ALPHA-DRIVEN FAST MAGNETOSONIC WAVE HEATING IN TOKAMAK PLASMAS

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An alpha-driven fast magnetosonic wave instability is investigated in tokamak plasmas for propagation transverse to the external magnetic field at frequencies several times the alpha gyrorate. A two-dimensional differential quasi-linear diffusion equation is derived in cylindrical $v_{\perp} - v_{\parallel}$ geometry. The quasi-linear diffusion coefficients in the small parameter $k_{\parallel} k_{\perp}$ are expanded and the problem is reduced to one dimension by integrating out the $v_{\parallel}$ dependence. Reactor relevant information is obtained using data from the one-dimensional formulation in a 1-1/2-dimensional tokamak transport code. Contour plots of the alpha threshold fraction are used to identify the instability regions in the $n_e - T_i$ plane. Alpha/background electron fractions as low as 10^{-6} to 10^{-4} may trigger the instability. For a typical reactor-size tokamak, an enhancement of the fraction of the alpha energy transferred to ions by as much as 1.5 can occur for $T_i = T_e$ at 7 keV. Still, due to the rapid equilibration of electron and ion temperatures, a <1 to 2% increase in fusion power occurs overall.

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Abstract--An alpha-driven fast magnetosonic wave instability is investigated in tokamaks. Contour plots of the alpha threshold fraction are used to identify instability regions in the n_e - T_i plane.

Introduction--We investigate the potential effect of an alpha-particle-driven fast magnetosonic (FMS) wave instability¹⁻⁴ in a D-T fueled tokamak. A perpendicularly propagating ($k_{\parallel} \ll k_{\perp}$) FMS wave is assumed. We employ an isotropic alpha birth distribution to calculate the energy released during quasi-linear diffusion in velocity space. Results are used in a 1 1/2-D transport code to scope potential reactor effects.

Quasi-Linear Diffusion--We take an FMS wave propagating at an arbitrary angle to the applied toroidal magnetic field, and employ circular cylindrical $v_{\perp}$ - $v_{\parallel}$ coordinates. We approximate a cylindrical tokamak with Cartesian geometry ($r \gg 1/\kappa_1$). With $\hat{x} = \hat{r}$, $\hat{y} = \hat{\theta}$, and $\hat{z} = \hat{z}$, the general quasi-linear diffusion equation is:⁵ (nomenclature follows Ref 4)

$$\frac{\partial f_{\alpha}}{\partial \tau} = \frac{\pi q_{\alpha}^2}{m_{\alpha}} \sum_L \int d\vec{k} \frac{1}{v_{\perp}} \hat{R}_1 \{ [\dots] (\hat{R}_2 f_{\alpha}) \delta(\omega(\vec{k}) - L\Omega_{\alpha} - k_{\parallel} v_{\parallel}) \}$$

$$\hat{R}_1 = \hat{R}_2 = \left(1 - \frac{k_{\parallel} v_{\parallel}}{\omega} \right) \frac{\partial}{\partial v_{\perp}} + \frac{k_{\parallel} v_{\perp}}{\omega} \frac{\partial}{\partial v_{\parallel}}$$

$$[\dots] = v_{\perp} \left| E_r \frac{L}{|\lambda|} J_L + i \eta_{\alpha} E_{\theta} J_L' + \frac{v_{\parallel}}{v_{\perp}} E_z J_L \right|^2 \quad (1)$$

$$\lambda = k_{\perp} v_{\perp} / \Omega_{\alpha} \quad \eta_{\alpha} = q_{\alpha} / |q_{\alpha}| \quad J_L = J_L(\lambda)$$

With $k_{\parallel} = 0$ the dispersion relation is:
$$\Delta = \frac{c^2 k^2}{\omega^2} - \frac{\epsilon_1 \epsilon_2 - \epsilon_x^2}{\epsilon_1} \quad (2)$$

where ϵ_1 , ϵ_2 , and ϵ_x are elements of the dielectric tensor. (We reject the second solution to $\Delta=0$ for $\vec{k}=\vec{k}_{\perp}$.) Equation 2 is formally inverted to give $\omega=\omega(k)$. We approximate the spectral density of the electric field by a box function in k space and substitute these forms into Eq. (1) and associate the delta function with the integral over $k_{\parallel}$.

