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By

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ABSTRACT

Pion condensates in neutron-star matter, formed either as new modes, or on states identifiable with those of the free pion are studied. A description of neutron-star matter at finite temperature is formulated upon a suitable basis of realistic interactions in a modified background field description, and leads to the onset of a pion condensate between the density of nuclear matter and the density of free hadrons. This condensate, however, is blocked when strange hadrons are incorporated in the description.

INTRODUCTION

There are two principal motives for interest in pion condensates in neutron matter. The first motive is that neutron star luminosities due to neutrino cooling processes, given by Bahcall and Wolff¹ as

$$L_{\nu}^{\pi n} \sim 2 \times 10^{52} (\text{erg/sec.}) \frac{n_{\pi}}{n_b} \frac{m_s}{m_{\odot}} T_{\text{MeV}}^6 \quad (1)$$

and

$$L_{\nu}^{\text{nne}} \sim 2 \times 10^{47} (\text{erg/sec.}) \left(\frac{\rho_N}{\rho} \right)^{1/3} \frac{m_s}{m_{\odot}} T_{\text{MeV}}^8$$

for $n + \pi \leftrightarrow n + l + \bar{l}$ and $n + n \leftrightarrow n + n + l + \bar{l}$ processes respectively, indicate that pions provide the dominant cooling channels for low temperatures. While the observational data is consistent with improved cooling calculations in the absence of pions, the rapid cooling possible in the pion channel might be necessitated by lower limits on the surface temperatures of neutron stars².

The second motive is that the pions are charged bosons. Thus a pion condensate might avoid the exclusion energy contributions which charged leptons and baryons would make to the pressure and energy. Such contributions could seriously affect the bulk properties of the matter, leading to observable

consequences for neutron stars³, and perhaps for large nuclei as well⁴. These contributions would be especially drastic if the dominant pion interactions were the attractive ones favoring condensate formation.

This paper is divided into two sections. The first section will discuss a perturbative argument, based upon the vacuum polarization contribution to the charged pion propagator, which suggests that new modes, analogous to zero sound, may be present in dense matter. The second section will present a realistic description of pion and nucleon interactions suitable for use in a modified background field description, and leading, in such a description, to the formation of a condensate in the negative pion mode.

NEW MODES

Interactions should modify the pion propagators in a density dependent fashion. This is strikingly demonstrated by the generation of new modes, similar to zero sound, if the pion is coupled through an intermediate axial vector field to neutrons and protons in neutral matter composed of neutrons, protons, electrons, and muons. The axial vector interaction is

$$L_{IA} = \sigma i \partial_\mu \pi_j A_j^\mu + A_j^\mu \bar{\psi} \gamma_\mu \gamma_5 \frac{1}{2} \psi. \quad (2)$$

With this interaction and using the finite density propagators given by Bowers and Zimmerman⁵, the strong one-loop contribution to the π^- vacuum polarization is:

$$\Pi(\omega, k) = \frac{g^2(K)}{\pi^2} m_n^2 R \left\{ \int_0^1 dz \ln[(1-4yR)/(1-ym_\pi^2/m_n^2)] \right\} \quad (3)$$

$$+ \frac{g^2(K)}{\pi^2} \int p^2 dp \left\{ \frac{\omega}{2m_n^2} (n_n - \bar{n}_n - n_p + \bar{n}_p) \right. \\ \left. - \frac{K^2}{4pkE} [Q^-(n_p + \bar{n}_n) + Q^+(n_n + \bar{n}_p)] \right\} \\ + \frac{ig^2(K)}{\pi} K^2 \int \frac{p^2 dp}{4EpK} \{ [\theta(\alpha^+ + 1) - \theta(\alpha^+ - 1)] \cdot [\theta(E + \omega)(n_p(E + \omega)$$

$$+ n_n(E) - 2n_p(E + \omega)n_n(E)) + \theta(-E - \omega)(\bar{n}_p(-E - \omega) + n_n(E) - 2\bar{n}_p(-E - \omega)n_n(E))]$$

$$+ [\theta(\alpha^- + 1) - \theta(\alpha^- - 1)] \cdot [\theta(\omega - E)(n_p(\omega - E) + \bar{n}_n(E) - 2n_p(\omega - E)\bar{n}_n(E))$$

