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ABSTRACT

It has recently been proposed that stochasticity resulting from toroidal coupling could lead to a saturation of the m = 1 internal mode in tokamaks. We present results from the nonlinear evolution of the m = 1 mode with toroidal coupling that show that stochasticity is not enough to cause saturation of the m = 1 mode.

I. INTRODUCTION

A characteristic of ohmically heated tokamak discharges are so-called "sawtooth" oscillations,[1-3] a repetitive pattern involving a gradual rise and sudden decline in the amplitude of soft x-ray signals. Earlier work has identified the $m/n = 1/1$ tearing mode as being responsible for the observed oscillations. The most frequently used explanation for the phenomenon is derived from single-helicity, cylindrical, linear, and nonlinear calculations.[4-6]

For discharges with a safety factor on axis, q_o, (the number of toroidal excursions needed to follow the magnetic field once around the poloidal direction) less than one, a helical perturbation at the $q = 1$ surface is linearly unstable for finite resistivity. An $m = 1$ magnetic island grows, continuously reconnecting flux within the central core until flattening the helical flux from the magnetic axis to the singular surface, thereby raising q everywhere to unity. A perturbation to the center of the discharge could initiate a thermal instability which would allow current to peak on axis again dropping q below unity and repeating the entire process.

Newer experimental results are not entirely consistent with the previously accepted explanation for sawtooth oscillations. The TFR group has noted the persistence of the $m/n = 1/1$ mode after the relaxation of the soft x-ray amplitude.[7] The TEXTOR group measures a q on axis of appreciably less than one, approximately 0.6. Both sets of results raise questions about the full reconnection picture.

Along with the newer experimental results have come more recent attempts to describe theoretically sawtooth oscillations. While earlier work was limited to pressureless plasmas in an infinite cylinder, recent numerical calculations have focussed on the effect of finite β (the ratio of thermodynamic pressure to magnetic field pressure) on both the linear and nonlinear evolution of the resistive $m/n = 1/1$ mode.[8] Using a reduced form of the MHD equations, the $m = 1$ island saturated for $q_o < 1$, but, sufficiently close to unity, and β sufficiently large. Cases studied include $q_o > 0.8$ and aspect ratio, $R_o/a = \varepsilon^{-1} > 4$.

Different results have been obtained by Lichtenberg.[9] Using a model equilibrium resembling TFR discharges, the disruptive phase of the sawtooth oscillation is attributed to the growth of a stochastic region surrounding the $m/n = 1/1$ mode. Nonlinear interactions between the toroidal equilibrium and harmonics of the 1/1 generate the stochastic field. Lichtenberg also suggests that for q_o sufficiently below unity ($q_o < 0.8$) the $m = 1$ does not completely reconnect all flux within the $q = 1$ surface. Rather, an increase in the size of the stochastic region inhibits the growth of the $m = 1$ island. The result is the survival of the central core of the discharge. This saturation is thought to be due to a diffusion of current caused by the stochasticity; and since he argues that high shear is more likely to have larger stochastic regions, low q_o will be more likely to saturate. This tendency is seen experimentally.

The purpose of our work is to extend the analysis performed previously to a consistent MHD treatment at low q_o. Using exact toroidal equilibria in conjunction with a consistent set of reduced MHD equations, we have studied the role of lower q_o and finite stochasticity in the growth and saturation of the $m = 1$ resistive tearing mode.

II. REDUCED MHD MODEL

We use the simplest set of reduced resistive MHD equations that have toroidal coupling and exhibit stochasticity. There are two such sets of equations. One, the so-called high beta set of Refs. 10 and 11, we reject because in experiments the pressure is quite low and does not satisfy the ordering assumed in their derivation. The other is that of Refs. 12 and 13, where the toroidal effects come through the variation in the moving magnetic field. In the derivation of these equations beta is assumed low, of the order of ε^2. Here, the expansion parameter is the inverse aspect ratio, $\varepsilon = a/R_o$ (Fig. 1). For completeness we display the equations below.

