs}} \right)^{-1/2} dx - \int_{-\infty}^{x_{ba}} \left(1 - \frac{1 + \alpha^* \delta x + \delta \sin x}{1 + \alpha^* \delta x_{bs}} \right)^{-1/2} dx \right],$$

where x_{bs} , the x position of the bounce point in the limit $\delta \rightarrow 0$, is used to parameterize the ripple-phase of an orbit. x_{ba} , the bounce point of the orbit in the asymmetric fields, is found by solving the transcendental equation:

$$\alpha^* x_{ba} + \sin x_{ba} = \alpha^* x_{bs}.$$

For small δ , we calculate the radial excursion, δr .

$$\delta r(x_{bs}) = \frac{v \sqrt{|\sin \theta_b|}}{\Omega_{ci} \sqrt{N}} \sqrt{\frac{g}{\epsilon}} \left[\int_{-\infty}^{x_{bs}} (x_{bs} - x)^{-1/2} dx - \int_{-\infty}^{x_{ba}} \left(x_{bs} - x - \frac{\sin x}{\alpha^*} \right)^{-1/2} dx \right].$$

$\Delta r(\alpha^*, x_{bs})$

Figure 11 shows a numerical evaluation of Δr vs. x_{bs} for a range of α^* . In the limit of $\alpha^* > 1$, Δr is sinusoidal in x_{bs} and can be shown to be

$$\Delta r = \frac{\sqrt{\pi}}{\alpha^*} \cos(x_{bs} - \pi/4) .$$

(c) Diffusion coefficient

For $\alpha^* < 1$, the mean value of Δr averaged over $0 < x_{bs} < 2\pi$ is positive (figure 12). This is because the local maxima "shield" the trajectories from some regions of x_{ba} , resulting in an average earlier return bounce, despite the divergence at $x_{bs} = s_{\max} + \sin x_{\max}/\alpha^*$. This is precisely the process that gives rise to the steady outward drift of banana orbits in ripple-injection geometry (Goldston and Jassby 1976). [Recently, it has been proposed to eliminate this outward drift by placing a set of ripple-injection coils on the top of the torus, in addition to the original coils on the bottom, but with the new coils spaced between the original ones in ϕ (Haegi and Bittone 1979).] In order to arrive at a diffusion coefficient due to the decorrelated radial excursions, we must subtract the systematic average radial motion, which is cancelled top and bottom in the usual ripple geometry.

$$D \approx \frac{v^2 |\sin \theta_b| q}{2\Omega_{ci}^2 \epsilon \tau_b N} \left(\langle \Delta r^2 \rangle_{x_{bs}} - \langle \Delta r \rangle_{x_{bs}}^2 \right)$$

where τ_b is the time between successive reflections in the toroidal field. Figure 13 shows the numerically computed $\langle \Delta r^2 \rangle - \langle \Delta r \rangle^2$, as well as its asymptotic form for $\alpha^* > 1$: $\langle \Delta r^2 \rangle - \langle \Delta r \rangle^2 = \pi/(2\alpha^{*2})$.

In the limit of large α^* (no local trapping) we have

$$D = \frac{\pi v^2 q |\sin \theta_p|}{4N\Omega_{ci}^2 \epsilon \tau_b \alpha^{*2}} .$$

[If we now average D over an isotropic Maxwellian distribution, we arrive precisely at Boozer's ripple-plateau diffusion coefficient (to be published).]

For typical beam parameters, and $\alpha^* \sim 1$, this diffusion process gives orbit loss times of the order of the beam ion thermalization time, so it can be an important effect in perpendicular injection, particularly when only shallow penetration is obtained. The diffusion falls, however, as α^{*-2} , so for $\alpha^* \gg 1$ over most of the cross-section, banana-drift diffusion becomes negligible. Thus once again we arrive at the conclusion that large symmetric trapping regions are required for good containment of beam-injected fast ions. For 3.5 MeV α -particles, however, the slowing-down times are much longer, the bounce times are shorter, and v^2 is much larger. This would suggest that banana-drift diffusion may still be an important loss process for banana-trapped energetic α 's even in highly symmetric devices. On the other hand, α -particles are not collisional enough to be in the "ripple-plateau" regime, so they tend to sample ripple phases systematically, giving rise to banana-drift islands (Chapin and Davidson 1977; Tsang 1977), and another regime of ripple transport, the "ripple-collisionless" regime, which remains to be investigated.