$$+ \theta(E - \omega)(\bar{n}_p(E - \omega) + \bar{n}_n(E) - 2\bar{n}_p(E - \omega)\bar{n}_n(E))]\}$$

where

$$K^2 = \omega^2 - k^2, \quad R = \frac{K^2}{4m_n^2}, \quad y = z(1-z), \quad E = (p^2 + m_n^2)^{\frac{1}{2}},$$

$$\alpha^\pm = \frac{K^2 + 2mE}{2pk}, \quad \text{and } Q^\pm = \ln \left[\frac{\alpha^\pm + 1}{\alpha^\pm - 1} \right].$$

The effective coupling constant $g^2(k)$ contains the axial vector propagator $g^2(K) = g_\pi^2 / (1 - m_A^2/K^2)$ and $n_p, n_n, \bar{n}_p, \bar{n}_n$ refer to the proton, neutron, anti-proton, and anti-neutron fermi distributions respectively. This is the relativistic counterpart of the fermion loop contribution which in earlier analysis of pion condensation⁶ was examined in a non-relativistic perturbation theory. Before considering the effects of the vacuum polarization it is noteworthy that the pole structures of the $Q^\pm$

terms (poles at $K^2 \pm 2\omega E = \pm 2pk$) immediately indicate the inappropriateness of a non-relativistic energy momentum relationship in the fermion variables.

The zeros of the inverse propagator, $G^{-1} = \omega^2 - k^2 - \Pi$, are the poles of the Green's function corresponding to $(\omega > 0, \frac{\partial G^{-1}}{\partial \omega} > 0)$ the negative pion, and to $(\omega < 0, \frac{\partial G^{-1}}{\partial \omega} < 0)$ the positive pion. Below the threshold for nucleon pair creation this vacuum polarization smoothly deforms the inverse propagator, implying that, with one exception, new states must appear in the propagator in pairs, at least one of which must have a ghost like residue. The exception is the dipole state for which $G^{-1} = \frac{\partial G^{-1}}{\partial \omega} = 0$, which is the tangent state with infinite residue. Numerical evaluations of the vacuum polarization indicate that the new states will enter at one-third of nuclear density.

The breakdown implied by the emergence of new modes is model dependent, and does not occur for the pseudo-scalar coupling. It also occurs in several other cases. The present calculation can be extended to the neutral pion, with similar results, and Pedigo⁶ has shown that the nucleon propagators, calculated to the same order, also develop ghosts at low density.

The mathematically indeterminate nature of these new states makes their interpretation impossible, and the absence of experimental verification of their presence has lead to newer calculations in which more effects have been included⁷. These safely remove the onset of these new states to densities beyond the current limits of observability. The densities to which the onset of these new states are removed however is a domain which should be dominated by the composite nature of hadrons. This

composite nature must be a salient feature of any description above two-and-one-third times nuclear density, and such calculations are thus inappropriate.

CONDENSATION IN MODIFIED FREE PION MODES

The second approach to pion condensation treats the pion as a bosonic competitor of the leptons⁸. Under this view it is not a new "zero-sound" state which is occupied, but a continuously modified free pion state. The simplest example of this possibility is the 139.6 MeV ceiling placed upon the electron chemical potential by the pion mass in non-interacting neutral matter composed of neutrons, protons, electrons, muons, and pions. This ceiling would be reached at four times nuclear density. Given the attractive component of the pion-nucleon interactions, the effects of interactions may lower this ceiling, and their effects should be examined in more detail.

The extent of a neutron star, and the extremely low density gradients which are expected make mean-field theory the most attractive many-body theory available. Framing such a description in a manner which includes pions poses problems, however. In the absence of a shift in the nucleon mass from scalar to pseudoscalar, a shift which does not adequately describe the nuclear dynamics, the simple Yukawa coupling leads to a background field equation

$$m_{\pi}^2 \langle g_{\pi} \pi \rangle = -g_{\pi}^2 |\langle \bar{\psi} i \gamma^5 \psi \rangle| = -g_{\pi}^2 \langle g_{\pi} \pi \rangle \int dp p^2 (n + \bar{n}) / (\pi^2 E) \quad (4)$$

which can only be satisfied by a vanishing pion expectation value. A second difficulty is that a mean-field description is a semi-classical description, and thus deals with only the two-point content of some more comprehensive, but unspecified theory.

