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Alpha Particle Density and Energy Distributions in Tandem Mirrors Using Monte-Carlo Techniques

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ALPHA PARTICLE DENSITY AND ENERGY DISTRIBUTIONS IN TANDEM MIRRORS
USING MONTE-CARLO TECHNIQUES

by

John A. Kerns

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LIST OF SYMBOLS

A_D	Constant in Spitzer's collision model
A_v	Constant in the radial drag gradient
$\vec{B}$	Magnetic field vector
$\vec{B}_v$	Vacuum magnetic field vector
B	Magnitude of the magnetic field
B_a	Magnitude of the magnetic field including diamagnetic effects
B_c	Magnitude of the magnetic field in the center cell of a Tandem Mirror
B_m	Magnitude of the magnetic field in the mirror of a Tandem Mirror
B_{max}	Maximum magnetic field in a mirror device
B_{min}	Minimum magnetic field in a mirror device
B_v	Magnitude of the vacuum magnetic field
B_x, B_y, B_z	Components of the magnetic field in Laboratory coordinates
C_0	Arbitrary constant in the theoretical energy distribution function
D-D	Deuterium - Deuterium
D-T	Deuterium - Tritium
$\hat{e}_1, \hat{e}_2, \hat{e}_3$	Unit vectors in the Guiding Center coordinate system
e_j	Energy per unit volume in radial bin j
$\vec{E}$	Electric field vector
$E_{ }$	Electric field parallel to the magnetic field
$\vec{F}$	General force field vector
$\vec{F}_D$	Drag force

f	Distribution function
f_{α}	Alpha distribution function
$f(\vec{r}, \vec{v}), f(\vec{r}, v_{\parallel}, v_{\perp})$	Velocity distribution function
$f(\vec{r}, \epsilon)$	Energy distribution function
$f(z)$	Function in the paraxial approximation defining B_z
$g(z)$	Function in the paraxial approximation relating the ellipticity to the magnetic field
j	Current density
k	Boltzman's constant
L	Typical length
L_c	Length of the center cell
L_{α}	Sink function for alpha particles
LnA	Coulomb logarithm
l_f	Speed of a plasma ion or electron
M	Mass in a.m.u.
M_j	Mass in a.m.u. of a plasma ion or electron in radial bin j
m_e	Mass of an electron
m_f	Mass of a plasma ion or electron
m_i	Mass of an ion
m_{α}	Mass of an alpha
N	Total number of particles
N_v	Total number of particles lost through velocity space loss channels
N_h	Total number of particles lost to the halo
$\dot{N}_{\alpha}$	D-T reaction rate
n	Density
n_c	Density in the center cell

n_D	Density of deuterium
n_{Dj}	Density of deuterium in radial bin j
n_e	Density of electrons
n_h	Density of alphas in the halo
n_i	Density of ions
n_j	Density in radial bin j
n_o	Density on axis
n_{oi}	Density of ions on axis
n_p	Density in the mirror plug
n_T	Density of tritium
n_{Tj}	Density of tritium in radial bin j
$n(i,j)$	Density in the velocity distribution function bin (i,j)
$\Delta n(r_i)$	Density in the energy distribution function bin r_i
P	Power from the alphas to the plasma
P_j	Power from the alphas to the plasma in radial bin j
P_r	Probability of an alpha being born at r
P_z	Probability of an alpha being born at z
P_x	Probability of an alpha being born at x
P_ψ	Probability of an alpha being born with pitch angle ψ
P_ϕ	Probability of an alpha being born with phase angle ϕ
p_o	Pressure on axis
$p_\perp$	Pressure perpendicular to $\vec{B}$
Q	Power generated by a nuclear fusion plasma divided by the power input into the plasma
q	Electric charge on a proton or electron

q_α	Electric charge on an alpha
$\vec{R}$	Guiding center position vector
$\vec{R}_D$	Guiding center drift velocity
R	Magnitude of the guiding center position vector
R_i	Random number
$\vec{r}$	Particle position vector
r	Magnitude of the particle position vector
r_c	Hot reacting plasma radius
r_p	Input plasma radius
r_{pD}	Scale length for density
r_{pT}	Scale length for temperature
S_α	Source function for alpha particles
T	Temperature
T_e	Electron temperature
T_i	Ion temperature
T_o	Temperature on axis
T_{oe}	Electron temperature on axis
t	Time
Δt	Time between collision sampling events
$\Delta t(i,j)$	Accumulated time of the alphas that passed through the velocity space bin (i,j)
$\Delta t(r_i)$	Accumulated time of the alphas that passed through the energy distribution bin (r_i)
ΔT	Accumulated time of all the alphas that contributed to the velocity distribution function
v	Velocity
$\vec{v}_D$	Drift velocity
$\vec{v}_\alpha$	Alpha velocity

$\langle \dot{\bar{v}}_D \rangle$	Radial drag drift acceleration
v	Magnitude of $\bar{v}$
v_0	Speed at $B_{\min}$
$v_\perp$	Speed perpendicular to $\bar{B}$
$v_{\perp 0}$	Speed at $B_{\min}$ perpendicular to $\bar{B}$
$v_\parallel$	Speed parallel to $\bar{B}$
ΔV_j	Volume of radial bin j
w	Speed in the particle coordinate system
$w_\parallel$	Speed in the particle coordinate system parallel to $\bar{v}$
$w_\perp$	Speed in the particle coordinate system perpendicular to $w_\parallel$
Z	Number of electric charges on a charged particle
Z_j	Number of electric charges on a plasma ion or electron

GREEK SYMBOLS

α	Alpha particle
β	Ratio of plasma pressure to magnetic field pressure
$\hat{\beta}$	Peak beta
$\langle \beta \rangle$	Average beta
$\beta(r)$	Beta as a function of r
γ	Variable denoting the power of r/r_p in density and temperature profiles
γ_D	Power of $(1 - \frac{r}{r_{pD}})$ in the modeled density profile
γ_T	Power of $(1 - \frac{r}{r_{pT}})$ in the modeled temperature profile
δ	Expansion parameter equal to p/L

ϵ	Energy
ϵ_{α}	Initial alpha energy (3.52 MeV)
ϵ_i	Mean energy in a radial bin
ϵ_h	Alpha energy lost to the halo
ϵ_v	Alpha energy lost through velocity space loss
$\Delta\epsilon$	Alpha energy change in a collision
$\Delta\epsilon_e$	Alpha energy given to electrons
$\Delta\epsilon_i$	Alpha energy given to ions
$\Delta\epsilon^*$	Width of an energy bin in the energy distribution
ϵ_0	Permittivity constant
$\eta(z)$	Ellipticity of the magnetic flux tube at z
μ	Magnetic moment
μ_a	Magnetic moment adjusted for diamagnetic effects
ν	Drag rate
ν_0	Drag rate at a reference point
$\vec{\rho}$	Larmor radius vector
ρ	Magnitude of the Larmor radius vector
ρ_0	Magnitude of the Larmor radius vector at a reference point
$\langle\sigma v\rangle$	Microscopic cross section for the D-T reaction
$\tau_{\alpha\alpha}$	Alpha-alpha 90° collision time
$\tau_{\alpha i}$	Alpha-ion 90° collision time
τ_b	Bounce time of an alpha between mirror point
τ_e	Confinement time of electrons
τ_{ee}	Electron-electron 90° collision time
τ_ϵ	Energy confinement time
τ_D	Drift time azimuthally around the plasma

$\tau_{\perp}$	Alpha 90° scattering time
$\tau_{\parallel}$	Alpha dispersion time
ϕ	Electrostatic potential
ϕ_{pc}	Plug to center cell potential
$\Phi(x)$	Error function
ϕ	Phase angle
χ	Azimuthal angle
ψ	Pitch angle
ψ'	Loss cone pitch angle
ψ_{ϕ}	Magnetic flux
ω_c	Cyclotron frequency

SUBSCRIPTS

a	Actual
b	Bounce
c	Cyclotron or center cell
D	Drift or Deuterium
D_j	Deuterium in bin j
e	Electron
ee	Electron-electron
f	Field particle
h	Halo
i	Ion
j	Radial bin j
max	Maximum
min	Minimum

o	On axis
oe	On-axis electron
oi	On-axis ion
p	Plasma
pT	Plasma temperature
pD	Plasma density
pc	Plug to center cell
r	Radial
T	Tritium
T_j	Tritium in j
v	Velocity space or vacuum
x,y,z	directions in the laboratory coordinate system
z	Axial

GREEK SUBSCRIPTS

α	Alpha particle
$\alpha\alpha$	Alpha-alpha
αi	alpha-ion
ϵ	Energy
χ	Azimuthal angle
ϕ	Phase angle
ψ	Pitch angle

SYMBOLS SUBSCRIPTS

$\perp$	Perpendicular
$\parallel$	Parallel

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ABSTRACT

Alpha particles, born at 3.52 MeV, are used to maintain the ignition temperatures in a deuterium-tritium (D-T) reacting plasma above 10 keV in tandem mirror reactors. The alpha energy is transferred to the plasma by Coulomb collisions and, after thermalization, the alphas can accumulate in the plasma and quench the reaction. We have simulated the alpha thermalization process using a Monte-Carlo technique, in which the alpha guiding center is followed between simulated collisions and Spitzer's collision model is used for the alpha-plasma interaction. Monte-Carlo techniques are used to determine the alpha radial birth position, the alpha particle position at a collision, and the angle scatter and dispersion at a collision. The plasma is modeled as a hot reacting core, surrounded by a cold halo plasma ($T \sim 50$ eV). Alpha orbits that intersect the halo lose 90% of their energy to the halo electrons because of the halo drag, which is ten times greater than the drag in the core. The uneven drag across the alpha orbit also produces an outward, radial, guiding center drift. This drag drift is dependent on the plasma density and temperature radial profiles. We have modeled these profiles and have specifically studied a single-scale-length model, in which the density scale length (r_{pD}) equals the temperature scale length (r_{pT}), and a two-scale-length model, in which $\frac{r_{pD}}{r_{pT}} = 1.1$. Alpha energy losses to the core and halo were determined by varying the vacuum magnetic field (B_v), peak beta

$\hat{\beta}$, and plasma radius (r_p) for each profile combination. The energy losses to the halo are a function of $\frac{\rho}{r_c}$ where ρ is $\frac{0.27}{B_v [1 - \langle \beta(r) \rangle]^{1/2}}$, and r_c is the reacting core radius. When $\frac{\rho}{r_c} > 0.3$ for the single-scale-length study, the energy losses to the halo are greater than 40%. This occurs for $\frac{\rho}{r_c} > 0.39$ for the two-scale-length study, but there is a 50% variation in the data for $\frac{\rho}{r_c} > 0.35$ in this case. This data scatter suggests that $\frac{\rho}{r_c}$ is not an exact scaling parameter. Until improvements are made to the scaling parameter, the alpha halo losses must be calculated using plasma density and temperature profiles that produce equilibrium alpha distributions.

PART 1

INTRODUCTION

1.1 Background

The prospect of power generation from thermonuclear fusion, which can operate on a virtually unlimited deuterium fuel supply has been the subject of extensive research and development efforts for over thirty years.^{1,2} In the fusion process, light nuclei are combined in a reaction that releases an amount of energy equal to the mass difference between the reactants and products. The light nuclei are brought together to distances equal to their nuclear radii by overcoming the Coulomb repulsive forces between them. Therefore, the reacting particles must be sufficiently energetic for the fusion reactions to occur. In a thermonuclear reactor, these energetic particles are to be contained at sufficiently high density and for times sufficiently long for significant fusion reactions to occur. Because these reacting particles are energetic, they will exist in a fully ionized state as a plasma. The ability to confine and sustain a plasma of light ions at thermonuclear conditions has been the subject of much of the efforts in controlled fusion research up to the present time.

Although nuclear fusion reactions involving hydrogen atoms are the source of energy of our sun and of all the stars in the universe, the fusion reactions involving hydrogen are too slow to duplicate in the laboratory for power generation.³ Other reactions, such as the deuterium-deuterium (D-D) reaction which has two reacting branches,

and the deuterium-tritium (D-T) reaction,

have much larger reaction cross sections and are therefore easier to produce. Of all the known fusion reactions, the D-T reaction has the largest maximum cross section ($5 \times 10^{-24} \text{ cm}^2$) which peaks at the lowest relative energy (100 keV). Deuterium is a hydrogen isotope that occurs naturally in seawater in the ratio of one part in 6500, and therefore there is an ample supply that is readily available. Tritium, which is radioactive with a half life of approximately nine years, does not occur naturally, but can be generated in the exothermic reaction,

Since the D+T reaction produces an energetic neutron and lithium is also an abundant element, it is possible to produce a power-generating fusion reactor plant based on the D-T reaction. Furthermore, the alpha particle (${}^4\text{He}$), which is the other reaction product, is born with 3.52 MeV energy and it will thermalize with the D-T plasma by Coulomb interactions. It is therefore possible to sustain a continuous burning D-T plasma at thermonuclear conditions by alpha heating, a condition known as ignition when no additional energy input is required. Because of these possibilities, the emphasis of the international program to develop fusion power is being directed mainly on reactor concepts operating on the D-T fuel cycle.⁴

At present, there are two principal approaches to controlled thermonuclear fusion. In one approach, known as inertial confinement, a small D-T fuel pellet is heated by either laser or particle beams to achieve a substantial D-T burn before the reaction is quenched by expansion cooling. The advantage of this approach is its relative simplicity. However, since the reaction time is limited to the disassembly time of the reacting plasma, the complete process, including initial heating, must be accomplished on a nanosecond time scale. The disadvantage of this approach, therefore, is the need to deliver a large amount of energy to the pellet on a very short time scale and at a rate fast enough to produce a significant amount of energy for power generation. Most of the research in inertial confinement fusion is based upon understanding the physics associated with the absorption of high energy pulses into the fuel pellet and in developing the necessary high energy sources for delivering these pulses.

The other approach, known as magnetic confinement fusion, attempts to confine the hot reacting plasma in vacuum, away from material walls, by using electromagnetic fields. Since the plasma particles are charged, their orbits, in terms of their position vector ($\vec{r}$) as a function of time (t), are governed by the equation of motion,

$$\ddot{\vec{r}} = \frac{q}{m} (\vec{E} + \vec{v} \times \vec{B}) , \quad (1.1.1)$$

where $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields in the plasma due to

both external (imposed) and internal (self-consistent) sources, q is the charge on the particle, and m is its mass. It is well known that in a slowly varying magnetic field, the magnetic moment,

$$\mu = \frac{1}{2} \frac{mv_{\perp}^2}{B}, \quad (1.1.2)$$

where $v_{\perp}$ is the velocity perpendicular to $\vec{B}$, is an approximate constant of the motion, i.e. an adiabatic invariant. Therefore, a charged particle can be effectively "tied" to a helical orbit about a field line, as shown in Fig. 1.1.

This fact suggests two types of magnetic field configurations for the confinement of a thermonuclear plasma. One is to bend magnetic field lines such that they close on themselves, and the system is effectively endless. This type of configuration is a "closed system", and the principal approach is the tokamak concept in which the magnetic flux bundle is bent into a torus to confine the plasma. The disadvantage of this configuration is that the curvature of the field lines gives rise to a particle drift out of the confinement system. These curvature effects are compensated by techniques which result in limiting the plasma pressure that can be stably confined. The other type of field configuration depends on μ being an adiabatic invariant and on the fact that a steady magnetic field can not impart energy to the particle; therefore, its energy,

$$\epsilon = \frac{1}{2} mv^2, \quad (1.1.3)$$

Figure 1.1 Charged particle helical orbit along a magnetic field line

is also constant. Particles with sufficiently small parallel motion along the field lines and situated in a region between two field maximums will be trapped in such a magnetic "well." This type of confinement system, which is usually linear to avoid the vertical drift, is known as a magnetic mirror, or "open system" because the field lines extend beyond the confinement region. The disadvantage of a magnetic mirror is that for a given mirror ratio,

$$R = \frac{B_{\max}}{B_{\min}}, \quad (1.1.4)$$

where $B_{\max}$ and $B_{\min}$ are the value of the maximum and minimum magnetic field in the system, only those particles with sufficiently large pitch angles at the field minimum are trapped. Particles that are magnetically trapped/confined have pitch angles (ψ) larger than ψ' , which is defined by

$$\sin^2 \psi' = \frac{v_{\perp}^2}{v^2} = \frac{1}{R} \quad (1.1.5)$$

where the perpendicular and parallel velocity components are defined with respect to the direction of the magnetic field. Therefore, the mirror system is leaky because particles are continuously scattered through all pitch angles, and they will be lost in an average sense in a characteristic scattering time. For this reason, simple magnetic mirror confinement is not expected to lead to an economically

attractive fusion reactor. However, interest in magnetic mirror confinement has been revived with the invention of the tandem mirror concept.^{5,6} In its most simple form, the tandem mirror consists of three linked magnetic mirror cells, with the center cell maintained at a lower potential than the two end cells. The two end cells serve as electrostatic end plugs for the plasma confined in the center cell. In principle, the confinement in the center cell can be arbitrarily scaled up by increasing the potential difference, and fusion yield can be also be arbitrarily raised by increasing the length of the center cell. With this enhancement, the tandem mirror is one of the mainline approaches for magnetically confined fusion.

In both the inertial and magnetic confinement approaches to controlled fusion, the plasma is heated by the 3.52 MeV alpha particles from the D-T reaction. This heating is a critical issue in determining the performance of the system. In the case of inertial confinement, the alpha particle energy deposition determines the fractional burn of the fuel pellet and, therefore, the energy gain that can be realized from each shot. In the case of magnetic confinement, the condition for ignition is determined by the balance of plasma heating by the alpha particles against energy losses through all channels. For this reason, the confinement and slowing down of 3.52 MeV alpha particles in a variety of magnetic confinement systems has been the subject of a number of previous studies concerned with both basic physics and specific fusion reactor designs.

The basic physics of energetic alpha particle slowing down in a homogeneous equilibrium plasma can be calculated from consideration of charged particle interaction via Coulomb forces. An approximate calculation for the fraction of energy transferred to the electrons and ions in a thermal hydrogen plasma as a function of the plasma temperature has been given by Rose,⁷ and his result is shown in Fig. 1.2. To sustain the fusion reaction in the plasma, it is necessary to maintain the ions hot. At temperatures in excess of 30 keV, a greater portion of the alpha energy is transferred directly to the ions. At temperatures below 30 keV, however, most of the energy is deposited with the electrons. The ions are then heated indirectly by the electrons. The partition of the alpha energy in a fusion reactor, therefore, depends on the transport and confinement lifetime of the electrons, ions, and alpha particles in the specific system.

There have been many studies of alpha particle confinement and thermalization in tokamak systems,⁸⁻¹⁵ ranging from determination of the energy distribution functions,¹² to effect of alpha bombardment on reactor walls^{9,15} and proposals for improving alpha energy confinement by changing the reactor designs.¹⁵ These studies, and many more, form a substantial body of knowledge on plasma heating by alpha particles in tokamaks. There have also been detailed studies of alpha thermalization in alternate fusion reactor concepts. Driemeyer¹⁶ made a complete study of fusion product effects in field reversed mirrors. Potok, et al.¹⁷ studied alpha particle confinement in torsatron type of reactors. Knowledge on the thermalization of alpha particles in tandem

Figure 1.2 Alpha energy fractions to plasma ions and electrons as a function of plasma electron temperature as determined by Rose⁷

mirror reactors, however, is much less complete. This is largely due to the fact that alpha particle confinement in tandem mirror reactors is a more complicated subject involving, in the most general case, three-dimensional fields in which many variations are possible. The three-dimensional character of the confinement field can lead to varying degrees of enhanced radial diffusion similar to neoclassical transport associated with field line curvature in closed confinement systems.

An initial study of alpha particle confinement in tandem mirrors has been performed at Lawrence Livermore National Laboratory (LLNL).¹⁸ This study evaluated basic considerations for alpha particle confinement in tandem mirror systems, but it neglected neoclassical diffusion effects. Neoclassical diffusion in tandem mirrors is important when there are significant nonaxisymmetric components of the magnetic field in the center cell. Effects of neoclassical diffusion on alpha confinement was later addressed by DeVoto and Ohnishi¹⁹ and by DeVoto, et al.²⁰ with the conclusion that that approximately 60-80% of the alpha energy is available for deposition in the plasma. These results were based on following alpha trajectories in a specific field configuration and determining a radial confinement life time by evaluating the radial displacement of the orbits. An analytical solution of the alpha energy deposition in a square-well magnetic mirror field with a uniform plasma background had been obtained by Santarius and Callen²¹ with similar numerical results.

In more refined models of tandem mirror reactors, it was recognized that the hot, reacting plasma core must be surrounded by a low temperature halo plasma.^{22,23,24} This halo plasma can be very effective in shielding the hot core from charge exchange losses with the background neutrals, and therefore has a profound effect on the equilibrium properties of the core. Furthermore, recent advances in the tandem mirror concept have led to designs of more axisymmetric center cells to minimize neoclassical transport losses and the introduction of thermal barriers to isolate the hot electron end plug from the reacting fusion plasma in the center cell. To evaluate the alpha particle thermalization in such systems, we need a basic understanding of the key physics processes affecting alpha energy deposition in tandem mirror systems, as well as a general technique for evaluating the effect of these processes in specific tandem mirror configurations. The present thesis is concerned with a study of alpha particle transport and energy deposition in tandem mirror reactors by using the Monte-Carlo technique. The objective is to identify and take into account all the important effects in the alpha energy deposition process and develop scaling relations useful for tandem mirror reactor designs. The technique, in the form of a computer code, is also sufficiently general to allow for evaluation of specific cases.

1.2 Basic Principles of Tandem Mirror Confinement

In order to address the issues of alpha particle confinement and energy deposition in tandem mirror reactors, we need to first have a clear understanding of the principles of tandem mirror confinement.

The present tandem mirror concept is a sophisticated embodiment of basic magnetic mirror confinement physics. Its complex design and many possible variations can be best understood by reviewing the evolutionary chain of magnetic mirror confinement devices. The magnetic mirror trap, in its most simple form, consists of two solenoidal coils separated along their common axis, as shown in Fig. 1.3a. This design produces an azimuthally symmetric field with an axial minimum at the midplane between the two coils. As has been noted, particles are trapped between the two mirror coils when their midplane pitch angle is greater than the critical value given by Eq. (1.1.5). The critical angle ψ' , rotated azimuthally, forms two opposing conical regions in velocity space as shown in Fig. 1.4a. These regions are known as the loss cone of a mirror trap within which particles are not confined. Particles with velocities outside the loss cone region will remain confined until their velocity is scattered into the loss cone at random by Coulomb collisions with other charged particles.

In addition to these velocity space losses, the simple mirror trap suffers from two serious problems. The first problem occurs because the surface of the plasma near the midplane, where the bulk of the plasma is confined, is concave. In the ideal MHD limit, the confining field lines conform to the plasma surface. Therefore, for concave surfaces, adjacent field lines in the vacuum region outside the plasma surface occupy a volume larger than the volume inside the surface which is occupied by the plasma. It can be shown that because

Figure 1.3 Evolution of mirror confinement devices, (A) simple mirror, (B) quadrupole field generated by Ioffe bars (C) quadrupole field generated by a baseball coil, (D) quadrupole field generated by Yin-Yang coils, (E) tandem mirror with a simple center cell connected to quadrupole end cells generated by baseball coils, (F) tandem mirror with thermal barriers, the center cell is a simple mirror, the end cells are made of Yin-Yang coils, and the thermal barrier is generated by baseball coils.