$$\vec{V}_\perp = \frac{R}{R_o} \nabla u \times \hat{\phi} \quad , \tag{1}$$

$$\vec{B} = \frac{R_o}{R} \nabla\psi \times \hat{\phi} + I_o \nabla\phi \quad , \tag{2}$$

$$\frac{Dp}{dt} = 0 \ , \tag{3}$$

$$\rho \frac{D}{\rho dt} \nabla^2 u - \frac{2}{R_o} \frac{\partial u}{\partial Z} \nabla^2 u = 2 \frac{\nabla R \times \nabla p}{R_o} \cdot \hat{\phi} + \vec{B} \cdot \nabla \Delta^* \psi \ , \tag{4}$$

$$\frac{\partial\psi}{dt} = \frac{R}{R_o} \left(\nabla\psi \times \hat{\phi} \cdot \nabla u + \frac{B_\phi}{R_o} \frac{\partial u}{\partial \phi} - \eta J_\phi \right) \quad , \tag{5}$$

$$J_\phi = \frac{-R_o}{R} \nabla^* \psi \quad , \tag{6}$$

$$\nabla^* \equiv R \frac{\partial}{\partial R} \frac{1}{R} \frac{\partial}{\partial R} + \frac{\partial^2}{\partial Z^2} \quad , \tag{7}$$

$$\nabla^2 \equiv \frac{1}{R} \frac{\partial}{\partial R} R \frac{\partial}{\partial R} + \frac{\partial^2}{\partial Z^2} \quad , \tag{8}$$

$$\frac{D}{dt} = \frac{\partial}{dt} + \vec{V} \cdot \nabla \quad . \tag{9}$$

Here ρ is the mass density, $\vec{v}$ is the fluid velocity, $\vec{J}$ is the plasma current, $\vec{B}$ is the magnetic field, p is the thermodynamic scalar pressure, γ is the ratio of specific heats, and η is the plasma resistivity. These equations have exact toroidal equilibria as steady-state solutions. They have been used to perform studies of the effect of toroidicity on resistive tearing modes with $m \geq 2$. A more complete discussion, as well as a derivation of these equations can be found in Ref. 12.

II. RESULTS

The linear and nonlinear evolution of the resistive $m = 1$ was studied for three cases. The q-profile is specified as

$$q(\psi) = q_o + (q_a - q_o)\psi^{\alpha}$$

with $\psi(o) = 0$ and $\psi(a) = 1$. Table 1 contains the values of q_o, q_a, α, and ε used in three different cases. Since, as was mentioned earlier, the beta is very low in these experiments and hence should play no role in the saturation, the pressure was taken as zero.

In Fig. 2 we display the equilibrium toroidal current density profiles for each case. Next we compute the linear eigenmodes for cases 1 and 2 and display the resulting magnetic field patterns. Successive field line traces are generated by arbitrarily increasing the amplitude of the linear eigenmode. Figure 3 shows stochastic regions generated by toroidal effects around the separatrix. There is a rather sudden onset of stochasticity in all cases (Fig. 4). We find the low q case to be more stochastic, as predicted by Lichtenberg. We point out that this is true when the island width is used as the relative measure. When the perturbation amplitude is used as the relative measure, the stochasticity is about the same. Both low and high q cases result in near total loss of good surface inside the island at approximately the same island width.

The main point we wish to discuss here, however, is the saturation of the islands, and for that we follow the nonlinear evolution of the m = 1 mode for case 1 and case 3. The results (Figs. 5 and 6) show no saturation of the m = 1 island; instead complete reconnection takes place. Stochasticity does not seem to affect the calculation. The much larger time required for reconnection at low aspect ratio (case 3) seems to be due to the decrease in the linear growth ratio. The stochasticity is seen to set in quite rapidly around the separatrix and could explain the rapid decrease in the large gradient in the temperature at the island edges as seen in the TFR experiment discussed by Lichtenberg.[9]

IV. CONCLUSION

Exact toroidal, pressureless equilibria have been analyzed using a self-consistent, reduced, MHD model. This model contains finite toroidal and stochastic field effects for resistive kink modes.

We conclude that a low q_o, while capable of generating large stochastic regions around the singular layer, will not produce saturated m = 1 islands nor will the discharge be able to sustain such a profile. Rather, the classical picture remains virtually intact, with the flux completely reconnecting and q_o increasing to unity. The saturation is more likely due to finite length effects seen in single helicity calculations of Ref. 14.

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Table 1. Equilibrium quantities for cases analyzed.

