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STREAKCAMERA APERTURE FUNCTION

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ABSTRACT

This paper derives the aperture function of a streaktube. It defines the input spectrum being a function of time and space. It is found that the zeros of the aperture function depend inversely on aperture size and sweep time.

INTRODUCTION

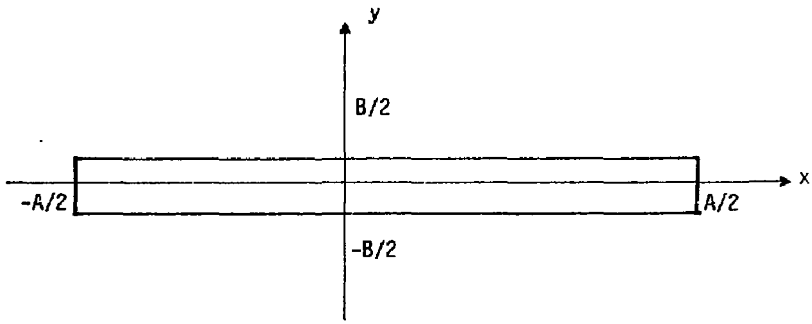
This paper derives the relation between the input slit size and its relation to the input spatial and temporial input spectra. The input slit height determines the maximum spatial variation and time variation of the input irradiance on the photocathode. If this limit on input frequency is exceeded, artifacts will result which will result in false data. Such data as high frequency data will result in low frequency data at the output of the phosphor. This should always be taken into consideration when a system is built using a streakcamera.

A streakcamera is a camera which uses a streaktube as an imaging device. A streaktube is an imaging tube device which can gate, turn on or off, or deflect the stream of electrons on their way to the phosphor screen. Such a device is a RCA C734535AK streaktube which is electrostatically focused and electrostatically deflected.

The time variation of light intensity of a point image or line image on the photocathode may be displayed on the phosphor screen as a streak. The original line image on the photocathode is moved across the phosphor screen causing a rectangular image composed of the original width times a height which is the original height moved from top to bottom on the screen. This moving of the input image photoelectrons on the phosphor represents a scanning process and is subject to aliasing effects.

COORDINATE TRANSFORMATION

The input slit imaged on the photocathode can be assumed to have a length and height of A and B units respectively and is shown below in schematic form.



Length of the slit is taken along the x direction while the height is taken along the y direction.

This input slit has no transmission outside the limits of the slit, $\frac{-A}{2} < x < \frac{A}{2}$, $\frac{-B}{2} < y < \frac{B}{2}$.

The input flux (watts) at the input to the photocathode is a function of both space and time and is designated as $\emptyset(x,y,t)$. Peak photocathode sensitivity is also a function of space and is given as $S_p(x,y)$. The current out of the photocathode at a point x_0, y_0 and time t_0 can be given as

$$I(x_0,y_0,t_0) = \emptyset,(x_0,y_0,t_0)\ S_p(x_0,y_0)\alpha \quad .$$

where α is the spectral matching factor. The average current from the photocathode can be obtained by integrating over the photocathode. For reasons which will become clear later, the integration should be done over y only

$$I(x,t) = \alpha \int_{\substack{\text{height of}\\ \text{photocathode}}} \phi(x,y,t)\ S_p(x,y)dy$$

Because the photocathode is masked by a slit, a rectangle function can be used to define the slit height, B.

So the combination of the slit and photocathode becomes

$$I(x,t) \left[U\left(y + \frac{B}{2}\right) - U\left(y - \frac{B}{2}\right)\right] = \alpha \int_{\frac{-B}{2}}^{\frac{B}{2}} \phi(x,y,t)\ S_p(x,y)dy$$

The rectangle function has changed the limits on the integral from the height of the photocathode to the height of the slit. In doing this we have made use of the following fact

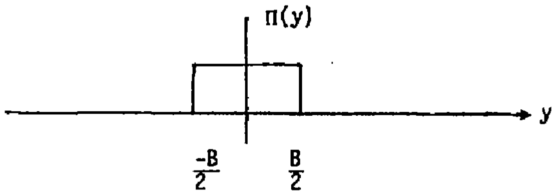
$$\int_{\frac{-B}{2}}^{\frac{B}{2}} f(y)dy = \int_{-A}^{A} f(y) \left[U\left(y + \frac{B}{2}\right) - U\left(y - \frac{B}{2}\right)\right] dy$$

where: $A > B$

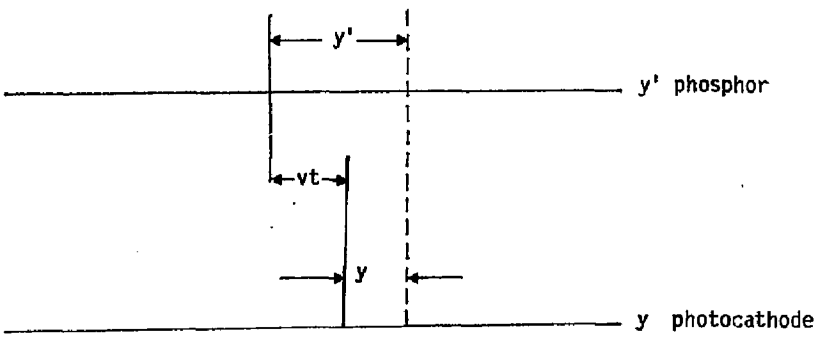
$U\left(y + \frac{B}{2}\right)$ is a unit step function starting at $y = \frac{-B}{2}$

$U\left(y - \frac{B}{2}\right)$ is a unit step function starting at $y = \frac{B}{2}$

$\left[U\left(y + \frac{B}{2}\right) - U\left(y - \frac{B}{2}\right)\right]$ is called a rectangle function of width B and centered at $y = 0$.