Figure 1.4 Velocity space losses for, (A) loss cone when the plasma potential is zero, (B) ion loss region, and (C) electron loss region when there is a plasma potential

of this difference, it is energetically possible for the plasma to exchange position with the vacuum region. This process can occur in the form of a hydromagnetic instability that leads to rapid loss of the confined plasma. The instability can be suppressed if the plasma surface is convex such that this interchange is energetically impossible.

Convex field lines imply that the magnitude of the magnetic field must increase away from the surface. This realization leads to the development of the minimum-B trap, the simplest form of which is obtained by adding a set of quadrupoles to the simple mirror, as shown in Fig. 1.3b. With this arrangement, the absolute magnitude of the confining field rises both axially away from the mid plane, because of the mirror coils, and radially away from the axis, because of the quadrupoles whose contributions are exactly cancelled on axis. MHD stable confinement by such a minimum-B mirror field was first demonstrated by Ioffe²⁵ in the U.S.S.R. A number of variations of the quadrupole minimum-B configurations are possible; most notable are the so-called baseball, shown in Fig. 1.3c, and the yin-yang coil system, shown in Fig. 1.3d. It is also possible to use higher order multipoles, which differ from the quadrupole fields, because the radial field cancellation occurs over a radial region around the axis, due to higher order cancellations of the field.

Although the minimum-B configuration is able to eliminate the MHD stability problem, there remains a second problem associated with magnetic mirror confinement. Because of the existence of the loss

cone, particles are confined, on average, in the mirror trap for only one collisional scattering time. The electrons have higher thermal speed, since their mass is smaller than the ions, and are scattered more frequently than the ions. As a result, the electrons are scattered into the loss cone at a higher rate, leading to the development of a positive space charge due to charge imbalance. In the presence of this space charge, electrons are electrostatically confined, and only the very energetic electrons escape while the ions are accelerated out by the positive potential, such that the low energy ions are not confined at all. By taking the electrostatic potential built up in mirror plasmas by this electron loss into consideration in the energy conservation, the loss boundary in velocity space is given by

$$\frac{v_{10}^2}{v_o^2} = \left(1 + \frac{2q\phi}{m}\right)/R, \quad (1.2.1)$$

where q is positive for ions and negative for electrons, and ϕ is the potential difference. This is the equation of a hyperboloid of a single surface for the ions and of two surfaces for the electrons, as shown in Figs. 1.4b and 1.4c, respectively. As a result, the electron loss cone is essentially eliminated, and the electron distribution can relax to closely approximate a Maxwellian distribution, while the ion loss cone is greatly enlarged, and the confined ion distribution is void of low energy particles. It has been shown by Pastukov²⁶ that

because of the presence of the potential well, electron confinement is enhanced by the exponential power of the ratio of the potential well depth to the temperature of the species; the confinement time is given by

$$\tau_e = \frac{\sqrt{\pi}}{2} \tau_{ee} \frac{2R+1}{R} \ln(4R+2) \frac{\phi}{T_e} \exp \frac{\phi}{T_e}, \quad (1.2.2)$$

where τ_{ee} is the electron-pitch-angle scattering time and T_e is the electron temperature. For the ions, however, their distribution possesses an inverted population and is potentially unstable. Instabilities of this type, which are referred to as microinstabilities because they depend on the distribution for their excitation, have been observed in virtually every relevant, simple, mirror experiment. A result of these instabilities, in which the free energy is released, is that the ions fill in the void space near the velocity space origin where they are not contained. Therefore, basic mirror confinement suffers from microinstabilities, which further reduces the confinement time that can be attained.

A technique for suppressing the microinstability was demonstrated in the 2X-IIB experiment²⁷ at LLNL by flowing a warm plasma stream into the plasma which fills in the loss cone. Although this technique suppresses the microinstabilities, it can be shown that the minimum-B confinement cannot be used by itself for a viable fusion reactor. The limitation for minimum-B confinement is that the energy gain, or Q, for this system is less than 2, whereas reactor Q values

should be greater than 5. Therefore, it was concluded in the mid 1970's that basic mirror confinement is not scalable to an economically attractive fusion power reactor and some alternative approach was needed.

The tandem mirror confinement concept was a timely invention conceived independently by Fowler and Logan⁵ in the U.S. and Dimov, et al.⁶ in the U.S.S.R. to meet this need. They reasoned that because the electrons are well confined by the potential well, their distribution is essentially in collisional equilibrium. Therefore, if one links three mirror cells together, as shown in Fig. 1.3e, the electron distribution will follow the Boltzmann well relation,

$$\phi = T_e \ln \frac{n_p}{n_c} . \quad (1.2.3)$$

If the center cell is maintained at a lower density than the two end cells, then the potential will have a similar axial profile, as shown in Fig. 1.5. Thus, while the electrons are electrostatically confined by the positive space charge potential throughout the system, the ions in the center cell are also electrostatically confined by the higher potential in the two end cells. According to the Pastukhov result, the ion confinement time in the center cell will be enhanced by the exponential factor of the ratio of the ion confinement potential, ϕ_{pc} , to the ion temperature, T_i , which can be arbitrarily scaled up by increasing the height of the potential barrier. The two end cells, which serve as electrostatic end plugs, still operate much as simple

Figure 1.5 Vacuum magnetic field axial plasma potential and plasma density for a tandem mirror reactor

mirror cells. By using minimum-B configuration in the end cells, the entire system can be made MHD stable. Ions confined in the center cell which are able to overcome the potential barrier will be lost by streaming through the end cells. This streaming plasma serves to fill the loss cone region of the plasma confined in the end plugs, which suppresses the excitation of microinstabilities. Therefore, the system can be sustained by heating and fueling the end plugs against Coulomb scattering losses. Furthermore, since the center cell has neutral MHD stability, it can be made arbitrarily long to increase the volume of the reacting fusion plasma, which increases the fusion power output. This system, which is known as a tandem mirror in the U.S. and as ambipolar trap in the U.S.S.R., is a very attractive basis for a thermonuclear fusion reactor.

Although in principle the tandem mirror concept is attractive for thermonuclear plasma confinement, there are several practical difficulties. First of all, the nonaxisymmetric quadrupole fields in the anchor, which are required for MHD stability, perturb the perfect axisymmetry of the solenoidal center cell field. The finite azimuthal magnetic field component created by the quadrupole fields causes large radial excursion of the confined particles. Ryutov and Stupekov²⁸ first pointed out that such excursions can give rise to enhanced radial diffusion, which limits the radial confinement time that can be realized in such tandem mirror configurations. In order to eliminate this enhanced diffusion, the magnetic field in the center cell must be made fully axisymmetric. Secondly, in order to develop a sufficiently

large potential barrier, the density in the plug must be much greater than in the center cell. Since the power required to sustain the tandem mirror configuration is proportional to the plug density, very high input power is required to develop the potential necessary for confinement. The magnetic field in the plug must also be high to contain the plug density, which complicates the quadrupole magnet design. Because the magnitude of the ambipolar potential in a mirror confined plasma is determined by the electron temperature, the barrier can be enhanced if the electrons in the plug can be heated independent of the center cell. This led Baldwin and Logan²⁹ to propose the thermal barrier tandem mirror, in which a potential minimum is introduced between the plug and the center cell, which limits the electron conduction between the two regions. A typical coil set for this mirror system is shown in Fig. 1.3f; the axial magnetic and potential profiles for this configuration, which became known as thermal barrier tandem mirrors, are shown in Fig. 1.6. To make the magnetic field in the center cell axisymmetric, a number of proposals have also been advanced. These include the use of choke coils to recircularize the quadrupole fields, as in most of the new tandem mirror designs, such as Gamma-10 in Japan and TARA and MFTF-B in the U.S. The use of more or fully axisymmetric anchors generated by using hexipoles, cusps, E-rings and pondermotive forces to provide MHD stability to the system have also been proposed. The current trend in tandem mirror reactor design, therefore, generally involves an

Figure 1.6 Vacuum magnetic field (A), axial plasma potential (B), and plasma density (C) for a tandem mirror with thermal barriers

axisymmetric center cell separated from an MHD anchor and includes electron isolation from the end plugs by a thermal barrier.

1.3 Physics Issues of Alpha Confinement in Tandem Mirror Reactors

Tandem mirror reactors are expected to operate in steady state with the center cell ignited by the 3.52 MeV alpha particle heating. We are therefore concerned with all aspects of the the lifetime of the alpha particles in the tandem mirror reactor from birth through slowing down to ultimate loss from the system. The issues involved are extremely complicated and the purpose of this thesis is to identify the more important ones and to study their role in determining the ignition condition. Specifically, we are concerned with modeling the distribution of the alpha particle source function in the equilibrium plasma, the thermalized alpha density, and the alpha energy deposition profile. The profiles are determined by the plasma condition along the alpha orbits in the specific confinement geometry. In this section, we shall discuss qualitatively the physics issues with the alpha birth and thermalization process.

The alpha particles are created by the fusion of deuterium and tritium ions in the plasma. The alpha particle source distribution function, therefore, is determined by the distribution of deuterium and tritium particles in the plasma. Therefore, one of the issues to be addressed in this thesis is the proper representation of the alphas particle source function in generic tandem mirror reactors.

Through momentum balance considerations, the alpha particle will be emitted with an average energy of 3.52 MeV. Because the alpha

particles are doubly charged, their motions will be affected by the electric and magnetic fields present. The electric and magnetic fields arise from both externally imposed fields, such as the vacuum magnetic field and self-consistent macroscopic fields, such as due to the plasma diamagnetism and ambipolar potential, as well as microscopic fields associated with the individual charged particles. The vacuum magnetic field is readily computed from specification of the confinement coils. The distribution of the plasma pressure confined by the externally imposed field is determined from solving the steady state MHD equation for momentum balance,

$$\nabla p_1 = \bar{\mathbf{J}} \times \bar{\mathbf{B}}_v, \quad (1.3.1)$$

where the current density is given by Ampere's Law,

$$\nabla \times \bar{\mathbf{B}}_v = \mu_0 \bar{\mathbf{J}}. \quad (1.3.2)$$

Combining the two equations, we obtain

$$\nabla \left(p_1 + \frac{B_v^2}{2\mu_0} \right) = \frac{1}{\mu_0} (\bar{\mathbf{B}}_v \cdot \nabla) \bar{\mathbf{B}}_v \quad (1.3.3)$$

where p_1 is the plasma pressure perpendicular to the local confinement magnetic field. The right side of Eq. (1.3.3) is the curvature which is exactly zero in a straight and uniform solenoidal magnetic field.

In actual magnetic field configurations, the approximation of setting the right side equal to zero, which is commonly known in mirror confinement theory as the "long, thin approximation," yields for the solution of the self-consistent magnetic field,

$$B_a = B_v \left(1 - \frac{2\mu_0 P_{\perp}}{B_v^2} \right)^{1/2} . \quad (1.3.4)$$

The effectiveness in the utilization of the vacuum magnetic field is often characterized by the parameter, β , defined as

$$\beta = \frac{2\mu_0 P_{\perp}}{B_v^2} , \quad (1.3.5)$$

which is the ratio of the plasma pressure to the vacuum magnetic field pressure. A characteristic of magnetic mirror confinement, which makes such confinement attractive, is that β -values approaching unity are possible. When beta approaches unity, the magnetic field inside the plasma goes to zero and, as a result, the Larmour radius, given by

$$\rho = \frac{mv_{\perp}}{qB_a} , \quad (1.3.6)$$

of the very energetic alpha particles can become comparable or even larger than the plasma radius. Therefore, plasma diamagnetism can have an important effect on alpha particle confinement.

As we have noted, the tandem mirror concept depends on the ambipolar potential associated with magnetic mirror confinement to plug the end losses. The presence of an ambipolar potential implies that there is a radial electric field which is associated with ion and electron radial density gradients. In the presence of a radial electric and axial magnetic field, the motion of the charged particles can be decomposed into three parts, a cyclotron gyration about the guiding center along a field line, motion of the guiding center along the field line, and a transverse drift perpendicular to both the electric and magnetic field, with velocity given by

$$\bar{V}_D = \frac{\bar{E} \times \bar{B}}{B^2} . \quad (1.3.7)$$

Therefore, in order to properly account for the motion of the alpha particles in the tandem mirror system, we need to also take into account the axial and radial electric fields.

The motion of the alpha particles is affected not only by the macroscopic but also the microscopic fields associated with individual particles. The individual charged particles interact through long-range Coulomb forces. In a dense plasma, each particle experiences the fields associated with many other particles in its vicinity simultaneously. As a result, the interaction is in the form of small perturbations to its thermal velocity and direction of motion. The effect of this type of "collisional" interaction on a particle

distribution, f , is described by the Fokker Planck collision operator,^{30,31}

$$\left. \frac{\partial f}{\partial t} \right|_c = - \frac{\partial \langle \Delta w_{\parallel} \rangle f}{\partial w} + \frac{\partial^2 [0.5 \langle (\Delta w_{\parallel})^2 \rangle f]}{\partial w^2} + \frac{\partial^2 [0.5 \langle (\Delta w_{\perp})^2 \rangle f]}{\partial w^2} + O(\Delta w^3), \quad (1.3.8)$$

where w is the velocity and $w_{\parallel}$ and $w_{\perp}$ denote the parallel and perpendicular components, respectively. The quantity Δw represents changes to the velocity as a result of the interactions. The brackets represent averages over the distribution of field particles in the vicinity of the test particle. Spitzer³² has physically interpreted the quantities $\langle \Delta w_{\parallel} \rangle$, $\langle (\Delta w_{\parallel})^2 \rangle$ and $\langle (\Delta w_{\perp})^2 \rangle$ as dynamical friction, velocity dispersion, and pitch angle scattering, respectively. He was also able to derive explicit expressions for these quantities on the basis of a test particle interacting with a background plasma, characterized by its ionic species, temperature, and density. Making use of these quantities, it is possible to define the following characteristic times:

$$\tau_D = \frac{w}{\langle \Delta w_{\parallel} \rangle} \quad (1.3.9)$$

for drag time,

$$\tau_c = \frac{w^2}{4 \langle (\Delta w_{\parallel})^2 \rangle} \quad (1.3.10)$$

for energy exchange time (thermalization time), and

$$\tau_1 = \frac{w^2}{\langle (\Delta w_1)^2 \rangle} \quad (1.3.11)$$

for the cumulative 90° pitch-angle scattering time.

The confinement and thermalization of the alpha in a tandem mirror reactor is a function of its velocity and radial position, along with the plasma properties at this position. Alphas are born isotropically at 3.52 MeV, which will be much greater than any confinement potential and if they are born in the loss cone, they will be lost immediately. Those that are born outside the loss cone will be confined initially and may be lost as they are dragged down in energy and pitch angle scatter on the background plasma. Alphas that scatter into the loss cone with energies greater than ϕ_{pc} will also be lost. In order to effectively utilize the alpha particles for sustaining the fusion plasma, the system must be designed such that the alpha particle confinement time is at least comparable with the drag time.

Once the alpha particles are thermalized, their average energy will be lower than the plugging potential, and they will be electrostatically confined. Furthermore, because they are doubly charged, they experience a potential barrier that is twice as high as for the hydrogenic ions. As a result, thermalized alphas will tend to accumulate in the burning plasma and displace the density of the reacting fuel. If such accumulation is not controlled, there will eventually be insufficient fuel to generate the fusion power needed to

sustain the plasma and the reaction will quench. Therefore, in addition to confinement and thermalization, removal of the alpha particles is also an important issue in the maintenance of an ignited fusion plasma.

Finally, we should also note that the reacting plasma is expected to be radially inhomogeneous in both density and temperature. Because the alpha particles are initially very energetic, their gyroradius will be large, particularly in a high beta plasma, and they will traverse regions of very different plasma density and temperature in each gyration. Since the drag rate depends on the local density and temperature, there is an uneven slowing down rate along each gyro orbit. As the alpha loses energy, its gyroradius becomes correspondingly smaller. It can readily be demonstrated from physical considerations that this uneven drag rate has the effect of pulling the alpha guiding center towards the region of high drag, i.e. regions of high density and low temperature. In general, for motion of a charged particle in a force field, $\vec{F}$ transverse to a magnetic field, there is a drift of the guiding center with velocity,

$$\vec{V}_D = \frac{\vec{F} \times \vec{B}}{B^2} . \quad (1.3.12)$$

In the case of present interest, the net force, $\vec{F}$, is the difference of the unequal azimuthal drag rates across the gyro orbit of the particle. For a cylindrical plasma with radially decreasing temperature and density profiles, typical of a hot reacting thermonuclear plasma

surrounded by a cool halo plasma, the resulting drift is radially outward. The effect of radial drag gradients, therefore, is to move high energy particles in the direction of the gradient. If the drift velocity is sufficiently high, core heating will be reduced and the alpha particles will deposit a significant fraction of their energy in the halo, which is generally poorly confined. This effect can also prevent the attainment of ignition condition and must be properly considered.

1.4 Methods of Analysis

The rigorous approach to determining the energy deposition of the alpha particles in the fusion plasma is to solve the Boltzmann equation for the alpha particle distribution, f_α , in the six-dimensional phase space,

$$\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \frac{\partial f_\alpha}{\partial \vec{r}} + \frac{\vec{F}}{m} \cdot \frac{\partial f_\alpha}{\partial \vec{v}} = \left. \frac{\partial f_\alpha}{\partial t} \right|_c + S_\alpha + L_\alpha, \quad (1.4.1)$$

where S_α and L_α are the alpha particle source and sink terms in real space and $\left. \frac{\partial f_\alpha}{\partial t} \right|_c$ is the Fokker-Planck operator for Coulomb collisions which may be interpreted as a source or sink in velocity space. The left side of the equation is just the total time derivative of the distribution function.

There are many difficulties that prevent the general analytical solution of Eq. (1.4.1). One problem is that this equation is

nonlinear. However, we expect the density of the alpha particles to be small compared to the density of the reacting plasma, and it is reasonable to only consider the alpha interaction with the distribution of the plasma electrons and ions. Therefore, we only need to be concerned with the solution of the linearized form of Eq. (1.4.1). There are still other difficulties. First of all, the distribution of plasma electrons and ions, which is determined by the fully nonlinear Boltzmann equation, is not known. Therefore, suitable approximations must be assumed to represent them. Secondly, the problem is multidimensional; for any realistic representation of a tandem mirror system, we generally need to consider at least one spatial (radial) and two velocity (parallel and perpendicular to the magnetic field) directions. It is extremely unlikely that analytical solutions can be found for specific confinement field configurations. In general, therefore, it is necessary to resort to computer solutions by numerical methods.

We note that the left side of Eq. (1.4.1) describes the phase space trajectory of particles in the distribution, while the right side accounts for source and sink terms in both physical and velocity space due to collisions. In considering the numerical solution of the Boltzmann equation, there are several characteristic times of interest. The first is the cyclotron gyration time,

$$\tau_c = \frac{1}{2\pi\omega_c} = \frac{1}{2\pi} \frac{m}{qB} \quad (1.4.2)$$

The second characteristic time is associated with the axial bounce period of particles as they are reflected from end to end in a mirror trap of length, L ,

$$\tau_b = \frac{L}{|\bar{v}|} . \quad (1.4.3)$$

There is also a characteristic time associated with the azimuthal drift period,

$$\tau_\phi = \frac{2\pi r_p}{|\bar{v}_d|} . \quad (1.4.4)$$

Finally we have the characteristic times associated with collisional interactions of the alpha particles with the bulk plasma, i.e. the drag, dispersion, and pitch-angle scattering times. In the case of 3.52 MeV alpha particles in a magnetically confined plasma, these characteristic times are ordered as follows:

$$\tau_c \ll \tau_b \sim \tau_\phi \ll \tau_d < \tau_\perp < \tau_e , \quad (1.4.5)$$

namely, the characteristic times associated with the terms on the right side of Eq. (1.4.1) are long compared to the characteristic times for periodic motion in phase space associated with terms on the left side of the equation. However, it is precisely these slow collisional processes that cause the alpha particles to thermalize and be

transported in phase space. Therefore, although these terms on the right side of the equation are much larger in magnitude compared to those on the left, they can not be ignored, since they govern the relaxation process. On the other hand, it is also evident that collisional relaxation is not a localized process and must be calculated by averaging over the phase space trajectory of the particles; these trajectories are determined by τ_c , τ_b , and τ_ϕ .

The method we have chosen to use in studying alpha particle thermalization is the Monte-Carlo technique, which constructs a solution based on random sampling. The technique has the advantage that the particle trajectory in phase space can be followed in detail. Furthermore, because the cyclotron gyration period is very short, compared to characteristic times for any other processes of interest, and the gyroradius is uniquely determined by μ , we need only to follow the motion of the guiding centers. The effects of the infrequent collisions are then evaluated by sampling from appropriate probability distributions. To evaluate the displacement of the guiding center due to collisions, it is necessary to know the exact position of the particle in relation to the guiding center. We can recover this information statistically by sampling. A significant portion of this thesis is concerned with developing reasonable models for representation of the system in a process that is computationally fast. The solution is then constructed from the results of a large number of samples. The accuracy of the solution is determined primarily by the number of alphas in the sample. In this thesis, the algorithm is then

applied to carry out parametric studies of alpha particle slowdown in a generic tandem mirror reactor.

The center-cell plasma parameters for a generic tandem mirror fusion reactor that have been used in this study are chosen to be consistent with present reactor designs. A comparison of the generic parameters with those of two current reactor design studies is made in Table 1.1.

TABLE 1.1

COMPARISON OF THE GENERIC TANDEM MIRROR REACTOR PARAMETERS
WITH THE SUPER-TARA AND OCTOPOLE END CELL DESIGNS.

<u>Parameter</u>	<u>Super-Tara</u> ³³	<u>Octopole End Cell</u> ³⁴		<u>Generic Design</u>
P (MW)	1000	500	1000	1000
r_p (m)	0.51	0.45	0.45	0.5
L_c (m)	24	50	100	60
B_c (T)	3.5	3.3	3.3	3.0
B_m (T)	25	24	24	24
$\hat{\beta}$ (Peak Beta)	0.84	0.75	0.75	0.8
T_i (keV)	40	40	40	40
T_e (keV)	28	27	27	27
$n_i \cdot 10^{14} \text{ cm}^{-3}$	3.01	2.3	2.3	1.65

The details of the numerical simulation and the computed results are presented in the following chapters. The specific magnetic and electrostatic field models are described in Chapter 2. The guiding center equations of motion for the alpha particles are developed in

Chapter 3. A detailed description of the Monte-Carlo scheme for treating collisional effects is presented in Chapter 4. The various types of density and temperature models for representing the bulk plasma system used in the study is described in Chapter 5. The results of the study and the various diagnostics used to verify the validity of the calculational technique are presented in Chapter 6. General conclusions from this study and suggestions for future research are given in Chapter 7.