Thus the embodiment of the mean-field description in a Lagrangian form must avoid explicitly non-Gaussian terms; the Lagrangian must be at most quadratic in the fields.

Neutron and proton fields may be combined quadratically to form isoscalar and isovector operators. The pion fields, when combined quadratically, match these with only an isoscalar scalar $\pi^i \pi^i$, and an isovector vector $\epsilon^{ijk} \pi^j \partial_\mu \pi^k$. The simplest description of pion-nucleon scattering will therefore contain only an intermediate isoscalar scalar, and an intermediate isovector vector, which imply only two low energy coupling parameters. If the effective interaction Lagrangian is:

$$L_{\text{eff.}} = \frac{1}{2} \alpha g_S \pi^a \pi^a (\bar{P}P + \bar{N}N) + g_I [\pi^+ \overleftrightarrow{\partial}_\mu \pi^- (\bar{P}\gamma^\mu P - \bar{N}\gamma^\mu N) - \sqrt{2} \pi^+ \overleftrightarrow{\partial}_\mu \pi^0 \bar{N}\gamma^\mu P + \sqrt{2} \pi^+ \overleftrightarrow{\partial}_\mu \pi^0 \bar{P}\gamma^\mu N] \quad (5)$$

then the cross sections for pion-nucleon scattering are:

$$\sigma(\pi^+P) = \frac{1}{8\pi(E+\omega)^2} \{ \alpha^2 g_S^2 (E^2+m^2) + 2g_I^2 [(2E\omega+p^2)^2 - p^2(\omega^2+\mu^2)] + 4\alpha g_S g_I (2E\omega+p^2) \} \quad (6)$$

$$\sigma_{\text{el}}(\pi^-P) = \frac{1}{8\pi(E+\omega)^2} \{ \alpha^2 g_S^2 (E^2+m^2) + 2g_I^2 [(2E\omega+p^2)^2 - p^2(\omega^2+\mu^2)] - 4\alpha g_S g_I (2E\omega+p^2) \}$$

$$\sigma_{\text{tot}}(\pi^-P) = \frac{1}{8\pi(E+\omega)^2} \{ \alpha^2 g_S^2 (E^2+m^2) + 6g_I^2 [(2E\omega+p^2)^2 - p^2(\omega^2+\mu^2)] - 4\alpha g_S g_I (2E\omega+p^2) \},$$

where p is the momentum in the center of mass frame, $E = (p^2+m^2)^{\frac{1}{2}}$ is the nucleon energy, and $\omega = (p^2+\mu^2)^{\frac{1}{2}}$ is the pion energy. These expressions fit the observed cross sections⁹, with the delta

resonances removed, for $\alpha g_s = 2.426 \times 10^{-3} \text{ MeV}^{-1}$ and $g_I = 3.075 \times 10^{-5} \text{ MeV}^{-2}$. If the intermediate vector particle is assumed to be the rho meson with

$$L_{I\rho} = g_\rho \rho_\mu^i \epsilon^{ijk} \pi \partial^\mu \pi + g_\rho \rho_\mu^i \bar{\psi} \gamma^\mu \frac{\tau^i}{2} \psi, \quad (7)$$

then the rho mass of 776 MeV implies that $g_\rho^2/(4\pi) = 2.95$. If the isoscalar interaction is similarly moderated by a particle, then it may be written

$$\alpha g_\sigma \sigma \pi^i \pi^i / 2 + g_\sigma \sigma \bar{\psi} \psi \quad (8)$$

with $g_s = g_\sigma^2/m_\sigma^2$. In this form it will contribute to $(\pi^+ \pi^+)$ scattering, an additional process which will be used to further improve the description. A four point interaction is already implied by higher order applications of the interactions under consideration, and should be given an independent strength, as it will require independent regularization. In order to satisfy the quadratic limitation upon the Lagrangian, and thus close the description at the level of the two-point Green's functions, this interaction will be included in the form

