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Mathematical Models for the Ringbearer Simulation Code

Frank W. Chambers

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MATHEMATICAL MODELS FOR THE
RINGBEARER SIMULATION CODE

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ABSTRACT

RINGBEARER is a particle simulation code for modelling an electron beam propagating in a gas characterized by a space and time varying conductivity generated by the beam. The code has two spatial dimensions, the beam is assumed to be axially symmetric. Equations for the beam driven pinch field, the plasma density, and the conductivity are given along with the appropriate initial conditions and boundary conditions. The equations governing particle energy loss, transverse motion, and scattering, in the beam generated pinch field are presented. The analytic models used in the code are presented with either references or derivations.

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INTRODUCTION

The RINGBEARER code is designed to simulate the propagation of an electron beam through a gas. The beam penetrating the gas will generate a plasma with a finite conductivity. This plasma will provide charge neutralization and partial current neutralization for the beam. The resulting radial electric field and tangential magnetic field will "pinch" or confine the beam. The beam description can be divided into three regions as indicated schematically in figure 1. At the beam head no pinch field is established and the beam undergoes free expansion. Further back in the beam a near equilibrium will be established with the beam particles executing oscillatory motion described as betatron orbits. The beam radius will expand slowly due to collisions with the gas molecules. This expansion is termed Nordsiecking. Between these regions there is a "pinch down" or necking region where the fields are established, and the transition from the freely expanding to the pinched section is made. As the beam propagates this necking region moves back in the beam, this process being termed erosion. In each region the particle dynamics, plasma production, and field generation must be solved self-consistently. The RINGBEARER code models are designed to describe this beam propagation problem.

The main components and flow of the RINGBEARER code are indicated in figure 2. The beam current is assumed to be cylindrically symmetric so only monopole ($m=0$) fields are

produced. Plasma generation is by direct ionization by beam electrons, and by avalanche ionization due to the ohmically heated plasma. Conductivity is determined from the plasma density assuming the plasma is collision dominated. The field equation is valid in both the low and high conductivity regions. The particle pusher includes energy loss and is correct for relativistic electrons within the paraxial approximation. The beam current is carried by individual particles which appear as rings in the axially symmetric system. The beam radial profile is computed from the particle positions and is therefore not constrained to a Bennett profile or a self-similar expansion. The major code sections to be described are 1) the pinch field equations, 2) the plasma density and conductivity calculations, and 3) the particle pusher.

The RINGBEARER code is well suited for calculating beam equilibrium propagation, head erosion, profile evolution, and Nordsiecking. Erosion rates can be studied for beams without the 1-d modelling used in Ref. 4. The growth of emittance due to anharmonic particle orbits (Ref. 1) can be verified. Beam profile hollowing can be studied. Further the program has the capability to study these phenomena with an arbitrary gas profile where 1-d and 2-d models with constrained radial profiles are not applicable. One can observe sausage instabilities where beam segments expand and contract in radius. One can not observe the hose instability which involves a lateral displacement of the beam. The particle pushing model does not permit the beam to

bunch longitudinally and the plasma is modelled as collision dominated. Hence one cannot observe the usual longitudinal two stream mode. Most of the computed results will be at pressures where the two stream interaction is suppressed.

In this report the analytic models employed for the major program sections are presented. The discussion of the implementation of these models in the actual program is deferred to a subsequent report.

PHYSICAL MODELS

Coordinate System:

The coordinates used to describe a propagating beam are Z, X, r, θ where Z is the propagation distance for a beam segment from its point of injection, X is the distance back from the beam head, r and θ are the transverse coordinates as illustrated in figure 3. For the cylindrically symmetric (monopole) case θ is ignorable. The beam longitudinal coordinate X , for a beam travelling at c , the free space velocity of light, is related to the time t since injection of the beam head by $X = ct - Z$.