PART 2

MAGNETIC AND ELECTRIC FIELDS

2.1 Introduction

An essential first step in studying alpha particle thermalization in magnetically confined fusion plasmas is to develop a method for accurately calculating the confining magnetic fields and electrostatic potentials. An exact analysis begins with a description of the vacuum magnetic fields. Next, the plasma diamagnetic effects and local fluctuations in the magnetic field would be generated by a self-consistent description of the confined plasma. The radial and axial electrostatic potentials would also be determined self-consistently. The development of a self-consistent field and potential model requires a major computational effort. Such a technique, although desirable, is not yet available for determining the magnetic confinement fields and electrostatic potentials that are associated with alpha particle thermalization in tandem mirrors. Even though a self-consistent calculation is unavailable, it is possible to determine the vacuum magnetic fields along with the plasma diamagnetic effects and the axial potential by using physically reasonable and computationally quick models. The radial potential which is generated by the plasma radial density and temperature profile is modeled based on radial potential control techniques³⁵ that are being tested on the TMX-U experiment. The vacuum magnetic field models along with the plasma diamagnetic and electrostatic potential models used in this study are presented in the following sections.

2.2 Tandem Mirror Vacuum Magnetic Fields

Vacuum magnetic fields in the plasma can be found by numerically integrating Ampere's Law over defined current elements. There are many 3-D electromagnetic codes, e.g. EFFI,³⁶ that can be used to calculate the vacuum fields from defined current elements. In using Monte-Carlo techniques to follow particle trajectories, the magnetic field is needed at every calculated particle position, and determining vacuum field values from three-dimensional codes would require an excessive amount of computer time. Analytical methods which approximate the magnetic fields can be used, but the derivative of the field, which is needed for particle drift calculations, is not smooth. We combine field values calculated at fixed nodal points from EFFI with the paraxial approximation,³⁷ given to order r^2 by

$$B_x = -\frac{x}{2} [f(z) - g(z)] \quad (2.2.1)$$

$$B_y = -\frac{y}{2} [f(z) + g(z)] \quad (2.2.2)$$

$$B_z = f(z) \quad (2.2.3)$$

to calculate the magnetic fields in the plasma. The value of $f(z)$ is found from EFFI calculations of B_z at points along the plasma axis. The function of $g(z)$ is determined from

$$g(z) = \eta'(z) f(z) . \quad (2.2.4)$$

where $\eta'(z)$ is the derivative of the magnetic flux tube ellipticity. The ellipticity is found from the conservation of magnetic flux given by

$$\begin{aligned} \psi_{\phi} &= r^2 f(0) \\ &= f(z) x^2 \exp\{-\eta(z)\} + f(z) y^2 \exp\{\eta(z)\} , \end{aligned} \quad (2.2.5)$$

where ψ_{ϕ} is the magnetic flux. When y equals zero, the ellipticity is

$$\eta(z) = \text{Ln} \frac{x^2 f(z)}{r^2 f(0)} , \quad (2.2.6)$$

with the derivative along z given as:

$$\eta'(z) = \frac{r^2 f(0)}{x^2 f(z)} f'(z) . \quad (2.2.7)$$

Values of $\eta'(z)$ are determined from EFFI calculations of B_z for a given x -displacement at points along the z axis. We assume that the ellipticity for a flux tube at arbitrary z is constant in the radial direction, and therefore we can find B_x , B_y , and B_z at any point in the plasma. Values of $f(z)$, $f'(z)$, and $\eta(z)$ can be found at any location

by using a cubic spline technique between nodal points. The cubic spline was incorporated into the calculation by Ohnishi and is described in Ref. 19.

The paraxial approximation can also be used to relate plasma parameters along a flux tube to the parameters at the plasma midplane. For example, the density and temperature are assumed to be functions of the flux coordinate (ψ_ϕ) and the radius. These variables are constant along a flux tube and are given by

$$n(\psi_\phi, r) = n(\psi_\phi(z=0), r(z=0)) , \quad (2.2.8)$$

$$T(\psi_\phi, r) = T(\psi_\phi(z=0), r(z=0)) . \quad (2.2.9)$$

The radial plasma potential (ϕ_r) is a function of the radial density and temperature profiles, and it may also be determined at any z by assuming that ϕ_r is a function of ψ_ϕ and r .

$$\phi_r(\psi_\phi, r) = \phi_r(\psi_\phi(z=0), r(z=0)) . \quad (2.2.10)$$

Therefore, once the plasma density, temperature, and radial potential profiles are defined at any z location, they can be determined at any other plasma location.

The calculation of the magnetic fields, plasma temperature, densities, and radial potentials can be further simplified for alphas

in axisymmetric fields and for a class of alpha guiding center trajectories, called "nonresonant" trajectories,²⁸ in nonaxisymmetric tandem mirrors. The bounce motion of these alphas can be neglected and their transport can be simulated at the plasma midplane. This simplifies the description of the vacuum fields, since only the midplane values are required. However, when the bounce motion cannot be neglected, the techniques described above must be used. Although the Monte-Carlo technique can perform detailed guiding center trajectory calculations throughout the plasma, they are not needed for the present axisymmetric reactor designs.

Particle motion in the central cell of an axisymmetric mirror can be described by knowing the density, temperature, and magnetic field at the plasma midplane. This is true because the curvature drifts generated in the axisymmetric mirrors of the center cell are azimuthal. Furthermore, this drift is generally small compared to $\bar{E} \times \bar{B}$ and grad B drifts, and since they do not affect alpha transport in axisymmetric tandem mirrors, the curvature drift can be neglected. By neglecting the curvature drifts, only the vacuum magnetic fields at the plasma midplane are required, and these are given by

$$B_x = B_y = 0, \quad (2.2.11)$$

$$B_z = |\bar{B}|. \quad (2.2.12)$$

These vacuum fields also satisfy the following conditions,

$$\nabla \cdot \vec{B} = 0 , \quad (2.2.13)$$

$$\nabla \times \vec{B} = 0 . \quad (2.2.14)$$

The vacuum magnetic field is altered by the plasma pressure, as shown by Eq. (1.3.4). Because there is a radial pressure profile, there is also a radial gradient in B. The long thin approximation for the magnetic field at the plasma midplane is dependent on the radial position, i.e.,

$$B_a = B_v [1 - \beta(r)]^{\frac{1}{2}} . \quad (2.2.15)$$

We define the pressure profile in terms of the density and temperature scale lengths r_{pD} and r_{pT} as

$$n = n_o f\left(\frac{r}{r_{pD}}\right) , \quad (2.2.16)$$

$$T = T_o f\left(\frac{r}{r_{pT}}\right) , \quad (2.2.17)$$

where the subscript "o" denotes the value at $r = 0$.

The pressure is given as

$$p_o = n_{oi} k T_{oi} + n_{oe} k T_{oe} \quad (2.2.18)$$

where the subscripts "i" and "e" refer to ions and electrons respectively. In a quasineutral plasma, the electron and ion densities are essentially equal throughout the plasma, therefore,

$$n_i \sim n_e . \quad (2.2.19)$$

The ion and electron temperatures can be different so the plasma pressure becomes

$$p = n_{eo} k (T_{oi} + T_{oe}) f\left(\frac{r}{r_{pD}}\right) f\left(\frac{r}{r_{pT}}\right)$$

or (2.2.20)

$$p = p_o f\left(\frac{r}{r_{pD}}\right) f\left(\frac{r}{r_{pT}}\right) .$$

where we have defined $p_o = n_{eo} k(T_{io} + T_{eo})$. Implicit in this relation is the assumption that the radial density profiles for ions and electrons are equal and the temperature profiles are assumed to have the same radial dependence. The radial gradient of B, which is needed to determine alpha drift motion, is

$$\nabla B_a = \nabla \{B_o [1 - B_o f\left(\frac{r}{r_D}\right) f\left(\frac{r}{r_T}\right)]^{\frac{1}{2}}\} ,$$

and this simplifies to

$$\nabla B_a = - \frac{\beta_0}{B_v(1-\beta(r))^{1/2}} \nabla \left(f\left(\frac{r}{r_{pD}}\right) f\left(\frac{r}{r_{pT}}\right) \right) . \quad (2.2.21)$$

2.3 Diamagnetic Effects on Magnetic Fields

Plasma diamagnetic effects as shown above lower the magnitude of the vacuum magnetic field. The diamagnetic effect is greatest in the tandem mirror center cell, where β is high, and small at the mirror throat where β is low. This effect creates magnetic wells that are deeper than the designed vacuum wells. Although the diamagnetic effect reduces the magnitude of B , it has a negligible effect on field line curvature, as discussed in Ref. 38. Therefore, at any point in the plasma, the direction of the magnetic field is given by the vacuum field unit vectors at that point.

Diamagnetism also affects the adiabaticity conditions of the plasma. Conon, et al.³⁹ have studied the conditions in which μ is an adiabatic constant in high β mirror machines. Confined particles are absolutely adiabatic when ρ/L , where L is the smallest magnetic scale length, is less than 0.1. In a high β plasma, the α Larmor radius can approach values of the magnetic scale length, so that the value of ρ/L can approach unity. Even when ρ/L approaches unity, the magnetic field can be designed to ensure conservation of μ . We have assumed that the axial variation of β is made smooth, so the α particles are adiabatic. Radial temperature and density profiles are made so that $\rho/r_p < 1$, such that the radial gradients do not affect the adiabaticity.

2.4 Axial and Radial Potential Profiles

Axial and radial potential profiles are modeled consistent with present reactor designs. The radial potential can be modeled using Eq. (2.2.6), but from radial transport considerations, it is desirable to have this potential at ground by using radial potential control plates.³⁵ Therefore, we set the radial potential to zero, and there is no $\vec{E} \times \vec{B}$ drift for the alpha particles. Elimination of the radial potential prevents the plasma ions from having large azimuthal drifts in nonsymmetric tandems to minimize plasma ion radial transport. Energetic alphas, however, have large azimuthal diamagnetic drifts which are in the opposite direction from the $\vec{E} \times \vec{B}$ drifts. Therefore, alphas in nonaxisymmetric high- β tandems which have no radial potential can still have resonant particle diffusion. Resonant diffusion of alphas is prevented by requiring the center cell fields to be fully axisymmetric. The radial potential in axisymmetric tandems affects only the alpha azimuthal motion and does not affect alpha transport.

Alpha particles that are scattered into the loss cone with energies less than the confining potential are mirror trapped, but energetic alphas will pass right over the potential peak. Potential peak values are approximately $4 T_i$, so that alphas with a plus two charge must have energies less than approximately $8 T_i$ or approximately 500 keV, which means that less than 15% of alpha energy is potentially confined. The majority of this 500 keV will be deposited in the center cell region, since the density in the transition and anchor regions is lower than the center cell density. Because of these considerations,

the axial potential is modeled as a square well with the axial peaks occurring at the center cell mirror peaks (see Fig. 2.1). The magnitude of the axial potential is given by $\kappa \cdot T_1$ where κ is an input parameter chosen as 4 for this study. The radial shape of the axial plugging potential follows T_1 .

Figure 2.1 Vacuum magnetic field (A), and axial plasma potential (B) for the generic reactor design

PART 3

GUIDING CENTER THEORY

3.1 Introduction

In general, alpha particle trajectories in a tandem mirror reactor can be described as a fast gyro motion, coupled with a drift perpendicular to the background magnetic field, and an acceleration parallel to the magnetic field. The alpha fluid motion is a part of the Boltzman equation, given by Eq. (1.4.1), and an individual alpha trajectory in velocity space can be found when f_α is defined as a single particle distribution function. The left side of Eq. (1.4.1), when multiplied by $m\bar{v}$ and integrated over velocity space for this single particle distribution, becomes

$$\frac{d\bar{v}_\alpha}{dt} = \frac{q_\alpha B}{m_\alpha} \left(\frac{\bar{E}}{B} + \bar{v}_\alpha \times \frac{\bar{B}}{B} \right) \quad (3.1.1)$$

where $\bar{v}$ is the actual particle velocity. This is the alpha equation of motion as given by Eq. (1.1.1).

The right side of the Boltzman equation contains the collision operator $\left. \frac{\partial f_\alpha}{\partial t} \right|_c$ plus source and sink terms. Part of the collision operator is the dynamical friction or drag which continuously slows the high energy alphas down to lower energies without affecting the alpha pitch angle. Because the drag is continuous, it can be treated in a similar manner as the electromagnetic forces during the slowing down process. Even though drag is continuous, it can be simulated by a

discrete collision in which accumulated drag effects shrink the orbit and the drag gradients cause radial guiding center drifts.

The alpha motion calculated in a Monte-Carlo technique must be effectively simulated by sampling the collision process many times and by following the alpha between collision sampling events. The alpha mean free path between sampling events is a fraction of $\tau_D |\bar{v}|$, which is approximately 10^5 to 10^7 m, and it is quite complicated. Even a fraction of this mean free path is long, and if it were computed exactly, it would require an excessive amount of computer time. Fortunately, μ is an adiabatic invariant between collisions, so the alpha guiding center can be followed and, in axially uniform and axisymmetric reactors, the bounce motion can be neglected, so that only the alpha drift motion at the reactor midplane needs to be followed. The guiding center trajectories derived from Eq. (3.1.1) are presented in the next section, followed by a discussion of the radial drag drifts.

3.2 Classical Guiding Center Theory

A charged particle in a uniform magnetic field gyrates about a magnetic field line as if it were attached to the line by a string. The point where the imaginary string attaches to the field line is the guiding center. The guiding center drifts off the field line when there are transverse electric fields, magnetic and plasma pressure gradients, and magnetic-field line curvature. These complex trajectories can be simplified by integrating over the fast gyro motion and following the guiding center motion.

The concept of guiding center motion can be used when the scale lengths for field changes are small over a Larmor orbit. This scale factor ($\frac{\rho}{L}$) can be found by non-dimensionalizing Eq. (3.1.1), given by

$$\frac{\rho}{L} \frac{d\tilde{v}}{d\tilde{t}} = \frac{\tilde{E}}{v_0 B_0} + \tilde{v} \times \tilde{B} \quad (3.2.1)$$

where

$$\tilde{v} = \frac{v}{v_0} ,$$

$$\tilde{t} = \frac{L}{v_0 t} ,$$

$$\tilde{E} = \frac{E}{E_0} ,$$

$$\tilde{B} = \frac{B}{B_0} .$$

The values of E_0 , B_0 , and v_0 represent typical values for the electric and magnetic fields and particle speed. When $\frac{\rho}{L}$ is small, the solution of Eq. (3.2.1) can be found by expanding the electric and magnetic fields in terms of this scale factor. This is an essential first step in deriving the guiding center equations of motion.

A formal derivation of the guiding center equations has been discussed by Northrop,⁴⁰ and a brief review of his results will be presented here. First, the radius vector ($\vec{r}$) is defined by a guiding center component ($\vec{R}$) and the Larmor radius ($\vec{\rho}$),

$$\vec{r} = \vec{R} + \vec{\rho} . \quad (3.2.2)$$

We use an orthogonal coordinate system with $\hat{e}_1$ equal to $\bar{B}/B$, $\hat{e}_2$ is perpendicular to $\hat{e}_1$, which defines $\hat{e}_3$ as $\hat{e}_1 \times \hat{e}_2$. In uniform fields we could substitute Eq. (3.2.2) into Eq. (3.1.1) and evaluate the fields at either $\bar{r}$ or $\bar{R}$. In the guiding center approximation we approximate the fields at $\bar{r}$ by expanding the fields at $\bar{R}$ using the expansion parameter $\delta = \frac{\rho}{L}$. Using this technique, the acceleration $\ddot{\bar{r}} = \ddot{\bar{R}} + \ddot{\rho}$ can be found to order δ . The resulting equation can be averaged over a Larmor radius, neglecting terms of order δ^2 or higher. The guiding center parallel and drift motion in the steady-state magnetic and potential fields of a tandem mirror are then given by

$$\frac{dv_{\parallel}}{dt} = -\frac{u_a}{m} \hat{e}_1 \cdot \nabla B_a + \frac{q}{m} E_{\parallel}, \quad (3.2.3)$$

$$\bar{v}_D = -\frac{\bar{B} \times \bar{E}}{B} + \frac{u_a}{qB_a} \hat{e}_1 \times \nabla B_a + \frac{mv_{\parallel}^2}{qB_a} \hat{e}_1 \times (\hat{e}_1 \cdot \nabla) \hat{e}_1. \quad (3.2.4)$$

The terms on the right side of Eq. (3.2.4) represent the drift motion caused by transverse electric fields, magnetic field gradients, and magnetic curvature, respectively. Implicit in these equations is the assumption that gravitational effects are negligible and the electric field parallel to B is of order δ .

3.3 Classical Drift Simulation

In a uniform plasma, the simulation of alpha guiding center motion depends on the plasma electrostatic potentials and the confining

magnetic fields. When the confining magnetic fields are nonaxisymmetric, some of the confined particles exhibit large, periodic, radial drift motions. Ryutov and Stupakov²⁸ were the first to show that particles in a nonaxisymmetric tandem mirror with azimuthal drifts per bounce equal to $\frac{n\pi}{2}$ when n is 1, 3, 5, . . . , exhibit large radial resonant excursions which enhance radial transport. It can be shown that the resonant width is finite in μ and particles near resonant conditions also have large radial excursions. Therefore, the parallel and drift motion of resonant or near resonant alphas trapped in a nonaxisymmetric tandem mirror must be followed between collision sampling events. Alphas that are far from resonant conditions have azimuthal drift motions that can be approximated by axis-encircling guiding-center trajectories whose bounce motion can be neglected. This is also true for alphas trapped in axisymmetric tandem fields; therefore, alphas with axis-encircling guiding-center trajectories are followed at the plasma midplane. Because mirror-confinement magnetic fields are still evolving, we have included both trajectory following schemes in this simulation.

Alpha particles in a nonsymmetric tandem mirror that are resonant or near resonant must be followed by integrating Eq. (3.2.3) and solving Eq. (3.2.4). This simulation is computed in the center cell, where the nonaxisymmetric field effects are negligible, by rotating the particle around the axis based on the drift velocity from Eq. (3.2.4) for the appropriate center cell length. When the particle enters regions where nonaxisymmetric fields affect the radial drift

motion, then the full solution of Eq. (3.2.3) and (3.2.4) is required. One method for solving these equations is to use conservation of energy to find $v_{\parallel}$ and Eq. (3.2.4) to find $\bar{v}_D$. This technique has been used in a leap-frog predictor-corrector scheme in which the guiding center velocity is known at time t_i and calculated at t_{i+2} , using the guiding center position and field information calculated at t_{i+1} .^{41,42} This algorithm has been used to track ions in a tokamak reactor, but it does not accurately track all trajectories through a mirror bounce because the direction of $v_{\parallel}$ is lost when conservation of energy is used. The direction of $v_{\parallel}$ must be found by integrating Eq. (3.2.3). Therefore, the guiding center algorithm that has been used is a leap-frog predictor-corrector scheme that solves Eq. (3.2.3) by Euler integration, calculates $\bar{v}_D$ from (3.2.4), and determines the guiding center position by Euler integration of the parallel and drift velocities. This new algorithm is also accurate and tracks particles through a mirror bounce with one predictor and corrector step. Plots of alpha trajectories determined by this algorithm are given in Refs. 19 and 20.

Particles that are nonresonant in nonaxisymmetric tandem mirrors, or all particles in an axisymmetric tandem, can be followed at the plasma midplane. By neglecting the bounce motion, the parallel motion is eliminated so that only $\bar{v}_D$ from Eq. (3.2.4) is calculated. The drift motion produced by electric, grad B, and curvature drifts is azimuthal, so the particle is simply rotated around the plasma axis between collisions. This azimuthal drift does not affect alpha

transport and can be neglected. We have chosen to neglect the curvature drift, since it is generally small compared to the other drift terms, but we include these other drifts to make the calculation more general. By neglecting the curvature drift, we can neglect the bounce motion, which allows us to use relatively large time steps between collision sampling events.

3.4 Radial Guiding Center Drift

The guiding center equations were derived under the assumptions that there is no interaction between the particle and the background plasma. This approximation is statistically valid for thermal particles in a uniform Maxwellian plasma where individual interactions do not affect the distribution function. But Coulomb collisions affect the alpha orbit and they also affect the thermalization process. Collisions can be simulated as a discrete process in which drag, angle scatter, and dispersion events occur (see Chapter 4). The angle scatter and dispersion events are modeled as hard sphere interactions, whereas the drag is treated as a continuous event. The drag force acting on an alpha can be defined by

$$\frac{d\vec{v}}{dt} = \nu \vec{v} \quad (3.4.1)$$

where $\nu = 1/\tau_D$. Although drag is continuous, it is not uniform because of the plasma temperature and density gradients. Large orbits, which are typical of energetic alphas, enable the particle to sample plasma regions which have different drag characteristics. Figure 3.1 shows

Figure 3.1 Assumed density and temperature profiles for a tandem mirror reactor with $r_p = 0.46$ m,
 $n(0) = 1.6 \times 10^{14} \text{ cm}^{-3}$, $T_i(0) = 40 \text{ keV}$, and
 $T_e(0) = 27 \text{ keV}$

assumed plasma density and temperature profiles for a tandem mirror reactor with a plasma radius of 0.46 m, $n = 1.6 \times 10^{14} \text{ cm}^{-3}$, $T_i = 40 \text{ keV}$, and $T_e = 27 \text{ keV}$. Figure 3.2 shows the corresponding alpha drag time as a function of radius. The drag in the halo region, i.e. the region where the plasma density and temperature is significantly less than the corresponding values at $r = 0$, is larger than the drag in the core because of the low halo temperature. The difference between the halo and core drag force that acts on an alpha orbit causes a radial guiding center drift, and this drift can be derived using the formalism developed by Northrop. The radial drift occurs in addition to the other guiding center drifts previously discussed.