$$L'_I = -A_\pi \pi^i \pi^i / 2 + A_\pi^2 / (2\lambda). \quad (9)$$

The differential cross section for $(\pi^+ \pi^+)$ scattering is then

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \left[\frac{\alpha^2 g_\sigma^2}{t - m_\sigma^2} + \frac{\alpha^2 g_\sigma^2}{u - m_\sigma^2} + 2\lambda + \frac{g_\rho^2 (s-u)}{m_\rho^2 - t} + \frac{g_\rho^2 (s-t)}{m_\rho^2 - u} \right]^2 \quad (10)$$

where s , t , and u are the usual Mandelstam variables. The zeroeth and second scattering lengths are then

$$8\pi m_\pi a_0 = (-\alpha^2 g_\sigma^2 / m_\sigma^2 + \lambda + 4g_\rho^2 m_\pi^2 / m_\rho^2), \quad m_\pi a_2 = 0.$$

The standard value¹⁰ $|2a_0 + a_2| = (.4 \pm .1) m_\pi^{-1}$, in conjunction with the value of g_ρ derived above then gives $\lambda - \alpha^2 g_\sigma^2 / m_\sigma^2 = .2339$.

In previous work¹¹ scalar and vector interactions were

used to describe infinite nuclear matter. The present scalar field may be associated with that scalar field, or a new field may be introduced. As the two schemes yield virtually identical results, only the former will be considered in this presentation. In that case the previous low energy coupling, fit to the binding energy and density of infinite nuclear matter, $g_S = g_\sigma^2/m_\sigma^2 = 3.023 \times 10^{-4} \text{ MeV}^{-2}$, implies through the scattering formulas that α is 8.025 MeV, and that λ is .2534. The previously determined value of $g_V^2/m_V^2 = 2.217 \times 10^{-4} \text{ MeV}^{-2}$ finishes the determination of the interaction Lagrangian:

$$\begin{aligned}
 L_I = & g_\sigma \alpha \sigma \pi^a \pi^a / 2 + g_\sigma \bar{\psi} \psi + g_V V_\mu \bar{\psi} \gamma^\mu \psi - \pi^a \pi^a \Lambda_\pi / 2 + A_\pi^2 / (2\lambda) \\
 & + g_\rho \rho_\mu^0 [\pi^+ i \vec{\partial}_\mu \pi^- + (\bar{P} \gamma_\mu P - \bar{N} \gamma_\mu N) / 2] + g_\rho \rho_\mu^+ [\pi^- i \vec{\partial}_\mu \pi^0 - \bar{P} \gamma_\mu N / \sqrt{2}] \\
 & + g_\rho \rho_\mu^- [\pi^0 i \vec{\partial}_\mu \pi^+ - \bar{N} \gamma_\mu P / \sqrt{2}] + g_\rho^2 [\rho_\mu^0 \rho_\mu^0 \pi^+ \pi^- - (\rho_\mu^+ \rho_\mu^- \pi^- \pi^- + \rho_\mu^- \rho_\mu^+ \pi^+ \pi^+) / 2 \\
 & - \rho_\mu^0 \pi^0 (\rho_\mu^+ \pi^- + \rho_\mu^- \pi^+) + \rho_\mu^+ \rho_\mu^- (\pi^+ \pi^- + \pi^0 \pi^0)].
 \end{aligned} \tag{11}$$

These interactions neglect the structure of the intermediate fields entirely. This would be catastrophic if scattering processes involving those states in final or initial states were contemplated, but in the thermodynamic regime under consideration, at temperatures and mode energies far less than the masses of the rho, epsilon, delta, or omega mesons, any effects based upon their internal structures should be subject to doubt. This scheme avoids the inevitable counting questions provoked by highly non-linear strong coupling effects, and accounts for all of the relevant interaction channels in an empirical manner.