Case No.	1	2	3
q_o	0.6	0.9	0.6
q_a	2.5	2.5	2.5
R_o/a	4.0	4.0	2.0
α	1.5	1.5	1.5

FIGURE CAPTIONS

FIG. 1 Toroidal coordinate system used in the initial value code HILO.

FIG. 2 Equilibrium toroidal current profiles for cases 1, 2, and 3 as described in Table 1.

FIG. 3 Puncture plot of magnetic field lines for q(o) = 0.6 (case 1), showing stochastic region around separatrix.

FIG. 4 Plot of stochastic width vs island width for q(o) = 0.6 (case 1) and q(o) = 0.9 (case 2).

FIG. 5 Puncture plots of magnetic field lines showing nonlinear growth of m = 1 island for case 1.

FIG. 6 Puncture plots of magnetic field lines showing nonlinear growth of m = 1 island for case 3.

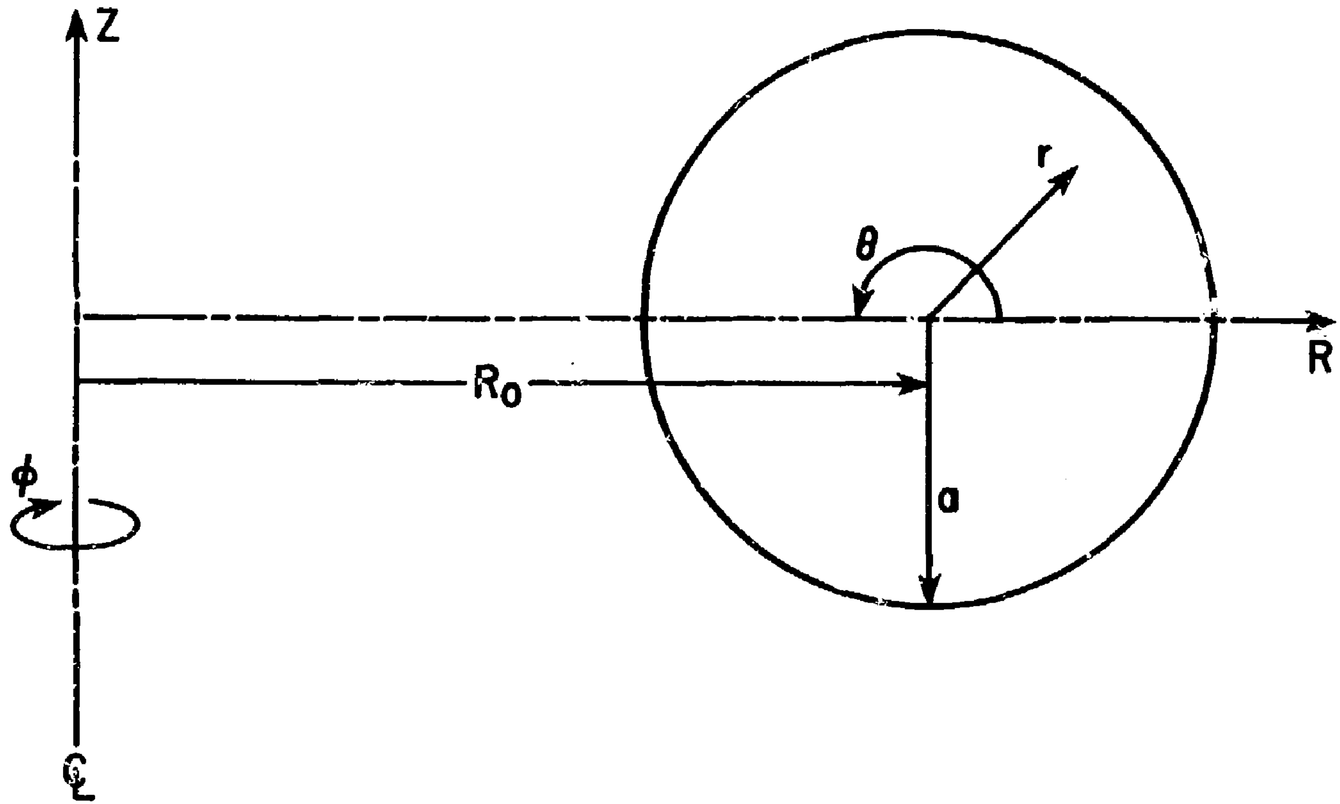

FIG. 1

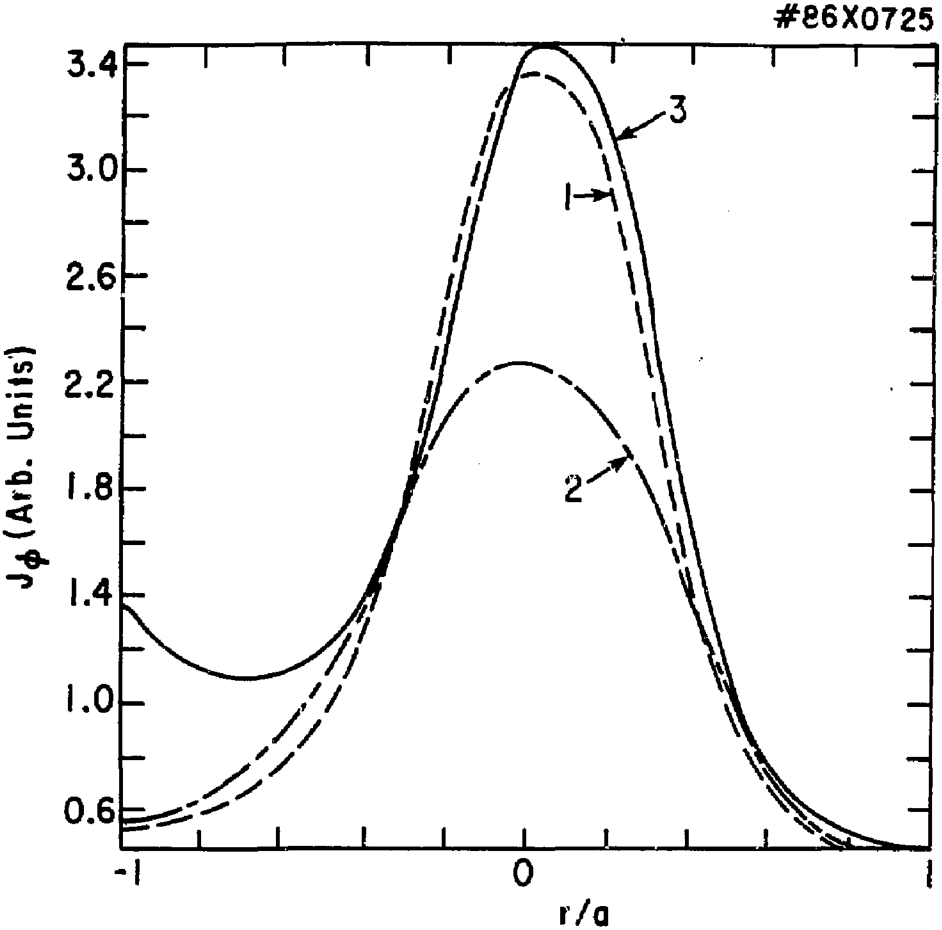

FIG. 2

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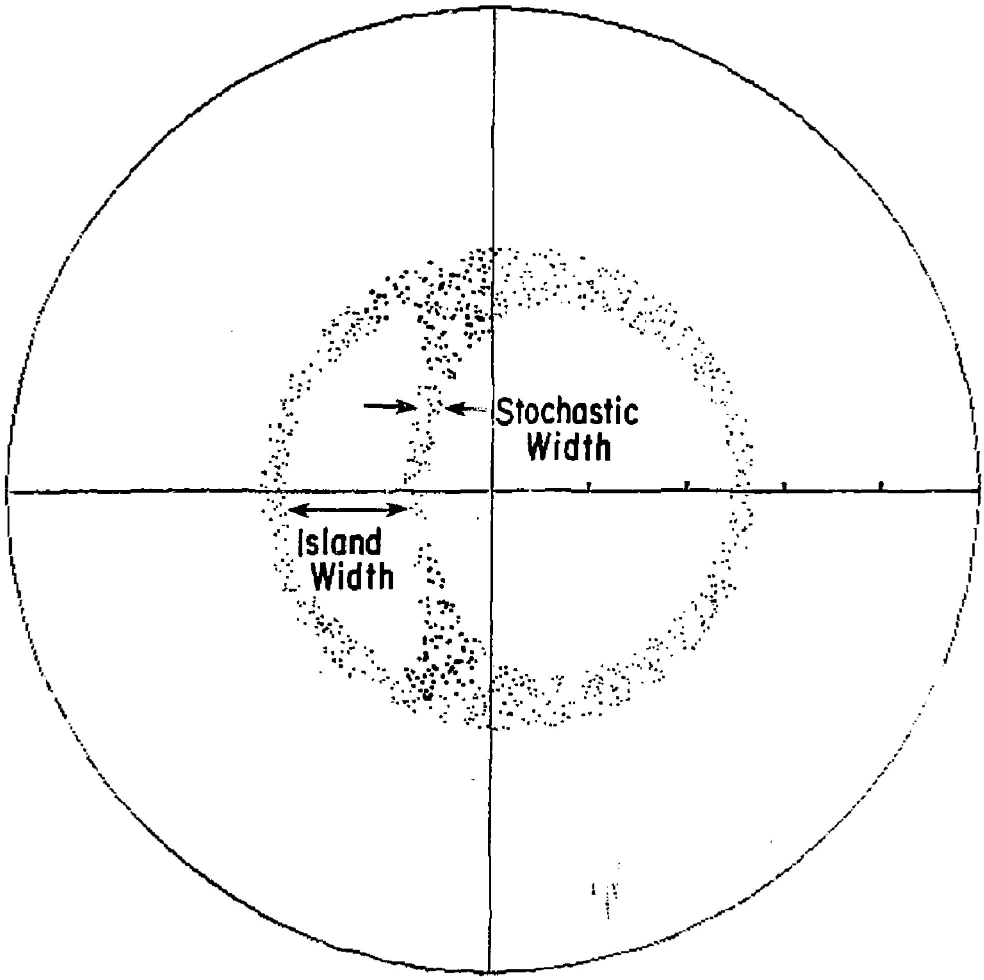
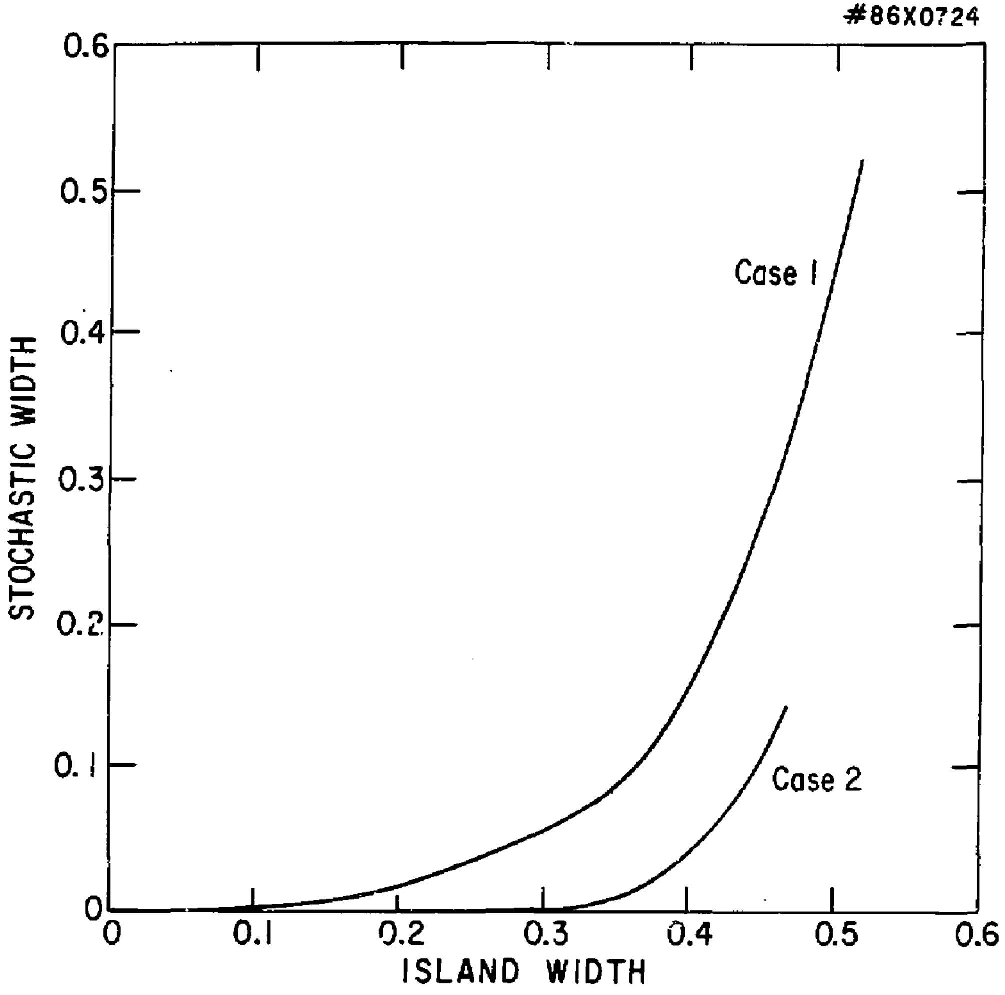