The derivation of the radial drift starts with the equation of motion given by

$$\frac{d\vec{v}}{dt} = \frac{q}{m} (\vec{v} \times \vec{B}) - \frac{\vec{F}_D}{m} \quad (3.4.2)$$

and neglects electrical, gravitational, and scattering collision forces. The drag rate across an orbit is not constant, so we expand v in the small parameter δ and keep the first order terms

$$v = v_0 + \frac{\partial v}{\partial r} \rho + O(\delta)^2 \quad (3.4.3)$$

The equation of motion in terms of the radius vector $\vec{r}$ and v is

$$\ddot{\vec{r}} = \frac{q}{m} \dot{\vec{r}} \times \vec{B} - (v_0 + \dot{\rho} \cdot \nabla v) \dot{\vec{r}} + O(\delta)^2 \quad (3.4.4)$$

Figure 3.2 Drag profile for the density and temperature profile of Fig. 3.1

The guiding center motion can be found by substituting $\bar{\mathbf{R}} + \bar{\rho}$ for $\bar{\mathbf{r}}$ and is

$$\ddot{\bar{\mathbf{R}}} + \ddot{\bar{\rho}} = \frac{q}{m} [(\dot{\bar{\mathbf{R}}} + \dot{\bar{\rho}}) \times \bar{\mathbf{B}}] - v_0 (\dot{\bar{\mathbf{R}}} + \dot{\bar{\rho}}) - \rho \frac{\partial v}{\partial r} \dot{\bar{\mathbf{R}}} \sin \phi - \rho \frac{\partial v}{\partial r} \dot{\bar{\rho}} \sin \phi + O(\delta)^2 \quad (3.4.5)$$

where $\bar{\rho} = \rho_j (\hat{\mathbf{e}}_2 \sin \phi + \hat{\mathbf{e}}_3 \cos \phi)$. The terms involving $\sin \phi$ arise because the gradient of v is in the radial direction parallel to the assumed $\hat{\mathbf{e}}_2$ direction and therefore $\rho \cdot \nabla v$ results in terms containing only $\sin \phi$. The guiding center motion is derived by averaging Eq. (3.4.5) over a gyroperiod. Terms containing $\sin \phi$ and $\cos \phi$ average to zero. Other terms which are not functions of ϕ remain unchanged in the average. The guiding center equation now becomes

$$\ddot{\bar{\mathbf{R}}} = \frac{q}{m} \dot{\bar{\mathbf{R}}} \times \bar{\mathbf{B}} - \frac{\rho^2 \omega_c}{2\pi} \int_0^{2\pi} \frac{dv}{dr} \sin^2 \phi \, d\phi \, \hat{\mathbf{e}}_3. \quad (3.4.6)$$

In order to discuss the parallel and perpendicular motion caused by the drag gradients, let the integral in Eq. (3.4.6) be equal to $\langle \dot{\bar{v}}_D \rangle$. The parallel motion is found by taking the scalar product of $\ddot{\bar{\mathbf{R}}}$ with $\hat{\mathbf{e}}_1$, i.e.,

$$\ddot{\bar{\mathbf{R}}} \cdot \hat{\mathbf{e}}_1 = \frac{q}{m} (\dot{\bar{\mathbf{R}}} \times \bar{\mathbf{B}}) \cdot \hat{\mathbf{e}}_1 - \langle \dot{\bar{v}}_D \rangle \hat{\mathbf{e}}_3 \cdot \hat{\mathbf{e}}_1 = 0. \quad (3.4.7)$$

Therefore, a radial drag force does not alter the parallel motion of the alpha guiding center. The drift motion is found from the vector product of $\ddot{\mathbf{R}}$ with $\hat{\mathbf{e}}_1$, i.e.

$$\ddot{\mathbf{R}} \times \hat{\mathbf{e}}_1 = \frac{q}{m} (\dot{\mathbf{R}} \times \mathbf{B}) \times \hat{\mathbf{e}}_1 - \langle \dot{\mathbf{V}}_D \rangle \hat{\mathbf{e}}_3 \times \hat{\mathbf{e}}_1. \quad (3.4.8)$$

The drift velocity then becomes

$$\dot{\mathbf{R}}_D = \dot{\mathbf{R}} - (\dot{\mathbf{R}} \cdot \hat{\mathbf{e}}_1) \hat{\mathbf{e}}_1 = -\frac{1}{\omega_c} \ddot{\mathbf{R}} \times \hat{\mathbf{e}}_1 - \frac{\dot{\mathbf{V}}_D}{\omega_c} \hat{\mathbf{e}}_2 + O(\delta)^2. \quad (3.4.9)$$

The first term on the right hand side of Eq. (3.4.9) is the radial acceleration of the guiding center. This acceleration, to zero order, can be produced by a radial electric field and has been considered in the guiding center theory. There are no acceleration terms of order δ or lower introduced by drag, so that $(\frac{1}{\omega_c}) \ddot{\mathbf{R}} \times \hat{\mathbf{e}}_1$ is $O(\delta)^2$ or higher.

The guiding center drift velocity is then reduced to

$$\dot{\mathbf{R}}_D = -\frac{\langle \dot{\mathbf{V}}_D \rangle}{\omega_c} \hat{\mathbf{e}}_2. \quad (3.4.10)$$

The radial direction of the drift is dependent on $\frac{dv}{dr}$ which depends on the density and temperature profiles.

3.5 Radial Drift Simulation

The radial guiding center drift occurs when there are gradients in drag over a Larmor orbit, as seen from Eq. (3.4.6). Besides this radial drift, the Larmor radius decreases due to the energy loss associated with the drag process. A sketch of the alpha orbits in the uniform core plasma and in the halo region is shown in Fig. 3.3. In simulating the alpha particle motion, the guiding center is moved radially between collision sampling events and the orbit shrinks at the collision event. The radial movement depends on evaluating the integral in Eq. (3.4.6), which is not well behaved since $\frac{dv}{dr}$ can have a large oscillation and the integrand is infinite at the end points. The end point singularity occurs whenever $\frac{dv}{dr}$ is functionally dependent on r . The deceleration $\ddot{R}$ is continuous and non-zero over the orbit; therefore, even though there are no radial forces acting on the particle when ϕ is $0, \pi, 2\pi, \dots$, there is a finite limit to $\frac{dv}{dr} \sin^2 \phi$ as ϕ approaches these critical values. A plot of $\frac{dv}{dr}$ for the profiles in Figs. 3.1 and 3.2 is shown in Fig. 3.4. The particle position $\bar{r}$ is defined in terms of $\bar{R}$ and $\bar{\rho}$ in Fig. 3.5 and the magnitude of $\bar{r}$ is

$$r = (R^2 + \rho^2 + 2R\rho \sin \phi)^{1/2}. \quad (3.4.11)$$

The integral can now be transformed into

Figure 3.3 Alpha particle orbits in the uniform core plasma and in the drag gradient region between the core and halo

Figure 3.4 Drag gradient profile for the density and temperature profile of Fig. 3.1

Figure 3.5 Relation between the particle position vector ($\vec{r}$) and the guiding center ($\vec{R}$) and Larmor radius ($\vec{\rho}$) vectors

$$\langle \dot{\bar{V}}_D \rangle = \frac{1}{2\pi} \frac{\rho}{R} \int_{R-\rho}^{R+\rho} \frac{dv}{dr} \frac{[r^2 - (R^2 + \rho^2)]^2}{4 R^2 \rho^2} \left(\frac{r dr}{1 - \frac{[r^2 - (R^2 + \rho^2)]^2}{4 R^2 \rho^2}} \right)^{1/2} \cdot \quad (3.4.12)$$

Because the integrand is infinite at the end points, a Gauss-Chevyshin numerical solution of Eq. (3.4.12) can be obtained when the limits of the integral are changed to ± 1 . Let the variable t equal

$\frac{r^2 - (R^2 + \rho^2)}{2 R \rho}$ and let $\frac{dv}{dt} = \frac{dv}{dr} \frac{dr}{dt}$. The guiding center drift becomes

$$\begin{aligned} \dot{\bar{R}}_D &= \frac{\langle \dot{\bar{V}}_D \rangle}{\omega} \\ &= \frac{\rho}{\pi} \int_{-1}^1 \frac{dv}{dt} \frac{(R^2 + \rho^2 + 2 R \rho t)^{1/2}}{R} \frac{t^2 dt}{(1 - t^2)^{1/2}} \cdot \end{aligned} \quad (3.4.13)$$

An exact evaluation of $\dot{\bar{R}}_D$ would require exact knowledge of $\frac{dv}{dt}$; but since a numerical scheme is used, the evaluation of $\frac{dv}{dt}$ is required at specified values of t . Therefore, $\frac{dv}{dr}$ can be evaluated for values of r equal $(R^2 + \rho^2 + 2 R \rho t)^{1/2}$. Using these relations, the above integrand can be evaluated if the function $\frac{dv}{dr}$ is well behaved.

The end point singularity in the integrand is compounded by a large swing in $\frac{dv}{dr}$ near the plasma boundary. This oscillation is caused by the assumed density and temperature profiles, which are discussed in Chapter 5. If this swing occurs over a short distance compared with the alpha orbit diameter, then the integral given by Eq. (3.4.13)

requires an excessive number of quadrature points. An example of the temperature and density profiles that produce large swings in $\frac{dv}{dr}$ is shown in Fig. 5.6. Large peaked swings indicate a sharp plasma halo boundary which may be physically impossible to maintain. Therefore, by choosing reasonable radial density and temperature profiles, the alpha radial drag drift can be calculated using a relatively small number of points.

The solution to Eq. (3.4.13) depends on determining the number of quadrature points to be used in the Gauss-Chevyshev numerical integration. A number of experiments were performed using the radial temperature and density profiles shown in Fig. 3.2 to determine the number of points needed to accurately calculate the integral. It was found that in the plasma core region (r less than $0.7 r_p$) four quadrature points are needed to obtain a radial drift within one percent of the determined asymptotic value. At larger radii where $\frac{dv}{dr}$ is fluctuating, the number of quadrature points increased to 16 to obtain the same accuracy.

PART 4

MONTE CARLO TECHNIQUE

4.1 Introduction

The Monte-Carlo technique is applied to alpha particle initialization and the alpha thermalization process with the plasma. Alpha particles are born in the plasma over a narrow energy band, and we assume that all alphas are born with the mean energy of 3.52 MeV. The initial alpha phase space coordinates are determined by Monte-Carlo techniques by sampling from appropriate distribution functions. This sampling technique is discussed in Sections 4.2 and 4.3. As the alpha particle slows down, it diffuses in a random manner through phase space. This random walk process, which changes the particles velocity and position coordinates, is modeled by the Boltzman equation using the Fokker-Planck collision operator. The collision operator contains a continuous slowing down or drag term plus scattering terms. These terms are an accumulation of many interactions over a relatively long time period, but they can be simulated as one event occurring over an infinitesimally short time. The details of modeling the collisions in a Monte-Carlo scheme are presented in Section 4.4.

4.2 Alpha Initialization

Alpha particles born in the plasma have their phase space positions which depend on the background plasma density, temperature, and the details of the D-T collision. The details of the fusion reactions are not needed to simulate the alpha birth position in phase

space. The alpha birth coordinates are selected from appropriate probability distribution functions. For example, since we know the speed of the alpha, we only need to specify the pitch angle ψ and an azimuthal or phase angle ϕ to determine the velocity space coordinates. There is no preferential birth direction, so the probability distribution for both pitch and phase angles is uniform. The pitch angle is chosen from 0 to π , and the phase angle is chosen from 0 to 2π . These velocity components lie on a velocity sphere of radius equal to $|\vec{v}|$, and the probability (P) that a particle has a velocity component between 0 and ψ and between 0 and ϕ is

$$P = \frac{\int_0^\phi \int_0^\psi v^2 \sin \psi d\psi d\phi}{\int_0^{2\pi} \int_0^\pi v^2 \sin \psi d\psi d\phi} . \quad (4.2.1)$$

Since the pitch and phase angles are independent of each other, we define the pitch probability (P_ψ) and the phase probability (P_ϕ) as

$$P_\psi = \frac{1}{2}(1 - \cos \psi) \quad (4.2.2)$$

and

$$P_\phi = \frac{\phi}{2\pi} . \quad (4.2.3)$$

Since neither ψ nor ϕ are known, the value of P_ψ and P_ϕ can be chosen from a uniform random distribution between 0 and 1, and the values of ψ and ϕ can be found from

$$\psi = \cos^{-1}(1 - 2 P_{\psi}) \quad (4.2.4)$$

$$\phi = 2\pi P_{\phi} \quad (4.2.5)$$

Therefore, by choosing two random numbers, the values of ψ and ϕ can be determined.

Particle position in a reactor is determined by a similar process. A cylindrical coordinate system is used, in which z lies along the axis of the machine. The probability of the alpha being born at any location is given by

$$P = \frac{\int_V \dot{N}_{\alpha} dV}{\int_V dV} \quad (4.2.6)$$

where the integral is performed over the volume and $\dot{N}_{\alpha}$ is the D-T reaction rate. The probability that the alpha is born in dV can be defined as the product of the probabilities of the alpha being born at any one of the three coordinate positions, i.e.,

$$P = P_r P_z P_{\chi} \quad (4.2.7)$$

where P_r , P_z , P_{χ} are the probabilities that the particle will be born between r and $r + dr$, z and $z + dz$, and χ and $\chi + d\chi$. The birth probability at any azimuthal position (χ) is similar to Eq. (4.2.5) and is given by

$$\chi = 2\pi P_{\chi} . \quad (4.2.8)$$

The alpha birth position along z can be set to one since the particle bounce time is significantly less than the collision sampling time, and the particle birth position along z is simulated at the plasma midplane.

The radial birth position is more complex because it is dependent on the D-T reaction rate. The reaction rate, given by

$$\dot{N}_{\alpha} = n_D n_T \langle \sigma v \rangle , \quad (4.2.9)$$

is a function of radius through the density and microscopic cross section $\langle \sigma v \rangle$. The cross section is temperature dependent, with the temperature being a function of radius. The probability of finding a particle between 0 and r is then

$$P_r = \frac{\int_0^r n_D n_T \langle \sigma v \rangle r dr}{\int_0^R n_D n_T \langle \sigma v \rangle r dr} . \quad (4.2.10)$$

By randomly choosing P_r from a uniform distribution between 0 and 1, we can solve for r analytically after performing the integrations. Solving for r in this manner is not trivial, since the plasma density and temperature, and therefore the reaction rate, are complicated functions. We solve this problem by numerically integrating

$$\int_{r_1}^{r_{i+2}} n_D n_T \langle \sigma v \rangle r dr$$

using Simpson's rule, where $i = 0, 2, 4, 6, \dots, N$, and N is even and is equal to

$$\frac{r_N}{r_{i+1} - r_1}.$$

Using this technique, we develop a table in which values of the integral, given by

$$S_{j+1} = S_j + \int_{r_1}^{r_{i+2}} n_D n_T \langle \sigma v \rangle r dr,$$

where j is $i/2$, are kept. The total integral,

$$S_{jN} = \int_0^{r_p} n_D n_T \langle \sigma v \rangle r dr$$

where $jN = \frac{N}{2}$, is therefore the last value in this table. Values of $P_r S_{jN}$ can be compared with the table values, and r can be found by linear interpolation. This technique is simple and accurate and only requires the numerical evaluation of the integral along with an interpolation scheme, whereas other root finding schemes require function evaluations, iterations, and the derivative of the function.

A typical radial alpha birth profile found from this technique is shown in Fig. 4.1 for the base case reactor parameters given in Table 1.1. This Monte-Carlo birth distribution was made using 7000 alpha particles and is compared to a theoretical birth distribution determined from Eq. (4.2.10), where P_r is calculated from $P_r = S_j/S_{jN}$. The results of numerical experiments show that simulations using 1000 particles produce energy deposition profiles that are smooth, but the particle birth distribution, as shown in Fig. 4.2, has more scatter than simulations using more alphas. This experiment also produced accurate thermal alpha particle distributions. Therefore, we chose to use 1000 alpha particles in the remaining alpha transport studies. The scatter that is seen in Figs. 4.1 and 4.2 is a function of the radial bin size compared to the number of alpha particles started, and it is also dependent on the seed value given to the random number generator.

Both Figs. 4.1 and 4.2 show that the fraction of alpha particles born on axis is zero. This value must be zero, since the probability density function is volume weighted and the reacting volume approaches zero as the radius goes to zero. The birth profile also goes to zero as the radius increases because the temperature and density of the plasma ions decrease, which causes the reaction rate to approach zero.

In order to check the birth distribution profile in velocity space, we can compare the expected number of particles born in the loss cone to that produced by the Monte-Carlo technique. By reducing the potential barrier to zero and maintaining high mirror ratios, we can show that the expected loss rate for a mirror ratio of 8 is

Figure 4.1 Alpha particle radial birth profile determined by Monte-Carlo techniques for 7000 alpha particles (x) and compared with a theoretical birth distribution (solid line)

Figure 4.2 Alpha particle radial birth profile determined by Monte-Carlo techniques for 1000 alpha particles (x) and compared with a theoretical birth distribution (solid line)

approximately equal to $1/2R$ or 6.25%; this compares with the loss rate of 5.93% produced by the code using 1000 alpha particles.

4.3 Alpha Particle Guiding Center and Drift

Once the particle position is specified, its guiding center position and drift velocities must be determined. A sketch of the alpha motion along with the pertinent coordinate systems is shown in Fig. 4.3. The particle position ($\vec{r}$) in the laboratory coordinate system is known along with the pitch and phase angle. The pitch angle in the laboratory system and in the guiding center (GC) system are equal at the plasma midplane since the flux lines are parallel to the z-axis. The velocity in the GC system is given by

$$\vec{v} = v \sin \psi \cos \phi \hat{i} + v \sin \psi \sin \phi \hat{j} + v \cos \psi \hat{k} . \quad (4.3.1)$$

This velocity is equal to the velocity in the laboratory system at the plasma midplane because the GC and laboratory systems are parallel. The velocity parallel to the magnetic field is found by taking the scalar product of $\vec{v}$ and $\vec{B}/B$ as

$$\vec{v}_{\parallel} = \left(\vec{v} \cdot \frac{\vec{B}}{B} \right) \frac{\vec{B}}{B} = (\vec{v} \cdot \hat{e}_1) \hat{e}_1 . \quad (4.3.2)$$

The perpendicular velocity is then given by

$$\vec{v}_{\perp} = \vec{v} - \vec{v}_{\parallel} = \vec{v} - (\vec{v} \cdot \hat{e}_1) \hat{e}_1 , \quad (4.3.3)$$

Figure 4.3 Coordinate systems in the center cell of a tandem mirror reactor, Laboratory system (x, y, z) , Guiding Center system $(\hat{e}_1, \hat{e}_2, \hat{e}_3)$, and Particle system $(w_{\parallel}, w_{\perp x}, w_{\perp y})$

and $\bar{v}_\perp$ can be used to find magnetic moment μ , and the Larmor radius ρ . Equations (4.3.2) and (4.3.3) can be used to find the drift velocity without drag effects, and these equations can be integrated to find the guiding center position,

$$\bar{\mathbf{R}} = \bar{\mathbf{r}} - \bar{\rho} . \quad (4.3.4)$$

The initial calculation of $\bar{\mathbf{R}}$ is sensitive to the magnetic field changes over the large alpha orbit. This effect occurs because the initial value of ρ is calculated using β at $\bar{\mathbf{r}}$, whereas the value of β at $\bar{\mathbf{R}}$ is needed to determine ρ . The velocities are not affected by β because, as argued in Chapter 2, the plasma pressure has little effect on field line curvature and, therefore, vacuum values of $\bar{B}/B$ accurately define the direction of the unit vector parallel to $\bar{\mathbf{B}}$. Since β can change over a Larmor orbit, a Newton iteration scheme is used to initially locate $\bar{\mathbf{R}}$. This iteration is performed by defining the function f_n as

$$f_n = |\bar{\mathbf{R}}| - \left| \bar{\mathbf{r}} - \frac{\bar{\rho}}{\left[1 - \beta_0 f\left(\frac{\mathbf{r}}{r_{pD}}\right) f\left(\frac{\mathbf{r}}{r_{pT}}\right) \right]^{1/2}} \right| , \quad (4.3.5)$$

where $\bar{\rho}$ is defined in terms of $\bar{v}_\perp$ and $B_v(R)$. The iteration is performed on $|\bar{\mathbf{R}}| - |\bar{\mathbf{r}} - \bar{\rho}_a|$ because the magnitude of the orbit determines the effect of β on the guiding center position. The iteration scheme is defined as

$$R_{n+1} = R_n - \frac{fn}{\frac{dfn}{dr}} \quad (4.3.6)$$

This method converges within ten iterations when an error criterion of 1×10^{-4} is used.

4.4 Particle Collisions

4.4.1 Velocity Transformation

After the guiding center position, parallel velocity, and drift velocity are known, the particle information is extraneous and is not kept except for statistical computations. The guiding center is tracked up to a collision sampling time, and the particle position is reconstructed using Monte-Carlo techniques. In this situation, $\bar{R}$ is known and we need to find $\bar{r}$. The particle phase angle, ϕ , is chosen from a uniform distribution given by Eq. (4.2.5) to statistically compensate for sampling the collisions at the plasma midplane. The magnetic moment μ is used to find $|\bar{v}|$ from Eq. (1.1.2) and the magnitude of the Larmor radius is found from Eq. (1.3.6). The particle position in laboratory coordinates can be found from $\bar{r} = \bar{R} + \rho_0 \cos \phi \hat{e}_2 + \rho_0 \sin \phi \hat{e}_3$.

The velocity is transformed to the particle coordinate (w) system in which the velocity before the collision sampling, defined as $w_{||}$ (Fig. 4.4), is given by

$$w_{||} = \bar{v} \quad (4.4.1.1)$$

Figure 4.4 Relation between the Laboratory and Particle Coordinate system before and after (primed) a collision

After the collision, the alpha has a parallel, $w_{\parallel}$, component and a component perpendicular to $w_{\parallel}$ or $w_{\perp}$. The perpendicular direction is defined in such a way that the transformation from the w to the laboratory system is computationally quick and accurate. Since we are at the plasma midplane, the laboratory and the guiding center systems differ only by the displacement $\bar{R}$. The perpendicular velocity ($v_{\perp}$) to $\hat{e}_1$ has an angular direction of $90 + \phi$. The parallel velocity before the collision in the laboratory coordinate system is

$$\begin{aligned} v_{\parallel x} &= w_{\parallel} \sin \psi \cos \phi , \\ v_{\parallel y} &= w_{\parallel} \sin \psi \sin \phi , \\ v_{\parallel z} &= w_{\parallel} \cos \psi . \end{aligned} \tag{4.4.1.2}$$

After the collision is sampled, the parallel velocity is changed to $w'_{\parallel}$ and the perpendicular components are $w'_{\perp x}$ and $w'_{\perp y}$. The angle from $w_{\perp}$ to the laboratory z-axis is $90+\psi$ where ψ is the initial pitch angle. This gives the transformation of $w'_{\perp x}$ into the laboratory coordinate system as

$$\begin{aligned} v_{\perp x} &= w'_{\perp x} \sin (90+\psi) \cos \phi = w'_{\perp x} \cos \psi \cos \phi , \\ v_{\perp y} &= w'_{\perp x} \sin (90+\psi) \sin \phi = w'_{\perp x} \cos \psi \sin \phi , \\ v_{\perp z} &= w'_{\perp x} \cos (90+\psi) = w'_{\perp x} \sin \psi . \end{aligned} \tag{4.4.1.3}$$

The direction of w'_{ly} is defined by $w'_{ly} = w'_{\parallel} \times w'_{lx}$ and the transformation to laboratory coordinates given by

$$v_{lyx} = w'_{ly} \sin \phi ,$$

$$v_{lyy} = w'_{ly} \cos \phi , \quad (4.4.1.4)$$

$$v_{lyz} = 0 .$$

This transformation in matrix form is then

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} v_{lxx} + v_{lyx} + v_{\parallel x} \\ v_{lxy} + v_{lyy} + v_{\parallel y} \\ v_{lxz} + v_{lyz} + v_{\parallel z} \end{pmatrix} = \begin{pmatrix} \cos \phi \cos \phi & -\sin \phi \sin \phi & \cos \phi \\ \cos \phi \sin \phi & -\cos \phi \sin \phi & \sin \phi \\ -\sin \phi & 0 & \cos \phi \end{pmatrix} \begin{pmatrix} w'_{lx} \\ w'_{ly} \\ w'_{\parallel} \end{pmatrix} \quad (4.4.1.5)$$

This transformation allows collisions to be sampled in the particle coordinate system and the resulting velocities transformed into laboratory coordinates.