Diffusion Coefficients--Next, we expand the Bessel functions in the small parameter $k_{\parallel} v_{\parallel} / L\Omega_{\alpha}$, perform the integral over $k_{\parallel}$ exactly and expand the evaluated integral again in the same small parameter. In $v_{\perp}$ - $v_{\parallel}$ geometry the quasi-linear diffusion coefficients are dependent on $v_{\parallel}$ only to first order in $k_{\parallel}/k_{\perp}$. Physically this occurs because the instability ($\omega - L\Omega_{\alpha} = k_{\perp} v_A$ and $k_{\parallel} \ll k_{\perp}$) acts on the perpendicular energy.

The instability is favored at $r \rightarrow 0$ where the fusion rate is maximum. Then

unstable harmonic numbers permitted in a tokamak magnetic field collapse to a single integer⁶; therefore, the diffusion coefficients are restricted to the same harmonics, and to zero order in $k_{\parallel}/k_{\perp}$:

$$D_{\perp} \approx k_{\parallel} v_{\perp} L \Omega_{\alpha} \{ \dots \} \Big|; \quad D_x \approx \frac{1}{2} k_{\parallel}^2 v_{\perp}^2 \{ \dots \} \Big|; \quad D_{\parallel} \approx \frac{1}{3} \frac{k_{\parallel}^3 v_{\perp}^3}{L \Omega_{\alpha}} \{ \dots \} \quad (3)$$

with

$$\{ \dots \} = \frac{v_A^2}{v_{\perp}^2} E_r^2 J_L^2 + E_{\theta}^2 (J_L')^2$$

The ordering of the diffusion coefficients is found to be:

$$D_{\perp} : D_x : D_{\parallel} \approx 1 : \frac{1}{2} 0 \left(\frac{k_{\parallel}}{k_{\perp}} \right) : \frac{1}{3} 0 \left(\frac{k_{\parallel}^2}{k_{\perp}^2} \right) \quad (4)$$

FMS Wave Growth and Damping--Next we require an expression for the alpha growth rate, γ_{α} , and plasma damping of the wave. Our derivation generalizes prior results³ for propagation in the $k_{\perp}$ plane at an arbitrary angle to the radial vector, and also includes the effect of relative polarization between the radial and poloidal electric fields vs. ω . Neglecting terms proportional to $k_{\parallel}$ we obtain:

$$\gamma_{\alpha} \approx \frac{-1}{\Delta' (1 + p_{r\theta}^2)} \frac{\omega_{p\alpha}^2}{k_{\perp}^2} \int d\vec{v} \frac{1}{v_{\perp}} \frac{\partial \hat{f}_{\alpha}}{\partial v_{\perp}} R(v_{\perp}, \omega) \quad (5a)$$

and

$$R(v_{\perp}, \omega) = \sum_L \frac{L \Omega_{\alpha 0}}{\omega} \frac{\left[J_L^2 + p_{r\theta}^2 \frac{v_{\perp}^2}{v_A^2} (J_L')^2 \right]}{\sqrt{\epsilon^2 - \left[1 - \frac{L \Omega_{\alpha 0}}{\omega} \right]^2}} \quad (5b)$$

with

$$\frac{\omega}{\Omega_{\alpha}} (1 - \epsilon) < L < \frac{\omega}{\Omega_{\alpha}} (1 + \epsilon) \quad (5c)$$

Dependence on $v_{\parallel}$ in Eqs. (5) appears only in the normalized alpha distribution function $\hat{f}_{\alpha}/n_{\alpha}$. If the instability operates predominantly along $v_{\perp}$, we may integrate out the $v_{\parallel}$ dependence. Such a condition exists for the isotropic alpha source without loss cones, which to a first approximation is assumed here.

Threshold Studies--We define threshold fraction, η_T , as the ratio of alpha-to-electron density at marginal stability, i.e., when the sum of the alpha-driven growth and damping rates equal zero. We assume the device parameters (major and minor radii and magnetic field), radial position and wave frequency are fixed and η_T is computed vs. density and ion temperature, giving threshold contours in the n_e - T_i plane. Typical results for a demonstration-sized reactor (FED⁷) are presented in Fig. 1. The threshold contours are incremented by units of -1 since we have plotted $\log_{10}(\eta_T)$. The heavy outer boundary labeled "0.0" is defined as the limit outside of which the FMS wave is stable. At a fixed n_e , η_T increases with an increased T_i due to Doppler broadening of the alpha source function. Strong variations in η_T along the n_e axis at fixed T_i arise from successive tuning and detuning due to the dependence of the growth and damping rates (γ_{α}) on the Alfvén speed. We find that the instability

heating is strongest where the threshold fraction is a minimum (~ 10 keV in Fig. 5). Thus, the effect will be strongest for a low $\langle \beta \rangle$ device, where the magnetic field remains relatively strong on axis.