Pinch Field:

The pinch field is described by the partial differential equation in X and r (Ref. 1):

(1)

$$\frac{1}{r} \frac{\partial}{\partial r} r \left(1 + \frac{c}{4\pi\sigma} \frac{\partial}{\partial X} \right) \frac{\partial A}{\partial r} - \frac{4\pi\sigma}{c} \frac{\partial A}{\partial X} - \frac{\partial^2 A}{\partial X^2} = - \frac{4\pi}{c} J_{BZ}$$

In this equation A is the "pinch potential" and $A = A_z - \varphi$ where A_z is the Z component of the vector potential and φ is the scalar potential (in a Lorentz gauge). J_{BZ} is the Z component of the beam current density. σ is the local scalar conductivity of the gas. The Z component of the electric field is:

$$E_z = - \frac{\partial A}{\partial X} \quad (2)$$

The radial component of the electric field is:

$$E_r = - \frac{c}{4\pi\sigma} \frac{\partial^2 A}{\partial r \partial X} \quad (3)$$

The corresponding θ component of the magnetic field is:

$$B_\theta = - \left(1 + \frac{c}{4\pi\sigma} \frac{\partial}{\partial X} \right) \frac{\partial A}{\partial r} \quad (4)$$

leading to a radial Lorentz force on a particle of charge q

moving in the Z direction at velocity c:

$$F_r = q \left(E_r - B_\theta \right) = q \frac{\partial \lambda}{\partial r} \quad (5)$$

Making the change of variables $Y = r^2$ equation (1) is rewritten as:

$$\frac{\partial}{\partial Y} \left(1 + \frac{c}{4\pi\sigma} \frac{\partial}{\partial X} \right) Y \frac{\partial \lambda}{\partial Y} - \frac{\pi\sigma}{c} \frac{\partial \lambda}{\partial X} - \frac{1}{4} \frac{\partial^2 \lambda}{\partial X^2} = - \frac{\pi J_{BZ}}{c} \quad (6)$$

Equation (6) is the field equation solved in RINGBEARER.

Initial conditions are that no pinch or E_z fields precede the beam since the beam is travelling at c.

$$\lambda(X=0) = 0 \quad (7)$$

$$\frac{\partial \lambda}{\partial X} (X=0) = 0 \quad (8)$$

Boundary conditions are that the pinch force vanish at $r = 0$ and that the pinch potential be constant (hence 0) at a maximum radius b where either conductivity vanishes ($\sigma = 0$) or a conducting wall is placed ($\sigma \rightarrow \infty$).

$$\frac{\partial A}{\partial r} (r=0) = 0 \quad (9)$$

$$A(r=b) = 0 \quad (10)$$

Note however that the simple condition in equation (9) must be modified when Y is the independent variable. Integrating equation (6) from 0 to ϵ we find:

$$\left\{ \left(1 + \frac{c}{4\pi\omega} \frac{\partial}{\partial X} \right) \frac{\partial A}{\partial Y} - \frac{\pi\sigma}{c} \frac{\partial A}{\partial X} - \frac{1}{4} \frac{\partial^2 A}{\partial X^2} \right\} \bigg|_{Y=0} = - \frac{\pi J_{BZ}}{c} \bigg|_{Y=0} \quad (11)$$

Equation (1) is equivalent to that solved by EMPULSE (Ref. 1), but we retain the second partial derivative term in X which the EMPULSE solution does not.

Plasma Generation and Conductivity

A plasma with density n_e is produced by the beam through two processes; direct and avalanche ionization, as described by the equation:

$$\frac{\partial n_e}{\partial X} = \kappa n_B + \frac{\nu_i n_e}{c} \quad (12)$$

Here κ is the direct deposition constant, n_B the beam density, and ν_i the avalanche ionization rate. The initial condition in equation (12) is that $n_e(X=0) = 0$. κ is related to observed beam and gas parameters as:

$$\kappa = \frac{\frac{dE}{dX} (\rho p)}{\Delta E} \quad (13)$$

dE/dX is the usual stopping power of the particular gas (Ref. 2) due to ionizing collisions, (ρp) is the gas density where ρ is the density at a particular pressure (say 1 torr) and p is the actual gas pressure. ΔE is the energy required to produce an ion pair. (Typical values are $dE/dX = 2.52 \text{ MeV cm}^2/\text{gm}$, $\rho = 1.7 \cdot 10^{-6} \text{ gm}/(\text{cm}^3 \text{ torr})$ and $\Delta E = 33.75 \text{ eV/ion pair}$).