4.4.2 Collision Sampling Time

Collisions are sampled at intervals determined by the alpha particle drag, scatter and energy exchange times. These characteristic times vary with alpha speed and plasma density and temperature, for example, τ_D in the core is more than ten times greater than τ_D in the halo. Since energetic alpha orbits can extend over large plasma radial distances, the collision sampling time (Δt) depends on characteristic

times around the alpha orbit. The value of Δt is computed using the smallest value of τ_D , τ_e , and τ_I , calculated at the particle position when

$$|\bar{R}| + |\bar{\rho}| \leq r_p \quad . \quad (4.4.2.1)$$

When Eq. (4.4.2.1) is not satisfied, the sampling time is computed from the smallest value of τ_D , τ_e and τ_I calculated at $\bar{r}$, τ_D calculated in the halo, and τ_e and τ_I calculated at $\bar{r} = 0$. The smallest characteristic sampling time is then multiplied by .025, which has been determined by numerical experiments to be an appropriate scaling factor, to determine Δt . This sampling time is then modified to coincide with the time the particle is at the plasma midplane by obtaining the integer value of $.025 v_{||} \Delta t / L$ and then multiplying this integer by the bounce time ($L/v_{||}$).

4.4.3 Mass Considerations

The plasma is composed of electrons, fuel ions, and alpha particles. We assume that an average ion atomic mass of 2.5 a.m.u. can be used to represent the fuel ions, since the deuterium and tritium are similar in charge, number density, and temperature. This assumption implies that the speed difference between the D-T ions is negligible in determining alpha-ion interactions. This assumption becomes exact for the drag collision process when the D-T densities are equal and when their speeds are equal. By averaging the D-T mass, the reacting plasma becomes a two-component plasma made of electrons and ions; this reduces

the computational time needed to follow alphas from birth to thermalization.

The alpha mass is not included in the average ion specie that makes up the plasma. We assume that the alpha is a separate fluid that interacts only with the background ion and electron plasma. This assumption is justified because typical alpha particle concentrations are expected to be approximately .05 in present tandem mirror reactor studies.³⁴ Since the alpha concentration is kept low in these studies, we have assumed that a low alpha concentration will be maintained in the generic reactor design. Therefore, the average plasma ion mass neglects the contribution due to the thermalized alpha particles.

Consistent with the assumption that the alpha concentration is low, alpha-alpha interactions are neglected. The alpha particle initially drags down on plasma electrons until it's energy is less than $15 T_e$.^{7,43} After this, the number of alpha-ion (α -i) and alpha-alpha (α - α) interactions increases. The ratio of α - α to α -i 90° scattering times is approximated by

$$\frac{\tau_{\alpha\alpha}}{\tau_{\alpha i}} \approx \frac{n_i}{Z_\alpha^2 n_\alpha} \quad (4.4.3.1)$$

Low alpha concentrations imply that the α - α scattering times are much longer than the α -i scattering time. When $\tau_{\alpha\alpha}$ is long compared to $\tau_{\alpha i}$ and since alpha-alpha interactions have little effect on the equilibrium alpha distribution functions, we neglect the alpha-alpha collisions.

4.4.4 Drag Collision Simulation

Alpha particles slow down by being dragged down in energy by the surrounding plasma and at the same time they suffer large angle scattering and parallel dispersion events. Spitzer³² has derived the average drag rate $\langle \Delta W_{\parallel} \rangle$ acting on a particle which is

$$\langle \Delta W_{\parallel} \rangle = - \sum_{f=i,e} A_D \ell_f^2 \left(1 + \frac{m}{m_f} \right) G(\ell_f w) \quad (4.4.4.1)$$

where the summation is over the background plasma ions and electrons. The subscript f refers to the field or plasma particles and ℓ_f is the reciprocal of the ion or electron velocity defined by

$$\ell_f = \left(\frac{2T_f}{m_f} \right)^{-1/2} . \quad (4.4.4.2)$$

In uniform plasmas, A_D , given by

$$A_D = \frac{8\pi q^4 n_f^2 Z_f^2 \text{Ln} A_f}{\epsilon_0^2 m^2 \alpha} , \quad (4.4.4.3)$$

is a constant relating the charges, density, and coulomb logarithm of the plasma and alpha test particle. The argument of the G is a dimensionless ratio of alpha to field particle speed, and G is

$$G(x) = \frac{\phi(x) - x\phi'(x)}{2x^2}$$

where

(4.4.4.4)

$$\phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy$$

In the limit as x goes to zero, i.e. as the plasma particle speed is much greater than the alpha particle speed, the value of G can be approximated by

$$G(x)_{x \rightarrow 0} = \frac{2}{3} \frac{x}{\sqrt{\pi}} \quad (4.4.4.5)$$

By substituting Eq. (4.4.4.5) into Eq. (4.4.4.1) and simplifying, it can be shown that the drag is proportional to $n_f m_i / T_f (m_f / m_\alpha \epsilon_\alpha / T_f)^{1/2}$. Therefore, as x approaches zero the drag becomes proportional to $n_f T_f^{-3/2}$.

As x becomes large $\phi(x)$ goes to zero and G is given by

$$G(x)_{x > 2} = \frac{1}{2x^2} \quad (4.4.4.6)$$

Substituting this result into Eq. (4.4.4.1) and simplifying shows that the drag is proportional to $n_f m_\alpha / \epsilon_\alpha$. Arguments for G along with the drag rate $\langle \Delta w_{||} \rangle$ for alphas at 3.52 MeV and thermal energies of 60 keV in the hot core, $T_i = 40$ keV, $T_e = 30$ keV, $n = 1.6 \times 10^{14} \text{ cm}^{-3}$, and surrounding halo plasma, $T_i = T_e = 50$ eV, $n = 5 \times 10^{12} \text{ cm}^{-3}$, are given in Table 4.1.

TABLE 4.1
VELOCITY CHANGES DUE TO DRAG FOR A 3.52 MeV AND 60 keV ALPHA
IN THE REACTOR CORE AND HALO

Plasma Specie	Alpha Energy	Plasma Core		Halo	
		(ℓ_{fw})	$\langle \Delta w_{ } \rangle$ m/sec	(ℓ_{fw})	$\langle \Delta w_{ } \rangle$ m/sec
Ions	3.52 MeV	7.41	5.4×10^6	209.7	6.5×10^4
Electrons	3.52 MeV	0.13	1.9×10^7	3.1	1.8×10^8
Ions	60 keV	0.97	1.3×10^8	27.4	3.8×10^6
Electrons	60 keV	0.02	3.0×10^6	0.4	4.8×10^8

The drag rate decreases as x approaches zero and as x becomes large. Intermediate values of x produce a maximum drag rate for a given plasma specie. The values of $\langle \Delta w_{||} \rangle$ in Table 4.1 also show that the energetic alpha drags down primarily on the background electrons, rather than on the ions. The alpha initially drags on electrons in the hot core, and ion drag becomes dominant as it slows down. Whereas in the halo, the alpha drags down on the electrons. This can be seen from Table 4.1 and Fig. 1.2.

The drag is simulated as an independent process from the angle scattering and dispersion events. At the collision sampling time, the guiding center position is determined by the appropriate azimuthal and radial drifts which have acted on the particle since the last sampling time. When the collision is sampled, the alpha orbit is first shrunk due to the drag effects between collisions. In this process the guiding center location is fixed and the particle position is moved in

toward the guiding center along the Larmor radius vector. The change in velocity in the particle coordinate system due to drag is given by

$$w_{\parallel D} = w_{\parallel} + \langle \Delta w_{\parallel} \rangle \Delta t \quad . \quad (4.4.4.7)$$

The drag rate $\langle \Delta w_{\parallel} \rangle$ is calculated using plasma conditions at the alpha position. The alpha position is used because the average drag rate in a nonuniform plasma may not correspond to the guiding center position. Sampling the collision process a number of times during thermalization at the alpha position will produce the same results as if the drag rate were computed with the average drag rate computed across the alpha orbit.

The justification that the alpha orbit shrinks in size with the pitch angle remaining constant can be shown when we solve the equations of motion in cylindrical coordinates. This analysis assumes that there is no motion along z and the electric fields are zero. If the magnetic field is in the z -direction and if only magnetic and drag forces act on the particle, then Eq. (3.1.1) can be written as

$$\ddot{\rho} - \rho \dot{\phi}^2 = \omega_c \dot{\rho} - \nu \dot{\rho} \quad \text{r-direction} \quad . \quad (4.4.4.8)$$

$$\ddot{\phi} + 2\dot{\rho}\dot{\phi} = -\omega_c \dot{\phi} - \nu \dot{\phi} \quad \phi\text{-direction} \quad . \quad (4.4.4.9)$$

The angular velocity ($\dot{\phi}$) equals ω_c and, since there is no acceleration in the azimuthal direction, $\ddot{\phi}$ is zero. The rate of change of the

Larmor radius (ρ) can be found from Eq. (4.4.4.9) and is given by

$$\dot{\rho} = -\frac{v}{3} \rho \quad . \quad (4.4.4.10)$$

The solution of Eq. (4.4.4.10) is a decaying exponential given by

$$\rho = \rho_0 e^{-\frac{v}{3} t} \quad . \quad (4.4.4.11)$$

This result is valid for uniform and non-uniform plasmas since the coordinate system is attached to the guiding center and the guiding center drift is not predicted by this analysis. This same result can be found by arguing that the drag decreases $w_{\parallel}$ and transforming this change into the Laboratory system does not change the ratio $v_{\perp}/v_{\parallel}$. The pitch angle is not affected by drag because ω_c is independent of the velocity and therefore only ρ decreases.

4.4.5 Angle Scatter and Dispersion Simulation

The collision sampling also includes angle scattering and dispersion. These velocity changes are described in the w -system of coordinates with angle scattering changing the direction of the particle velocity and dispersion changing the parallel motion. Angle scatter and dispersion are modeled as independent events where the resulting alpha direction depends on the collisional cross section and the resulting energy can either increase or decrease. Spitzer³² derives the angle scatter as

$$\langle (\Delta w_{\perp})^2 \rangle = \frac{A_D}{w} \{ \Phi(l_f w) - G(l_f w) \} \quad (4.4.5.1)$$

and dispersion as

$$\langle (\Delta w_{\parallel})^2 \rangle = \frac{A_D}{w} G(l_f w) \quad (4.4.5.2)$$

These terms can be viewed as a local standard deviation of the velocity in a collision for an alpha particle where the mean velocity change is given by Eq. (4.4.4.1). This deviation from the mean velocity is the probability that a particle can be angle scattered or dispersed around the mean during one collision (Fig. 4.5). A method of applying the Monte-Carlo technique to Eq. (4.4.5.1) and (4.4.5.2) was discussed by T. Rognlien⁴⁴ and is given by

$$w'_{\parallel} = w_{\parallel} + \langle \Delta w_{\parallel} \rangle \Delta t + [3 \langle (\Delta w_{\parallel})^2 \rangle \Delta t]^{1/2} R_1, \quad (4.4.5.3)$$

$$w'_{\perp x, y} = [1.5 \langle (\Delta w_{\perp})^2 \rangle \Delta t]^{1/2} R_2, R_3. \quad (4.4.5.4)$$

In these equations Δt is the collision sampling time, R_i are random numbers picked uniformly from 0 to 1, but transformed by

$$R_i \rightarrow 2 \left(\frac{1}{2} - R_i \right) \quad (4.4.5.5)$$

Figure 4.5 Velocity changes at a collision sampling include magnitude and direction changes (A), which are determined by dispersion (B), and angle scatter, (C) about the mean velocity which is determined by drag.

to a uniform distribution from -1 to 1, and the constants are necessary to obtain proper statistics.

4.4.6 Collision Model

Alpha particles initially drag down in energy before large angle scattering events occur. Drag processes dominate to alpha energies of approximately 500 keV, which corresponds to 86% of a trapped alpha's energy being transferred to the plasma by drag. Therefore, it may be possible to retain most of the alpha energy in the plasma and then, through angle scattering losses, let the alphas escape from the trap. It is, therefore, important to model both the drag process and the angle scattering events accurately in phase space to determine the alpha energy exchange, energy deposition, velocity space losses, and radial diffusion losses.

As mentioned previously, the collision process is a two-step procedure. In the first step, the particle is dragged down in energy, which represents a decrease in the $w_{\parallel}$. Although the magnetic moment changes, the pitch and phase angle of the velocity components remain unchanged and the particle is moved from its present radial position to a new location along the same radius (Fig. 4.6). The new values of Larmor radius, particle position, magnetic moment, and energy can be determined by calculating $w_{\parallel D}$ from Eq. (4.4.4.7).

After the new particle position is found, the angle scatter and dispersion events are applied using

$$w'_{\parallel} = w_{\parallel D} + [3\langle(\Delta w_{\parallel})^2\rangle\Delta t]^{1/2} R_1 \quad (4.4.6.1)$$

and Eq. (4.4.5.4). In this event, the particle remains fixed since the collision is simulated as an instantaneous event and the guiding center is allowed to move. Transforming the velocities from the w-system to the Laboratory system defines $v_{\parallel}$ and $v_{\perp}$. These velocities along with the particle position are used to find a new energy, magnetic moment, guiding center position, energy, and drift velocity (Fig. 4.6). The drag results cannot be affected by the scattering and dispersion events. It is important therefore to follow a two-step collision process to ensure that these processes remain separate.

This calculation was checked by simulating a collision process in which the angle scatter and dispersion were set to zero. In this drag dominated process, the guiding center remained fixed and the Larmor radius, $v_{\parallel}$, and $v_{\perp}$ changed to correspond to the new alpha energy. The slowing down rate has been compared with theoretical predictions,⁴³ and the results of these comparisons are discussed in Chapter 6 (Fig. 6.4).

Angle scattering results have been checked by following an alpha close to thermal energies and determining if final plasma average energy is $3/2T_e$. That is when a 40 keV alpha was started in a plasma with uniform ion and electron temperatures of 40 keV and when there are no loss mechanisms, then the average alpha energy becomes 60 keV, which is expected. These checks verified the accuracy of the angle scatter and collision operators.

The implementation of this simulation into a working code is shown by the block diagram of Fig. 4.7. The guiding center position is

Figure 4.6 The two step collision process first reduces the Larmor radius because of drag (A) and then incorporates angle scatter and dispersion (B)

Figure 4.7 Block diagram of the collision subroutine used in the Monte-Carlo simulation

known, so the initial step determines the particle position and velocities in the Laboratory and w-system. In the next step, the local density and temperature is determined at the particle position which is needed as input into the Spitzer collision terms. The change in particle velocity due to drag is computed using Eq. (4.4.4.7), and a new particle position, magnetic moment, energy, etc., are calculated. The angle scatter and dispersion terms are then calculated using the particle position found in the drag step. This step leads to a new guiding center location, Larmor radius, magnetic moment, and energy. Finally, a new collision interval is calculated.

During the collision calculation certain values are stored for future analysis. The velocity distribution function requires the parallel and perpendicular velocities before and after the collision. The energy change of the alpha for both the drag, and scattering process is also needed. The energy given to ions and electrons is stored along with particle position information at each collision. This information is tabulated and stored for plotting and statistical analysis.

PART 5

PLASMA DENSITY AND TEMPERATURE PROFILES AND DRAG DRIFT

5.1 Introduction

Plasma radial density and temperature profiles are specified at the plasma midplane and are made consistent with profiles from present reactor studies. These profiles must be continuous in r since the radial drift velocity depends on dv/dr where v is the drag rate defined by $\langle \Delta w_{||} \rangle / w$ and $\langle \Delta w_{||} \rangle$ is density and temperature dependent. Even though the radial drift rate is profile dependent, relevant plasma design information can be obtained from the general form of dv/dr and by studying several drag profiles.

5.2 Density Profile

Radial density profiles can be modeled by either Gaussian, parabolic, quartic, or higher power curves. The quartic profile is used in many reactor designs because it is flatter in the plasma core region, which produces higher fusion reaction rates. These density profiles are modeled by

$$n(r) = n_0 \left[1 - \left(\frac{r}{r_p} \right)^\gamma \right] \quad (5.2.1)$$

where r_p is the plasma radius, n_0 is the on-axis density, and γ can take on any value. The power profiles fit experimental data near the axis, but they are zero when r equals r_p and become negative when

$r > r_p$. Values of $n(r)$ for $r > r_p$ are therefore defined by another equation. One way of modeling $n(r)$ is to define

$$n(r) = n_o^* \left[1 - \left(\frac{r}{r_p} \right)^Y \right] + n_h \left(\frac{r}{r_p} \right)^Y ; \quad r \leq r_p \quad (5.2.2)$$

$$n(r) = n_h ; \quad r > r_p$$

where n_o^* is equal to $n_o + n_h$. In this model there is a discontinuity in the slope of the density at r_p .

A Gaussian profile can also be used to model the density profile, and it does not have a discontinuous slope at the plasma edge. The Gaussian, however, does not accurately model the halo region where the density is approximately constant, because $n(r)$ continues to decrease for $r > r_p$. The halo can be modeled by adding a flat density profile to the generalized form of the Gaussian given by

$$n(r) = A \exp [-x^Y] + B \quad (5.2.3)$$

The value of x , which is proportional to r/r_p , and Y are chosen so that $n(o)$ is the on-axis density and $n(r_p)$ is the halo density, which is several orders of magnitude smaller than $n(o)$. This Gaussian shape is very peaked and narrow, which is not consistent with present reactor design density profiles. Therefore, it is difficult to satisfy all the necessary conditions for the density profile using the simple Gaussian of Eq. (5.2.3).

The density profile model that is used is similar to Eq. (5.2.3) for the core region, but it has a smooth transition at $r = r_p$ where the density profile is flat and becomes equal to the halo density. The general form of this model is given by

$$n(r) = (n_o - n_h) \left\{ 1 - \exp \left[- \left(\frac{1 - \frac{r}{r_p}}{1 - f_D} \right)^{\gamma_D} \right] \right\} + n_h \quad ; \quad r \leq r_p \quad (5.2.4)$$

$$n(r) = n_h \quad ; \quad r > r_p$$

The density falls off at a rate determined by a scale length r_{pD} and an arbitrary function $(1 - f_D)$ where f_D is less than one. The value of γ_D is greater than one and also determines the profile shape. When γ_D equals 1.15 and f_D is 0.75, the density profile is very similar to a quartic profile, as shown in Fig. 5.1. The advantage of using Eq. (5.2.5) is that the first and second derivatives of the density are zero when r equals r_p . Therefore, this profile eliminates the discontinuity at the plasma halo interface and maintains core and halo density profiles consistent with present models.

5.3 Temperature Profiles

The temperature profile is modeled using the same functional form as Eq. (5.2.5) and is given as

Figure 5.1 Quartic density profile and the modeled quartic profile used in the Monte-Carlo calculation for the generic reactor design

$$T_i = T_{oi} \left\{ 1 - \exp \left[- \left(\frac{1 - \frac{r}{r_{pT}}}{1 - f_T} \right)^{\gamma_T} \right] \right\} + T_h ; r \leq r_{pT} \quad (5.3.1)$$

$$T_i = T_h ; r > r_{pT}$$

where T_{oi} is the on axis ion or electron temperature. The exact temperature profile is unknown; therefore, the constants r_{pT} , f_T , and γ_T can be chosen arbitrarily. We initially chose r_{pT} to equal r_{pD} because during initial plasma operation the temperature (r_{pT}) and density (r_{pD}) scale lengths should equal r_p . This scale length relation implies that the plasma temperature and density become equal to the constant halo values at the edge of the plasma. We also know from present experimental data that the plasma temperature has a flatter core profile than the density profile. It is expected that this condition will persist in reactor plasmas; therefore, γ_T and f_T are chosen so that the temperature profile is flatter than the density profile in the plasma core. These profiles are also chosen so that the gradient of the drag force across the alpha orbit is reasonable for alpha adiabaticity considerations.³⁹ Although small temperature gradients across an alpha orbit are desirable, this is not possible since plasma temperatures vary by three orders of magnitude over an approximate 50-cm radius. The drag gradient is therefore limited to occur over 30% length of an alpha Larmor radius when the beta corrected ρ is calculated with $\epsilon_\alpha = 3.52$ MeV. The alpha is assumed to be adiabatic because the axial fields are designed for mirror confinement,

the value of r_p/ρ is on the order of unity, and the density and temperature gradients occur over a relatively large fraction of a Larmor orbit.

The temperature and density profiles must be chosen so that a plasma temperature greater than 10 keV coupled with high plasma density is maintained over a significant portion of the core plasma. The high density and temperatures are needed to ignite the fusion reaction. Because of these considerations, the value of γ_T was selected to be 3.0 and f_T to be 0.75 for the reactor designs used in this study. The corresponding temperature profile along with the density profile is shown in Fig. 5.2.

The temperature and density profiles may not keep the same scale length characteristics, i.e., $r_{pD} \neq r_{pT}$. For example, if a large fraction of alpha energy is deposited in the halo because of the halo electron drag, then this will shift the thermal distribution of alphas to be weighted toward the plasma-halo boundary and the alpha energy will be concentrated in this region. This heating will increase the ion and electron temperature in the halo and therefore cause a shift in the scale lengths so that r_{pD} may not equal r_{pT} . When $r_{pD} \neq r_{pT}$, this is referred to as the two-scale-length problem. An example of the two-scale-length density and temperature profiles is shown in Fig. 5.9.

5.4 Drag Drift

The radial drag gradient is given by

Figure 5.2 Modeled density and temperature profiles for the single-scale-length generic reactor design used for the Monte-Carlo simulation

$$\frac{dv}{dr} = A_v \left\{ [\phi(x) - x\phi'(x)] \frac{dn_f}{dr} - \frac{n_f}{T_f} x^3 \phi'(x) \frac{dT_f}{dr} \right\} \quad (5.4.1)$$

where A_v is a constant for a given alpha energy and is given by

$$A_v = \frac{8\pi q^4 Z_f^2 Z^2 L n \lambda_f}{2 \frac{m_\alpha}{m_f}^3} \left(1 + \frac{m_\alpha}{m_f} \right). \quad (5.4.2)$$

There is no drag drift when dv/dr is zero, and this occurs whenever the term enclosed in square brackets is zero. It is readily apparent that this condition is satisfied whenever $dn_f/dr = dT_f/dr = 0$, which occurs on the plasma axis due to symmetry conditions and in the halo where the density and temperature are constant. However, the general condition for zero drag drift is given by

$$\frac{dn_f/n_f}{dT_f/T_f} = \frac{x^3 \phi'(x)}{\phi(x) - x\phi'(x)} \quad (5.4.3)$$

where the left side is the ratio of the density-to-temperature scale lengths and the right side is determined by the error function, its derivative, and the ratio of alpha speed to plasma thermal speed given by x . It is highly unlikely that the plasma density and temperature profiles will relax to a condition in which Eq. (5.4.3) is satisfied everywhere. Therefore, we expect dv/dr to take on positive or negative values, and this will lead to alpha particle radial drag drifts.

It has been argued that there will be at least two regions along the plasma radius where the drag drift is zero. The number of radial locations with $dv/dr = 0$ can be determined by solving Eq. (5.4.3) for the zero points or by plotting the left and right side of Eq. (5.4.3) as a function of r . In either method, it can be shown that the right side of this equation is well behaved with limits of 1.5 as x goes to zero and zero as x goes to infinity. The left side of the equation is not well behaved, being undefined at $r = 0$ and $r \geq r_p$. The density and temperature profiles, although smooth, determine whether there are two or more radial locations with $dv/dr = 0$.