5. Conclusions

In general we have found that the parameter $\alpha^* \equiv \epsilon |\sin \theta| / Nq\delta$ provides a good measure of the quality of fast-ion confinement in rippled tokamak fields. For $\alpha^* > 1$ local magnetic wells are washed out by the variation of B along the helical trajectory of the field lines in the symmetric $1/R$ toroidal field, and thus ripple-trapping effects are eliminated. For $\alpha^* < 1$, however, local magnetic traps return, and we calculate that suprathermal ions find these traps rapidly, if the ions have banana tips in the region where $\alpha^* < 1$. Two caveats remain, however. First, for cases where $\partial\delta/\partial y$ is small [$(NRq)^{-1}$], in some regions of the plasma the effect of toroidal precession on the collisionless ripple-trapping process can make the wells inaccessible collisionlessly, and difficult to reach even with collisions. Furthermore, the precession effect "pumps" trapped particles rapidly out of the local wells and onto confined banana orbits. The second caveat is that even for $\alpha^* > 1$, where local magnetic trapping has been eliminated, ripple-driven radial diffusion of fast ions still persists, due to the failure of fast ion banana orbits to close precisely. The resulting banana-drift diffusion is proportional to α^{*-2} , however, and becomes negligible for moderately large values of α^* . Thus we find, in general, that good fast ion confinement can be obtained when α^* is greater than unity over almost all of the plasma volume. For typical experimental conditions, this requires δ at the plasma surface to be $\sim 1\%$, resulting in a value of perhaps 3 to 5 for α^* at $\theta = \pm\pi/2$ and $r/a = 1/2$.

In the opposite limit, where $\alpha^* \ll 1$ across the plasma column, we may assume that fast ions which become banana trapped will be rapidly lost from the system. In this case, a simple model problem may be solved to estimate the heating efficiency of tangential neutral beam injection. First we write down the time-independent fast ion Fokker-Planck equation for an infinite homogeneous plasma (Cordey and Core 1974; Callen et al. 1974),

$$\begin{aligned} \frac{\partial f}{\partial t} = 0 = & -\frac{f}{\tau_{cx}} + \frac{1}{\tau_s v^2} \frac{\partial}{\partial v} (v^3 + v_c^3) f \\ & - \frac{v_{ii}}{2} \frac{\partial}{\partial \xi} (1 - \xi^2) \frac{\partial}{\partial \xi} f + \frac{S(\xi)}{2\pi v_o^2} \delta(v - v_o) \quad , \end{aligned}$$

where τ_{cx} is the charge-exchange loss time, τ_s is the Spitzer slowing-down time,

$$\tau_s = \frac{6.32 \times 10^8 A_f T_e(\text{eV})^{3/2}}{n_e(\text{cm}^{-3}) Z_f^2 \ln \Lambda_e} \quad ,$$

A_f is the atomic weight in A.M.U. of the fast ions, Z_f is their atomic number, and $\ln \Lambda_e$ is the usual Coulomb logarithm. The critical velocity, v_c , at which the ion and electron drags are equal is given by

$$\begin{aligned} v_c(\text{cm/sec}) &= 5.33 \times 10^6 \left([Z]/\bar{A}_i \right)^{1/3} T_e^{1/2}(\text{eV}) \\ W_c &= \frac{1}{2} m v_c^2 = A_f \left([Z]/\bar{A}_i \right)^{2/3} 14.8 T_e \end{aligned}$$

where

$$\frac{[Z]}{\bar{A}_i} \approx \sum_i \frac{n_i}{n_e} \frac{z_i^2 \ln \Lambda_i}{\ln \Lambda_e} .$$

$\ln \Lambda_i$ is the Coulomb logarithm appropriate for collisions between fast ions and thermal ions. ν_{ii} , the scattering frequency, is given by

$$\nu_{ii} = \frac{1}{\tau_s} \frac{\langle Z \rangle}{[Z]} \frac{\bar{A}_i}{A_f} \frac{v_c^3}{v^3} ,$$

$$\text{where } \langle Z \rangle = \sum_i \frac{n_i z_i^2 \ln \Lambda_i}{n_e \ln \Lambda_e} .$$

ξ is the cosine of the fast ion pitch angle with respect to the magnetic field, and $S(\xi)$ is the injected source function, normalized such that

$$\int_{-1}^1 S(\xi) d\xi = \dot{n} ,$$

where $\dot{n}$ is the fast ion source rate, in $\text{cm}^{-3} \text{sec}^{-1}$. In the absence of orbit losses and assuming τ_{cx} independent of v , this equation may be easily solved by expanding f in Legendre polynomials (Cordey and Core 1974; Callen et al. 1974). Furthermore, it is straightforward to then numerically evaluate the fraction of injected power delivered to the ions, (G_i) , and the fraction delivered to the electrons, (G_e) , as functions of the injection energy, W_0 , divided by the critical energy, W_c . The results may

then be parameterized in terms of τ_s/τ_{cx} (see figure 14). Note that pitch-angle scattering plays no role in the heating moments, G_i and G_e , since only the $l=0$ Legendre polynomial contributes.