Avalanche ionization occurs when the plasma electrons, having been ohmically heated by the beam driven electric fields, reach sufficient energies to ionize the neutral gas molecules. An expression for this rate based on the experiments of Felsenthal and Proud (Ref. 3) has been determined by E. P. Lee (Ref. 4):

(14)

$$\nu_i (\text{sec}^{-1}) = \frac{A p_A^3 S^3}{1 + B S^2 + C S^2 + D S^3}$$

$$A = 1.423 \cdot 10^{-4}$$

$$B = 9.179 \cdot 10^{-6}$$

$$C = 2.656 \cdot 10^{-10}$$

$$D = 2.820 \cdot 10^{-17}$$

(15)

$$S = \frac{E^2}{p_A^2}$$

E in statvolts/cm
p_A in atmospheres

RINGBEARER provides options for turning off avalanche ionization.

Assuming scalar conductivity σ is related to n_e by:

$$\sigma = \frac{n_e e^2}{m \nu_m} \quad (16)$$

where ν_m is the momentum transfer collision rate for plasma electron neutral molecule collisions, m is the electron mass, e is the electron charge. We use two models, the first assuming a constant collision rate:

$$\nu_m (\text{sec}^{-1}) = 4.864 \cdot 10^{12} p_A \quad (17)$$

To account for the variation of collision rate with plasma temperature we have the option of modelling ν_m as a function of $S = E^2/p_A^2$ as:

$$\nu_m (\text{sec}^{-1}) = 10^{12} p_A \left(\frac{a+b(S+KT_g)}{c+d(S+KT_g)} \right) \quad (18)$$

$$a = .01961$$

$$b = .04046$$

$$c = .9598$$

$$d = .008325$$

$$K = 205.5$$

The gas temperature in eV is T_g . For $N \rightarrow \infty$ equation (18) yields the same result as equation (17).

Thus the model equations for field solving in RINGBEARER are a coupled set of two equations; equation (11) being a partial differential equation in X, Y while equation (12) is an ordinary differential equation in X . The system is nonlinear because of the E^2 terms appearing in ϵ_1 and ϵ_m . An iterative approximation is employed to solve these equations.

PARTICLE PUSHING

The paraxial approximation that $v_{\text{perp}} \ll v_{\text{para}} \sim c$ is made to separate the transverse and longitudinal parts of the particle equations of motion. In the longitudinal direction (Z) the particle is assumed to move at $v_z = c$ but energy loss is included by permitting the particle γ , ($\gamma^2 = 1.0/(1.0 - v_z^2/c^2)$), to vary; the variation being described by an energy loss equation:

$$\frac{d\gamma}{dt} = - \frac{c\gamma}{\lambda_R} - \hat{\gamma}_{\text{DIRECT}} + \frac{qcE_z}{mc^2} \quad (19)$$

Here the first term involving the radiation length, λ_R , represents the energy loss due to Bremstrahlung. The second term refers to loss by ionizing collisions, and it can be related to the gas stopping power as:

$$\dot{\gamma}_{\text{DIRECT}} = \frac{c}{mc^2} \frac{(dE/dX)}{2} (np) \quad (20)$$

The third term represents ohmic losses in the induced longitudinal electric field.