FIG. 3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STOCHASTIC WIDTH
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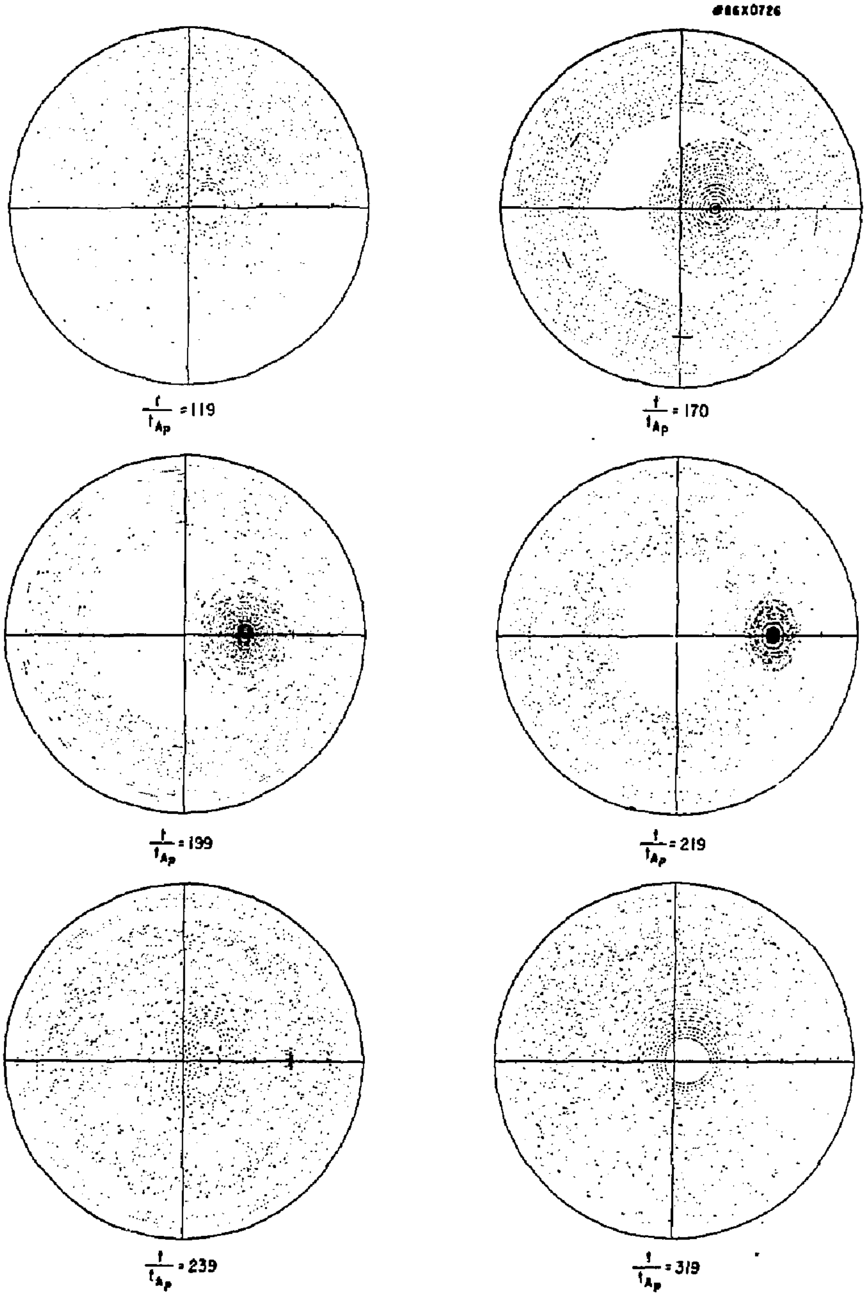
0.5
0.4
0.3
0.2
0.1
0
Case 1
Case 2
0
0.1
0.2
0.3
0.4
0.5
0.6
ISLAND WIDTH