The standard electromagnetic interactions are:

$$L_{em} = eA_\mu [\bar{P}\gamma^\mu P - \bar{e}\gamma^\mu e - \bar{\mu}\gamma^\mu \mu + \pi^{+\dagger} \overset{\leftrightarrow}{\partial}^\mu \pi^-] \quad (12)$$

$$+ e^2 A_\mu A^\mu \pi^+ \pi^- + 2eg_\rho A_\mu \rho_0^\mu \pi^+ \pi^- - eg_\rho A_\mu \pi^0 (\rho_+^\mu \pi^- + \rho_-^\mu \pi^+).$$

This Lagrangian will be used in the background field approximation, under the assumptions that the background fields are spatially isotropic and temporally static. The first variations of the Lagrangian lead to the background field equations:

$$-m_\sigma^2 g_\sigma \sigma + \alpha g_\sigma^2 \pi^a \pi^a / 2 + g_\sigma^2 \bar{\psi} \psi = 0 \quad (13)$$

$$m_\rho^2 (g_\rho \rho_0^0) + g_\rho^2 [\pi^{+\dagger} \overset{\leftrightarrow}{\partial}_0 \pi^- + 2(g_\rho \rho_0^0 + eA_0) \pi^+ \pi^- + (\bar{P}\gamma_0 P - \bar{N}\gamma_0 N) / 2 - g_\rho \pi^0 (\rho_+^0 \pi^- + \rho_-^0 \pi^+)] = 0$$

$$m_V^2 (g_V V^0) + g_V^2 [\bar{P}\gamma^0 P + \bar{N}\gamma^0 N] = 0$$

$$e^2 [\bar{P}\gamma^0 P - \bar{e}\gamma^0 e - \bar{\mu}\gamma^0 \mu + \pi^{+\dagger} \overset{\leftrightarrow}{\partial}_0 \pi^- + 2(eA^0 + g_\rho \rho_0^0) \pi^+ \pi^-] - eg_\rho \pi^0 (\rho_+^0 \pi^- + \rho_-^0 \pi^+) = 0$$

$$A_\pi = \lambda \pi^a \pi^a / 2.$$

As previously noted, the heavy meson fields will not participate dynamically. The electromagnetic field will participate, and its classical value will be written as A_c to distinguish it from its excitations, which will be written A_{th} .

The second variations of the Lagrangian with respect to the participating fields yield the following quasi-free inverse propagators:

$$G_P^{-1}(x-y) = [i\cancel{\partial} + g_I \cancel{\partial} / 2 + e\cancel{A} + g_V \cancel{V} - (m_N - g_\sigma \sigma)] \delta^4(x-y) \quad (14)$$

$$G_N^{-1}(x-y) = [(i\cancel{\partial} - g_\rho \cancel{\partial}_0 / 2 + g_V \cancel{V} - (m_N - g_\sigma \sigma)] \delta^4(x-y)$$

$$G_e^{-1}(x-y) = (i\cancel{\partial} - e\cancel{A} - m_e) \delta^4(x-y); \quad G_\mu^{-1}(x-y) = (i\cancel{\partial} - e\cancel{A} - m_\mu) \delta^4(x-y)$$

$$G_{\pi^+}^{-1}(x-y) = [(i\partial_\mu + eA_\mu^C + g_\rho \rho_\mu^0)^2 - (m_{\pi^+}^2 - g_\sigma \alpha \sigma + A_\pi - e^2 \langle A_\mu A^\mu \rangle_{th})] \delta^4(x-y)$$

$$G_{\pi^0}^{-1}(x-y) = [(i\partial_\mu)^2 - (m_{\pi^0}^2 - g_\sigma \alpha \sigma + A_\pi)] \delta^4(x-y)$$

$$G_{\mu\nu}^{-1}(\gamma)(x-y) = [(i\partial_\rho)^2 g_{\mu\nu} - i\partial_\mu i\partial_\nu - \xi^{-1} t_\mu t_\nu + 2e^2 \pi^+ \pi^- g_{\mu\nu}] \delta^4(x-y).$$

The thermodynamic propagator $\langle \pi^-(x) \pi^+(y) \rangle$ derived from the charged pion inverse propagator is mathematically bounded only if the vector contribution to the pion vector mode energy satisfies

$$(g_\rho \rho_0^0 + eA_C^0)^2 \leq m_{\pi^+}^2 - g_\sigma \alpha \sigma + A_\pi - e^2 \langle A_\mu A^\mu \rangle_{th}. \quad (15)$$

If this condition is not met by the thermal value of $A_\pi = \lambda(\langle \pi^+ \pi^- \rangle + \langle \pi^0 \pi^0 \rangle / 2)$, then charged pions will condense enforcing equality in this vector mode energy condition. The expectation value $\langle \pi^+ \pi^- \rangle$ will be composed of both thermal and condensed contributions, and the condensed contributions will naturally be included whenever $\langle \pi^+ \pi^- \rangle$ contributes, for example in the charged pion current, and in the effective mass of the electromagnetic fluctuations.