The fact that there can be more than two radial locations of zero drag drift is important because this implies that there are different alpha radial drift zones within the plasma. For example, the drag profile for Fig. 5.2 is shown in Fig. 5.3, and the drag gradient is shown in Fig. 5.4. These two profiles show that there are three drag drift regions inside this plasma. That is, when $r/r_p < 0.63$ and for $0.97 \leq r/r_p < 1.0$, the value of dv/dr is negative and the radial drift is inward. When $0.63 \leq r/r_p \leq 0.97$, the value of dv/dr is positive and the drift is outward. There is, therefore, a possibility of having radial zones that are rich in thermal alphas and other zones that are poor in thermal alphas. If the radial drift for given plasma profiles causes an excessive separation of thermal alpha particles from one radial zone to another, then the initial temperature and density profiles produce a nonequilibrium alpha distribution. The plasma density and temperature profile in this case will relax so that the

Figure 5.3 Drag profile for the density and temperature profiles of Fig. 5.2

Figure 5.4 Drag gradient for the drag profile of Fig. 5.3

plasma ions, electrons, and the thermal alphas are in an equilibrium distribution.

The relaxed plasma profiles cannot be obtained self-consistently, since the problem is too complex. Therefore, an iterative process is used in which density and temperature profiles consistent with present reactor designs are used to generate drag and drag gradient profiles. For example, the quartic density profile and step temperature profile shown in Fig. 5.5 are used in many reactor studies. Since the step temperature profile leads to a discontinuity in dv/dr , the step can be modeled using Eq. (5.3.1) with a continuous curve as shown in Fig. 5.6. This profile produces the drag and drag gradient profiles of Figs. 5.7 and 5.8, respectively. The sharp temperature boundary of Fig. 5.6 produces a large drag over a small radial region. In this case we felt that the sharp temperature boundary could not be maintained by the plasma and the temperature profile was relaxed to the profile shown in Fig. 5.2. These profiles with equal density and temperature scale lengths were then used to evaluate alpha transport in a tandem reactor (see Chapter 6).

The essential results of these alpha transport studies can be related to the drag drift, which is a function of the drag gradient profile of Fig. 5.4. In these studies, the drag drift in the various radial zones across the plasma caused a large fraction of alphas to thermalize at the plasma edge. This nonequilibrium distribution would eventually relax and cause the temperature and density scale lengths to change. This observation lead to the two-scale-length concept in which

Figure 5.5 Quartic density and step temperature profile used for some reactor designs

Figure 5.6 Modeled quartic density and step-temperature profile using Eq. 5.2.4 and 5.3.2, respectively

Figure 5.7 Drag profile for the density and temperature profiles of Fig. 5.6

Figure 5.8 Drag gradient for the drag profile of Fig. 5.7

the quartic density profile remained unchanged with the plasma density becoming equal to the halo density at r_p . The temperature scale length, however, was increased to correspond to a flatter temperature profile in the reactor core ($r < r_p$). By adjusting r_{pT}/r_{pD} to take on values greater than one, we can adjust the relation between the density and temperature profiles without changing the plasma axis parameters. An example of the two-scale-length density and temperature profiles is shown in Fig. 5.9. The associated drag and drag gradient profile is shown in Figs. 5.10 and 5.11. The drag gradient of Fig. 5.10 is significantly different from that of Fig. 5.4 in that the extreme values of dv/dr for the two-scale length problem are approximately five times smaller than that of the single-scale-length problem. This implies that the radial drag drift is much smaller for the two-scale-length problem. The next difference is that the large positive drift in the two-scale-length problem occurs for $r > r_p$. Therefore only those alphas that have orbits extending beyond r_p will have large positive radial drifts. Because of these observations, we can expect thermal alpha distributions that are closer to equilibrium from the two-scale-length profiles.

5.5 Conclusion

We have shown that continuous radial density and temperature profiles can be modeled using Eq. (5.2.5) and (5.3.1). These profiles are used to determine the radial drag and drag gradient profiles across the plasma. The drag gradient is used to determine the alpha transport and appropriate changes to the density, and temperature profiles are

Figure 5.9 Density and temperature profile for the two-scale-length problem

Figure 5.10 Drag profile for the density and temperature profiles of Fig. 5.9

Figure 5.11 Drag gradient for the drag profile of Fig. 5.10

made when alpha transport calculations lead to nonequilibrium distributions.

PART 6

ALPHA PARTICLE THERMALIZATION STUDIES

6.1 Introduction

The preceding chapters described the ingredients needed to study alpha particle transport in tandem mirror reactors. These ingredients were used to produce steady state information about the alpha thermalization process. Because there are many ways errors can be mixed into the calculation with the ingredients, there are several diagnostics which are used to verify the accuracy of the computed results. In the following section, the major diagnostics which are included as part of the calculation are discussed.

After discussing the diagnostics, a description of alpha particle thermalization results for the generic reactor design are presented. This includes a description of the alpha thermalization rates, along with energy deposition profiles and alpha power distribution profiles. The technique of converting the Monte-Carlo sums in radial bins to thermalization rates and power profiles is discussed. Pertinent statistical information is also presented.

The generic reactor results are too narrow and specific to generate generalized reactor design criteria. As shown in Chapter 5, the radial drift rate is dependent upon the plasma temperature and density profiles. Changes in the magnetic field (B_v), plasma radius (r_p), and β can also affect the alpha thermalization process. In order to systematically study these effects, a heuristic argument is given in

which the alpha energy deposition depends on $\frac{\rho}{r_p}$. This parameter can be varied for a given density and temperature profiles by varying r_p , peak beta ($\hat{\beta}$), or B_v . The results of varying $\frac{\rho}{r_p}$ for the generic density and temperature profiles are presented. A small perturbation in the density profile changes the value of $\frac{\rho}{r_p}$, and this effect, along with its implications, is discussed.

The single-scale-length temperature and density profiles limit the average plasma beta $\langle\beta\rangle$ to low values. Separating the scale lengths increases $\langle\beta\rangle$ and allows for more flexibility in studying temperature and density profile effects on alpha confinement. Results of varying these profiles and their associated scale lengths are also presented.

6.2 Calculation Verification

The calculations have been verified by including several diagnostics within the Monte Carlo simulation. Plots of energy decay of individual alpha particles as a function of time are made and compared with theory. Also included in this diagnostic is the fraction of alpha energy given to plasma electrons and ions. A second diagnostic is the velocity space distribution which shows velocity space information as the alpha particles slow to thermal energies. The velocity distribution is used to verify the slowing in distribution of alphas and to verify the loss cone calculation. The alpha energy distribution function is also computed, and the results are plotted and

compared with theory. Plots of alpha radial position as a function of the number of collision sampling events are also generated.

6.2.1 Alpha Energy Decay

Alpha particles slow to thermal energies by transferring energy to plasma ions and electrons. Alpha velocity changes and the corresponding energy changes ($\Delta\epsilon$) are calculated at every collision sampling. The total velocity change is made up of contributions from alpha-ion and alpha-electron interactions, which are computed individually. These individual velocity changes are then used to calculate the alpha energy given to plasma ions and electrons, which is used as a diagnostic tool.

The collision model is applied at discrete times, and the velocity changes are randomized by the Monte-Carlo technique. The alpha energy given to plasma ions ($\Delta\epsilon_i$) and electrons ($\Delta\epsilon_e$), divided by the total alpha energy, can be plotted as a function of time for individual alpha particles. We can compare these Monte-Carlo results with slowing down theory which has been derived by several authors.^{7,43,45,46} In these derivations, the average rate of alpha energy exchange with plasma ions and electrons is determined using appropriate drag cross-sections. We use the theoretical energy loss rate derived by Sivukhin⁴³ and given by

$$\left\langle \frac{d\epsilon}{dt} \right\rangle = \frac{4\pi Z^2 q^4}{\epsilon_w^2} \sum_{j=i,e} \ln \Lambda_j n_j Z_j^2 \left(\frac{\phi(x)}{M_j} - \frac{2x\phi'(x)(M+M_j)}{MM_j} \right) \quad (6.2.1.1)$$

where the subscript j refers to either the electron or ion species. In Eq. (6.2.1.1), the background plasma density and temperature are assumed constant, and the energy change ($\Delta\epsilon$) can be found by a simple Euler integration scheme when Δt is small. The result of this integration determines the total amount of alpha energy given to the plasma. Each Δt interval also determines the incremental amount of alpha energy that goes to the plasma ions and electrons, and this information is used to plot the theoretical values of $\frac{\Delta\epsilon_i}{\epsilon_\alpha}$ and $\frac{\Delta\epsilon_e}{\epsilon_\alpha}$ as a function of time.

The comparison between the Monte-Carlo and theoretical energy loss is shown in Fig. 6-1(a) through (d). A perfect agreement between the Monte-Carlo simulation and theory cannot be expected. The difference occurs because the theory only includes drag in the collision model and assumes that angle scatter and dispersion are negligible until the alpha energy is less than 15 Te. This, however, does not mean that angle scatter and dispersion can be neglected at higher energies, and when they are included, the individual alpha energy loss will vary from theory. Some alphas decay more slowly than theory [Fig. 6-1(a)] suffering up-scattering in energy as the alpha decays to thermal energies, while others decay faster [Fig. 6-1(b)], and still others compare favorably with theory [Fig. 6-1(c)].

Both the theory and the Monte-Carlo points in Fig. 6.1(a through c) were generated in a uniform plasma with constant density and temperature. In general, the plasma is not uniform, and alphas which sample the halo see a large variation in background plasma temperatures

(a)

(b)

(c)

(d)

Figure 6.1 Alpha particle energy decay in the hot reacting core plasma(a-c) and in the halo(d). Solid curve is a theoretical prediction given by Sivukhin⁴³

and densities. The energy decay for an alpha that samples the halo is approximately ten times faster than alpha energy decay in the core plasma, as shown in Fig. 6-1(d). Alphas that sample the halo drag down primarily on halo electrons and, as a consequence, more alpha energy is transferred to electrons than uniform plasma theory predicts.

Therefore, the theoretical predictions for $\frac{\Delta\epsilon_e}{\epsilon_\alpha}$ and $\frac{\Delta\epsilon_i}{\epsilon_\alpha}$ represent limiting values for alpha energy transfer in actual plasmas.

Although individual alphas will not always agree with theory, it is expected that the average energy given to the ions and electrons from all the alphas will match theoretical predictions. This numerical experiment has been performed, and the average energy to the plasma ions and electrons calculated from the code differs from Sivukhin's theory by less than 10% at plasma temperatures of 60 keV and less than 4% at plasma temperatures of 5 keV for 1000 alpha particles. A

comparison between the calculation's average fraction to ions ($\frac{\Delta\epsilon_i}{\epsilon_\alpha}$) and to electrons ($\frac{\Delta\epsilon_e}{\epsilon_\alpha}$) as a function of electron temperature is shown in

Fig. 6.2, where the solid line is derived from Sivukhin. We can compare the solid curve in Fig. 6.2 with the Rose approximation of Fig. 1.2. The difference between these curves occurs because Rose uses an approximation to the error function, whereas Sivukhin does not. The Monte-Carlo algorithm uses Spitzer's collision model, which can be used as a starting point to derive Eq. (6.2.1.1); therefore, we expect to have good agreement between the Monte-Carlo results and the theoretical

Figure 6.2 Fraction of alpha energy given to ions(o) and electrons (□) as a function of plasma electron temperature and compared with Sivukhin⁴³(—)

results predicted by Sivukhin. The small difference between the Monte-Carlo simulation and theory occurs because we include angle scatter and dispersion in the simulation, we use a low-order integration scheme to integrate Eq. (6.2.1.1), and the Monte-Carlo simulation produces an approximation to the exact fractional values.

6.2.2 Velocity Distribution Function

The velocity distribution function $f(\vec{r}, \vec{v})$, defines the steady-state phase space distribution of particles. The particle densities can be obtained by integrating the velocity distribution function as

$$n(\vec{r}) = \int_{\vec{v}} f(\vec{r}, \vec{v}) d\vec{v} , \quad (6.2.2.1)$$

where the velocities are given in cylindrical coordinates by $v_{\perp}$ and $v_{\parallel}$. In order to find the distribution function, we divide velocity space into small volumes, with dimensions $\Delta v_{\parallel}$, $\Delta v_{\perp}$, and $\Delta\theta$. The alpha distribution is isotropic in θ , and when $\Delta v_{\parallel}$ and $\Delta v_{\perp}$ are small, the distribution function is constant in the area $\Delta v_{\parallel} \times \Delta v_{\perp}$. Then Eq. (6.2.2.1) can be approximated over $\Delta v_{\parallel}$ and $\Delta v_{\perp}$ by

$$n(i,j) \approx 2\pi f(i,j) v_{\perp j} \Delta v_{\perp} \Delta v_{\parallel} \quad 6.2.2.2$$

The i and j refer to a bin location in velocity space where i corresponds to the x or $v_{\parallel}$ direction and j is in the $v_{\perp}$ direction. The total particle density (N) is found by summing the $n(i,j)$ densities as

$$N = \sum_i \sum_j n(i,j) \quad (6.2.2.3)$$

Next, the normalized distribution is found from

$$\hat{f}(i,j) = \frac{1}{\pi^2 v_{i,j} \Delta v_{\perp} \Delta v_{\parallel}} \frac{n(i,j)}{N} \quad (6.2.2.4)$$

where the ratio $\frac{n(i,j)}{N}$ is assumed to be proportional to the amount of time the alpha spends in each bin, i.e. $\frac{\Delta t(i,j)}{\Delta T}$ and ΔT is the total time required to follow all the alphas from birth to thermalization. The calculation of $\Delta t(i,j)$ for one particle can be found by knowing the time between sampling a collision event and the velocity space coordinates before and after the sampling. This information, shown schematically in Fig. 6.3, is used to mark the end points of the alpha's velocity space trajectory. This trajectory is assumed to be linear because the sampling time is small compared to the alpha drag and 90° scattering time. This assumption allows us to use linear interpolation to determine the amount of time the particle spends in each velocity bin. The total value of $\Delta t(i,j)$ is found by summing the contributions from all alpha trajectories that pass through bin (i,j) as they slow down. The value of $\Delta t(i,j)$ is associated with the coordinates at the center of the bin.

A velocity distribution function is actually associated with every point in plasma velocity space. In determining $f(\vec{r}, v_{\parallel}, v_{\perp})$ particle information is generated at many radial locations, but there

Figure 6.3 Model of alpha velocity space trajectory through one collision sampling

's not enough information at any one point to generate a distribution function for that point. In uniform plasmas, the radial position does not affect the velocity distribution, and an accurate distribution can be generated using information from collision sampling events occurring at different radial locations. Unfortunately, reactor plasmas are nonuniform and, in order to compensate for this problem, the plasma is divided into several radial zones. The plasma in each radial zone is assumed to be uniform, and a distribution function is generated for each zone. This assumption is especially valid for the innermost radial zones where temperature and density profiles are constant.

Before presenting the full velocity distribution function, it is informative to observe a single particle as it slows in velocity space. The simulated alpha velocity space trajectories are not straight, as shown in Figs. 6.4 and 6.5. The particle in Fig. 6.4 was born with a 90° pitch angle, while the particle in Fig. 6.5 was born on the loss cone with a 13° pitch angle. In both cases, the loss cone is neglected and the random motion is due to angle scatter and dispersion. We can compare the calculated ratios of $\frac{\tau_{\perp}}{\tau_D} = 5.9$, and $\frac{\tau_{\parallel}}{\tau} = 129.0$ where $\tau_{\parallel}$ is the dispersion time, with analytic approximations of 6 and 140, respectively. These ratios show that dispersion and scattering are negligible in the Monte-Carlo simulation for the high-energy alpha. But when we compare typical velocity changes, given by

Figure 6.4 Velocity space trajectory from birth to thermalization for an alpha with an initial pitch angle of 90° . Loss cone angle is set to 0°

Figure 6.5 Velocity space trajectory from birth to thermalization for an alpha with an initial pitch angle of 13° . Loss cone angle is set to 0°

$$\Delta w_{\parallel D} = \langle \Delta w_{\parallel} \rangle \Delta t = 1.7 \times 10^4 \text{ m/sec}$$

$$\Delta w_{\parallel} = (3\langle (\Delta w_{\parallel})^2 \rangle \Delta t)^{1/2} R_1 = 7.1 \times 10^4 \text{ m/sec } R_1 \quad (6.2.2.5)$$

$$\Delta w_{\perp} = (1.5\langle (\Delta w_{\perp})^2 \rangle \Delta t)^{1/2} R_2, R_3 = 2.3 \times 10^5 \text{ m/sec } R_2, R_3$$

for a collision sampling time (Δt) of 1×10^{-3} sec and a 3.52 MeV alpha, it is seen that the dispersion and angle scatter changes depend on the random numbers R_1 , R_2 , and R_3 . When a random number is large, i.e. approximately ± 1 , then the scatter or dispersion can be equal to the drag; this causes the scatter in Fig. 6.4 and 6.5. However, since R_1 , R_2 , and R_3 vary from -1 to $+1$, their average value is 0; therefore, the drag, in an average sense, dominates the slowing down process. Because of this average quality, we expect the particle trajectories, shown in Figs. 6.4 and 6.5, to show general movement toward lower velocities, but they will not follow a radial line joining their velocity space birth position to the origin.

Individual particle trajectories, such as those shown in Figs. 6.4 and 6.5, are summed to give a velocity distribution, shown in Figs. 6.6, 6.7, and 6.8. These distributions were made for the generic reactor parameters, and the distributions correspond to three radial zones across the plasma. The loss cone calculations have been included in these distributions, and the loss cone boundary is shown as the dashed line in the figures. Note that the loss cone varies from one radial zone to the next because the plugging electrostatic potential is proportional to T_1 , which varies with radius. As the potential

Figure 6.7 Alpha particle normalized distribution function for the generic reactor design. The distribution was produced by 1000 alphas followed from 3.52 MeV to 60 keV for $1/3 \leq r/r_p < 2/3$

Figure 6.8 Alpha particle normalized distribution function for the generic reactor design. The distribution was produced by 1000 alphas followed from 3.52 MeV to 60 keV for $2/3 \leq r/r_p < r_w/r_p$

decreases near the plasma edge, beta also goes to zero and the loss cone increases. The overlap of the distribution into the loss cone is caused by three factors. First, the loss cone boundary is determined for an average radial position inside each radial zone. Because the plasma is nonuniform, the velocity distribution in each radial zone is an approximation to the distribution function at the average radial position and, therefore, there will be some contribution to the distribution function that lies outside the calculated loss boundary. Second, the velocity bins described in Fig. 6.3 do not coincide with the loss cone boundary. This, coupled with a general conformal plotting routine, places some contours in the loss cone region. Finally, the method of calculation first samples a collision event, then determines the contribution of this event to the distribution function, and then checks to see if the particle is in the loss cone. This method will contribute information to the distribution function for that part of the particle trajectory in the loss cone. All of these problems can be corrected at the expense of a significant amount of additional computer time, but have not been incorporated in this calculation.

Although the distribution functions shown in Figs. 6.6 through 6.8 are only approximations, they are a useful diagnostic. First, they show that there is a plateau in the distribution at velocities corresponding to 3.52 MeV. This plateau rapidly drops to zero for higher velocities. There is also a peak in the distribution at thermal energies where the alpha ash accumulates. The contour lines suggest

that this peak gradually decreases to the high velocity plateau. Finally, the circular contour lines above the loss cone boundary suggest that drag is the dominant energy loss process.

6.2.3 Energy Distribution Function

The energy distribution, defined as $f(\vec{r}, \epsilon)$, defines the number of particles per unit volume, per unit energy, at some position in phase space. This distribution is calculated in the Monte-Carlo code by defining the distribution function in spherical coordinates as

$$n(\vec{r}) = \int_{\vec{v}} f(\vec{r}, v, \psi, \phi) v^2 dv \sin \psi d\psi d\phi \quad (6.2.3.1)$$

The integration over ψ and ϕ is performed and the speed is transformed into energy coordinates to give

$$dn(\vec{r}) = f(\vec{r}, \epsilon) \pi \left(\frac{2}{m}\right)^{3/2} \epsilon^{1/2} d\epsilon \quad (6.2.3.2)$$

The energy distribution is then defined as

$$f(\vec{r}, \epsilon) = \frac{\Delta n(r_i)}{\pi \left(\frac{2}{m}\right)^{3/2} \epsilon_i^{1/2} \Delta \epsilon^*} \quad (6.2.3.3)$$

where Eq. (6.2.3.2) has been discretized into small changes in density corresponding to respective energy changes. The alpha energy is

divided into radial zones Δr^* wide with a mean energy of ϵ_i in each zone. The density of particles $\Delta n(r_i)$ in each energy zone is proportional to the time the alpha spends in each zone. A linear interpolation technique, similar to the one used for the velocity distribution, is used to determine the time the alpha spends in each zone as it slows down during one collisional sampling time. The total accumulated time in each energy bin is proportional to $\Delta n(r_i)$.

The normalized distribution function is then found from

$$\hat{f}(\bar{r}, \epsilon) = \frac{f(\bar{r}, \epsilon)}{N(\bar{r})} \quad (6.2.3.4)$$

where $N(\bar{r})$ is proportional to $\sum_i \Delta t_i$ and the sum is the total thermalization time for all the particles making up the distribution. In terms of Eq. (6.8 and substituting Δt_i for $\Delta n(\bar{r}_i)$, Eq. (6.2.3.4) becomes

$$\hat{f}(\bar{r}, \epsilon) = \frac{1}{\pi \left(\frac{2}{m} \epsilon_\alpha\right)^{3/2} \left(\frac{\epsilon_i}{\epsilon_\alpha}\right)^{1/2} \frac{\Delta \epsilon^*}{\epsilon_\alpha}} \frac{\Delta t_i}{\Delta T} \quad (6.2.3.5)$$

where $\Delta T = \sum_i \Delta t_i$. A plot of $\pi \left(\frac{2}{m} \epsilon_\alpha\right)^{3/2} \hat{f}(\bar{r}, \epsilon)$ versus $\frac{\epsilon_i}{\epsilon_\alpha}$ for three radial zones of the plasma is shown in Figs. 6.9 through 6.11, where the Monte-Carlo calculated points are shown as crosses and the solid curve is a theoretical result.

Figure 6.9 Energy distribution function for the generic reaction design for 1000 alphas followed from 3.52 MeV to 60 keV for $0 \leq r/r_p < 1/3$ (x). Solid line is a theoretical approximation.