FIG. 4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$\frac{t}{t_{Ap}} = 219$
$\frac{t}{t_{Ap}} = 239$
$\frac{t}{t_{Ap}} = 319$

FIG. 5

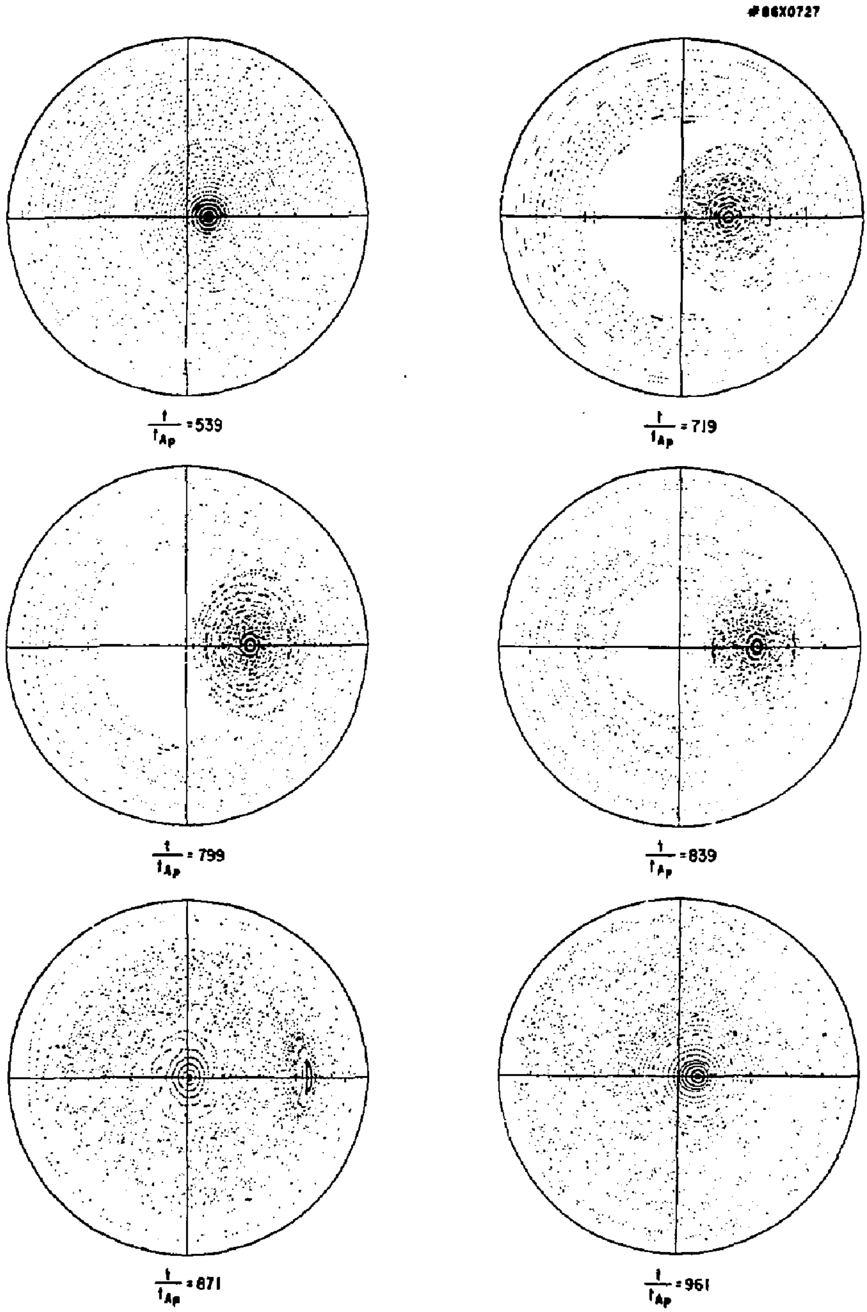

FIG. 6

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