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COUPLING BETWEEN EDDY CURRENTS AND RIGID BODY ROTATION:
ANALYSIS, COMPUTATION, AND EXPERIMENTS*

T. Q. Hua and L. R. Turner

Argonne National Laboratory
Argonne, Illinois 60439

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MASTER

T. Q. Hua[‡] and L. R. Turner

Abstract - Computation and experiment show that the coupling between eddy currents and the angular deflections resulting from those eddy currents can reduce electromagnetic effects such as forces, torques, and power dissipation to levels far less severe than would be predicted without regard for the coupling. This paper explores the coupling effects beyond the parameter range that has been explored experimentally, using analytical means and the eddy-current computer code EDDYNET. The paper also describes upcoming FELIX experiments with cantilevered beams.

INTRODUCTION

A conducting loop or plate normal to a changing magnetic field and parallel to another, constant magnetic field will experience an electromagnetic torque tending to rotate it. Such a situation has been studied in some detail, as it models a conducting plasma limiter in a tokamak fusion reactor, experiencing a torque from the decaying field of a plasma disruption and the constant field of the toroidal field coils. Computations [1,2] show that the coupling between the eddy currents and deflections makes the eddy currents, deflections, and other electromagnetic effects less severe than would be predicted without regard for the coupling. For example, with a given plate, restoring force constant, and decaying field, the peak deflection increases less than proportionally with increasing constant field.

In April and May 1984, a team of investigators from Princeton Plasma Physics Laboratory (PPPL) and Argonne National Laboratory (ANL) studied the effects of coupling between eddy currents and angular displacements with the FELIX (Fusion Electromagnetic Induction eXperiments) facility [3,4] shown in Fig. 1. Test pieces were square loops of copper and naval bronze and square plates of copper and stainless steel. Experimental conditions included dipole field levels of 25, 50, and 100 mT; dipole field discharge time constants of 10, 40, and 100 ms; solenoid field levels of 0.1, 0.2, 0.4, and 0.8 T; and restoring torques from the springs of 2.83 kN·m/rad and 8.70 kN·m/rad. The primary data recorded as functions of time were the angular displacement, measured by a rotational linear differential transformer (LVDT), and the total current in the test piece, measured by a Rogowski coil linking the test piece through a hole in the center of the loop or plate. The results of these experiments [5] clearly show decreased displacement through coupling effects and are in good agreement with computer calculations.

ANALYSIS

The simplest case for analysis is a conductor which electromagnetically can be characterized as a single current loop with fixed resistance R and inductance L , and which mechanically behaves as a rigid body with moment of inertia I_m able to rotate only about a fixed axis with a linear restoring torque, $-K\theta$. The conductor is initially parallel to a uniform, steady magnetic field B_x and perpendicular to a uniform, decaying magnetic field, $B_0 \exp(-\gamma_0 t)$.

Both authors are with Argonne National Laboratory, Argonne, IL 60439.

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[‡]Magnetic Fusion Energy Technology Fellow at Argonne from the University of Washington.

Fig. 1. The FELIX facility.

The current i and angular deflection θ obey, in the small angle limit,

$$L \frac{di}{dt} + Ri = A B_0 \gamma_0 \exp(-\gamma_0 t) + A B_x \frac{d\theta}{dt} \quad (1)$$

and

$$I_m \frac{d^2\theta}{dt^2} + K\theta = -i A [B_x + B_0 \exp(-\gamma_0 t)] \quad (2)$$

where A is the area bounded by the current loop. Equations (1) and (2) can be put in nondimensional terms, using the nondimensional current $i' = iL/AB_x$ and nondimensional time $t' = tR/L$.

$$\frac{di'}{dt'} + i' = C_1 C_2 \exp(-C_1 t') + \frac{d\theta}{dt'} \quad (3)$$

and

$$\frac{d^2\theta}{dt'^2} + C_3 \theta = -i' C_3 C_3 [1 + C_2 \exp(-C_1 t')] \quad (4)$$

where

$C_1 = \gamma_0 L/R$ is the ratio of L/R time to decay time.

$C_2 = B_0/B_x$ is the ratio of fields.

$C_3 = KL^2/I_m R^2$ is $4\pi^2$ times the square of the ratio of L/R time to the free oscillation period.

$C_4 = A^2 B_x^2 / LK$ is the ratio of the magnetic stiffness to the mechanical stiffness.