The self-consistent solution of the background field equations allows, in conjunction with the associated Green's functions, the computation of the stress energy tensor

$$T^{\mu\nu} = \frac{\delta L}{\delta \partial_\mu \psi} \partial^\nu \psi - g^{\mu\nu} L \equiv t^{\mu\nu} - g^{\mu\nu} L. \quad (16)$$

The assumed isotropy of the solutions imply that only the t^{00} and

t_j^j Legendre terms contribute to $T^{\mu\nu}$. These terms are:

$$t_j^j = \langle \bar{P} i \gamma^j \partial_j P \rangle + \langle \bar{N} i \gamma^j \partial_j N \rangle + \langle \bar{e} i \gamma^j \partial_j e \rangle + \langle \bar{\mu} i \gamma^j \partial_j \mu \rangle \quad (17)$$

$$+ 2 \langle \partial_j \pi^+ \partial^j \pi^- \rangle + \langle \partial_j \pi^0 \partial^j \pi^0 \rangle + e^{-2} \langle \partial_j A_\mu \partial^j A^\mu \rangle_{th}, \text{ and}$$

$$t^{00} = -t_j^j + m_{Ne} [\langle \bar{P} P \rangle + \langle \bar{N} N \rangle] + m_e \langle \bar{e} e \rangle + m_\mu \langle \bar{\mu} \mu \rangle + 2m_{\pi^+e} \langle \pi^+ \pi^- \rangle \quad (18)$$

$$+ 2m_{\pi^0 e} \langle \pi^0 \pi^0 \rangle / 2 + 2m_{\gamma e}^2 [\langle A_\mu A^\mu \rangle_{th} / (2e^2)] - g_V V_C^0 [\langle \bar{P} \gamma^0 P \rangle + \langle \bar{N} \gamma^0 N \rangle]$$

$$- g_\rho \rho_C^0 \{ (\langle \bar{P} \gamma^0 P \rangle - \langle \bar{N} \gamma^0 N \rangle) / 2 + \langle \pi^+ i \partial^0 \pi^- \rangle + 2(g_\rho \rho_C^0 + e A_C^0) \langle \pi^+ \pi^- \rangle \}$$

$$- e A_C^0 [\langle \bar{P} \gamma^0 P \rangle - \langle \bar{e} \gamma^0 e \rangle - \langle \mu \gamma^0 \mu \rangle + \langle \pi^+ i \partial^0 \pi^- \rangle + 2(g_\rho \rho_C^0 + e A_C^0) \langle \pi^+ \pi^- \rangle]$$

and the expectation value of the Lagrangian is:

$$\langle L \rangle = A_\pi (\langle \pi^+ \pi^- \rangle + \langle \pi^0 \pi^0 \rangle) / 2 - g_V V_C^0 [\langle \bar{P} \gamma^0 P \rangle + \langle \bar{N} \gamma^0 N \rangle] / 2 \quad (19)$$

$$- g_\rho \rho_C^0 \{ (\langle \bar{P} \gamma^0 P \rangle - \langle \bar{N} \gamma^0 N \rangle) / 2 + \langle \pi^+ i \partial^0 \pi^- \rangle + 2(g_\rho \rho_C^0 + e A_C^0) \langle \pi^+ \pi^- \rangle \} / 2$$

$$- g_\sigma \sigma [\alpha (\langle \pi^+ \pi^- \rangle + \langle \pi^0 \pi^0 \rangle / 2) + \langle \bar{P} P \rangle + \langle \bar{N} N \rangle] / 2 - \langle A_\mu A^\mu \rangle_{th} \langle \pi^+ \pi^- \rangle.$$

The energy density $E = T^{00}$ and the pressure $P = -T^i_i / 3$, are thus specified, and the Gibbs-Duheim relation implies that the entropy per baryon is $s = (E + P - \mu_B N) / (N \cdot T)$, where μ_B is the external baryon chemical potential, and N is the baryon number density.