To set up an equally simple model problem including loss effects, we first assume that all ions with $\theta = 0$ are instantly lost from the plasma. The loss region thus created is somewhat optimistic in that it requires fast ions to scatter completely around to perpendicular, rather than only to the banana trapping boundary, in order to be lost. To simplify the problem further, however, we will also assume that $S(\xi)$ is given by

$$S(\xi) = n_2 \xi \quad \text{for } \xi > 0$$

$$S(\xi) = 0 \quad \text{for } \xi < 0$$

This assumption is somewhat pessimistic, in that more ions are deposited close to the loss-region than would be the case for true tangential injection. In any event, the model problem is very easy to solve since it involves only the first Legendre polynomial, and we obtain, for $\xi > 0$,

$$f(v, \xi) = \left[\frac{v^3(v_o^3 + v_c^3)}{v_o^3(v^3 + v_c^3)} \right] \frac{\langle Z \rangle \bar{A}_1}{[Z] 3A_f} \left[\frac{v^3 + v_c^3}{v_o^3 + v_c^3} \right]^{\frac{\tau_s}{3\tau_{cx}}} \frac{n_2 \tau_s}{\pi (v^3 + v_c^3)}$$

and for $\xi < 0$,

$$f(v, \xi) = 0$$

We may now numerically evaluate the heating moments, G_i and G_e :

$$G_i = \frac{\frac{4\pi v_c^3}{\tau_s} \int_{-1}^1 d\xi \int_0^{v_0} f(v) v dv}{\dot{n} v_0^2}$$

$$G_e = \frac{\frac{4\pi}{\tau_s} \int_{-1}^1 d\xi \int_0^{v_0} f(v) v^4 dv}{\dot{n} v_0^2}$$

For the specific case of $\langle Z \rangle \bar{A}_1 / [Z] A_f = 1.5$, the results are displayed in figure 15.

In general we find that for this model case of tangential injection into a plasma with a ripple "loss cone", the electron heating efficiency of neutral beam injection is little affected by ripple loss, while ion heating is considerably reduced. As a result, injection at $W_Q \gg W_c$ results in little net ripple loss. When $\langle Z \rangle$ is increased, however, the losses become more severe, and higher beam energies must be employed. These simple results suggest that tangential neutral beam injection can be very effective in heating even highly nonaxisymmetric systems such as stellarators, so long as T_e/W_0 is small, and τ_{cx}/τ_s is large.

To obtain accurate predictions for specific experiments, this simple model notwithstanding, one must still employ large Monte Carlo Beam Orbit codes which simulate the detailed physics correctly. Hopefully the calculations presented here, however, will give new insight into the processes being treated in these codes and may also lead to much less time-consuming algorithms

for calculating the efficiency of neutral beam heating in
nonaxisymmetric toroidal devices.

Acknowledgments

The authors would like to thank Dr. Paul Rutherford for his continued encouragement and interest in this problem. We would also like to thank Dr. Rutherford and Dr. Allen Boozer for a number of enlightening discussions on the subject of collisionless ripple-trapping and banana-drift diffusion. Similarly, we acknowledge very helpful conversations with Dr. David Mikkelsen on the question of residence time in the ripple wells. Finally we are especially grateful to Dr. Marshall Rosenbluth for his elegant analytic solutions for N_n and $N_n(\rho)$ using Wiener-Hopf techniques.

This work was supported by U. S. Department of Energy Contract No. EY-76-C-02-3073.

References

- Boozer, A. H. (to be published).
- Callen, J. D., et al. 1974 in Plasma Physics and Controlled Nuclear Fusion Research, Proc. 5th Int. Conf. Tokyo; 1, 645.
- Chapin, C. C. and Davidson, J. N. 1977 in Bull. Am. Physical Society 22 9, 1179.
- Cordey, J. G. and Core, W. G. F. 1974 Phys. Fluids 17, 1626.
- Cordey, J. G. 1976 Nucl. Fusion 16, 499.
- Cordey, J. G., Core, W. G. F. and Gibson, A. 1979 in 9th European Conference on Controlled Fusion and Plasma Physics, Oxford, England.
- Davidson, J. N. 1976 Nucl. Fusion 16, 731.
- Dei-Cas, R. and Marty, D. 1974 EUR-CEA-726.
- Equipe TFR 1978 Nucl. Fusion 18, 1271.
- Fowler, R. H., Lee, D. K. Gaffney, P. W. and Rome, J. A. 1978 ORNL/TM 6293.
- Goldston, R. J. and Jassby, D. L. 1976 in 3rd International Meeting on Theoretical and Experimental Aspects of Heating of Toroidal Plasmas, Grenoble, France.
- Haegi, M. and Bittone, E. 1979 Frascati Report 79.16.
- Krommes, J. A. and Rutherford, P. H. 1974 Nucl. Fusion 14, 695.
- Lister, G. L., Post, D. E. and Goldston, R. J. 1976 in 3rd Symposium on Plasma Heating in Toroidal Devices, Varenna, Italy.
- Mirin, A. A., et al. 1977 J. Comp. Phys. 23, 23.
- Rome, J. A., et al. 1976 Nucl. Fusion 16, 55.
- Rutherford, P. H. and Goldston, R. J. 1977 in Bull. Am. Physical Society 22 9, 1106.
- Stringer, T. E. 1972 Nucl. Fusion 12, 689.
- Tani, K. Kishimoto, H. and Tamura, S. 1979 in 9th European Conference on Controlled Fusion and Plasma Physics, Oxford, England.
- Tsang, K. T. 1977 Nucl. Fusion 17, 557.