The transverse equations of motion are written in cartesian coordinates (note these coordinates are written with lower case x,y) for computational convenience.

$$\frac{dx}{dt} = v_x \quad (21)$$

$$\frac{dy}{dt} = v_y \quad (22)$$

$$\frac{d}{dt} (mv_x) = \frac{q}{m} \left(\frac{E_r - B_v}{r} \right) x + \frac{\delta F_x}{m} \quad (23)$$

$$\frac{d}{dt} (mv_y) = \frac{q}{m} \left(\frac{E_r - B_v}{r} \right) y + \frac{\delta F_y}{m} \quad (24)$$

The force quantity can be expressed in terms of λ as:

$$\left(\frac{E_r - B_v}{r} \right) = \frac{1}{r} \frac{\partial A}{\partial r} = 2 \frac{\partial^2}{\partial Y^2} \quad (25)$$

Scattering is included in the random scattering forces δF_x and δF_y .

The scattering forces δF_x and δF_y in the differential equation actually would contain the microscopic detail of individual random collisions. Since we will be stepping equations (23),(24) over time intervals during which many collisions will occur we can model the scattering as producing a distribution of momentum jumps. Each particle at each time step receives a random momentum kick; however, these random kicks are distributed in a fashion we can calculate. Collisions cause the particle to do a random walk in momentum space so the momentum change due to many collisions will have a Gaussian distribution.

$$p\left(\gamma v_x\right) d\left(\gamma v_x\right)=\frac{1}{\sigma} \exp \left[-\gamma^2\left(\gamma v_x\right)^2\right] \quad (26)$$

Here σ is a normalization constant, $\gamma^2=\left(\gamma^2/\gamma^2\right)$, and σ is the distribution width to be determined.

Considering for a moment only the scattering induced momentum change we can write:

$$\frac{d}{dt}\left(\gamma \vec{v}\right)=\frac{\delta \vec{F}}{m} \quad (27)$$

Forming the dot product with the momentum gives:

$$\frac{d}{dt}\left(\frac{\gamma^2 v^2}{2}\right)=\frac{\vec{p} \cdot \delta \vec{F}}{m^2} \quad (28)$$

Averaging over a time δT gives:

$$\frac{\left\langle \delta\left(\frac{\gamma^2 v^2}{2}\right) \right\rangle}{\delta t}=\frac{\overline{\vec{p} \cdot \delta \vec{F}}}{m^2} \quad (29)$$

The overscore on the right hand side indicates an average over many collisions, this average has been evaluated (Ref. 5) as:

$$\dot{v}_i = \frac{1}{\gamma m} \frac{d\vec{p}}{dt} = \frac{e^2}{\gamma m c^2} \frac{2C_N}{r_R} \quad (29)$$

where C_N is the Nordsieck constant ($C_N = 3.7 \cdot 10^{19}$ ergs/sec) and r_R is the radiation length. $\dot{v}$ is the rate of energy increase for a single particle due to scattering. The quantity on the left hand side of equation (29) can be computed for the distribution in equation (26); making these two substitutions in equation (29) we find:

$$\frac{1}{\gamma^2} = \frac{2C_N}{m} \Delta t \quad (31)$$

Thus the distribution of scattering events is completely specified.

The particle initial positions and velocities can be specified by the user.

SUMMARY

The RINGBEARER particle simulation code alternately performs two tasks, field solving and particle pushing. The field is described by equations (6),(12) with boundary conditions given in equations (9),(11) and initial conditions in equations

(7),(8). Particle pushing and scattering is described by equations (21)-(24),(26),(31).

While this report gives the full analytic models used in RINGBEARER the implementation of these models in the programming of the code is of equal importance and difficulty and will be discussed in a subsequent report.

ACKNOWLEDGMENT

The majority of the analytic results presented were originally derived by Edward P. Lee who has also contributed greatly to our understanding and implementation of these results. Jon A. Masamitsu has programmed the code and in so doing he has also contributed to the analytic work. He has verified results, corrected errors, and extended the analysis to be amenable to computation. Discussions with Simon Yu have been most helpful especially concerning the particle scattering and conductivity generation sections of the coding.

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Fig. 1. Schematic diagram of a propagating electron beam indicating the freely expanding, necking, and pinched regions.

Fig. 2. Main calculational section of the program RINGBEARER with the program flow, starting points, initial conditions indicated.

Fig. 3. Coordinate system used in describing the propagating beam.