Assume that the phosphor screen coordinate system is the prime system given as x',y'. The coordinate system model for the streaktube is given below



The coordinate transformation between the photocathode and phosphor is given below

$$y' = vt + y$$

$$B' = mB$$

$$x' = mx$$

where m is the demagnification factor between the photocathode and phosphor. This represents the decrease of image size at the phosphor.

Making these substitutions into the original equation for current and integrating over the interval that the input flux is present results in the following.

$$Q(x',y') = \int_{T_1}^{T_2} \left(I \frac{x'}{m}, t \right) \left[U\left(y' + \frac{B}{2m} - vt\right) - U\left(y' - \frac{B}{2m} - vt\right) \right] dt$$

where the limits on the integral, T_1 and T_2, are the times that the input irradiance starts and stops respectively. $Q(X',y')$ represents the charge incident on the phosphor. The height of the phosphor can be assumed to be C' units, so the velocity of the sweep can be given as

$$v = \frac{C'}{T_s}$$

where T_s is the time for sweeping the beam from the top to bottom of the phosphor. Making this substitution for velocity into the integral equation for current results in the following.

$$Q(x',y') = \int_{T_1}^{T_2} \left(I\,\frac{x'}{m},t\right)\left[U\left(y' + \frac{B}{2m} - \frac{C't}{T_s}\right) - U\left(y' - \frac{B}{2m} - \frac{C't}{T_s}\right)\right]dt$$

POWER SPECTRA

The power spectra of this charge can be obtained by taking the Fourier transform of this equation. If this is done the following results.

$$Q(fx',fy') = \iiint_{-\infty}^{\infty}\int_{T_1}^{T_2} I\left(\frac{x'}{m},\ t\right)\left[U\left(y' + \frac{B}{2m} - \frac{C't}{T_s}\right) - U\left(y' - \frac{B}{2m} - \frac{C't}{T_s}\right)\right]e^{-j2\pi\left(f_x'x' + f_y'y'\right)}dx'dy'dt$$

To make the problem easier the following substitution is made

$$Z = y' - \frac{C't}{T_s}$$

In making this substitution the following results

$$Q(fx',fy') = \iint_{-\infty}^{\infty}\left(I\,\frac{x'}{m},t\right)e^{-j2\pi\left(f_x'x' + f_y'\frac{C'}{T_s}t\right)}dx'dt\left[\int_{-\infty}^{\infty}\Pi\left(\frac{Zm}{B}\right)e^{-j2\pi f_y'Z}dz\right]$$

APERTURE FUNCTION

The term in the brackets is called the aperture function, $A(z)$. The double integral before the aperture function is the combined spatial and temporial spectra. Assume that $I(x'/m, t)$ can be separated into two functions

$$I(x'/m)\Big|_{t = K_1} \text{ and } I(t)\Big|_{x'/m = K_2} .$$

where:

$I(x'/m)\Big|_{t = K_1}$ is the current as a function of x'/m. when time is held constant.

$I(t)\Big|_{x'/m = K_2}$ is the current at a point on the phosphor, $x'/m = K_2$ and is allowed to be observed with time.

Using this fact of separability results in the following equation.

$$Q(f_x',f_y') = F(f_x')\ F(f)\ A(z)$$

where:

$$F(f_x') = \int_{-\frac{A}{2m}}^{\frac{A}{2m}} I\left(\frac{x'}{m}\right)\Big|_{t=K_1} e^{-j2\pi f_x' x'} dx'$$

$$F(f) = \int_{T_1}^{T_2} I(t)\Big|_{x'/m=K_2} e^{-j2\pi ft}\, dt$$

where $f = \frac{f_y'C'}{T_s}$

$$A(Z) = \int_{-\frac{B}{2}}^{\frac{B}{2}} e^{-j2\pi f_y'Z}\, dZ$$

The last integral does not depend upon the value of charge incident on the phosphor so can be evaluated and is given the name aperture function. The limits we obtained by the rectangle function. Evaluating the aperture function results in the following

$$A(Z) = \frac{B}{m} \text{Sinc}\left(\frac{\pi f_y'B}{m}\right)$$

Zeros of this function occur at $n\pi$, so the values of f_y' to obtain this are

$$f_y' = \frac{nm}{B}$$

The spatial frequency, f_y', can be given in terms of the input temporial frequency as follows:

$$f_y' = \frac{fT_s}{C'}$$

The zeros of the aperture function can be defined as follows:

$$f = \frac{nmC'}{T_s B}$$

Part of this equation represents the number of samples at the output phosphor screen specifically.

$$N = \frac{C}{\frac{B}{m}}$$

B/m is the size of the aperture at the phosphor so dividing this into the size of the screen results in the number of samples. The temporial frequencies which result in the aperture function going to zero therefore becomes:

$$f = \frac{N}{T_s}$$

This represents the sampling time for the structure. If T_s is the time required for one sweep then dividing this by the number of samples will result in the sampling time. If the input frequency of the input function is equal to the sample time then the output of the phosphor will be a constant value with no modulation.

CONCLUSION

The spatial frquency in the x' direction is windowed by the dimension of the input aperture projected onto the phosphor. So the limits on the integral become $-\frac{A}{2m} \leq X \leq \frac{A}{2m}$. $F(f_x')$ becomes the spatial frequency spectra of the input radiance, while $F(f)$ is the frequency spectra of the input irradiance. In order that aliasin[illegible] not a problem the input frequency should be one-half the sampling frequency. The degradation of input data is determined by the aperture function. Each harmonic of the input function is modified by this aperture function. In turn the aperture function is determined by the size of the slit's height and sweep time.