One-Dimensional Quasi-Linear Approximation--Another objective is to assess reactor level effects. We reduce the 2-D quasi-linear diffusion to 1-D with mild restrictions. The final set of 1-D quasi-linear equations is solved using a code package described elsewhere.⁶ We use representative reactor parameters:⁷ a toroidal magnetic field of 42 kG on axis, $n_e \sim 4.75 \times 10^{13} \text{ cm}^{-3}$ at 5 cm and $T_e \sim T_i \sim 10$ keV. Then $n_T \sim 3.55 \times 10^{-6}$ for $\omega = 6\Omega_\alpha$ and the total wave damping rate is $\gamma_d \approx 437 \text{ sec}^{-1}$. The enhancement factor, p_{iV} , is the ratio of the time averaged alpha ion heating rate in the presence of the instability vs. that with classical slowing. For present plasma conditions the maximum p_{iV} is ~ 1.3 .

A typical time history is shown in Fig. 2. The total growth rate γ_T is plotted along with the wave energy density and the response of p_{iV} . At time $\tau=0$, $\gamma_T=0$ and the wave electric field E_1 is set arbitrarily low so that quasi-linear diffusion is negligible. As the alpha densities increases, γ_T increases since the damping rate (dependent on the background plasma conditions) remains constant. Then, E_1 grows exponentially until D_1 , which depends on E_1 , becomes large enough to change the alpha profile. As the profile flattens, the growth rate which depends on the slope in $f_\alpha(v_1)$ drops. As the instability continues into saturation, $f_\alpha(v_1)$ diffuses sufficiently that $\gamma_\alpha < \gamma_d$ and the wave decays via cyclotron and collisional damping. After the distribution collapses, the process begins anew and repeats itself after ~ 15 ms.

Reactor Simulation--We use data from the lowest "lobes" of the contours in Fig. 1 (i.e., at $n_e \approx 5 \times 10^{13} \text{ cm}^{-3}$) to examine effects in a demonstration-type reactor. The data were incorporated in 1 1/2-D tokamak transport code and a typical result is shown in Fig. 3. Contours of the auxiliary power (P_{aux}) required to hold a given $\langle T \rangle$ are plotted. The instability occurs in the shaded region, i.e., during the startup phase at moderate density-temperature. The rather restricted instability region is encouraging from a control point of view and could be avoided with an appropriate startup trajectory.

Summary and Conclusion--Generally the threshold is smallest for relatively low electron density, ion temperature, and β , although values do extend over a fairly wide range of parameters (Fig. 1). Furthermore, cf Fig. 2, when the instability is triggered, a significant collapse of the alpha distribution occurs. A repeated buildup and collapse is observed with a period of ~ 15 ms. Since the energy fluctuation is $< 1\%$ of the background plasma energy, this is not viewed as disruptive. In fact, the increase in alpha energy transferred to background ions (versus electrons) is desirable. Also, the reduction in the time-averaged alpha pressure would be beneficial. However, cf Fig. 3, the region where the instability occurs is quite isolated.

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Fig. 1. Alpha threshold fraction (n_α) contours for the FMS wave in the FED: $\omega=6\Omega_\alpha=k_\perp v_A$, $k_\parallel \ll k_\perp$, $B_0 = 42$ kG, $r=10$ cm.

Fig. 2. Time histories of instability, $[\circ]=\text{total growth rate } \gamma_T=\gamma_\alpha+\gamma_D$, $[\square]=\text{wave energy } W_E(t)$, $\nabla=\text{density}$, $X=\text{relative error in system energy}$, $\Delta=\text{enhancement factor } p_{iv}$. The normalizations are $\log_{10}(\text{max. } W_E) = 1.992$ and $\log_{10}(\text{max. } \gamma_T) = 3.022$

Fig. 3. External power $p_{\text{aux}}^{\text{eq}}$ required to hold a given $\langle n \rangle$ - $\langle T \rangle$ operating point using Alcator scaling and $B_0 = 42$ kG. Alpha instability occurs in the shaded region.