Table 1. Ripple-trapped fractions for orbits starting at $\theta_i = 0$

$$r_i = 10 \text{ cm}, \quad \left. \frac{v_{\parallel}}{v} \right|_i = -0.3 \text{ in TFR-400}$$

		<u>Energy</u>			
<u>Ripple strength</u>		12.5 keV	25 keV	50 keV	
$\delta^0 = 2.5 \times 10^{-4}$			9.3%		Theory
			9.5%		Orbit Code
$\delta^0 = 5 \times 10^{-4}$		8.2%	13.1%	21.9%	Theory
		8.6%	13.0%	20.3%	Orbit Code
$\delta^0 = 1 \times 10^{-3}$			17.3%		Theory
			17.7%		Orbit Code

The ripple intensity in circular cross-section tokamaks is well described by $\delta(r, \theta) = \delta^0 \exp(\gamma/\lambda)$, where δ^0 is the ripple strength at the "ripple center" located at $R = R_{rc}$ and $y = 0$. λ is the ripple scale length $\approx (R+a)/N$, and γ is the distance from the "ripple center", $\gamma = \sqrt{(R-R_{rc})^2 + y^2}$. For TFR-400, $\delta^0 = 3.15 \times 10^{-4}$, $R_{rc} = 92.5 \text{ cm}$, $\lambda = 5.23 \text{ cm}$, and $N = 24$.

Table 2. Power distribution with near-perpendicular injection
into TFTR precompression plasma

<u>R</u>	<u>To ions</u>	<u>To electrons</u>	<u>To charge exchange</u>	<u>To lost orbits</u>	<u>Unionized</u>
310 cm	19%	14%	16%	30%	21%
285 cm	42%	26%	15%	3%	14%

796513
Figure 1: Example of collisional ripple trapping in a
TFR-400-like tokamak, with 3% uniform ripple.

796510
Figure 2: Magnetic field geometry for ripple calculations.

$$\tilde{B} = 1 + \alpha^* \delta x + \delta \sin x; \quad \alpha^* = \epsilon |\sin \theta| / N q \delta$$

803138
Figure 3: Dependence of the bounce point, x_b , of the marginally trapped orbit, on a^* .

796504

Figure 4: Dependence of the normalized ripple well length, L , on α^* .

796512

Figure 5: Example of collisionless ripple-trapping in TFR-400, using the true variation of the ripple strength over the cross-section.

796509

Figure 6: Dependence of the normalized bounce time, T , on a^* .

796505
Figure 7: Dependence of the normalized toroidal precession, P ,
on a^* .

796576
Figure 8: Fraction of ions remaining trapped in ripple wells after n bounces.

$$\rho = F_{c'less}/\sqrt{2\pi} F_{c'al}$$

796518
Figure 9: Example of banana-drift diffusion in TPR-400. Note that the radial motion arises from the "variable lingering period" near the banana tip.

796514
Figure 10: Continuation of figure 9. Example of the loss of a
fast ion by banana-drift diffusion in TFR-400.

796511
 Figure 11: Normalized radial excursion, Δr , versus ripple strength, a^* , and symmetric bounce point, x_{bs} .

796507
 Figure 12: Mean radial excursion, $\langle \Delta r \rangle$, versus α^* . Within the numerical precision, $\langle \Delta r \rangle = 0$ for $\alpha^* > 1$.

796506
 Figure 13: Mean square normalized random displacement per bounce
 versus α^* .

Figure 14: Beam energy transfer functions, G_i and G_e , without ripple losses, after Callen et al. (1974).

796515

796516
 Figure 15: Beam energy transfer functions, G_i and G_e , with
 ripple losses; $\langle Z \rangle \bar{A}_1 / [Z f A_f] = 1.5$.