Figure 6.10 Energy distribution function for the generic reactor design for 1000 alphas followed from 3.52 MeV to 60 keV for $1/3 \leq r/r_p < 2/3$ (x). Solid line is a theoretical approximation¹⁶

Figure 6.11 Energy distribution function for the generic reactor design for 1000 alphas followed from 3.52 MeV to 60 keV for $2/3 \leq r/r_p \leq r_w/r_p$ (x). Solid line is a theoretical approximation¹⁶

A theoretical velocity distribution function for high energy neutral particles injected into a plasma has been derived by Cordey and Houghton.⁴⁷ This derivation was done assuming that injected particles are dragged down to thermal energies in a uniform plasma by ions and electrons. Large angle scattering events are neglected and the derivation is valid in the region where $v > v_o$, where v_o is the velocity at which the energy transfer from hot ions to cold electrons and to cold ions is equal. The source function used in this derivation is typical of neutral injection systems. Driemeyer¹⁶ has transformed this distribution into an energy distribution function and incorporated an alpha particle source term. This normalized distribution function is given by

$$\hat{f}(\bar{r}, \epsilon) = \frac{C_o}{N(\bar{r})\epsilon_\alpha} \frac{\left(\frac{\epsilon}{\epsilon_\alpha}\right)^{1/2}}{\left(\frac{\epsilon}{\epsilon_\alpha}\right)^{3/2} + \left(\frac{\epsilon_o}{\epsilon_\alpha}\right)^{3/2}} \quad (6.2.3.6)$$

where $\frac{\epsilon_o}{\epsilon_\alpha}$ is $\left(\frac{3\pi^{1/2}}{4} \frac{m_e}{m_i}\right)^{2/3} \frac{m_\alpha}{m_e} \frac{T_e}{\epsilon_\alpha}$ and $\frac{C_o}{N(\bar{r})\epsilon_\alpha}$ is the normalization constant. The theoretical and Monte-Carlo energy distributions given by Eq. (6.2.3.5) and (6.2.3.6) are equal at energies near 3.52 MeV, and therefore the normalization constant is found from equating Eq. (6.2.3.5) to (6.2.3.6) and is given by

$$\frac{C_o}{N(\bar{r})\epsilon_\alpha} = \frac{1}{\pi \left(\frac{2}{m_\alpha}\right)^{3/2}} \frac{\left(\epsilon/\epsilon_\alpha\right)^{3/2} + \left(\epsilon_o/\epsilon_\alpha\right)^{3/2}}{\left(\epsilon/\epsilon_\alpha\right) \left(\Delta\epsilon^*/\epsilon_\alpha\right)} \frac{\Delta t_i^*}{\Delta T} \quad (6.2.3.7)$$

where Δt_i^* is proportional to the alpha density in the energy bin, where the constant is evaluated. To reduce statistical variations, we averaged the normalization constant over ten Monte-Carlo energy bins near 3.52 MeV.

A comparison between the energy distribution generated by the Monte-Carlo simulation and the theoretical distribution is shown in Fig. 6.9 through 6.11. From these plots, we see that the Monte-Carlo generated distribution has the same slope at high alpha energies, but varies from theory as the alpha thermalizes. The difference between the Monte-Carlo calculation and theory occurs for several reasons. First, the theory is valid for the case where $v_i \ll v_\alpha \ll v_e$. For alpha particles in a plasma with ion temperatures of 40 keV and electron temperatures of 28 keV, the ratio $\frac{v_i}{v_\alpha} = 0.14$ and $\frac{v_e}{v_\alpha} = 7.6$. These ratios do not explicitly satisfy the velocity condition given above. Another difference occurs because the Monte-Carlo calculation includes velocity dispersion and angle scatter. Although these processes become dominant for alpha energies less than 500 keV, they do affect alpha thermalization at higher energies. This is evident from the fact that velocity space collisionally lost alphas leave with an average energy of approximately 1.5 MeV (see Table 6.1). Finally, a simple drag model for the plasma shows that the energy distribution function is proportional to ϵ^{-1} , whereas the theoretical distribution function is proportional to $\epsilon^{-1} \left[1 + \left(\frac{\epsilon_c}{\epsilon_\alpha} \right)^{3/2} \right]^{-1}$. The effect of ϵ_c on the distribution function is shown in Fig. 6.12, where the alphas are

TABLE 6.1.
STATISTICAL ANALYSIS OF THE RESULTS FOR THE GENERIC REACTOR DESIGN

Description	Nine 1000-Particle Runs		7000-Particle Run
	Mean	Standard Deviation	
N_c/N_T $0 \leq \frac{r}{r_p} \leq 0.864$	0.51	0.01	0.51
N_h/N_T $\frac{r}{r_p} > 0.864$	0.35	0.01	0.35
N_L/N_T Lost immediately	0.03	0.01	0.03
N_{CL}/N_T Collisionally lost	0.10	0.01	0.11
ϵ_c/ϵ_T $0 \leq \frac{r}{r_p} \leq 0.864$	0.58	0.01	0.58
ϵ_h/ϵ_T $0 \leq \frac{r}{r_p} > 0.864$	0.32	0.01	0.33
ϵ_L/ϵ_T Lost immediately	0.03	0.01	0.03
ϵ_{CL}/ϵ_T Collisionally lost	0.04		0.05
ϵ_{th}/ϵ_T Thermal energy	0.02		0.02
$\langle \beta \rangle$	0.8		0.8
$\langle \beta \rangle$	0.4		0.4
B_v	3.0		3.0
r_p	0.5		0.5
r_c	0.43		0.43
ρ/r_c	0.27		0.27
$\langle u \rangle$ Collisionally lost α 's (MeV)	1.4		1.6
$\Delta \epsilon_i/\epsilon_\alpha$	0.21		0.21
$\Delta \epsilon_e/\epsilon_\alpha$	0.79		0.79

Figure 6.12 Energy distribution function predicted by Monte-Carlo techniques (x) and by Driemeyer¹⁶ (—) for an alpha in a uniform plasma at 10 keV

slowing down in a uniform plasma at 10 keV. This is similar to Fig. 6.11 where the alphas are near the edge of the plasma and the plasma temperature used in determining ϵ_c is 7 keV.

6.2.4 Guiding Center Drift Plots

This final diagnostic plots the absolute value of the guiding center coordinates at the plasma midplane after a predetermined number of collision sampling times have occurred. These plots have been incorporated as a visual picture of the guiding center movement under the influence of the gradient drag drift. Figures 6.13 through 6.15 show the guiding center radial movement in the generic reactor design after 1, 25, and 50 collision sampling times. Thermal particle positions are not shown on these plots, nor are the particles that are lost out the loss cone between plots. The important fact that these plots show is that the radial guiding center drift can be large and that alpha particles drift out to the plasma edge in less than 25 sampling times, which is on the order of milliseconds for this model. Therefore, for the generic reactor model a significant amount of alpha energy and ash accumulates at the edge of the plasma. Other temperature and density models can change this drift and will be considered in Section 6.4.

6.3 Equal Density and Temperature Scale Length Results

The reactor model that was first studied required the plasma density and temperature profiles to reach their halo values at the r_p , the edge of the plasma. The plasma radius therefore defined the

Figure 6.13 Alpha guiding center position after one collision sampling time for the generic reactor design

Figure 6.14 Alpha guiding center position after 25 collisions
sampling times for the generic reactor design

Figure 6.15 Alpha guiding center position, after 50 collisions sampling times for the generic reactor design

density and temperature scale length. This single-scale-length problem attempted to model reactor studies that place a sharp boundary between the plasma core and halo, with the core plasma being defined for $\frac{r}{r_p} < 1$. In this study, the sharp boundary is replaced by a smooth transition which requires the temperature and density to decrease in the core plasma. When the temperature and density decrease in the core, the alpha reaction rate ($\dot{N}_\alpha$) becomes negligible inside the plasma near $\frac{r}{r_p} \approx 1$.

6.3.1 Generic Reactor Design

Parameters for a generic reactor have been given in Table 1.1 and the density and temperature profiles are shown in Fig. 5.2. The alpha particle birth distribution was shown in Fig. 4.2 and 4.3. The radial guiding center drift causes particles to accumulate near the plasma edge, as shown in Fig. 6.13 through 6.15. The detailed Monte-Carlo results for this generic design are described in the following sections.

6.3.1.1 Alpha Particle Thermalization

The Monte-Carlo calculation follows particles from birth to thermalization and stores the thermal alpha location in a radial bin. As the number of alpha particles increases, the number in each bin increases and the radial distribution attains some distinct profile. By normalizing this distribution with the total number of particles that are started, a steady state thermalized distribution is found (Fig. 6.16). The ratio is zero at the origin since the bin size goes

Figure 6.16 Thermal alpha particle radial profile normalized to the total number of alphas followed, for the generic reactor design

to zero, and the ratio is zero at large values of $\frac{r}{r_p}$ since the $\dot{N}_\alpha$ goes to zero near r_p , and the alphas are limited in their radial diffusion.

The alpha density in each bin is given by

$$n_j = \frac{N_j}{\Delta V_j} \quad (6.3.1.1.1)$$

where $\Delta V_j = \pi(r_{j+1}^2 - r_j^2)L$ and L is the plasma length. The density ratio $\frac{n_j}{n_T}$ can be computed and is plotted in Fig. 6.17. The scatter near the origin is due to the fact that small deviations in the calculated distribution will cause large variations in the density since the bin volume is small. Note that the profile has a negative slope for $\frac{r}{r_p} \leq 0.7$, indicating an outward radial diffusion of the thermal particles, which should follow classical diffusion rates. There is also a large density spike at the edge of the plasma caused by the radial guiding center drift.

A particle balance on a radial bin can be used to find the thermalization ratio in each bin. The balance equation can be stated as

$$\left| \begin{array}{l} \text{Rate at which} \\ \text{alphas thermalize} \\ \text{in bin } j \end{array} \right| = \left| \begin{array}{l} \text{Rate at which} \\ \text{particles are} \\ \text{accumulated in } j \end{array} \right| - \left| \begin{array}{l} \text{Rate at which} \\ \text{particles are} \\ \text{lost from } j \end{array} \right|$$

Alphas are initially born in bin j at the rate of $n_{Dj}n_{Tj}\langle\sigma v\rangle_j$, and their trajectories enable them to sample many bins before they

Figure 6.17 Thermal alpha radial density profile normalized to the total number of alphas followed divided by the plasma volume, for the generic reactor design

thermalize. After an alpha is born in bin j , it can leave this bin because of its trajectory or because of velocity space losses. The flux of alphas through any bin can be written as $\langle \bar{v}_\alpha \rangle \cdot \nabla n_\alpha$ where $\langle \bar{v}_\alpha \rangle$ is the alpha fluid speed. The velocity space losses occur when the alpha is either born in the loss cone or when it is pitch angle scattered into the loss cone. The prompt or birth losses are given by $\frac{1}{2R_j} n_{Dj} n_{Tj} \langle \sigma v \rangle_j$ where R_j is the mirror ratio for bin j . By collecting these terms the thermalization rate in bin j becomes

$$\frac{\partial n_{\alpha j}}{\partial t} + \langle \bar{v}_\alpha \rangle \cdot \nabla n_{\alpha j} = n_{Dj} n_{Tj} \langle \sigma v \rangle_j \left(1 - \frac{1}{2R_j}\right) - C1 \quad (6.3.1.1.2)$$

where $C1$ represents the collisionally lost particles. By integrating this equation with respect to time, the thermalized alpha density in bin j can be found and the total number density is the sum of the bin densities.

$$n_\alpha = \sum_j n_{\alpha j} \quad (6.3.1.1.3)$$

The Monte-Carlo calculation integrates Eq. (6.3.1.1.2) by following alpha trajectories which produce the bin number densities, n_j . The thermalization rate for the j^{th} bin must be calculated using the solution to the integrated equation. We do this by assuming that the thermalization rate is proportional to the total alpha birth rate and is defined as

$$\frac{dn_{\alpha j}}{dt} = \frac{n_{\alpha j}}{n_{\alpha T}} (n_D n_T \langle \sigma v \rangle)_T \quad (6.3.1.1.4)$$

where the subscript T refers to an average value defined by $\frac{\int_v g(v) dv}{\int_v dv}$.

A plot of the alpha thermalization rate as a function of radius is shown in Fig. 6.18.

Finally, we observe that the thermalization rate for the generic design has a sharp spike near the edge of the plasma. The number of alphas that move from their birth position to the edge can be shown by plotting the alpha birth position minus the thermal position (Fig. 6.19). The majority of thermal alphas near the plasma edge come from near the center of the plasma ($\frac{r}{r_p} \sim 0.6$) where β is still large and the average Larmor orbit extends into the halo. The alpha thermal minus birth position is essentially zero for $\frac{r}{r_p} < 0.2$, which implies that the particles born near the axis see a uniform plasma and experience little radial motion during the thermalization process. This large spike in the thermal alpha density suggests that the plasma density and temperature profile models need changing so that a uniform thermal profile can be obtained.

6.3.1.2 Alpha Particle Guiding Center Thermalization

Guiding center distributions and thermalization rates are determined in the same manner as the alpha density distributions. A comparison between the guiding center and particle density distributions for thermal alphas is shown in Fig. 6.20. The alpha Larmor thermal radius is small compared to the characteristic scale

Figure 6.18 Alpha particle thermalization rate for the generic reactor design

Figure 6.19 Number of alpha particles born minus the number of alpha particles thermalizing in r_j

Figure 6.20 Alpha particle(x) and guiding center(o) normalized thermal densities as a function of radius

length (L) in the reactor core ($\frac{r}{rp} \leq 0.864$), but becomes comparable to the scale lengths at the plasma edge ($0.864 < \frac{r}{rp} \leq 1$).

In the region where $\frac{\rho}{L} \sim 1$, which occurs near the plasma edge, the relation between particle density and guiding center density is given by⁴⁸

$$n(r) = \frac{2}{\pi} \int_{r-\rho}^{r+\rho} \frac{N(R) R dR}{[4R^2 r^2 - (R^2 + r^2 - \rho^2)^2]^{1/2}} \quad (6.3.1.2.1)$$

where $N(R)$ is the guiding center density at R and r is the particle radius. This is a local relation which determines $n(r)$ over a Larmor diameter. If $N(R)$ equals a constant N_0 , then $n(r) = N_0$. If $N(R)$ is quadratic, such that $N(R) = N_0 \left(1 - \frac{R^2}{r_p^2}\right)$, then the above integral becomes

$$n(r) = N_0 \left(1 - \frac{r^2 + \rho^2}{r_p^2}\right), \quad (6.3.1.2.2)$$

which can be changed to

$$n(r) = N(R) + N_0 \left(\frac{R^2 - (r^2 + \rho^2)}{r_p^2}\right). \quad (6.3.1.2.3)$$

Note that $n(r)$ is less than the $N(R)$ when $R^2 > r^2 + \rho^2$ and $n(r)$ is greater than $N(R)$ when $R^2 < r^2 + \rho^2$. A similar phenomenon occurs near the plasma edge in the generic design and can be seen in Fig. 6.20 for

$0.9 < \frac{r}{r_p} < 1$. The difference between guiding center density and particle density when $\frac{\rho}{L} \sim 1$ can also be seen by comparing the alpha birth densities (Fig. 6.21).

When $\frac{\rho}{L}$ is small, the relation between particle density and guiding center density can be approximated by⁴⁹

$$n(r) = N(r) + \frac{\rho}{4} \nabla^2 N(r) + \dots \quad (6.3.1.2.4)$$

where $n(r)$ is the particle density, $N(r)$ is the guiding center density, and ∇^2 is the Laplacian perpendicular to $\bar{B}$. In Fig. 6.20, for $\frac{r}{r_p} < 0.864$, the guiding center density is nearly linear, and $\nabla^2 N(r) \sim 0$; therefore, $n(r) \approx N(r)$.

6.3.1.3 Alpha Particle Energy Distribution

The amount of energy given to each radial bin is determined as the alpha slows down. The total energy deposited in each bin (ϵ_j), divided by the total alpha energy available ($\epsilon_\alpha N_T$), is shown in Fig. 6.22 for the generic reactor design. This energy ratio is similar to the thermal alpha particle ratio since $\frac{\epsilon_j}{\epsilon_\alpha N_T}$ goes to zero at $\frac{r}{r_p} = 0$, and it is also zero for $\frac{r + 2\rho}{r_p} \geq 0$. The radial distribution of $\frac{\epsilon_j}{\epsilon_\alpha N_T}$ for the generic design has a peak near the plasma edge, caused by the radial guiding center drift.

Figure 6.21 Alpha particle(x) and guiding center(o) normalized birth densities as a function of radius

Figure 6.22 Normalized alpha energy deposition profile for the generic reactor design

The energy transferred per unit volume of the plasma, i.e., $e_j = \frac{\epsilon_j}{\Delta V_j}$ can be computed, and the ratio of $\frac{e_j}{e_T}$, where $e_T = \frac{\epsilon_{\alpha} N_T}{V}$, can be determined and is plotted in Fig. 6.23.

The ratio of $\frac{e_j}{e_T}$ can be used to determine the alpha power into each radial bin. This can be calculated in a similar manner as the particle thermalization rate, with the result given by

$$P_j = \frac{e_j}{e_T} \epsilon_{\alpha} (n_D n_T \langle \sigma v \rangle)_T \quad (6.3.1.3.1)$$

and shown in Fig. 6.24. The alpha power to the ions and electrons is shown in Fig. 6.25, and the fraction of energy to each species is given in Table 6.1. The alpha power to the plasma along with the alpha energy fraction to the plasma ions and electrons is necessary for designing future fusion reactors.

6.3.1.4 Statistical Analysis

The data and figures that have been presented are results from the generic reactor design. These results were generated using 7000 alphas and checked by using nine separate calculations consisting of 1000 particles per run, in which the seed given to the random number generator was changed in each run. In analyzing these results, we have divided the plasma into two regions. The first region is the hot reacting plasma core, $0 \leq \frac{r}{r_p} \leq 0.864$, which corresponds to the radial length over which the alphas are born. The second region, $0.864 < \frac{r}{r_p}$,

Figure 6.23 Alpha energy deposition profile per unit volume normalized to the total energy deposited in the total plasma volume

Figure 6.24 Alpha power given to the plasma in the generic reactor design

Figure 6.25 Alpha power given to plasma ions and electrons in the generic reactor design

corresponds to the edge plasma region where the density and temperature continue to decrease to $T_i = T_e = 50$ eV and $n_i = n_e = 5 \times 10^{12} \text{ cm}^{-3}$. The halo region is defined as the non-reactive plasma surrounding the hot core, i.e. $\frac{r}{r_p} > 0.864$.

The results of generic design study which are presented in Table 6.1 show that 51% of the alphas thermalize in the reactor core and become "ash" products. These alphas deposit 58% of their energy in the core, but a significant amount (32%) is deposited in the halo. This halo heating will eventually increase the halo temperature, which in turn increases the temperature-profile scale length with respect to r_p and r_{pD} .

Alpha particle prompt losses have been calculated to be 3% for the generic design and, therefore, 3% of the alpha energy is also lost by this mechanism. Beta corrected loss rates are 3% on axis and 6% near the plasma edge, and an average beta of 0.4 gives a loss rate of 5%. The calculated loss rate of 3% implies that the majority of alphas are born in the high beta region of the plasma, and these are the particles that contribute the most to the prompt losses.

The number of alphas that are collisionally lost during slowing down is 10%. These alphas remove approximately 4% of the total alpha energy from the plasma, and they leave the plasma with an average energy of 1.4 MeV. This implies that scattering events are becoming important at alpha energies near 1.4 MeV.

Another important quantity is the fraction of alpha energy going to ions and electrons. It has been shown (Fig. 6.2) that the Monte-

Carlo calculation accurately partitions the alpha energy to each plasma specie according to theory for uniform plasmas. In the present configuration the plasma is far from uniform and more alpha energy goes to the electrons, especially the electrons at the plasma edge. For this reason, the percent of alpha energy that goes to the electrons is 77%, whereas the ions receive 21%. These fractions do not add up to 100% because the final 2% of the alpha energy is equal to the plasma thermal energy of 60 keV.

Finally, we performed a statistical analysis for the nine 1000 particle calculations and compared these values with the one 7000 particle calculation. These results are also given in Table 6.1. The calculation for the 7000 particle run takes over 60 minutes of Cray C.P.U. time. This run time can be shortened by reducing the number of diagnostics available in the code and by further optimizing the computer program. These techniques can be effective, but the 1000 particle calculations can produce accurate results in approximately 12 C.P.U. minutes. We found that the averages for the 1000 particle runs agree with the 7000 particle run within one standard deviation, and therefore we concluded that 1000 alphas can be used for further reactor studies. Also, the values given in Table 6.1 show that the calculated densities and energies produced by a 1000 particle Monte-Carlo calculation will vary from the mean value by less than 3%, again verifying the accuracy of the 1000 particle runs.

6.3.2 Variations From the Generic Reactor Design

The generic reactor design parameters were chosen as representative values which could be used to illustrate the effect of temperature and density gradients on alpha particle confinement. The parameters that directly affect alpha particle confinement are the vacuum magnetic field in the solenoid, beta in the center cell, and the plasma radius. The importance of these three parameters will be illustrated in the following section. The ratio of ρ/r_p is a measure of the plasma's ability to contain the alpha energy, and changes in B , β , or r_p change the alpha energy confinement.

6.3.2.1 Scale Length Relation for Alpha Energy Confinement

Before varying any reactor parameter it is important to determine which of the parameters will affect the alpha energy deposition the most. An approximate way of determining these parameters is by making the following heuristic argument. The alpha deposits its energy along a radial line that extends over two times its Larmor radius and is given by $\frac{\epsilon_\alpha}{2\rho}$. Now the alpha energy which is deposited within the plasma along the plasma diameter is $\frac{N\epsilon_\alpha}{2r^*}$ where N is the total number of alpha particles and r^* includes only the alpha energy deposited inside the plasma. In plasmas where the alpha orbit is small compared to the plasma diameter, the energy deposited along the diameter can be approximated by $\frac{N\epsilon_\alpha}{2r_p}$. This ratio is proportional to $\frac{\epsilon_\alpha}{2\rho}$ so that

$$C \frac{\epsilon_{\alpha}}{N \epsilon_{\alpha}} = \frac{\rho}{r_p} \quad (6.3.2.1.1)$$

where C is an arbitrary constant. This ratio was used by Barr, et al.⁵⁰ in studying mirror-confined plasmas that had density temperature profiles that are shown in Fig. 5.5. When $\frac{(2m\epsilon_{\alpha})^{1/2}}{qB_v(1-\langle\beta\rangle)^{1/2}}$ is substituted for ρ in Eq. (6.20), then the scale length ratio for energy confinement becomes

$$\frac{\rho}{r_p} = \frac{0.27}{r_p B_v (1-\langle\beta\rangle)^{1/2}} \quad (6.3.2.1.2)$$

where r_p is in meters and B_v in Tesla. The ratio of $\frac{\rho}{r_p}$ defines the important parameters for alpha energy confinement to the plasma radius (r_p), the solenoid vacuum magnetic field (B_v), and the average plasma beta $\langle\beta\rangle$. Average plasma beta was chosen over $\hat{\beta}$ or $\frac{1}{2} \hat{\beta}$ (as given by Barr, et al.) because it makes ρ proportional to a mean alpha Larmor radius in the plasma. The fact that the plasma model used in this study is more complex, due to the temperature profile, the inclusion of scattering terms in the collision process, and the addition of the drag gradient drift, implies that the relation between the parameters in Eq. (6.3.2.1.2) may not be exact. Because of the complexity of finding an exact relation between these parameters, it becomes necessary to pick temperature and density profiles and then change r_p , B_v , or $\hat{\beta}$ to determine the alpha energy deposition in the plasma. Because r_p does

not necessarily correspond to the hot reacting plasma radius (r_c), we have substituted this value in place of r_p in the scale length relation, which becomes $\frac{\rho}{r_c}$.