The last term in (4) can generally be dropped since θ and C_2 are both small. Then the current i' can be eliminated from (3) and (4) to give the third-order differential equation:

$$\frac{d^3\theta}{dt'^3} + \frac{d^2\theta}{dt'^2} + C_3(1 + C_4) \frac{d\theta}{dt'} + C_3\theta = -C_1C_2C_3C_4 \exp(-C_1t') \quad (5)$$

In addition to the particular solution,

$$\theta_p(t') = f_1 e^{-C_1 t'} \quad (6)$$

where

$$f_1 = \frac{C_1 C_2 C_3 C_4}{(C_1^2 + C_3)(C_1 - 1) + C_1 C_3 C_4}$$

(5) generally has one real exponential solution $[e^{Bt'}]$ and two complex conjugate exponential solutions $[e^{(\pm j\omega)t'}]$. The homogeneous solution can then be restated in terms of real variables, $\cos \omega t'$ and $\sin \omega t'$:

$$\theta_h(t') = e^{at'} [f_2 \cos \omega t' + f_3 \sin \omega t'] + f_4 e^{Bt'} \quad (7)$$

Equations (6) and (7) combine to give the general solution for the angle of rotation

$$\theta(t') = f_1 e^{-C_1 t'} + e^{at'} [f_2 \cos \omega t' + f_3 \sin \omega t'] + f_4 e^{Bt'} \quad (8)$$

The constants f_2 , f_3 , and f_4 are determined from the initial conditions for θ and i' :

$$\theta = 0 \text{ and } \frac{d\theta}{dt'} = 0 \text{ at } t' = 0 \quad (9a)$$

$$i' = 0 \text{ at } t' = 0 \quad (9b)$$

From (4),

$$\frac{d^2\theta}{dt'^2} = 0 \text{ at } t' = 0 \quad (9c)$$

Solution for nondimensional current i' comes from (1):

$$i'(t') = e^{-t'} \left[\int_0^{t'} e^{t''} \left(C_1 C_2 e^{-C_1 t''} + \frac{d\theta}{dt''} \right) dt'' \right] \quad (10)$$

One can readily recognize in (10) two components of induced currents. The first component (terms involved $C_1 C_2 e^{-C_1 t''}$) is due to the decay of the time changing magnetic field, and the second component (terms involved $d\theta/dt''$) arise from the rotation of the conductor. The angular velocity $d\theta/dt'$ may be obtained from (8) and integrated to give the following expression for i' :

$$i'(t') = g_1 e^{-C_1 t'} + g_2 e^{Bt'} + g_3 e^{-t'} + g_4 e^{at'} \cos \omega t' + g_5 e^{at'} \sin \omega t' \quad (11)$$

where g_1 , g_2 , g_3 , g_4 , and g_5 are constant coefficients. Equations (8) and (11) show that the oscillations occur in both the mechanical and electrical solutions. The frequency of oscillation is dominated by the coefficient of the third term in (5):

$$\omega = [C_3(1 + C_4)]^{1/2} \quad (12)$$

Results

In the analysis we combine the nondimensional parameters C_1 , C_2 , C_3 , and C_4 in the forms they appear in the governing equations, (3) and (4). The new parameters are C_1 , C_{12} ($= C_1 C_2$), C_3 , and C_{34} ($= C_3 C_4$). Equations (8) and (11) were solved over a wide range of parameters within the small angle limit. For practical applications, the values of peak rotation θ_m , and frequency of oscillation ω are important quantities and are emphasized in this analysis. A prior prediction of these values would allow a designer to design the components suitable to the mechanical tolerances.

Because of the vast combinations among the four parameters C_1 , C_{12} , C_3 , and C_{34} , efforts were made to find a tractable characterization of θ_m for easy comprehension and predictions. The following may be seen from the results of our study:

1. For any combination of C_1 , C_3 , and C_{34} , θ_m increases linearly with C_{12} at a rate that depends on the former three parameters as illustrated in Fig. 2, i.e., θ_m may be written in the form

$$\theta_m(C_1, C_{12}, C_3, C_{34}) = C_{12} f(C_1, C_3, C_{34}) \quad (13)$$

Fig. 2. Maximum deflection for various combinations of C_1 , C_3 , and C_{34} .

2. For constant C_{34}/C_4 ($= C_4$), f is almost independent of C_3 . Therefore, the variation in θ_m may be understood from the relationship among f , C_1 , and C_{34}/C_3 . Figure 3 shows contour curves of f versus C_1 and C_{34}/C_3 at a contour increment of 0.25. The following observations may be drawn from this figure.

(i) For large fixed values of C_1 ($C_1 > 1$), f is generally small ($f < 0.75$) and increases weakly with C_{34}/C_3 , approaching a constant limit when $C_{34}/C_3 > 10$ (not shown in figure).