The isothermal and adiabatic equations of state are shown in figures one, two, and three. The vector mode energy conditions for condensation and the resultant species densities

Figure 1: The adiabatic pressure is shown as a function of the baryon number density for entropies per baryon of (A-K) .1,1,2,3,...10 k_B .

are shown in figures four through seven for temperatures of 1 and 100 MeV. Increased temperatures raise the density at which condensation occurs and decrease the range over which it is present, with the Curie point coming at a density of $10^{40.25}$ baryons per cm^3 , and a temperature of 224 MeV.

Figure 2: The adiabatic pressure is shown as a function of the energy density for entropies per baryon of (A-K) $.1, 1, 2, 3, \dots, 10k_B$.

These competitive condensates are not radical in their origins. It is not necessary to introduce new modes, but only to supply a vector mode energy for charged pions matching the effective charged pion mass. As the vector mode energy of the

Figure 3: The isothermal pressure is shown as a function of the baryon number density for temperatures of (A-K) 1, 10, 20, 30...100 MeV.

neutral pion is zero, and as the pion masses are constrained to remain above a minimum value for which the scalar attraction would cancel the nucleon mass, this form of condensate will never extend to neutral pions.

Figure 4: The electron vector mode energy, negative pion vector mode energy, the neutral and charged pion effective masses, and the effective mass of the electromagnetic field are shown as functions of the baryon number density at a temperature of 1 MeV.

If this description is extended to include the entire SU(3) baryon octet, however, then the competitive pion condensate proves to be insufficiently competitive. Under a variety of

Figure 5: The species current densities are shown as functions of the baryon number density at a temperature of 1 MeV. The species are: (A-E) neutrons, protons, electrons, muons, and negative pi mesons.

strong interaction schemes ranging from no interactions through entire nonets of scalar and vector mesons, the formation of negative sigma baryons competitively excludes the formation of a negative condensate of pions¹². The case with no interactions

Figure 6: The electron vector mode energy, negative pion vector mode energy, the neutral and charged pion effective masses, and the effective mass of the electromagnetic field are shown as functions of the baryon number density at a temperature of 100 MeV.

illustrates the nature of the competition. The proton potential must be at least 940 MeV, this implies that the neutron potential must be greater than 1080 MeV for the negative pion potential to

Figure 7: The species current densities are shown as functions of the baryon number density at a temperature of 100 MeV. The species are: (A-E) neutrons, protons, electrons, muons, and negative pi mesons.

be 140 MeV. The sigma-minus potential would then be 1220 MeV, greater than the sigma-minus mass. The presence of stable sigma-minus particles implied by this situation indicates that their neglect is critical to the formation of a condensate.

Considering this competitive situation in the context of hadron structure, the formation of strange quarks is favored over the formation of anti-up quarks.

There were two principle motives for interest in pion condensation in neutron star matter. Both of these motives can be met by a more detailed model using the full spectrum of available hadrons. The first motive was that the dominant cooling channels at low temperature were provided by pion processes. The larger phase space accompanying an expanded baryon spectrum would allow even more efficient cooling in the baryon-baryon channels which presently suffice to explain the observational limits on neutron star surface temperatures.

The second motive was that pions might avoid the exclusion energy contributions which would otherwise be made by charged fermions, thus leading to a softer pressure-energy relationship and a decreased upper limit for the masses of neutron stars. This motive is also substantially answered by an expanded baryon spectrum and phase space.

A third feature of a more complete model of hadrons, in particular one based on a constituent model, is a softening of the equation of state brought about by the lowering of the asymptotic relationship between the pressure and the energy. A constituent description of hadrons will be asymptotically free¹³, thus replacing the $P=E$ asymptotic limits expected for any description dominated by massive vector mesons with softer $P=E/3$ limits. Thus it appears that pion condensates may be unnecessary, and that they rest upon incomplete or mathematically indeterminate bases.

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