6.3.2.2 Variation of B , $\hat{\beta}$, and r_p for the Single-Scale-Length Analysis

The single-scale-length analysis encompasses the results of Monte-Carlo calculations in which the parameters B_v , $\hat{\beta}$, and r_p were changed when the temperature and density radial scale lengths are equal, i.e. $r_{pT} = r_{pD} = r_p$. These parameters were varied such that $0.085 \leq \frac{\rho}{r_c} \leq 0.52$ and $n_e(0)$, $T_i(0)$, and $T_e(0)$ were kept equal to the values given for the generic reactor design. The arbitrary β changes are inconsistent with these constant density and temperature values. This inconsistency enters into the Monte-Carlo simulation through the number of alphas born at any radial location and in Spitzer's collision model. The radial birth distribution is compensated by calculating alpha density and energy distributions as fractions of the total number of alphas or total alpha energy. As the alpha approaches thermal energy, variations in background plasma properties will affect the collision model, but the relative amount of alpha energy available for the plasma is small, less than 15% of the 3.52 MeV initial energy. Therefore, the collision model affects are negligible because small changes in plasma density and temperature do not significantly affect the drag, angle scatter, and dispersion of a high energy alpha. These arguments imply that the inconsistency of keeping the same values of $n_e(0)$, $T_i(0)$, and $T_e(0)$ for each run introduces second-order effects

into the calculation. Also by keeping the on-axis density and temperature fixed for each run, we eliminate the unknown effects in varying too many parameters in our study of alpha thermalization.

The results of the single-scale-length analysis using the density and temperature profiles presented in Fig. 5.2 are shown in Figs. 6.26 and 6.27. In Fig. 6.26, thermal alpha fractions in the core, halo, and velocity space losses are plotted as a function of $\frac{\rho}{r_c}$. Figure 6.27 shows the alpha energy fractions in each of these regions. Each data point in these figures represents a calculation in which 1000 alphas were followed from birth to thermalization. Best-fit curves to the data are also shown. The velocity space losses are modeled using linear regression between $0.085 \leq \frac{\rho}{r_c} \leq 0.52$, with the resulting straight lines given by

$$\% \frac{N_v}{N} = -33.6 \frac{\rho}{r_c} + 24.8 \quad (6.3.2.2.1)$$

$$\% \frac{\epsilon_v}{\epsilon} = -20.9 \frac{\rho}{r_c} + 14.9 \quad (6.3.2.2.2)$$

Halo losses are modeled using a fourth-order polynomial regression between $.085 \leq \frac{\rho}{r_c} \leq 0.4$, which are given by

$$\begin{aligned} \% \frac{N_h}{N} = & -7.97 + 279.78 \frac{\rho}{r_c} - 3042.61 \left(\frac{\rho}{r_c}\right)^2 \\ & + 13462.79 \left(\frac{\rho}{r_c}\right)^3 - 15411.04 \left(\frac{\rho}{r_c}\right)^4 \end{aligned} \quad (6.3.2.2.3)$$

Figure 6.26 Alpha particle distribution to the hot reacting core, halo, and velocity space losses as a function of ρ/r_c for the single-scale-length problem. The subscript j refers to the core, halo, or velocity space losses.

Figure 6.27 Alpha energy distribution to the hot reacting core, halo, and velocity space losses as a function of ρ / r_c for the single-scale-length problem. The subscript j refers to the core, halo, or velocity space losses.

$$\begin{aligned} \% \frac{E_h}{E} = & -9.67 + 71.87 \frac{\rho}{r_c} - 806.96 \left(\frac{\rho}{r_c}\right)^2 \\ & + 7708.13 \left(\frac{\rho}{r_c}\right)^3 - 10369.29 \left(\frac{\rho}{r_c}\right)^4 \end{aligned} \quad (6.3.2.2.4)$$

The fraction that is retained in the core is then

$$\% \text{ core} = 1 - \% \text{ halo} - \% \text{ velocity losses} \quad (6.3.2.2.5)$$

where the terms core, halo, and velocity losses refer to either the alpha or energy fractions.

After obtaining these results, several runs were made to determine the effect of changing the density and temperature profile shapes. In the first case, we varied the density profile as shown in Fig. 6.28 and kept the temperature profile constant. The values of B , $\hat{\beta}$, and r_p were chosen so that a comparison would be made with a calculation using a quartic density profile. These comparisons are given in Table 6.2 under the "quartic density" and "modified density" headings. The first difference occurs in $\langle \beta \rangle$, which is 7% greater for the modified density case, giving a 4% larger value of $\frac{\rho}{r_c}$. This density change also caused a 9% increase in the fractional amount of alphas and alpha energy into the core plasma and a corresponding 9% decrease in the halo. This is an opposite effect from what we expect, because, as seen from Fig. 6.26 and 6.27, as ρ/r_c increases the fraction of alphas and alpha energy going to the halo increases.

Figure 6.28 Modified density profile for the single-scale-length problem

Another study was made in which the density and temperature were modeled to correspond to an approximate tenth power (see Eq. 5.2.1) radial profile. These results are also presented in Table 6.2, and again $\langle \beta \rangle$ increases so that ρ/r_c is 0.327. As shown in the table, there are more alphas and more alpha energy going into the halo, as

TABLE 6.2
COMPARISON OF MONTE-CARLO SIMULATION RESULTS VARYING
THE DENSITY AND TEMPERATURE PROFILES

	Quartic Density and Generic Temperature Profiles	Modified Density and Generic Temperature Profiles	Tenth Power Density and Temperature Profiles
B (%)	3.0	3.0	3.0
$\hat{\beta}$ (T)	0.9	0.9	0.9
r_p (m)	0.5	0.5	0.5
$\langle \beta \rangle$	0.56	0.6	0.676
r_c (m)	0.45	0.43	0.48
$\frac{\rho}{r_c}$	0.305	0.330	0.327
$\% \frac{N_c}{N}$	40.3	44.0	37.6
$\% \frac{N_H}{N}$	46.2	42.0	51.6
$\% \frac{N_L}{N}$	13.5	14.0	10.8
$\% \frac{E_c}{E}$	48.7	53.0	43.3
$\% \frac{E_H}{E}$	43.5	40.0	50.5
$\% \frac{E_L}{E}$	7.8	7.0	6.2

compared with either the "quartic density" or "modified density" cases. The increased thermalization in the halo results in a lower thermalization rate in the hot reacting core.

6.4 Two-Scale-Length Calculation

The results of the single-scale-length calculation show that when ρ/r_c is greater than one-third, there is a significant amount of alpha heating in the halo. Monte-Carlo results show that the alpha energy heats the halo over a narrow radial region near r_p and this energy is initially transferred to the halo electrons. The concentration of a large fraction of alpha energy near r_p will cause the halo temperature to increase. This in turn will increase the temperature scale length making $r_{pT} > r_{pD}$, which leads to the two-scale-length problem.

As $\frac{r_{pT}}{r_{pD}}$ becomes larger than one the radial drift due to drag changes. The density effect (see Chapter 5) now occurs before the temperature effect, which can cause an inward radial drift. An inward drift keeps more of the alphas and more alpha energy in the core plasma. The increased temperature scale length also increases the radial region of the hot reacting core plasma to values nearer $\frac{r}{r_p} = 1$.

The effect of increasing $\frac{r_{pT}}{r_{pD}}$ greater than one changes the alpha energy deposition in the halo. Table 6.3 compares calculated energy and particle fractions in the hot reacting plasma core and halo for values of $\frac{r_{pT}}{r_{pD}}$ of 1.0, 1.1, and 1.2. These results were calculated

TABLE 6.3

COMPARISON OF THE RESULTS FOR THE TWO-SCALE-LENGTH PROBLEM

FOR SEVERAL VALUES OF r_{pT}/r_{pD}

	<u>Quartic Density Profiles</u>			<u>Modified Density Profile</u>
$\frac{r_{pT}}{r_{pD}}$	1.0	1.1	1.2	1.2
$\frac{N_G}{N}$ Core	52.0	74.0	78.0	78.0
$\frac{N_H}{N}$ Halo	35.0	9.0	4.0	5.0
$\frac{N_L}{N}$ Velocity Loss	13.0	17.0	18.0	17.0
$\frac{\epsilon_G}{\epsilon}$ Core	61.0	81.0	86.0	86.0
$\frac{\epsilon_H}{\epsilon}$ Halo	32.0	9.0	4.0	6.0
$\frac{\epsilon_L}{\epsilon}$ Velocity Loss	7.0	10.0	10.0	8.0
$\hat{\beta}$	0.8	0.8	0.8	0.8
B	0.8	0.8	0.8	0.8
r_p	0.5	0.5	0.5	0.5
$\langle \beta \rangle$	0.53	0.51	0.51	0.61
r_G	0.43	0.48	0.49	0.48
$\frac{\rho}{r_G}$	0.30	0.27	0.275	0.3

using the generic design parameters and the density and temperature profiles discussed in Chapter 5 (Fig. 5.2), except that $r_{pT} \geq r_{pD}$.

Also presented in this table are the results of a calculation in which

the modified density profile of Fig. 6.2 and $\frac{r_{pT}}{r_{pD}} = 1.2$ are used. The differences in particle and energy distributions between $\frac{r_{pT}}{r_{pD}} = 1.1$ and 1.2 are negligible, which implies that either of these values can be used to study alpha transport for $\frac{r_{pT}}{r_{pD}} > 1$. A closer look at these two cases (Figs. 6.29 and 6.30, respectively) shows that when $\frac{r_{pT}}{r_{pD}} = 1.1$ the power profile is close to equilibrium. In Fig. 6.29, the halo heating peak is low and it is near to the plasma core profile. Therefore, we chose to use the profile ratio of $\frac{r_{pT}}{r_{pD}} = 1.1$ for further study.

Finally, we note that the modified density profile compares favorably in alpha particle and alpha energy fractions given to the plasma core and halo. This comparison breaks down when we look at $\langle \beta \rangle$ and ρ/r_c . We note that the modified density profile has a 9% larger value of ρ/r_c , which implies that there should be a larger number of alpha particles and alpha energy in the halo, with a corresponding decrease in the halo as compared to the quartic density profile case. This result implies that the temperature and density profiles play an important role in the alpha thermalization process.

6.4.1 Variation of B , $\hat{\beta}$, and r_p for the Two-Scale-Length Problem

We have shown in Section 6.4 that values of $\frac{r_{pT}}{r_{pD}}$ equal to 1.1 produce alpha thermal distributions that are close to equilibrium. The next step is to vary B , $\hat{\beta}$, and r_p , using $n_e(0)$, $T_1(0)$, and $T_e(0)$ equal

Figure 6.29 Alpha power to the plasma for the two-scale-length problem when $r_{pT}/r_{pD} = 1.1$

Figure 6.30 Alpha power to the plasma for the two-scale-length problem when $r_{pT}/r_{p0} = 1.2$

to the generic reactor design values and $\frac{r_{PT}}{r_{PD}} = 1.1$. The results of these variations are alpha particle and energy fractions to the plasma core and halo that extended over the range $0.085 \leq \frac{\rho}{r_c} \leq 0.59$. This study was composed of 25 individual Monte-Carlo calculations, each containing 1000 particles. Because of the number of Monte-Carlo calculations performed and because of the scatter in the data, we will present the alpha velocity space losses and halo losses as functions of $\frac{\rho}{r_c}$ in two separate plots. The fraction of alpha particle and alpha energy deposition into the core can be found from the velocity space and halo losses.

The velocity space losses include the alphas born in the loss cone and those that pitch angle scatter into the loss cone. These losses are similar to the single-scale-length results and can be modeled using a linear regression technique. The percent of alpha lost through the loss cone in the range $0.085 \leq \frac{\rho}{r_c} \leq 0.59$ is

$$\% \frac{N_v}{N} = -29.43 \frac{\rho}{r_c} + 25.13, \quad (6.4.1.1)$$

and the corresponding energy loss is

$$\% \frac{E_v}{E} = -16.46 \frac{\rho}{r_c} + 14.08. \quad (6.4.1.2)$$

A plot of the energy loss as a function of $\frac{\rho}{r_c}$ is shown in Fig. 6.31, where Eq. (6.4.1.2) is shown as the solid line through the Monte-Carlo results. The alpha particle loss is not shown, since it is similar to Fig. 6.31.

The percent of alpha particles thermalizing in the halo and the alpha energy given to the halo for the two-scale analysis follows the same trend as the single-scale-length results. We can therefore model these losses using a fourth-order polynomial regression technique. The best-fit curve for the alpha particle loss to the halo for $0.085 \leq \frac{\rho}{r_c} \leq 0.55$ is

$$\begin{aligned} \% \frac{N_h}{N} = & -34.82 + 743.14 \frac{\rho}{r_c} - 5104.90 \left(\frac{\rho}{r_c}\right)^2 \\ & + 13980.97 \left(\frac{\rho}{r_c}\right)^3 - 11632.16 \left(\frac{\rho}{r_c}\right)^4, \end{aligned} \quad (6.4.1.3)$$

and the alpha energy loss is

$$\begin{aligned} \% \frac{E_h}{E} = & -30.49 + 646.23 \frac{\rho}{r_c} - 4370.61 \left(\frac{\rho}{r_c}\right)^2 \\ & + 12027.98 \left(\frac{\rho}{r_c}\right)^3 - 10039.95 \left(\frac{\rho}{r_c}\right)^4. \end{aligned} \quad (6.4.1.4)$$

Figure 6.31 Alpha energy losses out of the loss cone for the two-scale-length problem for $r_{p1}/r_{pD} = 1.1$

The Monte-Carlo calculated energy loss to the halo along with Eq. (6.4.1.4) is plotted as a function of $\frac{\rho}{r_c}$ in Fig. 6.32. The alpha particle loss to the halo is similar to Fig. 6.32 and is not shown.

The Monte-Carlo data in Fig. 6.32 has been plotted so that calculations made with equal values of B_v and r_p are marked by different symbols. We have also connected data points in which there were more than two calculations made with the same B_v and r_p . Other data, marked by points, either vary in B_v , β , or r_p from all the other data shown. We readily observe from this plot that whenever B_v or r_p changes, then equal halo losses do not have the same value of $\frac{\rho}{r_c}$. This implies that the alpha losses to the halo are not simple functions of $\frac{\rho}{r_c}$ when $\frac{r_{pT}}{r_{pD}} = 1.1$, and we expect this result to hold whenever $\frac{r_{pT}}{r_{pD}} \neq 1$.

Even though there is a lot of scatter in the data in Fig. 6.32, we can approximate the alpha particle and energy contributions to the plasma core. These can be calculated for the range of $0.085 \leq \frac{\rho}{r_c} \leq 0.55$ by

$$\% \text{ core} = 100 - \% \text{ halo} - \% \text{ velocity losses} \quad (6.4.1.5)$$

where the halo amount is calculated from Eq. (6.4.1.3) or (6.4.1.4) and the corresponding velocity losses from Eq. (6.4.1.1) or (6.4.1.2).

Finally, we made a comparison between a modeled sixth-power and tenth-power density and temperature profile shapes. These comparisons were made using equal values of B , $\hat{\beta}$, r_p , $n_e(0)$, $T_i(0)$, and $T_e(0)$ and

Figure 6.32 Alpha energy losses to the halo for the two-scale-length problem for $r_{pT}/r_{pD} = 1.1$

are given in Table 6.4. We can see that as the power of the profile increases, the value of $\langle \beta \rangle$ increases, which increases values of $\frac{\rho}{r_c}$. As $\frac{\rho}{r_c}$ increased, the fraction of alphas and alpha energy going to the halo increased and the corresponding fractions to the core decreased.

TABLE 6.4
COMPARISON OF THE RESULTS FOR THE TWO-SCALE-LENGTH PROBLEM USING
SIXTH- AND TENTH-POWER DENSITY AND TEMPERATURE PROFILES WITH $\frac{r_{pT}}{r_{pD}} = 1.1$.

	<u>Sixth Power Profile</u>	<u>Tenth Power Profile</u>
r_{pT}/r_{pD}	1.1	1.1
$\hat{\beta}$	0.9	0.9
B_r	3.0	3.0
r_p	0.5	0.5
$\langle \beta \rangle$	0.623	0.717
r_c	0.506	0.483
ρ/r_c	0.29	0.35
% N_c/N Core	76.1	44.4
% N_h/N Halo	9.7	44.2
% N_L/N Velocity Losses	14.0	11.4
% ϵ_c/ϵ Core	84.7	51.6
% ϵ_h/ϵ Halo	8.6	41.8
% ϵ_L/ϵ Velocity Loss	6.7	6.6

o	On axis
oe	On-axis electron
oi	On-axis ion
p	Plasma
pT	Plasma temperature
pD	Plasma density
pc	Plug to center cell
r	Radial
T	Tritium
T_j	Tritium in j
v	Velocity space or vacuum
x,y,z	directions in the laboratory coordinate system
z	Axial

GREEK SUBSCRIPTS

α	Alpha particle
$\alpha\alpha$	Alpha-alpha
αi	alpha-ion
ϵ	Energy
χ	Azimuthal angle
ϕ	Phase angle
ψ	Pitch angle

SYMBOLS SUBSCRIPTS

$\perp$	Perpendicular
$\parallel$	Parallel

solution of this problem using the Boltzman equation with the Fokker-Planck collision operator and three plasma species, i.e. electrons, ions, and alphas, is impossible in the three-dimensional tandem-mirror geometry. Therefore, we have had to rely on density and temperature models that are consistent with present reactor design studies. These profiles have been used to study the effect of varying B_v , $\hat{\beta}$, and r_p in a generic reactor design that defines $n_e(0)$, $T_i(0)$, $T_e(0)$, and other pertinent parameters.

We have investigated several density and temperature profile models, but we have concentrated the majority of the effort in two specific models. The first model, called the single-scale-length problem, uses a quartic density profile and a higher order temperature profile, both of which have the same scale length. The second model is called the two-scale-length problem because the temperature and density scale lengths differ, but the remaining parameters in these profiles are equal to their counterparts in the single-scale-length case.

In order to compare the results of these studies, we needed to find a common scaling parameter. We have presented a heuristic argument that relates the alpha particle and alpha energy distribution to the core and halo plasma as a function of ρ/r_c . We have defined ρ to be $\left(\frac{m}{2} \epsilon_\alpha\right)^{1/2} \frac{1}{qB_v[1 - \langle\beta(r)\rangle]^{1/2}}$ where $\langle\beta(r)\rangle$ is the average β in the plasma core and ϵ_α is 3.52 MeV. We can find r_c from the Monte-Carlo output, and it is equal to the radius of the hot reacting core

plasma. This can be approximated by determining the largest radius, which corresponds to a birth probability of .001.

After establishing that the alpha halo losses were a function of ρ/r_c , we plotted the alpha particle distribution at thermalization and the alpha energy deposition to the core and halo plasma versus ρ/r_c for the single-scale-length problem. We also included in this plot the velocity space losses, which includes the alphas that are immediately lost because they are born in the loss cone plus those that are collisionally lost out the loss cone. The results of this single-scale-length study show that ρ/r_c is a good correlation parameter. We also see that when $\rho/r_c > 0.3$, the alpha particle and energy losses to the halo are greater than 40%. This, coupled with alpha velocity space losses of 15% and corresponding energy losses of 9%, shows that there is less than 51% of the alpha energy available for core heating. Therefore, the single-scale-length results imply that tandem-mirror reactors should be designed with $\rho/r_c < 0.3$.

Fortunately, nature will probably not allow the single-scale-length profiles to exist in the plasma, especially after the D-T burn has started. We can say this because the results of the Monte-Carlo code, which produces equilibrium profiles, shows a very sharp peak in both alpha density and alpha energy that occurs in a narrow radial region of the halo for the single-scale-length problem. These sharp peaks will relax to some equilibrium condition which does not affect the D-T density profile, but it will increase the radial scale length of the temperature profile. We have investigated this possibility by

first choosing the ratio of temperature to density scale lengths ($\frac{r_{pT}}{r_{pD}}$),

such that alpha density and energy profiles are close to equilibrium.

This ratio of scale lengths used in this study was

$$\frac{r_{pT}}{r_{pD}} = 1.1.$$

Next, we changed values of B , $\hat{\beta}$, and r_p while keeping the other generic reactor parameters fixed and using $\frac{r_{pT}}{r_{pD}} = 1.1$ to study the alpha thermalization process. These results show that the functional relation between the alpha losses and ρ/r_c is not as simple as the single-scale-length case. Data scatter of 50% can be observed for some values of $\rho/r_c > 0.35$. We assume that this scatter is related to the alpha guiding center birth distribution because this guiding center distribution will also fluctuate depending on the size of the Larmor orbit compared to the radial scale length. Unfortunately, we have not been able to solve this problem at this time and we must leave it for future work.

Although we noted a significant amount of data scatter for the two-scale-length problem, we have nevertheless obtained a fourth-order best-fit curve to this data. From this curve we can generalize that when $\rho/r_c \geq 0.39$, then 40% or more of the alphas and alpha energy is lost to the halo. This is only a slight improvement in the parameter ρ/r_c as compared to the single-scale-length results. We must also point out that that large data scatter implies that exact results depend on the chosen values of B , $\hat{\beta}$, and r_p .

We have also done less extensive studies varying the density and temperature profiles for given values of B , $\hat{\beta}$, and r_p . We have done this for both the single-scale-length and two-scale-length problems. The results indicate that specific reactor designs are affected by the density and temperature profiles that are used. These profiles affect the alpha birth profile, β profile, the drag drift, and therefore the alpha thermal particle and alpha energy deposition profiles. We note that changes in the density and temperature profiles follow the same trends as discussed for the single- and two-scale-length problems, but more reliable results can only be obtained from specific calculations, especially when $\rho/r_0 > 0.3$.

We can summarize the results of this study with the following statements. The most general result is that values of ρ/r_0 greater than 0.3 produce excessive alpha energy loss to the halo. Secondly, the particular alpha particle and energy distributions after thermalization can only be determined by knowing the plasma density and temperature profiles along with the specific plasma parameters. Specific plasma profiles and parameters must be used as input to an alpha thermalization code, such as the one used in this study, to determine the alpha distributions. If the resulting alpha distributions are not in equilibrium, then changes have to be made to the original density and temperature profiles and then the calculation is performed again. This iterative process is necessary because of the complexity of solving the Boltzman equation self-consistently for this problem.

It is appropriate at this point to briefly review the areas where this study could be either improved or used. Theoretical work needs to be done to determine the limiting radial magnetic field gradients that can be used to meet adiabaticity conditions and therefore instill more confidence in the use of the guiding center approximation when radial gradients are present. Next, the radial drag drift rate is a complex calculation that requires an excessive amount of computer time; this scheme should be improved, if possible. Work is also needed to improve the alpha particle and energy correlation to the correct functional form of ρ/r_c so the data scatter in the two-scale-length problem is reduced. Finally, it is well known that any computer code generated to solve a complex problem cannot be guaranteed to be completely accurate in every application. Therefore, there are coding improvements and then further tests that could be made to guarantee the reliability and accuracy of this Monte-Carlo calculation.

As far as applying this Monte-Carlo code to other applications, we could modify this code to address the alpha thermalization process in tokamak fusion plasmas that are surrounded by an edge (halo) plasma. In a more related area, we could study alpha trapping rates into the thermal barrier and the associated alpha pumping problem. We could also address alpha transport in D-T driven mirror machines, which has applications toward advancing fusion technology. Other applications, such as studying fast ions, or other products in a tandem-mirror reactor, are also possible.

In conclusion, we have generated a tool that has been used to study alpha particle transport during the thermalization process in tandem-mirror reactors. This tool is a Monte-Carlo Code which incorporates the important physical processes that affect alpha thermalization.

PART 8

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