(ii) For $C_1 < 0.5$ and $C_{34}/C_3 > 1$, f increases rapidly with C_{34}/C_3 .

Fig. 3. Contour plot of f which appears in Eq. (13).

(3) The frequency of oscillation ω , which depends only on C_3 and C_{34} , is shown in Fig. 4. The contours are found to agree very well with (12).

Fig. 4. Contours of the angular frequency of rotation.

FELIX COUPLING EXPERIMENTS

In addition to the PPPL-ANL experiments, a series of experiments were performed on coupled effects with more complex test pieces. The test pieces were aluminum plates $1.0 \times 0.8 \times 0.01$ m thick, mounted perpendicular to the dipole field. One plate was solid; two had slits, and one had rectangular holes. The test pieces were placed in the support frame, and angles were measured by the rotational LVDT. Total current was measured by a Rogowski coil through a central hole in the plate. Current around one of the rectangular holes was measured by a Rogowski coil through this hole. Experimental conditions included dipole field levels, dipole field discharge time constants, and solenoid field levels similar to those used in the collaborative PPPL-ANL experiments. For a given dipole field level, the peak deflection was greater at an intermediate solenoid field (0.2 or 0.5 T) than for higher (0.9 T) or lower (0.1 T) values.

Another series of experiments is planned for the study of coupling effects between eddy currents and deflection in cantilever beams, which provide a better model for the limiter blades of a tokamak fusion reactor than do the rigid body experiments explained above. Bialek and Weissenburger [6] have analyzed this model. Test pieces will consist of half-hard copper, half-hard brass, phosphor bronze, and 6061-aluminum alloy. Several beams will be tested to allow variations in both mechanical and electrical characteristics. The beam will be clamped rigidly at one end on a nonconducting support frame (Fig. 5). The test piece is parallel to the solenoid field and perpendicular to the dipole field. Deflection along the beam will be measured with an Optron electro-optical tracking device model 561 [7]. Total current will be measured with a Rogowski coil through a hole in the center of the beam. Local current density may also be measured with a pair of differential search coils located above and below the test piece. Material stresses due to deflection will be measured with two-layer strain gauges which were designed to eliminate the spurious signal which arise from the time-changing magnetic field.

Fig. 5. A cutaway section of FELIX showing the assembly for the cantilever beam experiments.

COMPUTATIONS WITH EDDYNET

The computer code EDDYNET [8] solves eddy current problems by a network approach, modeling the conductor with a quadrilateral wire mesh. The code has been modified to model the coupled problem of eddy current and deflection for a flat plate [2] similar to those tested in the FELIX plate experiments. In this modified field code, rigid-body rotation of the plate is equivalent to a counterrotation of the applied magnetic field. Figures 6 through 9 are output from EDDYNET to illustrate the coupling effects for two different cases: weak and strong coupling. The initial B_y field is 0.025 T decaying exponentially with a time constant of 16 msec, with steady solenoidal fields of 0.05 and 0.25 T for an L/R time constant of 72 ms. The torsion constant is 8700 N·m/rad to restrain the maximum deflection to within 5 deg.

Results from Figs. 6 and 7 show the peak deflection increases by only a factor of 2.6 when the solenoidal field increases by a factor of 5. Without current-deflection interaction, the peak deflection would be expected to increase linearly with field intensity. The frequency of rotation is higher at higher B_x value, as predicted by (5), because the eddy

Fig. 6. (Weak coupling.) Deflection in flat plate for dipole field of 0.025 T decaying in 16 ms, a steady solenoid field of 0.05 T, a plate L/R time of 12 ms, and a restoring torsion constant of 8.7 kN·m/rad.

Fig. 7. Equivalent eddy current in a flat plate, all other parameters the same as in Fig. 6.

Fig. 8. (Strong coupling.) Deflection in a flat plate for a steady solenoid field of 0.25 T, and all other parameters the same as in Fig. 6.

Fig. 9. Equivalent eddy current in a flat plate for a steady solenoid field of 0.25 T, and all other parameters the same as in Fig. 6.

currents provide additional magnetic "stiffness". Figures 7 and 9 show the total current pattern. If there were no coupling, the current would decay smoothly to zero after reaching a peak value. For a weak coupling (Fig. 8) the current is only slightly perturbed; for a strong coupling (Fig. 9) the effects are more pronounced as the peak current has been reduced and oscillations have become obvious.

CONCLUSIONS

Analysis and experiments have shown that the coupling between eddy currents and motion is an important effect. Eddy current codes should provide the capability to take advantage of these coupling effects on the design of components for fusion applications. Experimental results will increase our confidence in the validity of analytical and computational results.

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